A Capacity Assessment Approach for Multi-Modal Transportation Systems

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Abstract – This article focuses upon multi-modal transportation systems (MMTS) and their capacity assessment. To perform this task, a linear programming model (LP) denoted by MMTS_CAP has been developed. This model determines the maximum flow of vehicles and commodities that can be accommodated, over a given time period, on different transportation modes within the MMTS. It chooses how to move commodities between different origin destination pairs (ODP) and facilitates the transfer of commodities across different modes. The proposed model can facilitate a variety of capacity planning and querying activities and provides a mechanism to quickly analyse the effect of structural and parametric changes within MMTS. The model has been applied to several scenarios, including a real life case study. Our numerical testing indicates that it is effective, highly flexible and extendable. Our analysis suggests that the proposed capacity assessment techniques are worthwhile and may help transportation planners build better MMTS.

Keyword: Transportation, capacity assessment, multi-modal transportation systems, congestion modelling

1. Introduction

1.1. Background

This article considers how to assess the capacity of an entire multi-modal transportation system (MMTS). The capacity assessment of transportation systems is an important task and needs to be performed for many practical decision making and planning activities.

MMTS are very important systems. They provide the primary means of transporting people and goods across most regions of our planet. A typical MMTS consists of many different modes of transportation. These occur on land, air and sea. There are many different types of MMTS. The three most typical systems are bulk material flow between mine and ports, urban transportation of passengers between different locations, and containerization movements between ports and urban locations. In the first scenario, the modes are rail and road. In the second, all modes occur. In the third, road, rail, sea modes are present. The items or objects transported in MMTS are called commodities. They include people, parcels, mail, containers, boxes, etc. Vehicles of different type typically transport the aforementioned commodities. However, vehicles may also be regarded as a type of commodity. Different clienteles utilize MMTS and the specific modes within it. These clienteles compete with each other, within a particular transportation mode or else across the entire MMTS.

For an MMTS to be regarded as efficient it is self-evident that the different modes should be highly connected, synchronised and complementary to each other in terms of capacity and utility. Unfortunately, this is rarely true due to the ad-hoc and independently considered expansion of the individual parts over time, as well as the changing nature of demand over time. Today many components of MMTS are highly congested. Road arterials are one example. They are a major component of many MMTS around the world and can only be expanded at great financial cost.

Due to rapid population growth in many areas of the world, there is a need to continuously expand MMTS to meet increasing demands. Evidently, this is not a simple task and is politically sensitive. Federal governments in many countries are actively proceeding with the expenditure of billions of dollars on intercity and regional passenger rail and road projects. Travel demand in urban

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transportation networks is continuously increasing and this is a challenging and important issue for decision makers. To our knowledge there is a lack of accepted tools for documenting the interrelationship between these modal investments. In addition, when planning activities are considered jointly, it is performed in an incomplete and limited way. In many parts of the world, rail and road investments are seen as complementary elements of a larger multimodal and intermodal public policy (The AASHTO, 2007).

A major issue in the operation of MMTS are incidents that cause the temporary closure and the reduction of flows within different links in the network. Typical incidents include accidents, construction activities, maintenance activities, weather and other national disasters, and even terrorist attacks. In these circumstances, it is vital to know whether the system can respond, and whether there are alternative routes with sufficient capacity. The capacity assessment approach introduced in this article is capable of assessing the impact of these types of incidents. It may also be used for expansion planning and evacuation planning. Our approach is well suited to future scenarios involving autonomous vehicles which can be operated at closer headway distances and do not exhibit behavioural nuances or anomalies. Human drivers in contrast are unpredictable and operate motor vehicles in many different ways, depending on the state of the weather, their level of experience, or on their emotion.

1.2. Research Approach and Methodology

In this article a capacity assessment approach has been developed for MMTS. Our approach is based upon the application of a kind of multi-commodity flow optimization model. That model is henceforth denoted as MMTS_CAP. Our MMTS_CAP has been developed because the evaluation of an entire transportation system is seldom addressed (Farahani et al., 2013; SteadieSeifi et al., 2014). The development of a more sophisticated capacity assessment approach that integrates all the different transportation modes and their flows and their inter connections is necessary.

The aforementioned model can be used to assess the structural attributes and characteristics of a MMTS and can help perform a variety of planning activities. The MMTS_CAP is useful because capacity assessments can be performed quickly and straightforwardly. The model is transparent and uncomplicated to use by planners and managers. The MMTS_CAP can identify existing bottlenecks within the system and can help decision makers with expansion activities. The effect of adding, removing, or even upgrading infrastructure can be evaluated effortlessly.

Our approach facilitates the transportation of both passenger and freight commodities. The model is particularly well suited to systems involving railways, roads, and busways. These modes are most common in all MMTS and have the highest demand. A typical scenario includes freight and passenger trains on railway lines, standard and articulated buses on busways, and a mixture of cars, trucks and buses on road links.

The performance of every system is limited by its capacity. Capacity can be defined in many different ways. In Burdett and Kozan (2006), Burdett (2015b), and Bevrani et al. (2015), the theoretical capacity of a railway corridor was analysed. Capacity was defined as the maximum number of trains that can travel across a bottleneck section in a specified period of time. The capacity of a road, according to Minderhoud et al. (1997), is the maximum traffic volume that can be achieved over a given time period. Road capacity is similarly defined in HCM (*HCM 2010 : highway capacity manual*) as the maximum hourly rate at which persons or vehicles can traverse a point or uniform section of a lane or roadway in a given time period.

In this article, a macro rather than a micro level approach is taken to capacity determination. Our approach is based upon the "theoretical" capacity paradigm. The theoretical capacity of an MMTS is a measure of the maximum number of vehicles or commodities that can be transported over a given time period. The value of evaluating theoretical capacity are many (see Burdett and Kozan (2006), Burdett (2015a),Burdett (2016)). First, it is a useful reference point, and second, it is straightforward to use in high level planning and decision making process. The assessment of operational capacity is

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an equally important and related topic. The operational capacity of transportation systems has been addressed in the HCM (*HCM 2010 : highway capacity manual*) and the transit capacity and quality of service manual (Kittelson et al., 2003). The detailed planning of vehicle flows and the creation of timetables is generally necessary to assess operational capacity. Those topics are however outside the scope of this article. It is important to note that our capacity assessment approach can be calibrated to real life and made more accurate using historical data, congestion functions and other behavioural models.

In the next section, prior research is discussed. The MMTS_CAP and the main technical developments are then introduced in Section 3. In that section, representations of MMTS and the calculation of arc capacities are discussed in detail. The optimization model and its advantages is also presented. In Section 4 the MMTS_CAP is first applied to several small scenarios to validate the model and to demonstrate that it can handle real life complexities. Each instance has a particular feature and complexity that occurs in real life MMTS. A case study of a real MMTS is later provided to verify the model is viable for assessing an entire real life MMTS. The results are discussed and reported. Finally, a summary and discussion of the key findings and accomplishments can be found in Section 5. A comprehensive listing of all parameters and notation can be found in Appendix A.

2. Literature Review

Recent approaches for MMTS capacity assessment are reviewed in this section. MMTS have been focused upon with regularity in the literature. Examples include (Arnold et al., 2004; Boyac & Geroliminis, 2011; Lai & Barkan, 2011; Li et al., 2007; Park & Regan, 2005). The article by Park and Regan (2005) is most noteworthy as they presented a conceptual framework to evaluate the capacity of multimodal freight transportation system. They introduced a bi-level problem. They incorporated multiple modes and commodities, behavioural aspects of network users, external factors as well as the physical and operational conditions of a network. Their model estimates the capacity of a multimodal network, and also identifies the existing capacity "gaps" over all facilities in the network. However, the model focuses solely upon freight, and the applicability of the model is not tested on a real life case study. In addition, their approach is not sufficiently generic for general capacity querying activities. In contrast our approach considers both passenger and freight and the transfer of commodities between modes.

In prior research the effect of toll costs on the capacity of transportation networks has been studied. For example, Xiao et al. (2007) studied the competition and efficiency of private toll roads. Their paper focused on toll and capacity competition among different private roads. The presence of congestion was also included. Their study considered both pricing and capacity choices in order to achieve equilibrium within the network. They reported that more competition does not necessarily lead to less congestion on the roads.

Specific modes like roads and rail have been addressed more often. Yang et al. (2000) developed for road networks a bi-level optimization model to efficiently determine the maximum number of trips from each origin. Their model can assist in identifying whether an existing transport network is capable of supporting future urban growth. Chen et al. (2002) proposed a model to determine the capacity reliability of a road network. Their model can identify the probability that the network can facilitate a certain traffic demand. Abril et al. (2008) reviewed different methods of evaluating railway capacity in the literature. The considered methods are characterised as either analytical, optimisation, or simulation based. In conclusion analytical methods were found to be good reference point to identify capacity and to identify bottlenecks. Cancela et al. (2015) introduced a mathematical programming formulation to define the number and itinerary of bus routes and their frequencies for public transportation systems. Other examples include Burdett and Kozan (2006), Burdett (2015b), and Bevrani, et al. (2015). Those articles are quite relevant to the determination of capacity in MMTS. The models developed in Burdett and Kozan (2006) and Kozan and Burdett (2005) are essential techniques for determining the theoretical capacity of a railway. Those capacity models can be used to identify how the infrastructure can best be used and whether it can support an intended future traffic load. A generic multi-objective approach has recently been devised in Burdett

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(2015b). That approach facilitates a more in depth analysis of different capacity metrics and competitions between entities.

Litman (2011) introduced practices for multi-modal transportation planning and found that more research is needed to analyse the connectivity between modes in transport systems. This apparent gap is a source of motivation for this article. Prior research on MMTS has also considered a number of other decision problems. The problem of finding shortest paths between specific origin-destinations in urban systems has gained popularity. sFor instance Modesti and Sciomachen (1998) introduced an intermodal path planning model that minimizes the overall cost, time and users discommodity associated with the required paths. The shortest path study was continued by Choi et al. (2006), to find out a feasible area with an effective time range and effective cost range in order to get multiple Pareto optimal solutions. They also tested the efficiency of the heuristic algorithm for constrained shortest path problem. They found that it can be useful for third party logistics. Later, Zografos and Androutsopoulos (2008) presented a new formulation and an algorithm for solving the itinerary planning problem to determine an optimized set of criteria for the itinerary (i.e. total travel time, number of transfers, and total walking and waiting time while departing from the origin and arriving at the destination within specified time). Their model provided fast and accurate solution for the real-life itinerary planning problem.

Strategic planning is an important way of increasing the efficiency of MMTS. A MMTS can carry thousands of containers and commuters on railways and roads. A large number of commuters and goods travel by MMTS daily. Trucks for example need to return home within a specified time, and empty containers have to be accessible in the right place at the scheduled time. Yamada et al. (2009) proposed a strategic transport planning model for freight terminal development. Their model can be used to efficiently expand and design a multimodal freight transportation system. Jansen et al. (2004) studied these repositioning aspects of MMTS in order to get a cost-efficient solution. They introduced a daily planning approach for MMTS system which is both flexible and adaptable. It supports the operations of a MMTS. It should be noted that they only focused upon freight, however, passenger or various vehicle modes are not considered in their model. Xie et al. (2012) developed a location and routing model for hazardous materials. It should be noted that this model does not maximise the flow of vehicles and commodities. This model determines where transfer yards should be placed and how hazardous materials should be routed in the most cost effective manner, through a multi-modal network containing road and rail links. Their model was applied to a case study with approximately 600 ODP and 1200 arcs. The model was solved to optimality in 40 minutes using CPLEX.

Simulation methods are widely used to evaluate the performance of urban and freight transportation systems. Many articles in this area acknowledge the fact that simulation needs to be adapted to different applications and environments (Abril et al, 2008). (Abril, et al., 2008) have reviewed this field and highlight that a variety of macroscopic, mesoscopic and microscopic approaches have been proposed of varying capability and sophistication. These are traditionally applied independently but are increasingly applied as part of intelligent systems. Commercial software is primarily but not exclusively used to facilitate the aforementioned simulations. Cellular automata is often cited favourably amongst the different simulation methods available. Boyac and Geroliminis (2011) estimated the capacity of MMTS using a simulation platform. They studied the effect of network geometry, operational characteristics, and congestion. Their simulation platform includes a large time space diagram with many links. Mishra et al. (2012) proposed a graph theoretical approach for measuring the connectivity within a MMTS. Their connectivity measures are used for many different purposes, but primarily to evaluate the overall performance of MMTS. They measure the connectivity of rail lines, bus lines and major roads as well as their integration. These measures have been tested on a real life case study by extensive simulations. Abadi et al. (2015) proposed an approach for estimating traffic flows within a transportation network. Their approach consists of a dynamic traffic simulator, an optimization methodology to adjust origin to destination routing, and an auto regressive prediction model. In order to account for random effects and uncertainties they used Monte Carlo simulations. They identified that the prediction of traffic flow rates up to 30 min ahead of time can be accurate. Van der Gun et al. (2015) proposed a methodology to simulate MMTS during emergency

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conditions. They simulated choice behaviour of individuals in MMTS and proposed a macroscopic and mesoscopic multimodal dynamic network loading model. An activity-based escalation model for the choice behaviour of individuals in the network was utilised. Wall et al. (2015) combined a discrete event based logistics simulator for a freight trucking terminal and a discrete time-step-based traffic micro-simulator for the MMTS serving the terminal. Their numerical testing indicates their approach captures the dynamic interaction of the two systems being modelled.

To conclude this literature review, it is worth pointing out that relatively few innovations have been proposed in actual simulation techniques applied to MMTS. Our analysis of the literature indicates that prior approaches have a number of limitations. Many approaches are too simplistic and do not accurately portray the characteristics of the actual system. Few approaches have integrated different transportation modes contained in urban MMTS. Additionally, few approaches facilitate or discuss expansion planning and other what if analysis. Those activities are more important than traditional capacity assessment activities, because they consider how the current system can best be extended. A new system cannot simply be built nor can the existing system be easily or cheaply dismantled.

3. The Capacity Model (MMTS_CAP)

In this section the details of the MMT_CAP are presented. The model's purpose is to perform an assessment of a given transportation system. The model determines the maximum flow of commodities on different arcs and between different origin destination pairs (ODP) using different modes. The total commodity flow is called the capacity of the MMTS. The model is also able to determine the unused capacity on different links. The second purpose is to perform *capacity querying*. The model can test whether the network can facilitate and support specific flows of vehicle and commodities. Additionally, it can be used to identify the duration of time that is required to move a specific number of commodities (i.e. demand) throughout the network on each ODP. The model can be applied to different intervals of time and for different demands within a day.

The proposed approach is macroscopic and deterministic. The model provides the theoretical capacity. Hence some levels of realism are not incorporated. This is not a weakness, but a choice, as our focus is improved strategic planning. In order to determine the actual operational capacity, low level microscopic details such as human psychology and behavioural phenomena must be included. Simulation techniques are best suited for those circumstances. To our knowledge, it is difficult to mathematically model the complex behaviours of people and vehicles in a typical MMTS, in a viable and credible way.

We believe that the model is straightforward to use and its data requirements are relatively minimal. The model is autonomous and more transparent in comparison to other approaches like simulation. The primary input for this model is a detailed description of the MMTS. The MMTS representation is described in the next section.

3.1. MMTS Representation and Data Requirements

MMTS are large complex systems. A sizeable amount of information is required to analyse an MMTS. The following is a list of necessary inputs:

- ODP: List of (origin, destination) pairs
- Nodes: List of locations
- Input Output Nodes: List of locations where vehicles and commodity flows originate or terminate
- Arcs: source, destination, mode type, number of lanes/tracks, length, headway distance, vehicle mix
- Vehicles: type, length, speed limit, number of commodities carried.
- Demands: Number of vehicles or commodities required on each ODP.

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Fortunately, much of this information can be obtained from websites and freely accessible documents. The accessibility of demand data between different ODP and on different modes is most difficult to obtain. It should be noted that headway distance is the separation between adjacent vehicles to maintain safety. Arc length is the length of the associated road or rail segment. Another important piece of data not previously mentioned is the mix of different vehicle types that traverse MMTS arcs (i.e. links). For instance, on roads there is typically a mix of cars, trucks, taxis, motorbikes, buses, etc. Each of these vehicle types move a different number of commodities and has different physical characteristics. The proportion of these vehicles affects the number of passengers that can ultimately be moved.

MMTS can be modelled in a variety of different ways and different levels of detail may be specified. Without loss of generality it is necessary to model MMTS using networks and network diagrams. A transportation network is hence a collection of nodes connected by arcs. Nodes demonstrate origins and destinations (i.e. places where travel begins and ends), and arcs define existing links between nodes. In other words, arcs are used to portray roads, busways and railway tracks. Directed arcs imply the direction of travel, and undirected arcs may be used to imply travel in both directions. Exact guidelines for generating a network are described below:

1. Directed arcs imply direction of travel, i.e. $(a, a') \Rightarrow a \rightarrow a'$. Undirected or double ended arcs imply travel in both directions, i.e. $(a, a') \Rightarrow a \rightarrow a' \wedge a' \rightarrow a$.
2. Different modes are represented by different arcs. Multiple modes may be available from a location. This possibility may be represented by a single node or by several nodes (see Fig 1).
3. Parallel lanes that have different directions of travel require several different arcs for different directions (see Fig 2). This situation can be separated into several nodes. Several nodes are required if a return trip is not permitted. In Fig 2b, A-B-A is possible.
4. Parallel lanes of size *N* (i.e. with the same direction of travel) can be represented by one arc of capacity *N* or *N* separate arcs of capacity one (see Fig 3). If change or choice of lane is not permitted, then node A should be divided into two separate nodes.
5. Intersections and the shared areas within it can be represented by nodes or arcs as shown in Fig 4. Fig 4b is a simpler representation and models the shared area as a node. In Fig 4c, there are four different shared areas.

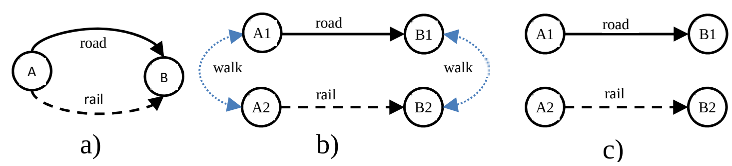

Figure 1: Network representations of multiple mode access from a location

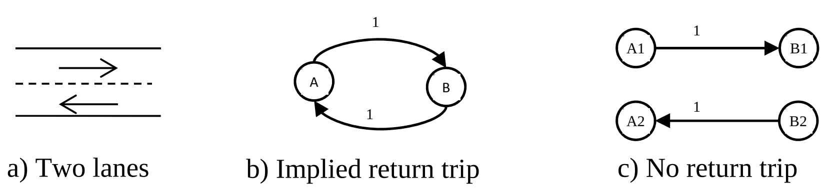

Figure 2: Choice of network when considering two sided lanes

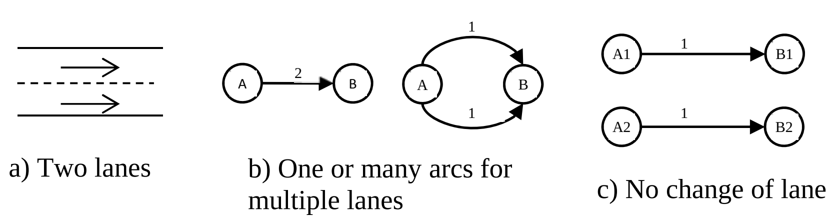

Figure 3: Choice of network when considering parallel lanes

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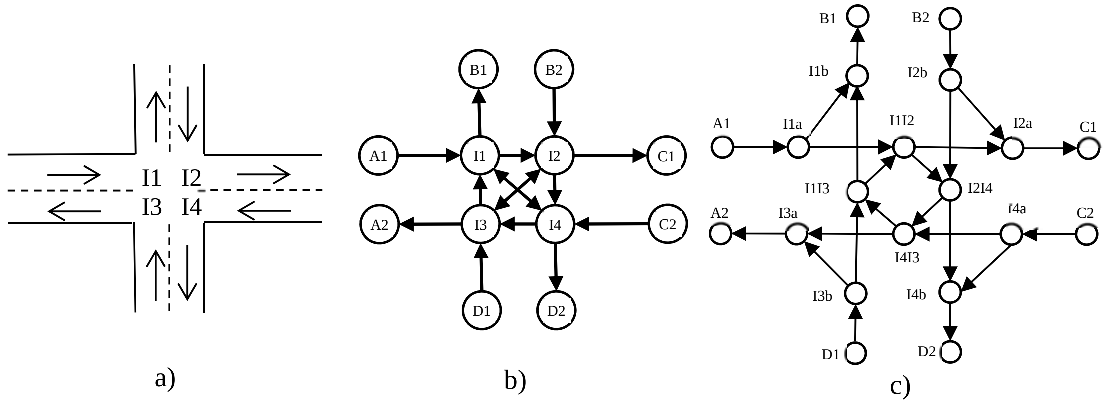

Figure 4: Representation of intersections.

Fig.1a implies that the either of the two modes may be taken from location A and that the two modes originate from the same position. Those modes are also to some extent parallel. Fig 1b is an equivalent representation. In Fig.1c links between A1 and A2 and between B1 and B2 are not present. Hence transfers between the modes are not deemed permissible. The network in Fig.2c reveals nothing about the proximity of A1 and A2 or B1 and B2. Only their position on paper implies that perhaps they are adjacent and in close proximity.

Intersections are important feature on many roads. Their purpose is to regulate the competing flows occurring at a specific location. Their presence causes interruptions and delays. These delays can significantly affect the capacity of the system. There are many ways to model intersections using nodes and arcs. Two alternatives are shown in Figure 4. Figure 4c provides the greatest level of detail. Further details about intersections will be provided in Section 3.4.

3.2. The MMTS_CAP

The following linear programming formulation is proposed to perform a capacity assessment of an MMTS.

Maximize $\sum_{k\in K} \omega_k . \Gamma_k$ (1)

Subject to

$\Gamma_k = \sum_{p\in P} \gamma_{p,k} \quad \forall k \in K$ [Number of commodities] (2)

$\gamma_{p,k} = \sum_{a\in A|o_a=\tilde{o}_p} \mathbb{F}_{p,a,k} \quad \forall p \in P, \forall k \in K$ [Flow from the origin] (3)

$\gamma_{p,k} = \sum_{a\in A|d_a=\tilde{d}_p} \mathbb{F}_{p,a,k} \quad \forall p \in P, \forall k \in K$ [Flow into the destination] (4)

$\sum_{a\in A|o_a=n} \mathbb{F}_{p,a,k} = \sum_{a\in A|d_a=n} \mathbb{F}_{p,a,k} \quad \forall p \in P, \forall k \in K, \forall n \in N | n \notin \{\tilde{o}_p, \tilde{d}_p\}$ [Conservation of commodity flow] (5)

$\mathbb{F}_{p,a,k} \leq \mathbb{V}_{p,a} . \overline{CPV}_{a,k} \quad \forall p \in P, \forall k \in K, \forall a \in A$ [Commodity flow limitation] (6)

$\sum_{p\in P} \mathbb{V}_{p,a} \leq \mathbb{T} . C_a . \tau_a \quad \forall a \in A$ [Capacitated vehicle flows] (7)

$\gamma_{p,k} \geq \mathbb{D}_{p,k} \quad \forall p \in P, \forall k \in K$ [Required demands] (8)

$\mathbb{V}_{p,a} \geq 0, \mathbb{F}_{p,a,k} \geq 0 \ \forall a \in A, \forall p \in P, \forall k \in K$ [Positivity of flows] (9)

This model maximises the total weighted flow of commodities, over all origin destination pairings (ODP) in the MMTS. The time horizon for the capacity assessment is denoted by $\mathbb{T}$. The capacity is determined in equation (1) as the weighted sum of the different commodity types. As different commodities are not always comparable, the priority weighting ω_k may be introduced. Additional priorities may be designated on ODP using $\omega_{p,k}$ if needed. Each origin destination pairing is defined by the following pair $(\tilde{o}_p, \tilde{d}_p)$. Each pairing may be loosely called a path or a corridor. It should however be noted that an exact path from the origin to the destination, is not explicitly defined. This gives the model more freedom to choose how commodities are moved. The primary decision variable

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of this model is denoted by $\mathbb{F}_{p,a,k}$. It is the number of commodities of type k that are be transported across arc a for ODP p. The number of commodities that can be transported is directly related to the number of vehicles that can be moved. This is denoted by $\mathbb{V}_{p,a}$. The total number of commodities of type k is denoted by Γ_k. Similarly the total number of commodities of type k that can be moved on a specific ODP p is denoted by $\gamma_{p,k}$. The total number of commodities of each type is summed over all ODP as shown in Eq. (2). Constraint (3) and (4) defines the number of commodities that can be moved from the origin and to the destination of each ODP. It is important that flows originating at $\tilde{o}_p$ arrive at destination $\tilde{d}_p$. Constraint (5) has been formulated to ensure the conservation of commodity flow at each node. It is only applied to nodes which are not origins or destinations of ODP's. Constraint (6) provides the necessary linkage between vehicle and commodity flows on each arc. The number of commodities carried by vehicles of type $i \in I$ on arc a is defined as $CPV_{a,i,k}$. The average number of commodities is denoted as $\overline{CPV}_{a,k}$. It is calculated as $\overline{CPV}_{a,k} = \sum_i \mu_{a,i}.CPV_{a,i,k}$ where $\mu_{a,i}$ is an input parameter that describes the proportion of vehicles of type i on arc a. It should also be pointed out that all commodities are moved by vehicles. If the commodity moves on its own (i.e. pedestrians) then the vehicle is the commodity and $\overline{CPV}_{a,k} = 1$. The commodity flow is less than or equal to the vehicle flow multiplied by the average number of commodities per vehicle. An exact equality is not possible because in some circumstances a network may not facilitate the flow of certain commodities from or to specific origins or destinations. In those circumstances certain commodity flows are zero. An example of this circumstance occurs when an attempt is made to move freight through a busway. As busways only move passengers, the commodity flow variable for freight must be zero.

Constraint (7) ensures that the flow of vehicles on each arc must be less than or equal to the capacity of the arc multiplied by the number of lanes or tracks (i.e. τ_a). Positivity of vehicle and commodity flows is enforced by Eq. (9). Specified demands for commodity flows on different ODP (namely $\mathbb{D}_{p,k}$) are enforced by constraint (8). When demands are added, two things can happen. First the model may solve and will report flows over and above the specified demands. Otherwise the model will not solve. In that event, one or more demands will be unable to be met. How near the MMTS is to having sufficient capacity is not known. To determine this, and to identify the actual flows that are achievable, the model should be solved with an alternative objective function and the removal of constraint (8).

$$\text{Maximize } \sum_{p \in P} \sum_{k \in K} \omega_{p,k}.\left(\gamma_{p,k} - \mathbb{D}_{p,k}\right) \quad (10)$$
$$\text{Maximize } \sum_{p \in P} \sum_{k \in K} \omega_{p,k}.\mathbf{min}\left(0, \gamma_{p,k} - \mathbb{D}_{p,k}\right) \quad (11)$$

Objective function (10) is an inferior metric, as unmet demand can be balanced with oversupply elsewhere. In contrast objective function (11) only counts unmet demand. The objective function value is zero when all demand is met. If $\left(\gamma_{p,k} \geq \mathbb{D}_{p,k}\right)$ then $\mathbf{min}\left(0, \gamma_{p,k} - \mathbb{D}_{p,k}\right) = 0$. Otherwise the value will be negative. The unmet demands may be weighted to facilitate prioritized flows between different ODP. As objective function (11) is non-linear, the following linearization is necessary:

$$\text{Maximize } \sum_{p \in P} \sum_{k \in K} \omega_{p,k}.\delta_{p,k} \quad (12)$$
$$\delta_{p,k} \leq \gamma_{p,k} - \mathbb{D}_{p,k} \;\; \forall p \in P, \forall k \in K \quad (13)$$
$$\delta_{p,k} \leq 0 \;\; \forall p \in P, \forall k \in K \quad (14)$$

If $\left(\gamma_{p,k} - \mathbb{D}_{p,k} < 0\right)$ then $\delta_{p,k} = \gamma_{p,k} - \mathbb{D}_{p,k}$. Otherwise if $\left(\gamma_{p,k} - \mathbb{D}_{p,k} > 0\right)$ then $\delta_{p,k} = 0$. Unmet demand is defined as $-\delta_{p,k}$. It follows that the total unmet demand is $-\sum_{p,k} \delta_{p,k}$. It is possible to combine objective (12) with the former:

$$\text{Maximize } \sum_{k \in K} \omega_k.\Gamma_k + \sum_{p \in P} \sum_{k \in K} \omega_{p,k}.\delta_{p,k} \quad (15)$$

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This combined objective function provides a more generic model. The unused capacity on each arc (i.e. in terms of commodities) can be determined by the model directly or else determined afterwards in the following way:

$$\mathbb{U}_{a,k} = \left(C_a.\tau_a.\overline{CPV}_{a,k} - \sum_{p\in P}\mathbb{F}_{p,a,k}\right) \quad \forall a \in A, \forall k \in K \qquad \text{[Unused capacity]} \qquad (16)$$

This unused capacity is shared amongst the traffic moving between a variety of different ODP. The free capacity that exists for each ODP is computed in the following way.

$$\mathbb{u}_{p,k} = UB_{p,k} - \gamma_{p,k} \;\; \forall p \in ODP; \forall k \in K \qquad (17)$$

To evaluate this equation the upper bound (i.e. $UB_{p,k}$) for each ODP must be determined separately.

3.3. Benefits of MMTS_CAP

The MMTS_CAP is a type of multi commodity flow (MCF) formulation. Background information on MCF formulations can be found in Ahuja et al. (1993). The traditional MCF has four technical constraints, namely link capacity, flow conservation on nodes, flow conservation at the source and flow conservation at the destination. Our model differs from the aforementioned MCF formulation in a number of ways. Traditional formulations do not determine how commodities are actually transported. Those formulations just assume commodities can be moved independently and that the total number of commodities that can be moved on each arc is restricted by a given upper bound. Previous formulations also assume that every commodity can travel on every arc. This is not so in MMTS. If vehicles per chance were commodities, then the MCF formulation is unable to integrate or link the flow between different transportation modes in a MMTS. In contrast, commodities can be transported via different types of vehicles and hence by different modes of travel in this article's MMTS formulation. Our MMTS_CAP determines how commodities should be moved between different ODP. That is, it determines the path that should be taken. In addition, arc capacities are given for the number of vehicles that can travel and not the number of commodities. In the MMTS_CAP there is an additional ODP index in the flow variable $\mathbb{F}_{p,a,k}$. This is advantageous because the MMTS_CAP can determine where the flow on specific arcs comes from. The flow may originate from a variety of different sources. This is not identified by traditional MCF formulations. A greater number of decision variables are however required than the traditional MCF.

The aforementioned MMTS_CAP formulation is an LP and has no binary or integer decision variables and no non-linearity. Consequently, it can be solved for very large networks.

3.4. Arc Capacity Calculations and Other Details

In this section arc capacity calculations are presented. The capacity of each arc C_a must be pre-computed in order to solve this model. Those values depend upon the mode of the arc and a variety of different system parameters, however in essence it is the travelling time and travelling speed that is most influential in their calculation. Roadways are first discussed.

***Roadways*:** The theoretical capacity of a single lane road can be approximated by a function of vehicle speed (V), headway time (H_t), headway distance (H_d), length of vehicle ($\mathbb{L}$) and the proportional mix of vehicles (η_i) that use it (*HCM 2010 : highway capacity manual*). In other words:

$$C_a = F(V, H_t, H_d, \mathbb{L}, \eta_i) \;\; \forall a \in A \qquad (18)$$

The theoretical capacity of a section of road that carries a single vehicle type is $C_a = 1/H_t$ per unit of time. In that equation headway time H_t is equal to $H_t = (H_d + \mathbb{L})/V$. The headway time depends on

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vehicle speeds and length. If there are multiple vehicle types, then the headway time can be approximated by the following, $H_t = (H_d + \bar{\mathbb{L}})/\bar{V}$, where the average length and average speed of vehicles are denoted by $\bar{\mathbb{L}}$ and $\bar{V}$ respectively. Hence, the capacity of the arc is calculated as $C_a = \bar{V}/(H_d + \bar{\mathbb{L}})$.

Intersections: Intersections are required whenever several roads meet or cross each other. The flow of vehicles is reduced at intersections because flows in different directions are handled one at a time, in a repeating cycle. When assessing the capacity of an MMTS with intersections, some additional requirements must be added to the aforementioned model. Figure 5 describes a detailed representation of an intersection.

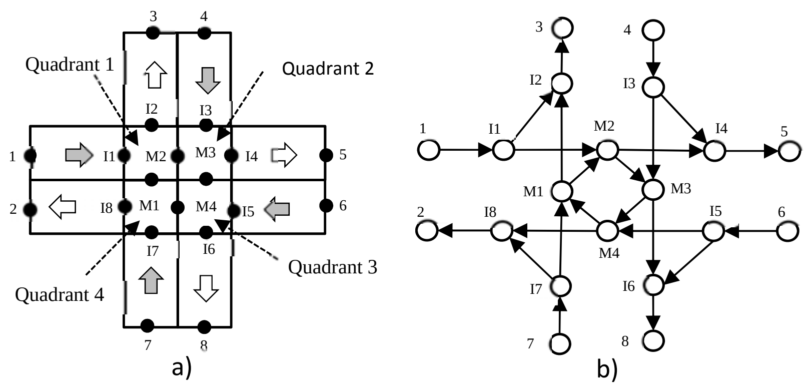

Figure 5: Modelling an Intersection

In a typical intersection there are four quadrants that road users may travel through. The flow through these four quadrants must be regulated so that collisions do not occur. Each quadrant represents a single space. The network representation for an intersection is shown in Fig 5b. There are three arcs associated with each quadrant. In the first quadrant for instance the flows are: (I1,M2),(M1,I2),(M1,M2) and the sources of those flows are I1 and M1. These flows cannot happen simultaneously. This network representation is problematic for the MMTS_CAP, as it allows three separate flows through the quadrant at the same time. To correct this problem, the concept of a group of arcs is introduced:

Quadrant 1: *in* = {I1, M1}; *out* = {I2, M2}; ⇒ Group 1 = {(I1,I2),(I1,M2),(M1,I2),(M1,M2)};
Quadrant 2: *in* = {I3, M2}; *out* = {I4, M3}; ⇒ Group 2 = {(M2,I4),(M2,M3),(I3,M3),(I3,I4)};
Quadrant 3: *in* = {I5, M3}; *out* = {I6, M4}; ⇒ Group 3 = {(I5,I6),(I5,M4),(M3,I6),(M3,M4)};
Quadrant 4: *in* = {I7, M4}; *out* = {I8, M1}; ⇒ Group 4 = {(I7,M1),(I7,I8),(M4,M1),(M4,I8)};

The definition of "arc groupings" provides us with a mechanism to regulate the flow of vehicles in each quadrant. The set of groups is denoted by G. Let the arcs present in group *g* be denoted by A_g. The capacity of a group is the minimum capacity of the arcs within the group, i.e. $CAP_g = \min_{a \in A_g} C_a$. The following constraint is then added:

$$\sum_{a \in A_g} \sum_p \mathbb{V}_{p,a} \leq CAP_g \quad \forall g \in G \tag{19}$$

Each intersection has different phases of traffic flow. Each phase facilitates the movement of traffic through the intersection from a specific direction (i.e. a green period). Each phase has a cycle of red, green and yellow lighting that is repeated. There may be a different number and ordering of phases at each intersection. The time to perform "green" periods for all the phases at an intersection and their lost time is the cycle time. The time for each of these green periods is not necessarily the same; this is dictated by the phase split. The magnitude of the splits greatly affects the capacity of an MMTS (Burdett et al., 2014). To include this in our MMTS_CAP, we note that apart from the lost time incurred

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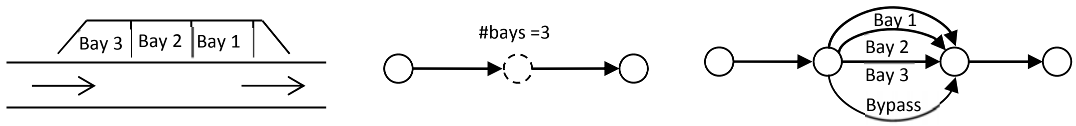
(i.e. to facilitate safety), each quadrant is fully utilised. Hence the capacity of each quadrant should be reduced by some amount. Let $\mathbb{T}_a$ be the time that arc *a* is available during the analysis period. Under normal conditions, $\mathbb{T}_a = \mathbb{T}$. For arcs present on intersections however, $\mathbb{T}_a \leq \mathbb{T}$. To compute $\mathbb{T}_a$ historical and empirical information can be used. Let λ_a be a reduction factor. Hence, $\mathbb{T}_a = \lambda_a \mathbb{T}$. The aforementioned arc capacity limit can therefore be revised as the following equation:

$$\sum_{p \in P} \mathbb{V}_{p,a} \leq \mathbb{T}_a . C_a . \tau_a \quad \forall a \in A, \forall p \in P \qquad \text{[Revised arc capacity]} \qquad (20)$$

Railways: If we assume that homogenous train types are used, the calculations are identical, except that the headway distance is equal to the length of the section that is being traversed (i.e. $H_d = L_a$). If the railway track is used by different types of trains of different lengths, then an average headway time must be used, for example $H_t = (H_d + \bar{\mathbb{L}})/\bar{V}$. The average length of trains is calculated using a weighted average $\bar{\mathbb{L}} = \sum_i \eta_i L_i$, where η_i is the proportion of trains of type i, and L_i is the length of train i.

Busways: Busways are dedicated roads for buses to travel on. They typically have a single lane and homogenous vehicles generally use them. This is not the case on other types of roads. Platforms occur at locations along the busway, to load and unload passengers. Each platform contains space (i.e. bays) for one or more buses. Bus platforms can be represented in MMTS networks in several different ways. For instance, they may be represented as nodes or as arcs. When represented as arcs, the number of loading bays must be incorporated as multiple arcs (see Fig. 1c).



a) Natural layout **b)** Platform on node **c)** Platform on arcs

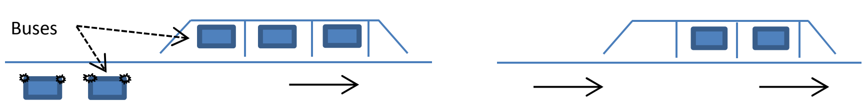
Figure 6: Bus platform representations

Busway capacity calculations are similar to roadway. It is however necessary to take into account the time that is lost at stations/platforms when passengers board and dismount because these times may reduce the capacity of busway links. The capacity of a busway link is a function of the number of loading bays N_l at the preceding platform and the lost time δ. In other words:

$$C_a = G(T, H_t, \delta, N_l) \qquad \text{[Busway capacity]} \qquad (21)$$

It is worth noting that at best, the flow into a bus platform is equal to $\mathbb{T}/H_t$ over time $\mathbb{T}$. If the lost time is δ then the flow out of a bus station can be determined by $(\frac{\mathbb{T}}{\delta})\times\min(\frac{\delta}{H_t}, N_l)$.

PROOF: The number of buses that arrive in a time period of duration δ is δ/H_t. If that number of buses is less than the number of loading bays (i.e. $\delta/H_t < N_l$), then the capacity of the bus platform is $\frac{\mathbb{T}}{\delta}.\frac{\delta}{H_t} = \frac{\mathbb{T}}{H_t}$. Otherwise, the flow of buses is greater than the number of bays, and a queue of buses will develop, and buses will accumulate before the platform. In this situation the capacity is $(\mathbb{T}/\delta).N_l$. As a result the capacity is $(\frac{\mathbb{T}}{\delta})\times\min(\frac{\delta}{H_t}, N_l)$ in general. These situations are shown in Fig.6.



a) $\frac{\delta}{H_t} > N_l$ b) $\frac{\delta}{H_t} < N_l$

Figure 7: Dwell time and its effect on usage of bays within a bus station

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3.5. Modelling Congestion

Traffic congestion is a phenomenon that reduces the flow of vehicles on roads and other transportation infrastructure. The primary cause of road congestion is human behaviour. It has been found in many studies and observations that traffic congestion increases as the number of vehicles (i.e. vehicle density) increases. As congestion reduces road capacity, the presence of congestion should be included somehow in the MMTS_CAP. There are various mathematical functions for traffic congestion. These are generally piecewise linear or non-linear (Bliemer et al., 2015). Previous congestion functions have been defined in terms of flow versus density. In other words, it is assumed that flow is a function of density. To use these functions, some modification is required, as there is no density variable in the MMTS_CAP. Upon reflection it is apparent that those congestion functions can be reposed in terms of intended flow (i.e. f_a) versus corrected flow (i.e. f_a') such that $f_a' = \boldsymbol{\mathcal{F}}(f_a, S_a, L_a)$. This function includes the speed of vehicles S_a and the length of the road or rail, namely L_a. Different congestion effects may occur for different speeds and lengths and on different roads. Our translation of the "traditional" functions is possible because intended flow maps directly to density and vice versa. For instance if the arc length L_a, headway distance H_d and speed S_a are provided, then the density is $den_a = L_a/H_a^{dist}$, the headway time is $H_t = H_d/S_a$ and the intended flow is $f_a = \mathbb{T}/H_t = \mathbb{T}.S_a/H_d = \mathbb{T}.S_a.L_a/den_a$. Evidently f_a and den_a are interconnected.

In this paper, we utilise a triangular function to demonstrate that congestion can be included in our approach. The triangular function is shown in Figure 8. It has two linear segments. It is a simplification of a more general concave non-linear function.

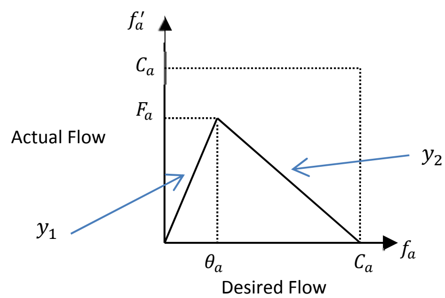

Figure 8: Traffic congestion diagram

To incorporate this function, the parameters and decision variables shown in Table 1 are defined.

Table 1: Parameters and other values needed for modelling congestion

Variable	Description
$\boldsymbol{C_a}$	Maximum flow possible when vehicles are positioned at a specified headway distance
$\boldsymbol{\theta_a}$	Intended flow at which maximum actual flow is realised
$\boldsymbol{F_a}$	Maximum value of actual flow. Requirement: $F_a \leq \theta_a \ \forall a \in A$
$\boldsymbol{\alpha_a}$	Binary variable to choose first or second interval (i.e. y_1, or y_2)
$\boldsymbol{f_a}$	Desired "uncorrected" flow. Note: $0 \leq f_a \leq C_a$
$\boldsymbol{f_a'}$	Actual "corrected" flow. Note: $0 \leq f_a' \leq F_a$ and $f_a' \leq f_a$

The purpose of this function (hereby denoted by $\wp$) is to correct chosen flow levels to actual flow realities. This function demonstrates that the highest theoretical flow level of C_a is not reachable. The highest flow that is attainable is F_a at θ_a. It is noteworthy to mention that $F_a \leq \theta_a$. As $\wp$ comprises two separate linear functions it is necessary to add the following constraints: $f_a \geq \theta_a + (\alpha_a - 1)C_a$ and $f_a \leq \theta_a + \alpha_a M$. These constraints ensure that if $\alpha_a = 0$ then $0 \leq f_a \leq \theta_a$ and if $\alpha_a = 1$ then $\theta_a \leq f_a \leq C_a$. The combined congestion function is as follows:

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$$f'_a = \wp(f_a) = \left(\frac{F_a}{\theta_a}\right) f_a - \left(\frac{C_a F_a}{\theta_a - C_a}\right) \alpha_a + \left(\frac{F_a}{\theta_a - C_a} - \frac{F_a}{\theta_a}\right) \alpha_a f_a \quad (22)$$

The derivation can be found in Appendix B. This function is non-linear as $\alpha_a f_a$ is a product of a binary and a real valued decision variable. Fortunately this term can be linearized using a technique from AIMMS 3.13 (2014). Let $\gamma_a = \alpha_a f_a$ where $0 \le f_a \le C_a$ and $0 \le \alpha_a \le 1$. The following constraints can then be added:

$$\gamma_a \ge 0 \quad (23)$$
$$\gamma_a \le \alpha_a C_a \ \ \forall a \in A \quad (24)$$
$$\gamma_a \le f_a \ \ \forall a \in A \quad (25)$$
$$\gamma_a \ge f_a - C_a(1 - \alpha_a) \ \ \forall a \in A \quad (26)$$

It should be noted that if $\alpha_a = 0$ then $\gamma_a = 0$. Also if $\alpha_a = 1$ then $\gamma_a = f_a$.

The incorporation of the triangular congestion causes the model to change from an LP to a mixed integer programming model (MIP). The inclusion of α_a only results in the addition of $|A|$ binary decision variables. For most networks the total number of arcs is not excessive and the incorporation of congestion will not cause any additional computational complexity and overhead to solve the model.

Other congestion functions can be facilitated by replacing constraint (22) with appropriate replacements. If the number of segments in the congestion function is small then only a handful of binary decision variables need be added. A more generic approach is to define f'_a as a piecewise linear function and to use separable programming techniques to solve the resulting model. The testing of additional congestion functions however is outside the scope of this article and will be considered in future studies.

4. Numerical Investigations

In this section, the MMTS_CAP is first applied to some simple examples before a full sized case study is presented. The case study occurs in Brisbane, Queensland, Australia. The MMTS considered includes road, rail and busways south of the Brisbane CBD. Our data was obtained from the Department of Transport and Main Roads (TMR) in Queensland. The exact details of this case study will be discussed in due course. All of the results presented were obtained by IBM's CPLEX Studio. We used a 64 bit, quad core, Dell personal computer with a 2.5 GHz processor and 4 GB memory. No CPU times have been described in this section as the MMTS_CAP model was not time consuming to solve. This is because the model is an LP when there are no congestion functions or a MIP with a relatively small number of binary variables.

4.1. Basic MMTS Examples

The MMTS_CAP was applied to a number of smaller instances to demonstrate its applicability and capability and second to verify the model provides the correct answers for different MMTS scenarios. These test instances are shown in Figure 9 and have features that are prevalent in full sized, real life MMTS. The label of each arc defines the mode and the number of lanes/tracks therein. For example, if the label is [Rd,2] then the arc is a road and it has two lanes. These examples also demonstrate how an MMTS network should be defined in order to apply the MMTS_CAP. The details of these examples are as follows:

Example 1 (Fig 9a): In this example passengers may travel by bus and car between location 1 and 4. There are two alternative paths for ODP (1, 4) and only a single direction of travel.

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Example 2 (Fig 9b): In this example there are two ODP, namely (1, 3) and (3, 1). Passengers also have to use two different modes to get from 1 to 3 and 3 to 1. No delay is considered when transferring from one mode to another here.
Example 3 (Fig 9c): This example is similar to Example 1. Travel in both directions however is included. There are two ODP namely (1, 3) and (3, 1).
Example 4 (Fig 9d): In this example there is a single ODP namely (1, 4) and two modes of travel can be used. This example involves travel by road, busway and railway. There is a pedestrian link between location 2 and 3.
Example 5 (Fig 9e): This example involves four ODP, namely (1, 5), (1, 3), (4, 5), (4, 3), and two different destinations. Two modes of travel can be used. Location 2 is an intersection of the different paths.
Example 6 (Fig 9f): This example involves 20 ODP. The mode of all is roadway. In this example, the given data for each arc is the same. Different green time ratios are incorporated.
Example 7 (i.e. Fig 9g): In this scenario congestion functions are specifically tested. This example involves four ODP and all arcs are roads. We have used: θ_a=(1500,1000,1200,1000,1200) and F_a=(1100,800,900,800,900). We also assume two people per vehicle.

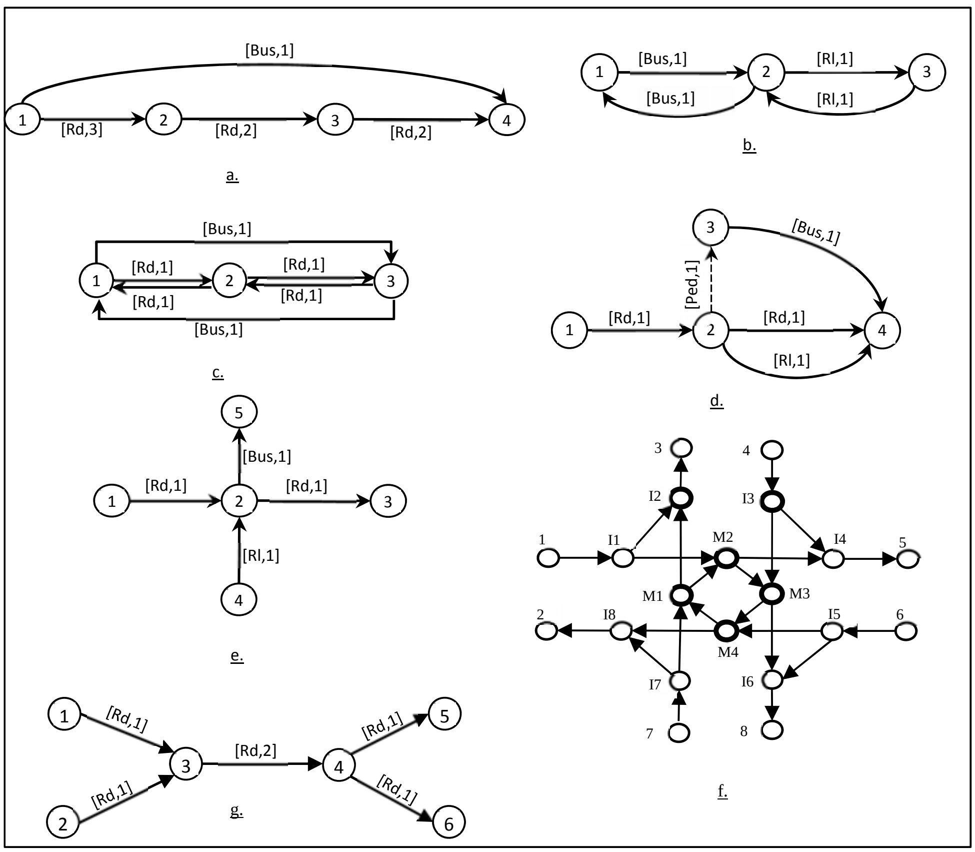

Figure 9: Simplified MMTS scenarios used in model verification

The demand on each ODP is considered to be zero in these examples. In other words, we let the model choose the amount of flow between each ODP. The MMTS_CAP results for the given examples are shown in Table 2 to 8. These solutions can be verified by hand. The flow of commodities and vehicles as well as unused capacity for commodities is shown. In Table 7 free capacity is not displayed because the network is totally utilised. Table 8 shows how the desired flow levels are scaled down to more attainable levels.

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Table 2: Example 1 results

a	$\mathbb{F}$	$\mathbb{V}$	$\mathbb{U}_{a,k}^{com}$
1-2	[16211,352]	5286	[8105,176]
2-3	[16211,352]	3524	[0,0]
3-4	[16211,352]	3524	[0,0]
1-4	[50163,0]	1475	[0,0]

Table 3: Example 2 results

a	$\mathbb{F}$	$\mathbb{V}$	$\mathbb{U}_{a,k}^{com}$
1-2	[6702,0]	[1311,0]	[37888,0]
2-3	[6702,0]	[38,0]	[0,172]
3-2	[67020,0]	[38,0]	[0,172]
2-1	[6702,0]	[1311,0]	[37888,0]

Table 4: Example 3 results

a	$\mathbb{F}$	$\mathbb{V}$	$\mathbb{U}_{a,k}^{com}$
1-2	[8824,72]	1446	[0,0]
2-3	[8824,72]	1446	[0,0]
1-3	[50163,0]	1475	[0,0]
3-2	[8824,72]	1446	[0,0]
2-1	[8824,72]	1446	[0,0]
3-1	[50163.93,0]	1475	[0,0]

Table 5: Example 4 results

a	$\mathbb{F}$	$\mathbb{V}$	$\mathbb{U}_{a,k}^{com}$
1-2	[8824,72]	1446.65	[0,0]
2-3	[0,0]	5000	[5000,0]
2-4	[8824,72]	1446.7	[0,0]
2-4	[0,0]	39.81	[7962,119]
3-4	[0,0]	1475.41	[50163,0]

Table 6: Example 5 results

a	$\mathbb{F}$	$\mathbb{V}$	$\mathbb{U}_{a,k}^{com}$
1-2	[8824,0]	1446	[0,72]
2-3	[7962,72]	1446	[862,0]
4-2	[7962,72]	39	[0,47]
2-5	[8824,0]	259	[41339,0]

Table 7: Example 6 results

ODP	$\mathbb{F}$	$\mathbb{V}$	ODP	$\mathbb{F}$	$\mathbb{V}$
1-3	[4824,39]	954	6-8	[4824,39]	954
1-5	[1000,8]	163	6-2	[1000,8]	163
1-8	[1000,8]	163	6-3	[1000,8]	163
4-5	[4824,39]	954	7-2	[4824,39]	954
4-8	[1000,8]	163	7-3	[1000,8]	163
4-2	[1000,8]	163	7-5	[1000,8]	163

Table 8: Example 7 results

a	f_a	f'_a	$\mathbb{V}_a$	$\mathbb{V}'_a$
1-3	[3000,0]	[2200,0]	1500	1100
2-3	[2000,0]	[1600,0]	1000	800
3-4	[4800,0]	[3900,0]	2400	1800
4-5	[2000,0]	[1600,0]	1000	800
4-6	[2000,0]	[1800,0]	1200	900

These small scenarios verify that the model is viable and satisfies the technical conditions that are present. The model successfully satisfies flow in both directions of travel and can successfully link different modes.

4.2. Real life case study

In this section, the MMTS_CAP is applied to a full sized MMTS. Our MMTS (i.e. MMTS_SEB) is located in Brisbane, Australia in the south east corner. This transportation network covers a large area of land. It includes 73 major zones between Brisbane and the Gold Coast, and there are approximately 66 major origin destination pairs (ODP). Brisbane and Gold Coast have the highest population in Australia after Sydney and Melbourne. On a daily basis many people travel between these two major cities. Commuters travel to three major universities, namely Griffith University, University of Queensland, and Queensland University of Technology. Many commuters also travel to major hospitals such as Princess Alexandra Hospital (PAH), Mater Hill Hospital, and Greenslopes Private Hospital. Thousands residents located in the suburbs also travel to the Brisbane central business district (CBD) daily. The MMTS_SEB includes railway, busway, and roadway.

The network is drawn with undirected arcs in Figure 10. In reality the flows occur in two directions and there are two different "connected" networks, an inbound and an outbound. They have similar structure and links, but there are differences at a number of places. For instance, the on ramps and off ramps are positioned differently depending on whether travel is to or from the CBD. In this article we considered the inbound flows and the capacity of that associated MMTS network. Outbound flows

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could also be separately investigated or the whole network could be jointly analysed. This may be performed in a later study but is outside the scope of this article. Some of the more advanced modelling has not been included in the case study. Intersections for example were not included. No pedestrian links were modelled per se. The effect of bus platform number was also ignored. For the sake of interest, there are room for three bays at each platform in our networks busway.

In practice the demand originating from each suburb varies and depends on the population size in an area/suburb (see Table 9). For added realism we have identified the population of each suburb. This information was obtained from the Brisbane City council, Logan City council, and City of Gold Coast. This data was used as an input to the model.

Table 9: Population of south east Queensland's suburbs

Suburb	Population	Suburb	Population	Suburb	Population	Suburb	Population
Robina	20,522	Underwood	5,328	Moorooka	9,984	Buranda	4000
Helensvale	15,984	Springwood	8,991	Yeerongpilly	1,983	Coorparoo	14,944
Beenleigh	8244	Rochedale	1,092	Holland Park	7,849	Norman Park	6,003
Edens Landing	5,176	Runcorn	14,075	Yeronga	5,539	Morningside	9,399
Waterford	3,930	Sunnybank	8,090	Greenslopes	8,565	Tingalpa	8,540
Loganlea	6,173	Eight Mile Plains	13,378	Carindale	15,577	Cannon Hill	4,507
Loganholme	6,126	Coopers Plains	4,208	Belmont	4,595	Murarrie	3,958
Kingstone	10,183	Mt Gravatt	3,238	Fairfield	2,554	Eagle Farm	4,719
Woodridge	12,786	Salisbury	6,095	UQ	1000	South Brisbane	5,416
Daisy Hill	6,204	Rocklea	1,256	Dutton Park	1,471	Brisbane CBD	7,889
Kuraby	7,776	Mackenzie	1,845	Park Road	1,987	Fortitude Valley	5,615

In the first step of our analysis, we have separately analysed each ODP. The results are shown in Table 11 for a time period of four hours. This sensitivity analysis has identified the maximum flow of commodities (i.e. $UB_{p,k}$) that can be achieved with no interference from other sources of traffic, for instance that are travelling between other ODP. In the second step of our analysis, we have run the model and included all ODP at the same time. In this case specific demands are not defined. The results are shown in Table 10. The reduction in flow that occurs for commodity k and ODP p when all ODP's are jointly and not individually considered, is represented by $r_{p,k}$.

Table 10: The maximum flow of commodities in MMTS_SEB

No.	ODP	$\gamma_{k,p}$	$r_{k,p}$	No.	ODP	$\gamma_{k,p}$	$r_{k,p}$
1	Robina-Central Station	[6840,72]	[80.2,80.3]	22	Griffith Uni-Elizabeth St	[3924,0]	[97.7,0]
2	Robina-Eagle Farm	[6840,422]	[87.9,65.7]	23	Holland Park-Elizabeth St	[3924,0]	[97.7,0]
3	Helensvale-William St	[7992,173]	[85.9,85.9]	24	Greenslopes B-Elizabeth St	[4282,0]	[97.4,0]
4	Beenleigh-William St	[4122,89]	[92.7,92.8]	25	Greenslopes-William St	[4282,93]	[91.1,81.7]
5	Edens Landing-Elizabeth St	[2588,254]	[94.6,0]	26	Carindale-Fortitude Valley	[15577,164]	[34.9,35.4]
6	Waterford-William St	[3930,85]	[92.3,84.4]	27	Moorooka-Central Station	[2217,23]	[93.6,93.7]
7	Loganlea-William Street	[3086,67]	[94,87.7]	28	Buranda B-William Street	[1333,28]	[97.6,97.7]
8	Loganholme-William St	[6126,133]	[88.3,88.3]	29	Buranda B-Elizabeth Street	[1333,0]	[99.2,0]
9	Kingston-William Street	[5091,110]	[89.4,78.4]	30	Coorparoo-Central Station	[14944,158]	[56.7,56.7]
10	Woodridge-William St	[4262,92]	[92.5,87.9]	31	Morningside-Central Station	[9399,99]	[72.7,72.9]
11	Daisy Hill-William St	[6204, 134]	[88.2,88.2]	32	Tingalpa-Fortitude Valley	[8540,155]	[84.9,82.3]
12	Kuraby-William St	[3888,84]	[91.9,83.5]	33	Boggo Road-Elizabeth St	[993,0]	[99.4,0]
13	Springwood-Elizabeth St	[8991,0]	[84.1,100]	34	Dutton Park B-Elizabeth St	[735,0]	[99.6,0]
14	Rochdale-Elizabeth St	[1092,0]	[99,100]	35	UQ-Elizabeth Street	[1000,0]	[99.4,0]
15	8 Mile Plains-William St	[5164,112]	[90.9,90.9]	36	Mater Hill B-Elizabeth Street	[1083,0]	[99.4,0]
16	8 Mile Plains B-Elizabeth St	[6689,0]	[96,0]	37	Mater Hill-Fortitude Valley	[1083,11]	[98.7,98.7]
17	Mt Gravatt-Eagle Farm	[1079,763]	[98.1,0]	38	South Bank-Elizabeth Street	[1083,0]	[99.4,100]
18	Garden City-William St	[1079,23]	[98.1,98.1]	39	South Brisbane-Central Stn	[1083,11]	[96.9,97]
19	Upper Mt Gravatt-Elizabeth St	[1079,0]	[99.4,0]	40	Murarrie-Fortitude Valley	[1979,20]	[96.5,97.7]
20	Mackenzie-Fortitude Valley	[1845,42]	[96.7,95.2]	41	Eagle Farm-Fortitude Valley	[4719,50]	[80.3,80.3]
21	Belmont-Fortitude Valley	[4595,48]	[91.9,94.5]				

An incremental analysis whereby different proportions of the population are taken as demands was also performed. This mimics an evacuation planning process or the planning of a major event in the CBD whereby a large number of people need to attend. We first assumed that 50 percent of the

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population needs to travel from their suburb to the Brisbane CBD. In this situation the model did not solve. Hence we can conclude that the MMTS does not have sufficient capacity to move so many people (i.e. 146645 people) in such a short period of time. The next question that arises is, if cannot move 50%, what number of people can we move? To answer this question, the objective function is altered appropriately. In other words, we use the objective at (Eq.12). Constraint (13) and (14) are also added to the model. Additional constraints concerning the flow are also required when the model is used in query mode. The following constraint is added to limit the number of people:

$$\gamma_{p,k} \leq UB_{p,k} \quad \forall p \in P, \forall k \in K \qquad \text{[Limit the flow to suburb population]} \qquad (27)$$

The results of our query are shown in Table 12. The network can move 115064 people (i.e. 40% of the population) as well as 3919.8 containers in four hours throughout the network.

As previously mentioned, 50 percent of the population could not be moved through the network in four hours. A new query hence arises. If four hours is insufficient then what is the minimum required time to move 50% of the population? The objective was altered to facilitate this query. The new objective function is to minimise $\mathbb{T}$. Constraint (28) is also added to ensure than $\mathbb{T} = \min_a \sum_{p \in P} \frac{\sum_{p \in P} \mathbb{V}_{p,a}}{C_a . \tau_a}$:

$$\mathbb{T} \geq \frac{\sum_{p \in P} \mathbb{V}_{p,a}}{C_a . \tau_a} \quad \forall a \in A, \forall p \in P \qquad \text{[Number of hours]} \qquad (28)$$

The revised model was solved. It produced a solution of 7.84 hours.

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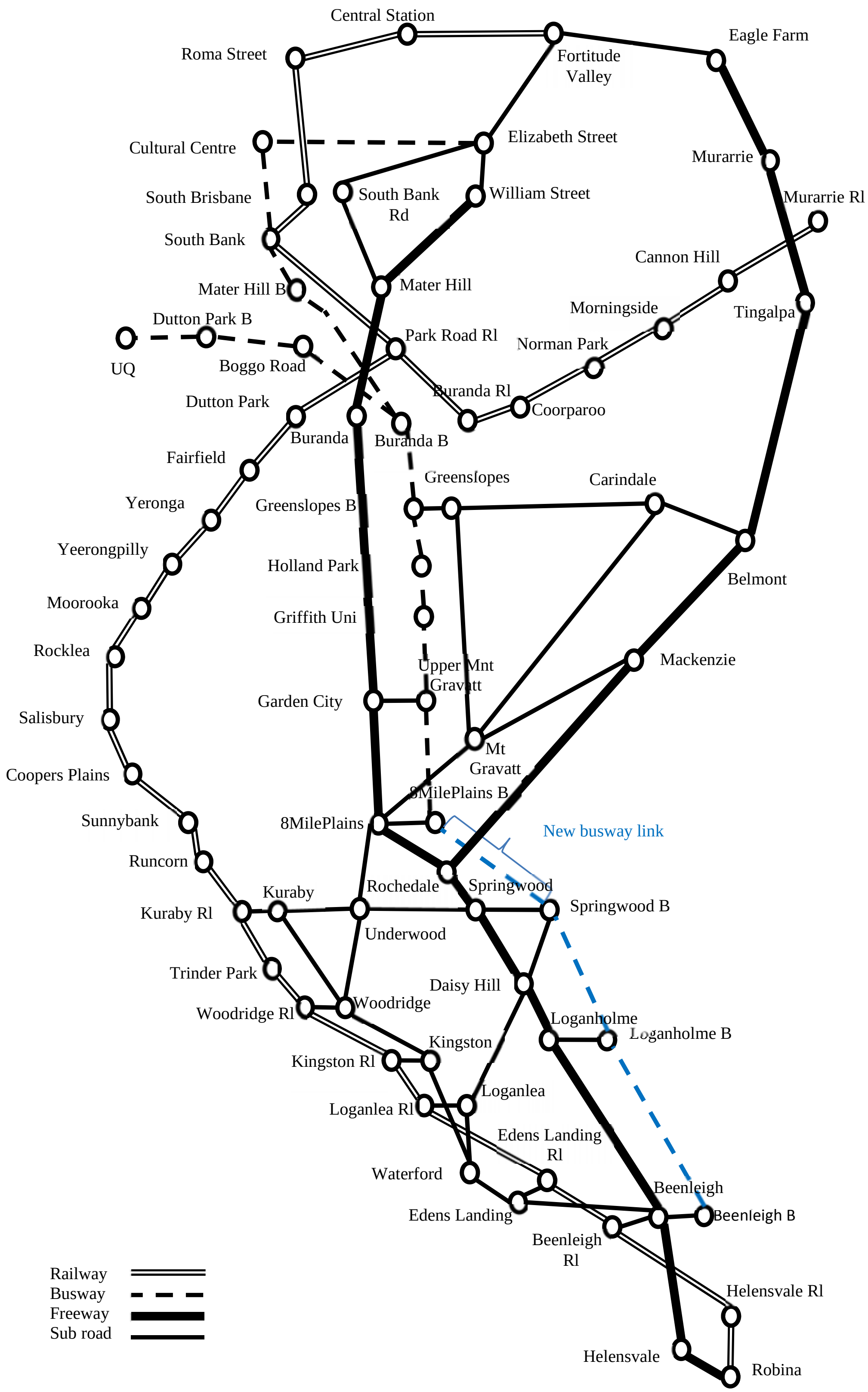

Figure 10: MMTS of south Brisbane

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Table 11: The number of commodities that can be moved through a specific ODP

No.	ODP	[People, Goods]	No.	ODP	[People, Goods]	No.	ODP	[People, Goods]
1	Robina-Central Station	[34484,365]	23	Springwood-Elizabeth Street	[56561,254]	45	Fairfield-Central Station	[34484,365]
2	Robina-Eagle Farm	[56561,1229]	24	Rochdale-Elizabeth Street	[113123,254]	46	Dutton Park-Central Station	[34484,365]
3	Robina-William Street	[56561,1229]	25	Runcorn-Central Station	[34484,365]	47	Buranda-William Street	[56561,1229]
4	Helensvale-William Street	[56561,1229]	26	8 Mile Plains-William Street	[56561,1229]	48	Buranda B-Elizabeth Street	[167869,0]
5	Helensvale Rail-Central Station	[14229,150]	27	8 Mile Plains B-Elizabeth St	[167869,0]	49	Buranda Rail-Central Station	[34484,365]
6	Beenleigh Rail-Central Station	[34484,365]	28	Sunnybank-Central Station	[34484,365]	50	Coorparoo-Central Station	[34484,365]
7	Beenleigh-William Street	[56561,1229]	29	Coopers Plains-Central Station	[34484,365]	51	Norman Park-Central Station	[34484,365]
8	Edens Landing-Elizabeth Street	[47854,254]	30	Mount Gravatt-Eagle Farm	[56561,763]	52	Morningside-Central Station	[34484,365]
9	Edens Landing Rail-Central Stn	[34484,365]	31	Garden City-William Street	[56561,1229]	53	Cannon Hill-Central Station	[34484,365]
10	Waterford-William Street	[51272,545]	32	Upper Mnt Gravatt-Elizabeth St	[167869,0]	54	Murarrie Rail-Central Station	[34484,365]
11	Loganlea Rl-Central Station	[34484,365]	33	Mackenzie-Fortitude Valley	[56561,874]	55	Tingalpa-Fortitude Valley	[56561,874]
12	Loganlea-William Street	[51272,545]	34	Belmont-Fortitude Valley	[56561,874]	56	Park Road Rail-Central Station	[34484,365]
13	Loganholme-William Street	[52371,1138]	35	Griffith Uni-Elizabeth Street	[167869,0]	57	Boggo Road-Elizabeth Street	[167869,0]
14	Kingston Rail-Central Station	[34484,365]	36	Holland Park-Elizabeth Street	[167869,0]	58	Dutton Park B-Elizabeth Street	[167869,0]
15	Kingston-William Street	[47854,509]	37	Greenslopes B-Elizabeth Street	[167869,0]	59	UQ-Elizabeth Street	[167869,0]
16	Woodridge Rail-Central Station	[34484,365]	38	Greenslopes-William Street	[47854,509]	60	Mater Hill B-Elizabeth Street	[167869,0]
17	Woodridge-William Street	[56561,763]	39	Carindale-Central Station	[23927,254]	61	Mater Hill-Fortitude Valley	[82338,874]
18	Daisy Hill-William Street	[52371,1138]	40	Salisbury-Central Station	[34484,365]	62	South Bank-Elizabeth Street	[191796,254]
19	Trinder Park-Central Station	[34484,365]	41	Rocklea-Central Station	[34484,365]	63	South Bank-Central Station	[58411.3,365
20	Kuraby Rail-Central Station	[34484,365]	42	Moorooka- Central Station	[34484,365]	64	South Brisbane-Central Station	[34484,365]
21	Kuraby-William Street	[47854,509]	43	Yeerongpilly-Central Station	[34484,365]	65	Murarrie-Fortitude Valley	[56561,874]
22	Underwood-William Street	[47854,509]	44	Yeronga-Central Station	[34484,365]	66	Eagle Farm-Fortitude Valley	[23927,254]

Table 12: The commodities flow closest to 50% of each suburb population in a given peak hours

No.	ODP	[People, Goods]	No.	ODP	[People, Goods]	No.	ODP	[People, Goods]
1	Robina-Central Station	[3420, 36]	23	Springwood-Elizabeth Street	[4495,0]	45	Fairfield-Central Station	[0,0]
2	Robina-Eagle Farm	[3420, 444]	24	Rochdale-Elizabeth Street	[546,0]	46	Buranda-William Street	[666, 14]
3	Robina-William Street	[3420, 74]	25	Runcorn-Central Station	[7037, 74]	47	Buranda B-Elizabeth Street	[666, 0]
4	Helensvale-William Street	[3996, 86]	26	8 Mile Plains-William Street	[3344, 538]	48	Buranda Rail-Central Station	[0,0]
5	Helensvale Rail-Central Stn	[0, 0]	27	8 Mile Plains B-Elizabeth Street	[3344, 0]	49	Coorparoo-Central Station	[7472, 79]
6	Beenleigh Rail-Central Stn	[0, 0]	28	Sunnybank-Central Station	[4045, 42]	50	Norman Park-Central Station	[0,0]
7	Beenleigh-William Street	[2061, 44]	29	Coopers Plains-Central Station	[0, 0]	51	Morningside-Central Station	[4699, 49]
8	Edens Landing-Elizabeth St	[2588, 13]	30	Mt Gravatt-Eagle Farm	[539, 763]	52	Cannon Hill-Central Station	[0,0]
9	Edens Landing Rail-Central Stn	[0, 0]	31	Garden City-William Street	[539, 11]	53	Murarrie Rail-Central Station	[0, 0]
1	Waterford-William Street	[1965, 42]	32	Upper Mt Gravatt-Elizabeth St	[539, 0]	54	Tingalpa-Fortitude Valley	[4270, 196]
11	Loganlea Rail-Central Station	[1543.25, 16]	33	Mackenzie-Fortitude Valley	[922, 9]	55	Boggo Road-Elizabeth Street	[496,0]
12	Loganlea-William Street	[1543.25, 33]	34	Belmont-Fortitude Valley	[2297,319]	56	Dutton Park B-Elizabeth Street	[367, 0]
13	Loganholme-William Street	[3063, 66]	35	Griffith Uni-Elizabeth Street	[1962, 0]	57	Dutton Park-Central Station	[0, 0]
14	Kingston Rail-Central Station	[0, 0]	36	Holland Park-Elizabeth Street	[1962, 0]	58	UQ-Elizabeth Street	[500, 0]
15	Kingston-William Street	[2545, 55]	37	Greenslopes B-Elizabeth Street	[2141, 0]	59	Park Road Rail-Central Station	[48, 0]
16	Woodridge Rail-Central Stn	[0, 0]	38	Greenslopes-William Street	[2141, 46]	60	Mater Hill B-Elizabeth Street	[541.6, 0]
17	Woodridge-William Street	[2131, 46]	39	Carindale-Central Station	[7788, 82]	61	Mater Hill-Fortitude Valley	[541, 229]
18	Daisy Hill-William Street	[3102, 67]	40	Salisbury-Central Station	[0, 0]	62	South Bank-Elizabeth Street	[541,0]
19	Trinder Park-Central Station	[0, 0]	41	Rocklea-Central Station	[0,0]	63	South Bank-Central Station	[0, 0]
20	Kuraby Rail-Central Station	[0, 0]	42	Moorooka-Central Station	[4992,52]	64	South Brisbane-Central Station	[0, 0]
21	Kuraby-William Street	[1944, 42]	43	Yeerongpilly-Central Station	[0,0]	65	Murarrie-Fortitude Valley	[989, 10]
22	Underwood-William Street	[2664, 57]	44	Yeronga-Central Station	[2769, 29]	66	Eagle Farm-Fortitude Valley	[2359, 25]

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5. Conclusions

In this article how to apply quantitative methods to measure and assess the capacity of a multi-model transportation system was considered. To fill the current gap in analytical capacity modelling, a generic "optimization based" approach has been developed. Our model named MMTS_CAP determines the theoretical capacity of an entire MMTS. It is a multi-commodity flow formulation. It integrates different mixes of vehicles and is suitable for MMTS with unidirectional and bidirectional traffic flows. It is well suited to MMTS with rail, busways and roadways. It is important to note that our approach is a macro rather than a micro level approach. It determines the maximum capacity of a MMTS from a "structural perspective". The maximum capacity is a very useful reference point. While the maximum capacity cannot be achieved in practice, it is sufficiently accurate to facilitate a variety of different planning activities which would not otherwise be possible or would be timely to perform. It should be noted our deterministic approach to calculate the maximum flow of MMTS is well suited to scenarios involving autonomous vehicles. These types of vehicles behave more precisely than human drivers. Our approach however can facilitate different congestion functions. This is a powerful feature of the MMTS_CAP.

The primary innovation of our model is that it links and integrates a variety of different modes. To our knowledge, this is not simple to do and past research has failed to do this. Often individual transportation systems are separately analysed. Additional benefits of the MMTS_CAP are listed below:

i. An assessment of MMTS capacity can be performed quickly and straightforwardly
ii. Our model is transparent and straightforward to use by planners and managers.
iii. The structural attributes and characteristics of a MMTS can be assessed.
iv. Bottlenecks within the system can be identified which can help decision makers perform expansion planning activities and other alterations.

To test the validity of the model it was applied to a test bed of different scenarios. Those scenarios exhibit characteristics and complexities inherent in many real life MMTS. The proposed model was also successfully applied to a full sized case study. Our numerical investigation has determined how that MMTS can best be utilised and what number of passengers can be transported.

In conclusion this article indicates that quantitative techniques are worthwhile and may help transportation planners build better MMTS. Quantitative approaches can in theory be integrated into existing information technology platforms and their data requirements are no longer unrealistic in this day and age. Our approach can facilitate a variety of capacity planning and querying activities and provides a mechanism to quickly analyse the effect of structural and parametric changes within MMTS. MMTS decision makers need to answer many "what if" questions in practice and regularly need to perform the aforementioned activities.

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Appendix A: Nomenclature

Indices	
m, p, a, n, k	Modes, OD pairings, arcs, nodes, commodities
Sets	
N, A, O, D, P, K	Set of nodes, arcs, origins, destinations, OD pairings, commodities
M	Set of modes. $M = \{road, rail, busway, pedestrian\}$
Parameter	
o_a, d_a	Origin and destination of arc a
$\tilde{o}_p, \tilde{d}_p$	Origin and destination of pairing p
$\mathbb{T}$	Time period duration for the capacity analysis
H_t, H_d	Headway time (seconds) and headway distance (metres)
L_a	length of arc a
τ_a	Number of lane/track on arc a
C_a	Capacity of arc a in terms of vehicles per unit of time
$CPV_{a,i}$	The number of commodities in each vehicle of type i on arc a
δ_a	The density of vehicle flow on arc a
S_a	Speed on arc a
$CPV_{a,k}, \overline{CPV}_{a,k}$	The number of commodity k per vehicle for arc a
$V, \bar{V}$	Speed of commodity k, average speed of commodities
$\mathbb{L}, \bar{\mathbb{L}}$	Length of vehicle type i, average length of vehicles for mode m
η_i	Proportional mix of vehicles of type i
$\mathbb{D}_{p,k}$	Demand for commodity k on ODP p
F_a	Congestion function: Maximum value of actual flow
θ_a	Congestion function: Maximum actual flow occurs at this intended flow
$UB_{p,k}$	An upper bound on the number of commodities of type k that can travel on ODP p
Decision variable:	
Γ_k	Total flow of commodity k throughout the network
$\gamma_{k,p}$	Number of commodity k on ODP p
$\mathbb{F}_{p,a,k}$	Flow of commodity k for ODP p on arc a
$\mathbb{V}_{p,a}, \mathbb{V}'_{p,a}$	Number of vehicles that travel on arc a for ODP p
$\mathbb{U}^{com}_{a,k}$	Unused capacity on arc a for commodity type k
$\mathbb{u}_{p,k}$	Unused capacity on ODP p for commodity k
α_a	Binary variable used in separable programming to choose 1st or 2nd interval
f_a, f'_a	Desired "uncorrected" flow; Actual "corrected" flow
$r_{k,p}$	Reduction in flow for commodity k and ODP p when all ODP's are jointly and not individually considered. Note: $r_{k,p} = 100.\left(UB_{p,k} - \gamma_{k,p}\right)/UB_{p,k}$

Appendix B: Derivation of traffic congestion function

$$y_1(f_a) = \left(\frac{F_a}{\theta_a}.f_a\right) \text{ and } y_2(f_a) = \left(\frac{F_a}{\theta_a - C_a}\right) f_a - \left(\frac{C_a.F_a}{\theta_a - C_a}\right) \quad (1B)$$

$$f'_a(f_a) = (1-\alpha_a) y_1(f_a) + \alpha_a y_2(f_a) \quad (2B)$$

$$f'_a(f_a) = (1-\alpha_a)\left(\frac{F_a}{\theta_a}.f_a\right) + \alpha_a\left[\left(\frac{F_a}{\theta_a - C_a}\right) f_a - \left(\frac{C_a.F_a}{\theta_a - C_a}\right)\right] \quad (3B)$$

$$f'_a(f_a) = \left(\frac{F_a}{\theta_a}.f_a\right) - \alpha_a\left(\frac{F_a}{\theta_a}.f_a\right) + \alpha_a\left(\frac{F_a}{\theta_a - C_a}\right) f_a - \alpha_a\left(\frac{C_a.F_a}{\theta_a - C_a}\right) \quad (4B)$$

$$f'_a(f_a) = \left(\frac{F_a}{\theta_a}\right) f_a - \left(\frac{F_a}{\theta_a}\right)\alpha_a f_a + \left(\frac{F_a}{\theta_a - C_a}\right)\alpha_a f_a - \left(\frac{C_a F_a}{\theta_a - C_a}\right)\alpha_a \quad (5B)$$

$$f'_a(f_a) = \left(\frac{F_a}{\theta_a}\right) f_a - \left(\frac{C_a F_a}{\theta_a - C_a}\right)\alpha_a + \left(\frac{F_a}{\theta_a - C_a} - \frac{F_a}{\theta_a}\right)\alpha_a f_a \quad (6B)$$

$$f'_a(f_a) = \left(\frac{F_a}{\theta_a}\right) f_a - \left(\frac{C_a F_a}{\theta_a - C_a}\right)\alpha_a + \left(\frac{F_a}{\theta_a - C_a} - \frac{F_a}{\theta_a}\right)\gamma_a \quad (7B)$$

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