Remaining Useful Life Prediction of Rotating Equipment Using Covariate-based Hazard Models - Industry Applications

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Abstract

The ability to estimate the expected Remaining Useful Life (RUL) is critical to reduce maintenance costs, operational downtime and safety hazards. In most industries, reliability analysis is based on the Reliability Centred Maintenance (RCM) and lifetime distribution models. In these models, the lifetime of an asset is estimated using failure time data; however, statistically sufficient failure time data are often difficult to attain in practice due to the fixed time-based replacement and the small population of identical assets. When condition indicator data are available in addition to failure time data, one of the alternate approaches to the traditional reliability models is the Condition-Based Maintenance (CBM). The covariate-based hazard modelling is one of CBM approaches. There are a number of covariate-based hazard models; however, little study has been conducted to evaluate the performance of these models in asset life prediction using various condition indicators and data availability. This paper reviews two covariate-based hazard models, Proportional Hazard Model (PHM) and Proportional Covariate Model (PCM). To assess these models' performance, the expected RUL is compared to the actual RUL. Outcomes demonstrate that both models achieve convincingly good results in RUL prediction; however, PCM has smaller absolute prediction error. In addition, PHM shows over-smoothing tendency compared to PCM in sudden changes of condition data. Moreover, the case studies show PCM is not being biased in the case of small sample size.

Keywords: Hazard function, Reliability, Remaining useful life, Proportional hazard model, Proportional covariate model

1. Introduction

Lifetime analysis of engineering assets is a significant field of research within engineering asset health management (Le Son et al., 2013). In lifetime analysis of assets, condition data are often available alongside failure time data. Condition data provide more and prompt information about the health status and the level of degradation of an asset. This sort of data are the output of a stochastic process generated by an asset under study and observed only as long as the asset is operational (Kalbfleisch and Prentice, 2002). In general, condition indicator can be classified into two groups: direct and indirect condition indicators (Wang et al., 2000, Wang and Christer, 2000, Christer and Wang, 1995, Si et al., 2011). The thickness of a brake pad, the sectional loss and wear in a component are common examples of direct condition indicators. The vibration of fitted rotating machinery and the level of metal particles in engine oil analysis are familiar examples of indirect condition indicators.

Recently, interest has been expanded in most industries regarding the use of condition indicators in life prediction models (Si et al., 2011). The condition indicator process for an asset generally takes its values subsequent to the operating and environment parameters in which the asset operates. Therefore, such condition indicators are responsive since their values could have already been influenced by the operating condition (Kalbfleisch and Prentice, 2002). There are several methods in literature that have arisen to take the advantage of condition indicator data into the modelling of asset life (Jardine et al., 2006, Heng et al., 2009b, Si et al., 2011, Gorjian et al., 2009b, Gorjian et al., 2009a). One of these techniques is the covariate-based hazard model (Si et al., 2011, Gorjian et al., 2009b). Gorjian et al. (Gorjian et al., 2009b, Gorjian et al., 2009a) presented a state-of-the-art review of the existing literature on covariate-based hazard models in reliability modelling and discussed about each model merits and limitations. In general, the major advantage of the covariate-based hazard models is employing data that provide asset condition information in addition to failure and maintenance history. While there are a variety of these models, little is investigated on the model performance in asset prognostics using various condition indicators and field data availability. In using covariate-based hazard models, care should be exercised in interpreting estimated regression coefficients since condition indicators may exclusively carry information about the degradation level and health status of an asset (Jewell and Kalbfleisch, 1996). Therefore, statements about survival probabilities require a model for the condition indicator process as well as the covariate-based hazard model (Jewell and Kalbfleisch, 1996).

The majority of existing covariate-based hazard models employ operating environment indicators (or external covariates) in lifetime analysis of assets (Gorjian et al., 2009b). Two of these covariate-based hazard models were mainly applied for using condition indicators (or internal covariates) in lifetime analysis of assets. One is the widely used Proportional Hazard Model (PHM) which was initially developed to study the effects of external covariates in lifetime analysis of an individual and/or asset (Cox, 1972). Later, it was applied to model the degradation of an asset using the age of the asset monitored and its condition information obtained (Banjevic and Jardine, 2006, Jardine et al., 1987, Vlok et al., 2002, Gorjian et al., 2009b, Jardine et al., 2001, Jardine et al., 1989, Jardine and Tsang, 2006). Another one is the Proportional Covariate Model (PCM) which was developed and applied for lifetime analysis of an asset using both the failure event data and condition indicator data (Sun et al., 2006, Sun, 2006). While these models have been applied to model asset life using both failure event data and condition indicator data; however, the yet more important question is about the performance of these models regarding different types of condition indicators, event histories, and data availability. To this end, in this paper, the performance of PHM and PCM is evaluated through application to real life data using various sorts of condition indicators (features) and event histories, data availability (small and large sample size) as well as different operating and environment conditions.

The remainder of the paper is organised as follows. Section 2 briefly presents the formulation of PHM and PCM as well as their associated statistical inference procedures. It also explains the calculation of the expected Remaining Useful Life (RUL). Section 3 describes two real life case studies. A study using pump vibration data (RMS) with large event sample size from a pulp and paper mill and a study using pump vibration data (kurtosis) with small event sample size from the Liquefied Natural Gas (LNG) are discussed in this section. Performance of these models is verified by aforementioned cases. The results of RUL prediction for these studies are shown in Section 4. The conclusions are given in last section.

2. Background Theory

PHM assumes that the effects of different covariates (or risk factors) accelerate or decelerate the lifetime of an individual and/or asset. PHM supposes that the hazard is taken to be a function of the explanatory variables (or independent variable) and unknown regression coefficients multiplied by an arbitrary and unknown baseline hazard function (Cox, 1972). PHM with time-dependent covariates is given by (Prentice and Kalbfleisch, 1979):

$$h(t; \vec{z}(t)) = h_0(t) \exp(\vec{\gamma}\vec{z}(t)) \quad \textbf{(1)}$$

Here, $h_0(t)$ is the unspecified baseline hazard function. The positive log-linear function, $\exp(\vec{\gamma}\vec{z}(t))$, is dependent on the effects of covariates, which have multiplicative effect on the baseline hazard function. The likelihood function in PHM is given by Equation (2).

$$L = \prod_{i \in \Theta_F} h(t_i; \vec{z}(t_i)) \quad \times \prod_{j \in \{\Theta_F \cup \Theta_C\}} R\left(t_j; \vec{z}(\tau) \middle| 0 \le \tau < t_j\right) \quad \textbf{(2)}$$

Where, $h(t_i; \vec{z}(t_i))$ is the hazard function and $R\left(t_j; \vec{z}(\tau)\right)$ is the reliability function. Here, Θ_F indexes the set of failure times and Θ_C denotes the set of surviving times. In the preceding equation, t_i is the failure time of the i^{th} item, and t_j is either the failure time or the surviving time of the j^{th} item. All of the unknown parameters are obtained by maximising the log-likelihood function using a nonlinear optimisation method (e.g. Nelder – Mead's algorithm).

PCM was originally proposed by Sun in the reliability field (Sun, 2006). PCM has only been applied with a laboratory test data since its inception (Sun et al., 2006). This model assumes that condition indicator data are responsive as their values reflect the health status of an asset. PCM requires two steps for the modelling of hazard. Sun (Sun, 2006) described all of these steps in great details. The baseline covariate function in PCM can be obtained using the following equation (Sun et al., 2006):

$$c(t_i) = \frac{\zeta(t_i)}{h_0(t_i)} \qquad i = 1,2, \ldots, n \quad \textbf{(3)}$$

Where, n is number of failure times; $\zeta(t_i)$ is a vector of historical condition indicators; $h_0(t_i)$ is an initial hazard which is estimated using historical failure time data. Thus, a set of discrete hazard values, $h_0(t_i)$, at time t_i , $i = 1, \ldots, n$ can be generated. The likelihood function in PCM is given by:

$$L = \prod_{i \in \Theta_F} h_0(t_i) \quad \times \prod_{j \in \{\Theta_F \cup \Theta_C\}} R\left(t_j\right) \quad \textbf{(4)}$$

Where, $h_0(t_i)$ is the initial hazard function and $R\left(t_j\right)$ is the reliability function. Here, Θ_F indexes the set of failure times and Θ_C denotes the set of surviving times. In Equation (4), t_i is the failure time of the i^{th} item, and t_j is either the failure time or the surviving time of the j^{th} item.

After calculating the baseline covariate function, the posterior hazard, $h(t)$, can be updated via the current condition indicator, $\vec{z}(t)$, using the following equation:

$$h(t) = \frac{\vec{z}(t)}{c(t)} \quad \textbf{(5)}$$

Once the reliability function is estimated for each model, the RUL function can be calculated. The RUL function is the expected remaining life, $T - t$, given that an asset has survived up to time t (Banjevic, 2009). The expected RUL, $(t) = E(T - t | T \ge t)$, can be predicted as:

$$r(t) = \int_t^{\infty} \exp\left(-\int_t^{\tau} h\left(y; \vec{z}(y)\right) dy\right) d\tau \quad \textbf{(6)}$$

For RUL prediction using the preceding equation, the new condition indicator $\{\vec{Z}(\tau) | t < \tau < \infty\}$ needs to be predicted first. There are many statistical approaches such as time series analysis (e.g. autoregressive model, moving average model, and autoregressive moving average model) and nonlinear model fitting method to predict the new condition indicator beyond the current time (Liao et al., 2006, Banjevic and Jardine, 2006). In this study, for simplicity, the nonlinear model fitting method is applied to predict the condition indicator beyond the current time t .

Heng et al. (Heng et al., 2009b) and Sikorska et al. (Sikorska et al., 2011) claimed that PCM would be suitable in cases of sparse historical failure data while PHM requires a large number of historical failure data. However, both models have limitation of assuming that hazard changes proportionately with covariates and the proportionality constant is the same at all time (Heng et al., 2009b, Si et al., 2011).

3. Case Study Descriptions

Two real life case studies using different types of condition indicators (features) and event data availability (sample size) were conducted in order to evaluate performance and effectiveness of PHM and PCM at RUL prediction in aforementioned scenarios.

The data in the first case include the pumps vibration data (RMS) and failure histories of 18 centrifugal pumps in a pulp and paper mill industry. This case study conducted where a large sample size was available. The second case uses another industrial field data that include the pumps vibration data (kurtosis) and failure histories of two centrifugal pumps in a LNG industry. This case study included a small sample size. In this section, these cases are introduced and explained in more detail.

3.1 Pulp and Paper Mill Case

For this case study, the failure histories and RMS of vibration signals of 18 identical centrifugal pumps (i.e. Gould 3175L) in a pulp and paper mill was provided by the Centre for Maintenance Optimisation and Reliability Engineering (C-MORE) at University of Toronto (Canada). Figure 1 shows a 3D solids model of this typical centrifugal pump. Sundin et al. (Sundin et al., 2007) explained vibration and event data collection and data processing for these pumps in details. The data collection intervals were varied based on different operation conditions over four years. The measurement frequency ranged from 25 to 60 days during normal operation and 3 to 20 days when degradation indicators were observed. The data were collected from early 2001 to early 2005.

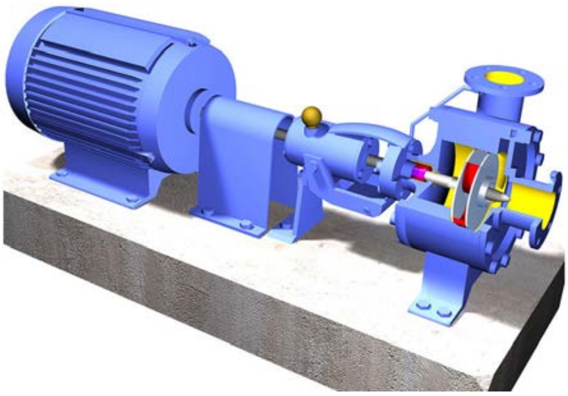

Figure 1: A 3D-solids model of the centrifugal Pump

These centrifugal pumps are business critical rotating equipment in this pulp and paper mill and have a high incidence of unpredicted failures. They are used to pump various liquids from one processing station to another in the paper making process. This pulp and paper mill operates 24 hours every day except for during the planned maintenance shutdowns. Unplanned shutdowns could be very costly due to production loss. The pulp and paper mill has implemented the condition monitoring technique for these pumps to assist for preventing unplanned maintenance shutdowns.

The non-drive end bearing and mechanical seal failures were identified as two common failure modes for these centrifugal pumps (12 bearing failures and 6 mechanical seal failures). Due to criticality of pumps in production, the asset owner would not allow these pumps to run to failure and hence the bearing was replaced when a degradation defect was detected. These pumps were subject to an extensive condition-monitoring programme, the most important of which was the collection of vibration readings using a portable device placed at eight pre-defined locations on the pump.

The eight locations correspond to four places on the pump at which horizontal measurements were taken, three places at which a vertical measurement was taken, and one axial measurement. The data used in this study consisted of the raw vibration signal pre-processed into five frequency bands, an overall summary of the five bands, and an acceleration value. The seven values were reported at each of the eight locations, resulting in a total 56 covariates. Sundin et al. (Sundin et al., 2007) stated that two covariates (horizontal and vertical measurements) were found to be significantly related to bearing failures using covariate analysis. Heng et al. (Heng et al., 2009a) asserted that these condition indicators are highly correlated and might introduce redundancy in the description of the bearing degradation since they correspond to the same frequency band of the signals from the same bearing end. Figure 2 shows the RMS data related to the bearing on pump 4165 in two bearing life cycles. Here, each life cycle refers to the operating time of the bearing until its replacement. The bearing which installed on this pump was replaced after 473 operating days. Then, the bearing was again replaced on this pump after 282 operating days from the first replacement.

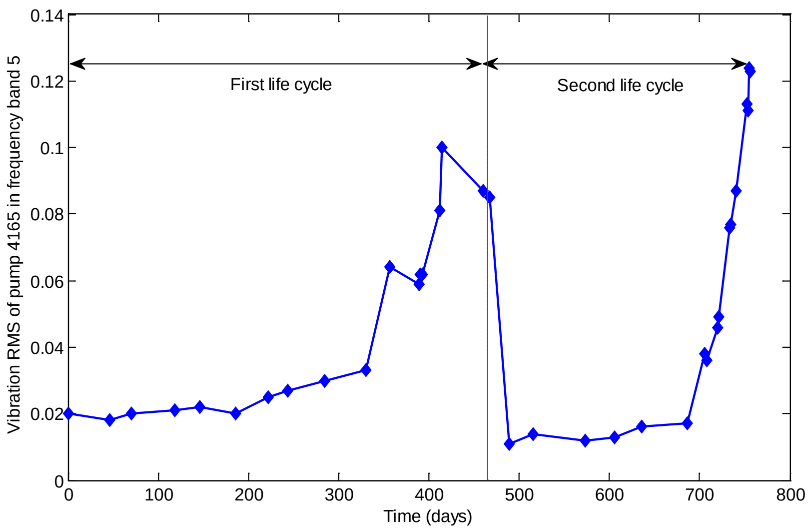

Figure 2: RMS of vibration signals related to the bearing on pump 4165

3.2 Liquefied Natural Gas Case

For this case study, kurtosis of vibration signals from two identical centrifugal pumps was provided by a LNG industry. This type of pump is critical rotating equipment in the LNG industry. A sudden breakdown of this pump would decrease the amount of LNG at the receiving terminal and cause performance dropdown of the whole plant. The LNG centrifugal pump is enclosed within a suction vessel and mounted with a vessel top plate. There are three ball bearings installed to support the entire dynamic load of the integrated shaft of a pump and a motor. The three bearings in the LNG centrifugal pump are self-lubricated at both sides of the rotor shaft and tail using LNG. The three bearings operate at a high speed. Moreover, the bearings are poorly lubricated, due to the low viscous value of LNG. Therefore, bearings installed in these pumps are failure-prone. In order to monitor the degradation of these bearings, three accelerometers were installed on the housing near the bearing assembly in horizontal, vertical, and axial directions respectively. Figure 3 shows the pump schematic and vibration measuring points. Vibration signals were measured in the horizontal direction. Vibration readings were collected during 4356 operating hours at inspections with irregular intervals. At the beginning and the last phases of lifetime, vibration signals were measured more frequently. However, at the middle phase of lifetime, the vibration signals were measured at relatively longer intervals. This type of data collection strategy is often used in real-life situations since it is not essential to measure vibration signals regularly when a bearing is working smoothly.

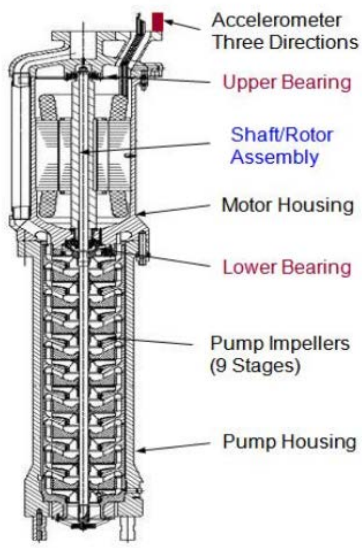

Figure 3: Pump schematic and vibration measuring points (Kim, 2010)

In preventing unexpected failures and minimising overall maintenance costs, the asset owner would not allow these pumps to run to failure and hence the bearing was replaced when a degradation defect was detected. Based on the historical event data, there were two failure times of the bearing. The bearing which installed on pump P301C was replaced after 4356 operating hours due to the outer raceway spalling. Another bearing that installed on pump P301D was replaced after 3452 operating hours due to the inner raceway flaking. Kurtosis of vibration signals was used for this case as it is a widely used degradation feature for detection and diagnosis of the inner and outer raceway defects on bearings (Tao et al., 2007, Heng and Nor, 1998). Figure 4 shows kurtosis data related to the bearing on pump P301C.

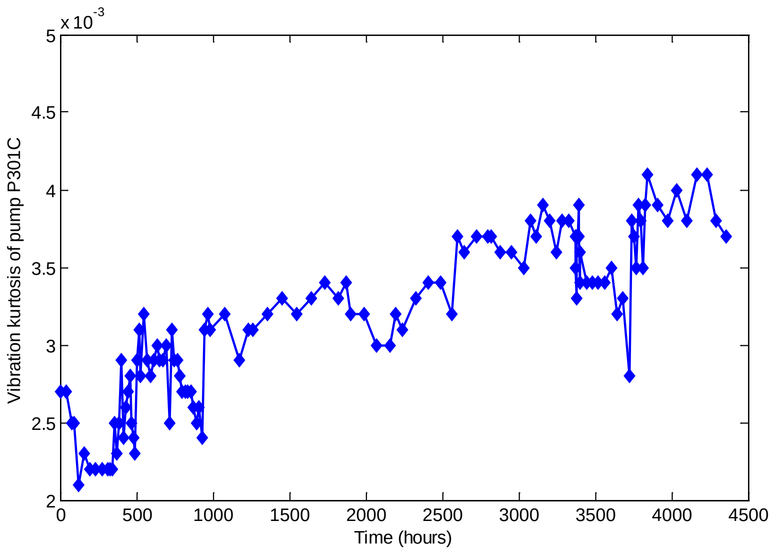

Figure 4: Kurtosis of vibration signals related to the bearing on pump P301C

4. Results

In this section, the hazard, reliability and RUL prediction using both PHM and PCM are presented for the above-mentioned cases. In order to appraise the performance of PHM and PCM, the predicted RUL are compared to the actual RUL. The absolute prediction error (the difference value between the predicted and actual RULs) validates this comparison.

4.1 Bearing Life Prediction for the Pulp and Paper Mill Pump

Based on the parameter estimation algorithm of PHM, the shape and scale parameters of the Weibull distribution as well as the regression coefficient parameter are respectively estimated as: $\hat{\beta} = 2.40$, $\hat{\eta} = 2360$, and $\hat{\gamma} = 41.40$. The shape and scale parameters of the Weibull distribution in PCM are estimated as: $\widehat{\beta} = 1.520$ and $\widehat{\eta} = 809.478$. By knowing these parameters then hazard, reliability, and RUL estimations of the bearing on pump 4165 in two life cycles are calculated using PHM and PCM. Hazard and reliability estimations of the bearing are shown in Figures 5 and 6, respectively. Due to using a large number of failure event data in this case, both models attain the expected results in hazard and survival probability estimates.

As it can be seen in these figures, the reliability of the pump decreased to 0.049 using PHM and 0.69 using PCM after 473 operating days. The actual data on pump shown the bearing was replaced in this time due to detecting the degradation defect by condition monitoring data (refer to Figure 2). After changing the bearing the condition of the pump returned to as good as new. However, the bearing again replaced after 282 days due to intense degradation defect. Figure 6 shows the reliability of the pump reduced to 0.002 using PHM and 0.44 using PCM at the end of bearing life.

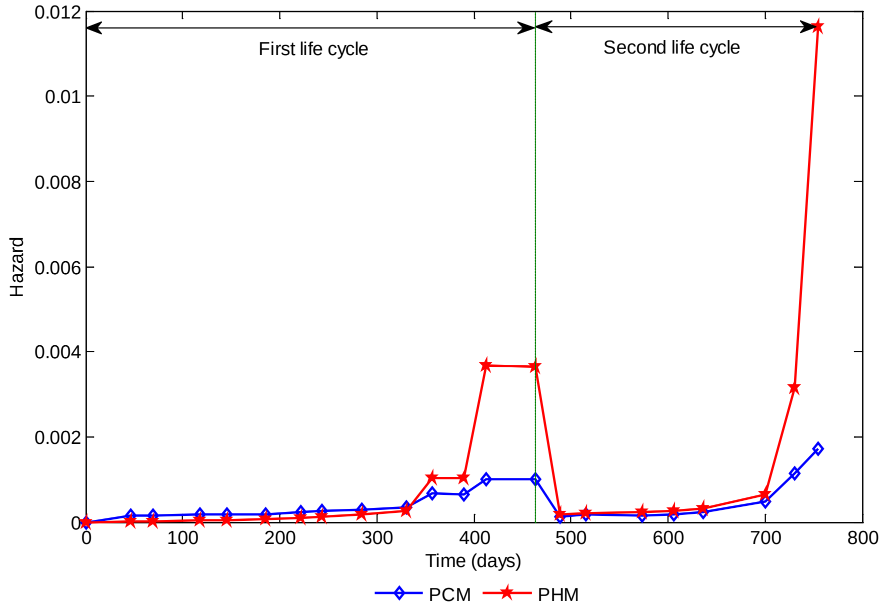

Figure 5: Hazard estimate of the bearing on pump 4165

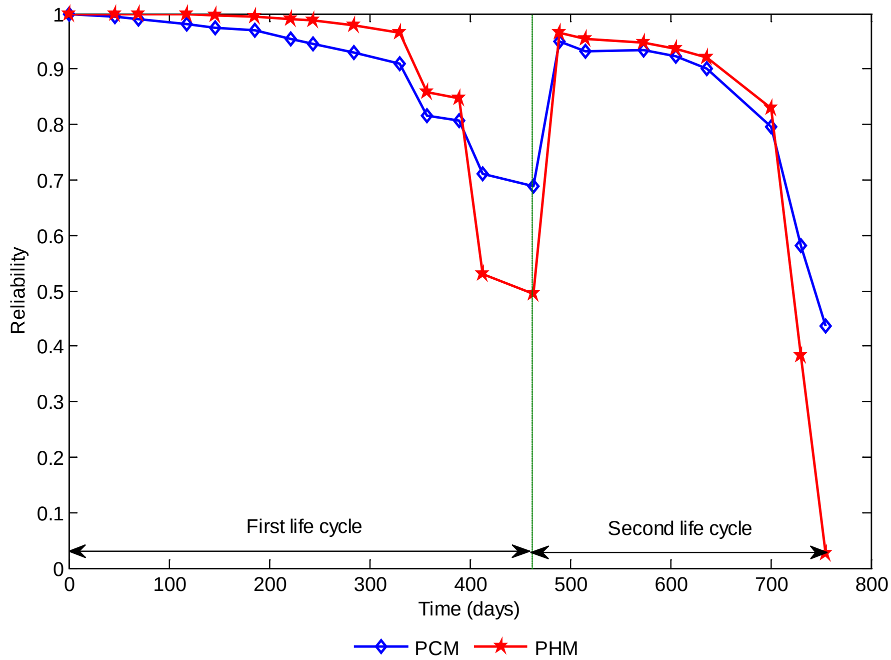

Figure 6: Reliability estimate of the bearing on pump 4165

To evaluate the performance of PHM and PCM for this case, the predicted RUL is compared with the actual RUL. 35 RMS data values were available for two life cycles of the pump bearing. The nonlinear model fitting approach is used to predict new RMS values. The RMS data values and actual life data of last three time points in each life cycle kept as the test data set, while the remaining RMS data values were used as the training data set during the nonlinear model fitting. The new RMS values are predicted for the last three time points; then, RUL is predicted for these time points. The expected RUL and its prediction error are shown in Table 1. According to the set of complete life data of the bearing on pump 4165, this bearing failed after 473 and 755 operating days at the first and second life cycles, respectively.

Table 1: Expected RUL results of the bearing on pump 4165

Bearing Life Cycles	Time (days)	Actual RUL (days)	Predicted RUL (days)		Prediction error (days)	
			PHM	PCM	PHM	PCM
First	**390**	83	74.316	79.922	8.648	3.078
	413	60	50.650	58.017	9.350	1.983
	463	10	8.002	9.914	1.998	0.086
Second	**700**	55	53.240	53.908	1.760	1.092
	730	25	23.425	24.580	1.575	0.420
	753	2	1.947	1.995	0.053	0.005

According to the actual RUL, Table 1 depicts that the absolute RUL prediction error in PCM is smaller than PHM, consistently. A possible reason is that PCM calibrates itself from the early period of prediction while PHM calibrates itself in the later time of prediction.

4.2 Bearing Life Prediction for the Liquefied Natural Gas Pump

The shape and scale parameters of the Weibull distribution and the regression coefficient parameter in PHM are respectively estimated as: $\widehat{\beta} = 10$, $\hat{\eta} = 4345$, and $\widehat{\gamma} = 183.6$. The estimated values of shape and scale parameters of the Weibull distribution in PCM are: $\widehat{\beta} = 10.3$ and $\hat{\eta} = 4107$. By knowing these parameters, hazard, reliability, and expected RUL of the bearing on pump P301C using both PHM and PCM can be estimated. Figures 7 and 8 illustrate the calculated hazard and reliability of this bearing using these covariate-based hazard models. The estimated survival probability in Figure 8 shows the over-smoothing tendency of PHM. A possible reason would be using only few failure event data in the modelling and parameter estimation of this model.

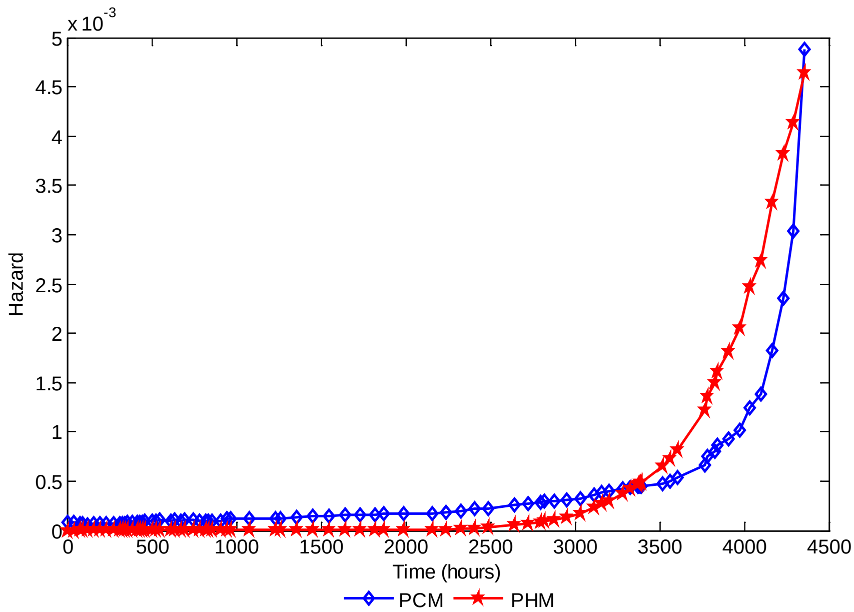

Figure 7: Hazard estimate of the bearing on pump P301C

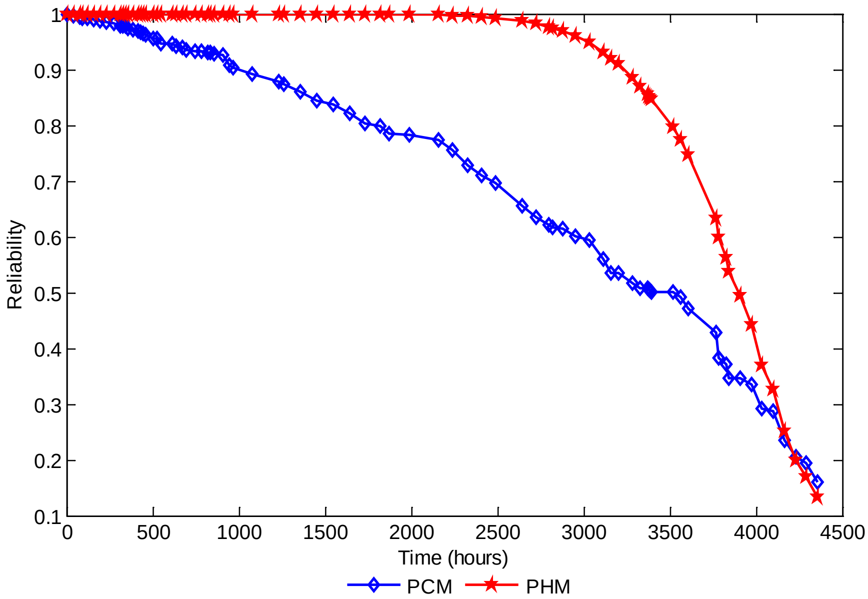

Figure 8: Reliability estimate of the bearing on pump P301C

In this case, the predicted RUL using PHM and PCM is compared with the actual RUL to assess the performance of these models. There are 121 Kurtosis data values for the bearing on pump P301C, then we could have more point estimations of RUL compared to the previous case. Similar to the prior case, the new condition indicator can be predicted beyond the current time using the nonlinear model fitting method. 80 kurtosis data values are used as the training data set during the nonlinear model fitting, while the remaining kurtosis data values and actual life data keeps as the test data set. After forecasting of the new kurtosis data, RUL can be predicted. The expected RUL and its prediction error are shown in Table 2. According to the set of complete life data of the bearing on pump P301C, this bearing was replaced after 4356 operating hours.

Table 2: Expected RUL results of the bearing on pump P301C

Time (hours)	Actual RUL (hours)	Predicted RUL (hours)		Prediction error (hours)	
		PHM	PCM	PHM	PCM
3158	1198	805	914	393	284
3285	1071	700	815	371	256
3394	962	621	741	341	221
3563	793	503	616	290	177
3603	753	478	589	275	164
3783	573	374	459	199	114
3842	514	340	412	174	102
3971	385	279	318	106	67
4099	257	227	225	30	32
4163	193	205	182	23	11
4228	128	181	130	53	2

Table 2 demonstrates that the expected RUL using PHM and PCM converges towards the actual RUL; however, the absolute prediction error of PCM is significantly smaller than PHM.

5. Conclusions

Traditional reliability models are based on lifetime distributions that need historical failure data. However, in reality, statistically sufficient failure histories are often difficult to attain due to the fixed time-based replacement and sometimes small sample size of the similar assets. Hence, asset life prediction methods using both condition indicator data and failure time data become more desirable than those relying on failure time data alone. Covariate-based hazard approach is one of such life prediction methods. Amongst covariate-based hazard models, Proportional Hazard Model (PHM) and Proportional Covariate Model (PCM) are two fundamental models that would have significant application potential for predicting hazard using both failure event data and condition indicator data. This work attempts to investigate and evaluate the performance and robustness of these models in Remaining Useful Life (RUL) prediction using different types of condition indicators (features) and data availability. To this end, two industry case studies were conducted.

Results show that performance of PHM in the above cases varies due to type and quantity of the data. The absolute prediction error in LNG case is more significant than the pulp and paper mill case. A likely reason is that only few failure event data were used in the modelling and parameter estimation of this case. In fact, the estimated values of parameters in PHM are biased in the case of a small sample size (Oakes, 1981, Nachlas, 2005). Thus, care should be taken in applying PHM for asset life prediction when only small sample size is available. The both case studies demonstrate that the predicted RUL using PCM regardless of the number of failure event data and sample size always have smaller prediction error. A possible reason is PCM calibrates its prediction as the posterior hazard function in this model updated via the current condition data (see Equation 5).

The advantage of using both condition indicator data and failure event data in modelling is to improve the accuracy of prediction. This work shows PHM and PCM have achieved this expectation about using such data in asset life prediction. Both models have promised more to the life prediction challenges than the classical failure history based reliability distribution approach. Further studies are required to investigate the performance of these models and other covariate-based hazard models using various condition indicators and data availability. Moreover, PCM should compare to the Bayesian regression model due to using posterior distribution to obtain the hazard function.

To explicitly model the impact of condition indicators on asset life, it is suggested to incorporate three types of data (i.e. population characteristics, condition indicators, and operating environment indicators) into a covariate-based hazard model so as to achieve more effective forecasting results. This would be a complicated study and challenge since operating environment indicators (e.g. speed or load) will impact on the observed condition indicators. As a result, over-fitting could occur in the expected RUL which may decrease the robustness of the developed estimation model. The authors are undertaking in depth research in this direction.

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