INQUIRY ACTIVITIES IN A CLASSROOM: EXTRA-LOGICAL PROCESSES OF ILLUMINATION VS LOGICAL PROCESS OF DEDUCTIVE AND INDUCTIVE REASONING. A CASE STUDY

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The paper presents results of the research, which was focused on studying students' inquiry work from a psychological point of view. Inquiry activities of students in a classroom were analysed through the evaluation of the character of these activities within learning process with respect to mathematician's research practice. A process of learning mathematical discovery was considered in details as a part of inquiry activities of students in a classroom.

INTRODUCTION

Different questions dealing with the nature of mathematical discovery and inquiry activities have always been in the focus of researchers' interest (Krutetskii, 1976; Polya, 1962; Tall, 1980). Among the outstanding mathematicians, who paid great attention to the topic, were Hadamard (1945) and Poincaré (1952), though they had mostly relied upon personal experience. Historical analysis of the process of mathematical discovery was given by Lakatos (1976). Nowadays the phenomenon of mathematical discovery, its mechanism and mental processes remain into the educational research limelight (Barnes, 2000; Burton, 1999; Devlin, 2000; Okada & Simon, 1997; Sinclair, 2002). Indeed, the concepts of inquiry and mathematical discovery have quite many common features with learning process for being considered together. Nevertheless, much of the previous work on the process of mathematical discovery in the mathematics education literature had often been concentrated upon mathematicians and their research practice without clear indication to the needs and objectives of learning process and subjects involved in. Moreover, most of the contributions concerned the illumination stage of mathematical discovery. In this respect a recent paper by Liljedahl (2004) emphasizes the situation:

> Mathematical discovery and invention are aspects of 'doing' mathematics that have long been accepted as standing outside of the theories of "logical forms". That is, they rely on the extra-logical processes of insight and illumination as opposed to the logical process of deductive and inductive reasoning. (p.256)

At the same time, it is obvious that within any educational process the great part of it should be provided by the teacher and carried out by the students on the base of using logical processes of deductive and inductive reasoning and links between them. This contrast raises the main question of our research: To analyse relationships between extra-logical processes of illumination and logical processes in the scope of students' inquiry activities in a classroom and what are the ways of evaluation of students' inquiry work in such situation? Also, we would like to lift up other questions related to the mentioned above: Whether is it possible to develop students' skills and understanding of different mathematical ideas up to advanced level through the appropriately designed inquiry tasks and environment in classroom activities? How much of students' argumentation in inquiry work do the logical links take in?

We shall attempt to answer these questions in the context of using different forms of students' inquiry activities in a classroom.

THEORETICAL FRAMEWORK

At first, we need to define more precisely a main object of the research. This is a process of learning mathematical discovery in a classroom. We understand *learning mathematical discovery in a classroom* as a short-term active learning process aimed at the development of students' abilities to assimilate new knowledge through the use and interpretation of their existing knowledge structures with the help of a teacher or with considerable autonomy and only teacher's control of the direction of the inquiry activities within the topic studied. We would like to note that the term *inquiry work in a classroom* or *inquiry activities* is usually used in the mathematics education literature. However, we have intentionally introduced the term *learning mathematical discovery in a classroom* to emphasize the difference between it and the above terms. A process of learning mathematical discovery in a classroom has a completed and local character. Moreover, it is restricted with some questions of curriculum and short time limits. Therefore, we would like to study a small, though the most important part of inquiry activities in zoom.

The most important point was to find out how natural for students was a process of learning mathematical discovery in a classroom from a psychological point of view, i.e. whether it meant when students had revealed some property of the topic studied they had been mentally prepared in advance that it happened only due to illumination and intuition, without logical links to their argumentation in inquiry activities. To answer this question, we had to know how we could evaluate the relationship between logical and extra-logical processes, of which mathematical discovery consists of, within the students' inquiry work in a classroom. Thus, it was necessary to introduce quantitative and qualitative characteristics, which would describe a process of learning mathematical discovery in a classroom.

We took the position that AFKS (Active Fund of Knowledge of a Student, Yevdokimov, 2003) had the most relevant structure to introduce such characteristics for studying this process. We understand AFKS as student's knowledge of definitions and properties for some mathematical objects of a certain topic and skills to use that knowledge in inquiry work. The key point of using AFKS in the context of learning mathematical discovery was the following: where, when and for what mathematical objects a student would apply a certain mathematical property or action and whether it would be necessary to apply that property or action in that case in general. It was obvious that with respect to the process of learning mathematical discovery in a classroom AFKS represents, first of all, student's using of logical process of deductive and inductive reasoning. Also, we followed Edwards' idea *conceptual territory before proof* (1997) in investigating the process of learning mathematical discovery in a classroom and taking into account that exploration and explanation constituted the main elements that preceded formal discovery.

METHODOLOGY

Five 10 Grade classes with mathematics profile from different schools (students' age 16 years) were involved in the study in January 2005. All students had been proposed the same advanced course of plane geometry (college geometry), which was conducted by five experienced teachers. The content of the course had been unknown to all students before, though they were familiar with mathematical concepts, objects, their definitions and basic properties related to the theme. The course was organised in the following way: During the first month teachers presented new material one time (45 minutes) per week, with significant amount of questions and problems of different complexity levels for students' work on their own. The next month teaching was focused on using different forms of students' inquiry work in a classroom: there were two lessons (90 minutes) two times per week. At the end of this month two tasks were proposed to students in the scope of a certain form of learning mathematical discovery. We indicated this procedure as Phase 1 of the study. After that the second cycle of two months was carried out with corresponding Phase 2 in the end. The same cycle with corresponding Phase 3 completed the first part of our project. We had intentionally separated Phases 1, 2 and 3 with months of teachers' presentation. We had tried to provide conditions, where students' thinking was not concentrated on a certain form of learning mathematical discovery and students' using stereotyped approaches to inquiry work was minimal. Before Phase 1, two most successful students in each class had been distinguished by the teachers for the study. We regarded two students as most successful in a class (not necessarily talented or genius), if they had dominated over the rest of the students in the same class during the time period before Phase 1 at least in 2 points from the following ones: deep understanding theoretical material given by a teacher, solving/proving complex problems, posing non-trivial problems related to the theme. Starting from that point inquiry activities of those 10 students had been under peer observation of the teachers. Thus, taking into account specification of the theme of our study we took the best part of the students for evaluation their work in learning mathematical discovery in a classroom. At the same time, to provide real results of students' achievements, all 5 pairs of students were in their usual social classroom environments during the study. The rest of the students in each class played the technical role being involved in inquiry activities in a classroom. However, such information was for teachers' use only. For analysis we used students' protocol sheets of the corresponding phase tasks and teachers' commentaries to them, teachers' notes concerning students' inquiry work in a classroom and audio-files of fragments of the lessons. Finally we took short students' interviews about their beliefs on the process of mathematical discovery and attitudes to different forms of learning mathematical discovery in a classroom.

For quantitative evaluation of student's conscious involvement in the process of learning mathematical discovery in a classroom we determined an index *I* using AFKS in that process, i.e. *I* served an indicator how much AFKS was used in doing each task. We studied the character (logical or non-logical) of using AFKS within learning mathematical discovery in a classroom. Analysing the data received we tried to highlight the factors, which contributed to students' successful display in the process of learning mathematical discovery in a classroom, and obstacles of their work in the same process. Protocol sheets consisted of students' step by step description of their suggested actions, students' explanations of their preference for every action performed and teachers' commentaries on students' real actions and explanations. There were two evaluation columns for teacher's use only. The teachers used a dual code of {0, 1} for marking students' progress in both evaluation cases. The first one concerned *students' explanations* about the reasons why they used a certain suggestion or carried out a certain action on each step of the task. The teachers evaluated students' explanation with 1, if it was logically presented using appropriate argumentation. In the opposite case the corresponding explanation was marked with 0. In the second case teachers dealt with *students' actual suggestion or action* on each step of the task. They evaluated students' actual suggestion or action with 1, if teachers accepted it had been logically performed, even in the case when students were not able to provide satisfactory explanation for their decision. Again, in the opposite case the corresponding suggestion or action was marked with 0. For calculation *I* of each student for a certain task we used a formula of elementary probability for the finite number of events (in our terminology steps): $I = \frac{\sum 1}{N}$, where *N* was a number of steps for a certain task.

THE CASE OF TEN STUDENTS

Now we would like to characterise briefly three different forms of learning mathematical discovery in a classroom with presentation data received and findings of the research. These forms had been used for quantitative and qualitative evaluation in Phases 1, 2 and 3 correspondingly. It is important to stress that we have paid great attention and rigour to the procedure of phase tasks selection. In Phase 1 we used students' individual work on protocol sheets without help of a teacher. Phase 2 consisted of students' collaborative work in pairs with help of a teacher. There was one protocol sheet for a pair of students for each task. In Phase 3 we used students' individual work again, but with the help of a teacher.

Phase 1

Gray et al. (1999) pointed out that "didactical reversal - constructing a mental object from 'known' properties, instead of constructing properties from 'known' objects causes new kinds of cognitive difficulty" (p.117). We used the idea of "didactical reversal" for our tasks at this stage. We called it *didactical chronology of discovery*, i.e. we proposed students to build up a successive chain of their argumentation, which would lead them to the revealing of a certain property. A characteristic feature of our tasks was the condition that a property and its full proof were included in the body of the tasks and were available for students' studying from the first moment of their work on the tasks. Two tasks of this phase were devoted to discovery of a circle of nine points and Euler line correspondingly. The main peculiarity of Phase 1 was the fact that students needed to explain the ideas and actions, which had been proposed by another person (e.g. famous Euler).

Phase 2

Main components of the second phase of the research were open problems and the help of a teacher, who was a guarantee for creating environment suitable to exploration and inquiry work. Thus, a teacher was a person, who had to regulate directions of students' inquiry work in the process of learning mathematical discovery and adapt it to the classroom needs. For open problems proposed for students we followed Arsac et al (1988) characterisation of:

> The statement of the problem is short, so that it can be easily understood, it fosters discovery and all students are able to start the solution process. The statement of the problem does not suggest the method of solution, or the solution itself, but it creates a situation stimulating the production of conjectures. The problem is set in a conceptual domain, which students are familiar with. Thus, students are able to master the situation rather quickly and to get involved in attempts of conjecturing, planning solution paths and finding counter-examples in a reasonable time.

We called this phase *learning discovery within open problem*. Taking into account that the illumination stage of mathematical discovery is accompanied by a feeling of certainty (Poincaré, ibid.) and positive emotions (Burton, ibid.; Rota, 1997) it was teachers' responsibility to manage a process of learning mathematical discovery. In other words, building up and supporting the corresponding learning environment aimed at the conditions for the best display of students' abilities in the process of mathematical discovery was the chief objective of teacher's work at this phase. At the same time we took into account that "open problems promote the devolution of responsibility from the teacher to students" (Furinghetti & Paola, 2003, p.399). Two tasks of this phase concerned discovery of different properties for Brocard and Lemoine points.

Phase 3

Following Brown and Walter (1990) we proposed "situation", an issue, which was a localised area of inquiry with features that can be taken as given or challenged and modified. We called this phase *learning discovery over situation*. We would like to note that there were different directions of students' inquiry work in this phase. To control situation by the teachers we used Mercer's idea (1995) of "the sensitive, supportive intervention of a teacher in the progress of a learner, who is actively involved in some specific task, but who is not quite able to manage the task alone". Situations for two tasks of this phase were based on Simson line and Morley triangle properties. It is important to stress it was more generalised phase than the first two ones.

Findings

We processed 60 protocol sheets (20 ones for each phase), 5 of them had not been completed (1 sheet of the Phase 1, 1 sheet of the Phase 2 and 3 sheets of the Phase 3 correspondingly). Generalised data are given in Table 1 below.

The process of learning mathematical discovery in a classroom	1^{st} Task		2^{nd} Task		Average *I* on the base of students'		Character of the process on the base of students'	
	Evaluation *I* on the base of students'		Evaluation *I* on the base of students'					
	explanations	actions	explanations	actions	explanations	actions	explanations	actions
Phase 1	0,81-0,86	0,83-0,94	0,78-0,89	0,87-0,97	0,84	0,91	logical	logical
Phase 2	0,7-0,86	0,81-0,89	0,71-0,84	0,79-0,91	0,78	0,86	non-logical	logical
Phase 3	0,72-0,79	0,79-0,91	0,69-0,81	0,8-0,89	0,76	0,84	non-logical	logical

Table 1: Data of quantitative and qualitative characteristics of the process

Values of *I* on the left side of the table indicate the limits, in which *I* changed for a certain task on a certain phase of the study, e.g. on the Phase 1 values *I* of all students for the 2^{nd} Task on the base of their actions were from 0,87 (the worst result) to 0,97 (the best result). In the case of obtaining average value of *I* close to 1 either on the base of students' explanations or students' actions on a certain phase of the study, we could state about logical character of the process. The number of steps in the proposed tasks was up to 10, therefore empirically we considered the character as non-logical, if more than one step of the task was evaluated as non-logical (with mark 0). Thus, we decided to define the character of such process as non-logical, if $I \leq 0{,}8$. Of course, it requires further discussion and specification. However, the most important results are the values of *I* for the corresponding phases. They indicate the tendency that illumination stage diminishes to 0 in the scope of the process of learning mathematical discovery in a classroom.

We have found out that logical processes of deductive and inductive reasoning play significant role within the three different forms of the process, which were considered in the study. We can construct a set of key tasks with indicated in advance quantitative scale of using extra-logical processes in students' inquiry activities in learning mathematics. Thus, we can distinguish and regulate the illumination stage of mathematical discovery within *learning mathematical discovery*, we can adapt it to the needs of classroom activities or to the thinking process of a certain student involved in these activities. We would like to stress the crucial meaning of the students' successful work on the first phase of the study. Despite the apparent simplicity of the tasks, we observed that students experienced other kinds of cognitive difficulty than on the stage of studying a property and its proof. We had the similar situation in the third phase of the study, however, in the conceptual context. We observed that students commented, argued and explained thoughts and ideas of other students, teachers and mathematicians (e.g. see the task of famous Euler above) much better than their own suggestions. Therefore, non-logical character of the process on the base of students' explanations for the second and third phases was connected, first of all, with semantic difficulties of students, when they were to communicate with others using symbolic and usual language simultaneously, often some of the best students could not express their thoughts in a correct way or quite clearly. At the same time teachers' observations and short students' interviews showed much more students' interest in the second form of learning mathematical discovery than in others. It emphasized the role of a teacher in managing this process. In our study we presented a teacher as a provider of knowledge on a foreseen in advance level, who was able to regulate the illumination stage of students' mathematical discovery additionally to the tasks proposed. Though, we found out that students only intuitively differ logical and extra-logical processes in learning mathematical discovery, their opinion was practically unanimous that teacher's contribution to students' successful work was invaluable. This tendency was confirmed with the least differences of changing I in Phase 2, i.e. students intuitively felt that teachers most of all contributed to development of their skills to explain logically their ideas and actions based on logical process of deductive and inductive reasoning. In all tasks of the forms of learning mathematical discovery we distinguished key didactical situations, which we called *hills of discovery*. Students' success or fail in each phase task depended mainly on their abilities to go through these hills of discovery. From our point of view they were additional regulators of using extra-logical processes in students' inquiry activities in a classroom.

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