Ponder this

Oleksiy Yevdokimov
University of Southern Queensland
<yevdokim@usq.edu.au>

The focus of this set of problems is Euclidean geometry. Since the ancient times, many magnificent results and theorems have been discovered and proved in this area of mathematics. Nevertheless, Euclidean geometry continues to serve as a source of wonderful properties and interesting problems. The geometry problems here were all posed by Igor Sharygin, whose talent in geometry problem posing is well known to mathematicians all over the world.

Problem set 3

1. Let M be a point in the interior of quadrilateral $ABCD$ such that AMB and CMD are isosceles triangles with the angle at vertex M equal to $120°$. Prove that there exists a point N such that triangles BNC and DNA are equilateral.

2. Let $ABCD$ be a cyclic quadrilateral such that $CD = AD + BC$. Prove that angle bisectors at vertices A and B meet at the side CD.

3. Let $ABCD$ be a cyclic quadrilateral with perpendicular diagonals. Prove that mid-points of all sides and the perpendiculars dropped from the diagonal intersection point to the sides lie on a circle.

4. Let AC be a diameter of a circle. Point E lies on AC. Draw chord BD through the point E in a way such that the area of quadrilateral $ABCD$ will have the greatest value.

5. An angle bisector at vertex A of the parallelogram $ABCD$ meets the straight lines BC and CD at the points K and L respectively. Prove that the centre of a circle which contains points C, K and L lies on a circle which contains points B, C and D.