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# A review of the state-of-the-art methods in estimating crash modification factor (CMF)

highlighted in this review.

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A R T I C L E I N F O	A B S T R A C T
<i>Keywords:</i> Safety Safety effectiveness Evaluation Crash modification factors	Road authorities and road safety experts are involved in estimating the expected outcomes from road safety treatments. Information derived from proposed treatments enables planners to make comparisons between the expected savings from crash reductions and associated treatment costs. This review aims to provide direction to agencies and practitioners interested in estimating safety effectiveness. Specifically, this study discusses the main methods for developing CMFs, including an overview of each method, data considerations, and their strengths and weaknesses. It also discusses the techniques of estimating combined CMFs resulting from multiple safety treatments. The review showed that observational Before–After (BA) studies with the Empirical Bayes (EB) and Full Bayes (FB) approaches provide enhanced consistency and precision for the estimated safety effectiveness. Alternatively, the cross-sectional method can be adopted in cases where observational BA studies are not practical due to data restrictions. Five additional techniques for estimating combined CMFs were also reviewed. The study notes that while there has been substantial research in the broad area, few studies have reported comparative methods of combined CMF estimation. Future research directions and research gaps are also

## 1. Introduction

The Crash Modification Factors (CMFs) quantify the change in road safety (i.e. crash risk) outcomes from implementing a single or combined treatment. A typical treatment may incorporate modifications to roadside elements, changes to traffic operational arrangements, or the introduction of any suitable countermeasures. A CMF value of 1.0, for example, represents zero effect on safety in the evaluation process, whereas a CMF value greater than 1.0 indicates that the treatment results in a higher crash risk, while a CMF value less than 1.0 indicates a lower crash risk. Over the last three decades, several research studies validated CMF values for single and multiple treatments. The availability of CMFs has also been considered good indicators in forecasting the expected outcome, therefore, experts started using them in economic analysis of safety improvement projects worldwide. In early 1980 s, using before and after studies to estimate CMFs received more popularity, as a result there are many methods developed in estimation. The Highway Safety Manual (HSM) Volume 3 Part D (AASHTO, 2010) and other studies (Al-Marafi, 2019; Al-Marafi et al., 2021; Bahar, 2010; Bonneson & Pratt, 2009; Elvik et al., 2022; Galgamuwa and Dissanayake, 2018; Gross et al., 2010; Li et al., 2010; Persaud et al., 2010; Wang et al., 2017) used the following state-of-the-art methods to estimate the CMFs: Observational Before-After (BA) studies and Cross-Sectional method. The observational BA method adopts a range of approaches, for example, Comparison Group (CG) approach, Empirical Bayes (EB), and Full Bayes (FB) method. On the other hand, the cross-sectional method, where development of full regression models in the name of crash prediction models were introduced and recommended for crash forecasts (Washington et al., 2005). Subsequently, it was also found as logical approach in the Highway Safety Manual and SafetyAnalyst software due to its ability to produce satisfactory outcomes with fewer data requirements than observational BA studies.

The present studies give priority to estimating safety improvements from multiple treatments, and currently the following five techniques received popularity in evaluating combined CMFs such as: HSM technique, Selecting the most effective CMF value (NCHRP, 2008), Systematic reduction of a subsequent CMFs technique (NCHRP, 2008), Turner (2011) technique, and Bahar (2010) technique. For example, to estimate the safety effectiveness of combined treatments, HSM method proposed multiplying the CMF values of each treatment. However, this

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suggestion is based on the assumption that the road safety effect of each treatment is independent. Therefore, the HSM warns that the multiplication of the CMF values may result in over-estimating or underestimating the combined effects of multiple treatments. A similar dilemma zone exists within each of these methods when estimating the safety effectiveness of multiple treatments.

The advancement in several overlapping areas with road safety analysis, such as image mapping, artificial intelligence, and machine learning, has also played a vital role in selecting more innovative methods and the following relatively more sophisticated techniques, such as extreme value theory and isolation forecast method, have become its latest additions. These innovative methods recently gained popularity among transportation professionals and academics. However, they also have a range of specific challenges that limit their usage, for example, the availability of expertise within the road safety team to undertake the innovative analysis and adequate data collected to support the analysis. In particular, to carry out road safety analysis, it is necessary to solicit input or assistance from those who are more familiar with these methods. If the research team does not have the expertise or adequate data, then they need to request outside expertise or use an alternative state-of-the-art method to investigate the outcome. Therefore there is a need to review the resources to identify the latest and state-of-the-art techniques because using the most effective and practical methods to evaluate the safety effectiveness of the treatments has received close attention from road safety experts.

This study aimed to review the different methodological aspects of estimating CMFs to provide an overview for road safety stakeholders. The approaches within each selected method were discussed, including their main advantages and disadvantages. This is a necessary forerunner before employing much more complex analytical tools such as Machine learning and Artificial intelligence as tools for analysing road safety outcomes. The remainder of the study is organized as follows. The following section discusses the five approaches employed to implement observational BA studies. Then, the cross-sectional method is evaluated as an alternative to observational BA studies for estimating CMFs. Five techniques to estimate the CMFs for multiple treatments are next discussed. The study concludes with conclusions and recommendations.

## 2. Observational before-and-after studies

The observational before-after (BA) studies involve estimating either the number of crashes or some other risk measure before and after a given treatment is implemented on either one or several sites (Gross et al., 2010). The observational BA studies account for the Regression-To-the-Mean (RTM) bias. The RTM phenomenon occurs due to the tendency of sites (e.g., roadway segments) that have a high crash frequency in a particular year to regress to a lower crash frequency the following year. In other words, when considering a site with a high crash frequency or rate during a particular year, the random nature of crashes occurring indicates that it is likely that the crash frequency may decrease next year to follow the long-term mean value even without treatment and without a change in traffic conditions. The following five approaches that were employed in observational BA studies; (i) Comparison Group (CG) approach, (ii) Yoked Comparison (YC) approach, (iii) Naïve (simple) approach, (iv) Empirical Bayes (EB) method, and (v) Full Bayes (FB) method (Abdel-Aty et al., 2014; Elvik et al., 2017; Harwood et al., 2003; Hauer, 1997; Lan et al., 2009; Park et al., 2015; Persaud et al., 2010; Shen, 2007; Wang et al., 2017). Each of these approaches was discussed below in detail.

#### 2.1. Comparison group approach

In this method, crash data from a Comparison Group (CG) is used to estimate crashes that would have occurred at the treated sites if the treatment had not been performed. The CG is carefully selected to compensate for the external contributing factors that may affect the change in the crash frequencies (Mbatta, 2011; Park, 2015; Shen, 2007). Mountain et al. (1992) reported that the accuracy of the outcome increases as the similarity between treated sites and comparison sites increases. The CG approach is based on two basic assumptions (Shen, 2007):

- Factors affecting safety had changed in the same way from the before period to the after a period (where treatment had been applied) on both treated sites and comparison sites, and
- The changes in the other factors that affect the safety of treated sites and comparison sites are identical.

Using this approach, the expected crash frequencies after period for the treated sites without performing safety improvement,  $N_{a}$ , can be estimated as follows (Hauer, 1997):

$$N_a = N_b \times R_c \tag{1}$$

where,  $N_b$  is the recorded crash frequencies in the before period for the treated group, and  $R_c$  is the ratio of after-to-before recorded crash frequencies at the comparison sites. The CMF can thus be estimated at a particular site as the ratio between the recorded crash frequencies after the improvement was performed and the expected crash frequencies before the improvement was performed using Eq. (1). Pendleton (1991) stated that the sample size of the comparison sites should be at least five times larger than the treated sites. Likewise, Hauer (1997) stated that the crash frequencies in the comparison sites should be significant compared with the crash frequencies in the treated sites. Furthermore, the length of before-and-after periods for the treated sites and comparison sites should be the same. Fig. 1 illustrates the conceptual outline employed by the CG approach. However, as indicated by Hauer 1997 and Park (2015), the CG approach does not take into account the naturally expected reduction in crash frequencies after a period for treated sites with high crash rates. Thus, this approach does not account for the RTM bias associated with crash data.

#### 2.2. Yoked comparison approach

The Yoked Comparison (YC) approach is a special case of the CG approach where a single treatment site is matched to each comparison site (i.e., one-to-one matching) on the basis of similar traffic and geometric conditions. Fig. 2 illustrates the conceptual outline employed by the YC approach. According to Gross et al. (2010), the strengths and weaknesses of the YC approach are similar to those of the CG approach with a couple of exceptions. The main benefit of the YC approach, in relation to the CG approach, is that it does not require as much data. This is also a weakness of the YC approach since it limits the amount of data for evaluating safety benefits, as well as this approach cannot deal with

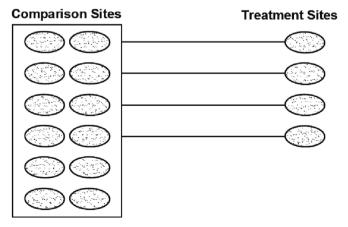


Fig. 1. Conceptual outline of the CG approach.

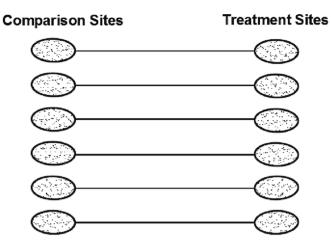


Fig. 2. Conceptual outline of the YC approach.

#### RTM bias.

Harwood et al. (2003) evaluated the safety effectiveness of right-turn lane and left-turn lane improvements using Empirical Bayes (EB), Yoked Comparison (YC), and Comparison Group (CG) approaches. The authors recommended using YC and CG approaches only if the results of the EB approach are not statistically significant. This is because the YC and CG approaches cannot account for the RTM effect. In addition, the study showed that the CG approach results were more precise than the YC approach results as the CG approach employs more than one comparison site for each treated site.

## 2.3. Naïve approach

The main assumption of the naïve (simple) approach is that the crash frequencies before the treatment implementation will be as expected (Abdel-Aty et al., 2014). In this approach, the expected crashes are calculated by using the ratio of road crashes to the number of years before treatment and converting that ratio to the expected after crashes using only the number of years after treatment (Isebrands & Hallmark, 2012; Liu et al., 2011; Persaud & Lyon, 2007). Fig. 3 illustrates the conceptual outline employed by the naïve approach. According to Gross et al. (2010) and Abdel-Aty et al. (2014) the naïve approach tends to over-estimate the effect of the treatment due to the RTM problem. In another work, Lan et al. (2009) found that the naïve approach incorrectly predicted a total reduction in crashes after a hypothetical treatment was performed without any effect. The reason that this is incorrect is due to RTM bias which is not accounted for in this approach.

## 2.4. Empirical Bayes approach

The Empirical Bayes (EB) approach was introduced by Hauer (1997)

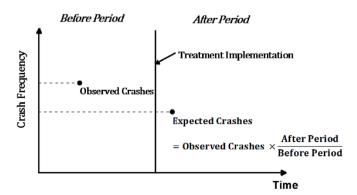


Fig. 3. Conceptual outline of the naïve approach.

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and Hauer et al. (2002) to estimate road safety. This approach increases the precision of estimation to address the main limitation of the CG and Naïve approaches by accounting for the RTM effect (Khan et al., 2015; Saccomanno et al., 2007; Shen & Gan, 2003). This is achieved by estimating a weighted average of the observed and predicted crashes (Abdel-Aty et al., 2014; Elvik et al., 2017; Hauer et al., 2002; Persaud & Lyon, 2007; Tegge et al., 2010). Eq. (2) incorporates the weighted combination of the recorded and predicted crashes number to provide an expected crash frequency for a specific site.

Estimate of the expected crashes for an entity

= weight  $\times$  predicted crashes on the entity +(1 - weight)

$$\times$$
 observed crashes on the entity (2)

The value of *weight* varies from 0.0 to 1.0 and is obtained as follows:

$$weight = 1/(1 + K \times \text{predicted crashes on the entity})$$
(3)

Where *K* represents the over-dispersion parameter of a crash prediction model (CPM). This parameter specifies the systematic variation in the crash frequencies which is unexplained by the model. When the predicted model explains all systematic variations in the crash frequencies, the over-dispersion parameter will have a zero value (Elvik et al., 2017). In such case, the value of *weight* will be equal to 1.0.

The EB approach accounts for the effects of traffic volumes and time trends on crash occurrence and safety (Persaud & Lyon, 2007). Therefore, it is considered better than the comparison group approach. According to Ko et al. (2013) the EB approach estimates the safety at treated sites based on comparison with reference sites (intersections or roadways) with similar features and crash history. Fig. 4 illustrates the conceptual outline employed by the EB approach.

Persaud and Lyon (2007) compared CG, and EB approaches in estimating safety benefits at treated sites had treatment not been implemented. Data of crash frequencies were collected from 1669 stop control intersections during a six-year period (1994–1999) in California. The dataset was divided into two groups. The first group included the crashes between 1994 and 1996 and the second group between 1997 and 1999. The expected crash frequencies for the after a period (1997–1999) were estimated using both CG and EB methodologies and then compared with actual crashes in the after period. The results showed that the CG approach systematically overestimated the crash frequencies for sites In contrast, the EB approach appeared to be unbiased in that it sometimes under-estimated and sometimes overestimated the crash frequencies for the sites. The superiority of the EB approach based on cumulative residuals is illustrated in Fig. 5.

The same study made a comparison between Naïve and EB approaches where data were incorporated from previous studies such as Hauer and Persaud (1987); Lyon et al. (2005); Persaud et al. (2005); Persaud et al. (1984); Persaud et al. (1997); Persaud et al. (2001); Persaud et al. (2004). The results showed substantial differences between the naïve and EB estimated in terms of actual reduction.

## 2.5. Full Bayes approach

The Full Bayes (FB) approach is similar to the Empirical Bayes (EB) in using non-treated reference sites to make inferences and to account for possible influences unrelated to the treatment. Lan et al. (2009) stated that the main difference between the FB and EB approaches is that the predicted crash frequencies without treatment were obtained by the CPM estimated using data from both before the period of treated sites and reference sites. On the other hand, the CPM was estimated using only data from reference sites for the EB approach.

More recently, researchers have recommended using FB approach to evaluate the impact from safety treatments (El-Basyouny & Sayed, 2010; Lan et al., 2009; Persaud et al., 2010; Sacchi & Sayed, 2015). This approach has shown several advantages over other approaches, including the ability to account for all uncertainties in the data used. It

## **Reference Sites**

Treatment Sites

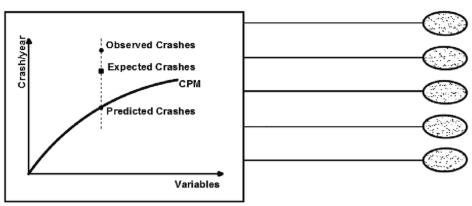
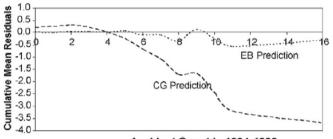


Fig. 4. Conceptual outline of the EB approach.



Accident Count in 1994-1996

**Fig. 5.** Cumulative residuals based on the crash frequencies during 1994–1996 Source: Persaud and Lyon (2007).

requires less data, while providing more flexibility in selecting crash frequency distributions, more detailed causal inferences, and the ability to consider the effect of one site's proximity to other sites (i.e., spatial correlation) in the model formulation. Sacchi and Sayed (2015) compared the results of naïve, EB, and FB approaches in estimating the treatment effectiveness. Two types of the hypothetical treatment sites selection were adopted to perform the analysis: random selection to reduce the selection bias effect; and non-random selection by selecting sites with abnormal crash frequency (black spots). For sites selected randomly, the results revealed that all approaches provide reasonable results. In addition, the results revealed that the FB approach better approximated performance than the naïve and EB approaches on the basis of non-random sites selection. It is worth noting that the complexity of the FB approach makes the EB approach more attractive to researchers for its use (Khan et al., 2015; Persaud et al., 2010).

#### 3. Cross-sectional method

Although observational BA studies are considered to be the preferred approach in estimating CMFs, there are a few practical limitations in using them. For example, the treatment date should be known in order to determine the before and after periods for evaluation; and several years have to elapse after implementing any treatment in order to collect the required data. It is also difficult to distinguish safety effects when implementing more than one treatment at a specific site (Al-Marafi et al., 2021; Hauer, 1997; Persaud et al., 2010; Wood et al., 2015). In such cases, the cross-sectional method can be employed to estimate CMFs because of its simplified approach to obtaining data compared to observational BA studies. The cross-sectional method has been widely used in recent years to overcome some of these issues (Al-Marafi et al., 2021; Anjana & Anjaneyulu, 2014; Gross et al., 2010; Li et al., 2010; Park et al., 2015; Wu & Lord, 2016). The cross-sectional method is also known as a CPM or Safety Performance Function (SPF), which relates crash numbers with geometric characteristics and traffic volumes of a roadway. In this method, CMF can be estimated directly from the coefficient of the variable associated with the proposed treatment as follows (Al-Marafi, 2020):

$$CMF_i = e^{\beta_i [X_i - X_{ib}]} \tag{4}$$

Where  $X_i$  – observed value for the variable *i*,  $X_{ib}$  – base condition for the variable *i*, and  $\beta_i$  – model parameters for the variable *i*. The standard Error (SE) for each treatment also can be calculated as follows:

$$SE_{i} = \frac{\left(e^{\beta_{i}[X_{i}-X_{ib}]+SE_{ji}} - e^{\beta_{i}[X_{i}-X_{ib}]-SE_{ji}}\right)}{2}$$
(5)

Where  $SE_i$  – standard error of the CMF<sub>i</sub> and  $SE_{\beta i}$  – standard error of the model parameter  $\beta_i$ .

It is worth mentioning that the cross-sectional method does not take into account the effects of factors that were not included in the analysis, i.e. external causal factors (Al-Marafi et al., 2020; Gross et al., 2010; Hauer, 2013). Another criticism is that sufficient sample size is particularly required when large explanatory variables are included in the developed model. According to Gross (2006), the cross-section method is conducted in the case where an observational BA study is impractical. AASHTO (2010) also indicated that the cross-sectional method might be appropriate when observational BA studies are not practical due to data limitations (e.g., prior period crash data are not available). This method is used when comparing the road safety performance of a site with certain specific features to another site lacking these features (Li et al., 2010).

Ideally, assuming a systematic safety effect for all treated sites with different characteristics would not make sense. For example, greater benefits of safety improvements may be obtained at sites with high traffic volume. Thus, as a part of the cross-sectional method, the crash modification function (CMFunction) method has been employed recently to derive CMFs at a specific site. The CMFunction method uses the coefficients of prediction models to estimate the safety benefits after improvements (Al-Marafi et al., 2020; Gross et al., 2010; Lee et al., 2015; Lord & Bonneson, 2007; Park et al., 2014; Park et al., 2015; Sacchi et al., 2014; Wood et al., 2015). Wood et al. (2015) compared the CMFs obtained from observational BA studies (using the EB approach) and the cross-sectional method (using the regression approach). Their study revealed that the cross-sectional method appears to yield results consistent with the EB approach results. Therefore, using the crosssectional method will yield an acceptable result where data is not available for after treatments. Likewise, Sacchi et al. (2014) and Park et al. (2015) proposed using CMFunctions based on a cross-sectional approach to identify the relationship between safety effects and roadway characteristics.

Sacchi et al. (2014) indicated that estimation of CMF as a single value might not be adequate to represent how safety treatment affects crash frequency over time. Therefore, the authors developed CMFunctions which incorporate the variation in safety effectiveness of treatment over time. Elvik (2009) developed a framework to evaluate CMFunctions for the same treatment type based on *meta*-analysis for several studies. Elvik estimated CMFunction for the installation of a bypass road and conversion of a signalised intersection to a roundabout on the basis of population changes. The author found that CMF values increased with the population for both treatments. However, the author recommended using a fairly large sample size to develop more accurate CMFunctions. In summary, Table 1 provides a listing of methods used to estimate CMF along with their advantages and disadvantages.

## 4. CMFs for multiple treatments

There are a number of techniques proposed to estimate the value of combined CMFs for multiple treatments. The sections below highlight the state-of-the-art methods that are popular among safety experts and practitioners.

#### 4.1. HSM technique

In this technique, the CMFs from individual treatments are multiplied to estimate combined CMFs (Park et al., 2014; Wu & Lord, 2016). This technique was initially adopted by the HSM (AASHTO, 2010), and this technique assumes that the road safety effect of each treatment is independent. According to Gross and Hamidi (2011), this assumption of independence provides a straightforward computational technique but lacks a consistent theoretical justification. For instance, adding a single lane and increasing shoulder width are treatments that both address crash frequency and the implementation of one of these two treatments may have an influence on the safety effectiveness of the other.

## 4.2. Turner technique

The second technique proposed by Turner (2011) applied a specific weighting factor of two-thirds to estimations of combined CMFs from two or more treatments. Turner developed this weighted factor after analysing combined CMFs for multiple safety treatments using data exclusively from New Zealand. Turner introduced this weightage after a comparison study demonstrated that all techniques had over-estimated the actual crash reductions. The validity of this technique for other regions requires further verification.

#### Table 1

Summary of methods used for estimating crash modification factors.

### 4.3. Systematic reduction of subsequent CMFs technique

A third technique was proposed by the US State of Alabama (NCHRP, 2008). It assumes that the safety effects of the less effective safety treatment are systematically reduced. This means that the full effect of the most effective safety treatment among all treatments is used and had an added benefit of additional treatments, i.e. less effective treatments, as detail given in Table 2. Moreover, this technique recognizes that additional safety treatments are likely to add an additional benefit, but not the full benefit, due to the potential interrelationships between treatments.

## 4.4. Applying only the most effective CMF technique

The fourth technique applies only the most effective safety treatment, which is the lowest CMF among all treatments. This selective technique was proposed by NCHRP (2008) based on the results of a survey. The disadvantage of this technique is that it underestimates the combined effect of safety treatments if the additional safety treatments provide additional benefits (Gross & Hamidi, 2011; Park et al., 2014).

## 4.5. Bahar technique (meta-analysis)

The fifth technique was proposed by Bahar (2010), where a weighted average of CMF values for the same treatment from various studies was identified using *meta*-analysis. It is important to note that this technique was not developed to estimate the combined impact of different treatments. Instead, it was developed to combine CMF values estimates for the same treatment. However, Gross and Hamidi (2011) assert that this technique can be applied to combine CMF values for different treatments.

## 4.6. Summary

Table 2 summarizes the main existing techniques for combining individual CMFs. This study revealed that there are very few studies on estimating the combined effect from multiple treatments. In a study by Pitale et al. (2009), the CMF values for individual and combined treatments were estimated using before-after evaluation. This study found that the safety impacts of paving aggregate shoulders, installing shoulder rumble strips, and widening paved shoulders from 0.6 to 1.2 m (2 to 4 feet) on rural two-lane roadway segments are 16%, 15%, and 7% reductions in crash frequencies, respectively. The study also found that a 37% reduction in crash frequencies resulted from multiple (combined) treatments, consisting of paving shoulders + installing shoulder rumble strips. In other work, Bauer and Harwood (2013) investigated the safety

Method Type	Advantages	Disadvantages	Note
Comparison Group (CG)	Control the effects of external causal factors.	Does not account for RTM bias; difficulty to find an adequate number of similar sites without treatment.	Produces more accurate estimates than a naive comparison method.
Yoked Comparison (YC)	Simplicity of applying, no need for a large number of reference sites.	Does not account for RTM bias; limits the amount of data for evaluating safety benefits; difficulty dealing with zero crash frequency.	A single treatment site is matched to each comparison site.
Naïve Comparison	Simplicity of applying.	Does not account for RTM bias; over-estimate the effect of the treatment; no control of the effects of external causal factors.	The crash frequencies before the treatment implementation would be expected.
Empirical Bayes (EB)	Mitigating the RTM bias; no need for a large number of reference sites.	Difficult to collect reasonable data	Produces more accurate estimates than a CG and naive comparison method.
Full Bayes (FB)	Mitigating the RTM bias; ability to account for all uncertainties in the data used; no need for a large number of reference sites; capable of accounting for the temporal and spatial variations.	Complexity of applying; difficult to collect reasonable data.	Can be used as complex alternative to the EB approach.
Cross-Sectional	Mitigating the RTM bias, accounts for variation in safety effectiveness of treatment over time.	It does not take into account the effects of elements that are not included in the analysis; sufficient sample size is especially required when large explanatory variables are included in the developed model.	The precision is affected by how closely a developed model expresses the relationship between explanatory variables and crash frequency.

## Table 2

Summary of existing techniques for combining individual CMFs.

Number	Techniques	Description
1	$CMF_{combined,i} = CMF_{f1} \times CMF_{f2} \times \cdots \times CMF_{ij} \times \cdots \times CMF_{in}CMF_{combined,i}$ : combined CMF at the $i^{th}$ site.	Proposed by USA's HSM (AASHTO, 2010) and assumes independence of separate treatments.
2	$\text{CMF}_{in}$ : CMF associated with treatment $j$ ( $j = 1, 2,, n$ ) at $i^{\text{th}}$ site. $\text{CMF}_{\text{combined},i} = 1 - [\frac{2}{3} (1 - (\text{CMF}_{i1} \times \text{CMF}_{i2} \times \cdots \times \text{CMF}_{ij} \times \cdots \times \text{CMF}_{in}))]$ CMF <sub>combined,i</sub> :	Proposed by Turner (2011) and is based on multiple weighted factors.
	combined CMF at the <i>i</i> <sup>th</sup> site. CMF <sub><i>in</i></sub> : CMF associated with treatment <i>j</i> ( <i>j</i> = 1, 2,, <i>n</i> ) at <i>i</i> <sup>th</sup> site.	
3	$CMF_{combined,i} = CMF_{i1} - \frac{1 - CMF_{i2}}{2} - \dots - \frac{1 - CMF_{ij}}{j} - \dots - \frac{1 - CMF_{in}}{n} CMF_{combined,i}$ combined CMF at the <i>i</i> <sup>th</sup> site.	Proposed by US state of Alabama (NCHRP, 2008) and is assume safety impacts of second treatment is systematically reduced.
	CMF at the <i>i</i> site. CMF <sub>in</sub> : CMF associated with treatment <i>j</i> ( <i>j</i> = 1, 2,, <i>n</i> ) at <i>i</i> <sup>th</sup> site.	
4	Only the lowest value of CMF is applied (i.e., the most effective safety treatment).	Apply only the most effective CMF.
5	$CMF_{combined} = \frac{\sum_{r=1}^{n} CMF_{imbiasedr}/S_{r}^{2}}{\sum_{r=1}^{n} 1/S_{r}^{2}} S = \sqrt{\frac{1}{\sum_{m=1}^{n} 1/S_{m}^{2}}} CMF_{combined}$ : combined unbiased CMF	Proposed by Bahar (2010) and is based on <i>meta</i> -analysis (weighted average of multiple CMF values).
	value.	
	$CMF_{unbiased}$ : unbiased CMF value from study <i>r</i> .	
	n: number of CMF to be combined. S: standard error for the combined CMF.	

#### Table 3

Examples of single and combined CMFs using existing methods.

Method	Treatment	Study by	Estimate CMF (Actual CMF)	Standard error	Number of sites <sup>a</sup>
Comparison group	Improve pavement friction on curves	David et al. (2015)	0.607 <sup>b</sup>	0.067	35
	Add a new freeway in Norway	Elvik et al. (2017)	0.976 <sup>c</sup>	0.113	47
	Install single left-turn on a major road to Urban unsignalised intersection	Harwood et al. (2003)	0.730 <sup>b</sup>	N/A	20
Yoked comparison	Install road diet	Huang et al. (2002)	0.940b	N/A	25
F	Lane reduction from four-lane undivided roadway to a three-lane with two-way left-turn lanes	Noyce et al. (2006)	0.630 <sup>b</sup>	N/A	9
	Install single left-turn on a major road to Rural unsignalised intersection	Harwood et al. (2003)	$0.720^{\mathrm{b}}$	N/A	61
Naïve comparison	Improve pavement friction on curves	David et al. (2015)	$0.502^{\mathrm{b}}$	0.052	43
	Add a new freeway in Norway	Elvik et al. (2017)	0.661 <sup>c</sup>	0.077	47
Cross sectional	Install a bike lane	Park et al. (2015)	$0.680^{b}$	0.083	227
	Install a median island on minor approaches	Al-Marafi et al. (2021)	$0.720^{\circ}$	0.099	106
	Increase lane width from 10ft to 12 ft	Park and Abdel-Aty (2016)	$0.986^{\mathrm{b}}$	0.005	6420
Empirical Bayes	Install a bike lane	Park et al. (2015)	$0.829^{\mathrm{b}}$	0.029	227
	Add a new freeway in Norway	Elvik et al. (2017)	0.971 <sup>c</sup>	0.112	47
	Install roadside barriers	Park et al. (2016)	0.960 <sup>b</sup>	0.040	147
Full Bayes	Install roadside barriers	Park et al. (2016)	0.940 <sup>b</sup>	0.040	147
•	Use adaptive signal control technology	Kodi et al. (2021)	$0.922^{b}$	0.028	60
HSM	Shoulder widening + Install shoulder rumble strips	Park et al. (2014)	0.588 <sup>c</sup>	N/A	316
	<b>0</b>		(0.608)		
	Implement safety edge treatment $+ \mbox{ adding 2 ft paved shoulders}$	Galgamuwa and Dissanayake (2018)	0.344 <sup>b</sup>	N/A	12
	Shoulder widening $+$ Install shoulder rumble strips	Gross and Hamidi (2011); Pitale, et al. (2009)	0.730 <sup>b</sup> (0.630)	N/A	180
Most effective	Shoulder widening $+$ Install shoulder rumble strips	Park et al. (2014)	0.763 <sup>c</sup> (0.608)	N/A	316
	Shoulder widening $+$ Install shoulder rumble strips	Gross and Hamidi (2011); Pitale, et al. (2009)	0.850 <sup>b</sup> (0.630)	N/A	180
Systematic reduction Turner, 2011	Shoulder widening $+$ Install shoulder rumble strips	(Park et al., 2014)	0.653 <sup>c</sup> (0.608)	N/A	316
	Shoulder widening $+$ Install shoulder rumble strips	Park et al. (2014)	0.726 <sup>c</sup> (0.608)	N/A	316
	Implement safety edge treatment $+ \mbox{ adding 2 ft paved shoulders}$	Galgamuwa and Dissanayake (2018)	0.414 <sup>b</sup>	N/A	12
Meta-Analysis	Shoulder widening $+$ Install shoulder rumble strips	Gross and Hamidi (2011); Pitale et al. (2009)	0.860 <sup>b</sup> (0.630)	0.045	180
	Shoulder widening $+$ Install shoulder rumble strips	Park et al. (2014)	0.767 <sup>c</sup> (0.608)	0.038	316

<sup>a</sup>For intersection-based treatments the sites are intersections, and for segment-based treatments the sites are segments.

<sup>b</sup>CMF calculated based on all crashes.

<sup>c</sup>CMF calculated based on injury crashes.

N/A: No information is available regarding the standard error.

impact of the combination of percent grade (vertical alignment) and horizontal curvature on rural two-lane highways in Washington State. CPMs of five types of vertical and horizontal alignment combinations for severe crashes and property damage only crashes were developed using crash histories from 2003 to 2008. In this study, CMFs representing safety performance were estimated as the ratio of the predicted crashes for a given grade and horizontal curve combination to the predicted crashes for the level tangent (grade less than 1%) that defined a base condition.

Park et al. (2014) examined the existing combining techniques. They confirmed that both the HSM and the fourth technique were effective safety treatment techniques because they yielded CMF values that were close to the actual values of CMF. Similarly, Park and Abdel-Aty (2017) evaluated the performance of several existing techniques and developed an alternative technique based on exploratory analysis. The values of CMF were estimated for various roadway types in Florida using observational BA studies (with EB and CG approaches) and the cross-sectional method. In this study, the data on roadway treatments (single and combined) were obtained from previous studies (Park & Abdel-Aty, 2015; Park et al., 2014). The treatments included widening shoulder width, installing shoulder rumble strips, and combining both treatments. The results of the comparison of the combined techniques have identified the third technique (systematic reduction on the second treatment) as the best-combined technique. Gross and Hamidi (2011) used the result from two earlier studies by Hanley et al. (2000) and Pitale et al. (2009) to examine the techniques that were employed to estimate combined CMFs. The study used two individual treatments (widening shoulders and installing shoulder rumble strips) to achieve the objective. Combined CMFs that were estimated using the techniques adopted by the HSM and that introduced by the State of Alabama were close to actual CMFs.

In summary, previous studies confirm that the values of CMF are likely to vary for the same treatment type and are dependent on the study area. Thus, combining the values of CMF obtained from different study areas and comparing the results with actual values of CMF for multiple treatments do not clearly identify the best technique for combining multiple treatments.

## 5. Evaluation of CMFs using the reviewed methods

Table 3 shows a sample of CMFs for individual and combined treatment methods from previous studies. As shown in Table 3, for individual treatment methods, cross-sectional, EB, and FB methods provided more reliable estimates of crash modification factors for all crash categories (i.e. lower standard error) than the CG, naïve comparison, and YC methods. This result was expected since the CPMs used in cross-sectional, EB, and FB methods included more sites than the other methods.

Notably, the study found that very little research has been done on the CMFs for combinations of treatments. Table 3 shows examples of different methods for combining individual CMFs when considering multiple treatments. The examples illustrate that most researchers used the actual values of combining individual CMFs to give an indication of whether estimated CMFs based on multiple treatments accurately capture the true combined effect. This approach can help other researchers to use the best method in their analysis. In Table 3, from the comparison between the estimated and actual combined CMFs, methods, HSM, Turner (2011), and systematic reduction produced the combined CMFs closest to the actual safety effects for multiple treatments. These methods apple to estimate the safety effects of multiple treatments at reasonable accuracy since the ratio of actual CMF to estimated CMF is closer to 1.

#### 6. Discussion and conclusions

Crash Modification Factors (CMFs) can provide a quick and easy

arithmetic method for estimating crash reductions, therefore, they are a useful tool for quantifying of a particular treatment. For example, the countermeasures to treat run-off-the-road crashes on two-lane local and rural roads include providing road delineation signs, installing edge line striping, widening shoulders, flatten horizontal curves. After implementing one or more of the countermeasures, the selected estimation method can estimate the CMFs for reducing run-off-the-road crashes. This help to gain an understanding of safety treatment effectiveness by comparing CMFs that reduce the occurrence of selected crash severity and crash type. The reduction in crash costs and treatment costs yields the greatest return on road safety investments. Therefore, CMFs assist economic analysis to identify the most beneficial treatments for safety improvements and allow prioritization of safety improvement projects.

Observational Before-After (BA) studies and the cross-sectional method are the two main existing methods for estimating safety effectiveness and calculating the CMFs of treatment sites. Several studies have estimated CMFs using observational BA studies. This study explained the following five state-of-the-art approaches use observational BA data: (i) CG approach, (ii) YC approach, (iii) Naïve approach, (iv) EB approach, and (v) FB approach. Among them, the EB and FB approaches help to account for the RTM bias and are more precise than the other three approaches. Further, the review showed some practical limitations associated with these approaches, such as countermeasures or treatment implementation dates must be known to determine the before and after evaluation periods, sufficient years must pass after treatments are implemented, and safety effects are difficult to distinguish when more than one treatment has been implemented at a specific site. As a result, the cross-sectional method has been widely used in recent years to estimate CMFs. In this method, the CMF value is estimated for a specific site based on its characteristics before the implementation of the treatment by using the coefficients of the prediction models. According to previous studies, the results from the crosssectional method seem to be consistent with the observational BA study results. The basic issue with the cross-sectional method is that it does not consider crash risks that would be attributed to external causal factors that are not included in the analysis. Whereas the difference in estimated safety levels may be the result of unknown factors or known but unmeasured factors. However, relevant studies have recommended this method as a viable alternative method that can be adopted in cases where observational BA studies are not practical due to data restrictions.

Most previous studies estimate CMF as a single value by ignoring the variation of CMF values among different site characteristics. In most cases, it is not realistic to assume a uniform safety impact for all treated sites with different characteristics. Recently, a few studies estimated CMF values by developing CMFunctions to overcome this limitation. A CMFunction allows the value of CMF to change based on site characteristics. In practice, however, using this method is often difficult because more data is needed to detect such differences.

The review also showed that the values of CMF are likely to vary according to the study area, even for the same treatment type. Thus, combining the values of CMF obtained from different study areas and comparing the results with actual values of CMF for multiple treatments do not precisely identify the safety effect of combining multiple treatments. Many researchers have pointed out that very few studies have been carried in order to estimate CMFs for the combined effect of several safety treatments, especially within the same study area. Moreover, it can be observed that the combined CMFs results from the five existing techniques are different. The related studies did not identify which of these techniques provide a more reliable estimate of the effects of multiple treatments. Thus, future studies should try to evaluate the accuracy of the current techniques for determining reliable combined CMFs for multiple treatments. The evaluation can be performed by comparing the actual CMFs calculated using observational before-andafter studies with combined CMFs estimated using current techniques.

Lastly, in the previous studies, the focus was only on developing CMFs and applying these factors to identify the appropriate treatments

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on the basis of the crash reduction percent achieved. Therefore, it is recommended to incorporate traffic simulation models with CMFs to evaluate the effect of the proposed safety treatments on both traffic operation and crash reduction achieved.

#### **Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

No data was used for the research described in the article.

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