Sum-rate Optimal Communication under Different Power Constraints

Mohammad Kaisb Layous Alhasnawi[1,2] and Ronald G. Addie[2]
[1]*Faculty of Administration and Economics, Sumer University, Thi-Qar, Iraq*
[2]*School of Agricultural, Computational and Environmental Science, University of Southern Queensland,*

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Abstract: In this paper the problem of optimal allocation of power to different devices and spectrum when communication takes place in the same region, using shared spectrum, is investigated. We assume that there must be constraints on the power, or EMF, used at each device participating in the shared communication. We consider different forms of power/EMF constraint and compare the sum-throughput achieved by all devices, under these different constraints.

1 INTRODUCTION

Since the introduction of CDMA more than twenty years ago, it has been understood that efficient use of spectrum resources is to a high degree connected with power management, i.e. the choice of how much power is used by each device, in each part of the available spectrum. In the commercial deployment of CDMA, nearly orthogonal codes were used, which gives the impression that efficient power management relies on the shared use, i.e. overlapping use, of spectral resources. However, in this paper we argue that efficient power management is actually better explained by the concept that meeting the power constraint is inherently a shared responsibility. Even when different devices use orthogonal resources, such as transmission at different times, or in different frequencies, the collection of devices communicating in the same geographical region at approximately the same time share responsibility for keeping the total field strength of transmitted signals below a regulated level.

Orthogonal Frequency Division Multiple Access (OFDMA) has strong support as the radio transmission technology for the next generation of cellular mobile wireless systems (Yang, 2010; Yadav et al., 2017). OFDMA is a variant of OFDM which also implements frequency division multiple access, using the orthogonal sub-frequencies. This scheme is used in several generations mobile systems such as 3GPP Long Term Evolution (LTE), and IEEE 802.16m advanced WiMAX.

The paper is organized as follows with the arrangement; Section 2 provides the background about OFDM system and explains the mathematical model by using Shannon Bound theory to a model wireless system. Section 3 compares throughput under the five different configurations; time-segregated transmission, OFDMA, EMF constrained, SS-OFDM, and mutually interfering. The maximum sum-rate throughput for each of the power allocation and sharing those five configurations will determine at Section 4. Section 5 displays the throughput model implemented in Netml. The conclusion is set out in Section 6.

2 BACKGROUND

2.1 Relationship between OFDMA and Sum-rate Optimality

OFDMA is one of the most important multiple access schemes for wireless networks (Yang, 2010; AlSabbagh & Ibrahim, 2016). It has all the communication advantages of OFDM together with efficient sharing of spectral resources (AlSabbagh & Ibrahim, 2016; Castro e Souza et al., 2016).

In broadband multiple access, a significant performance measure is the sum-rate capacity. An important question which is investigated in this paper is whether, and in what sense, is OFDMA sum-rate optimal, i.e. does it acheive, under the appropriate con-

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straints, the optimal total throughput achievable by a given collection of communicating devices?

(Li & Liu, 2007) investigated sum rate optimality of an OFDMA system in an uplink. They found conditions under which OFDMA is sum-rate optimal. They found that the gap between OFDMA and the optimal solution is very small when the number of subchannels is large. Also, they investigated maximizing the sum rate of an OFDMA system in the uplink multi-carrier situations with a limited number of subchannels.

2.2 System Model

The model used here is similar to that of (Chen & Oien, 2008) except that as well as n separate power constraints, we also consider a uniform constraint on total EMF. This constraint also applies at all of the nodes (both origins and destinations) of the network, but it is reasonable to suppose that the constraint is now *the same* at all nodes. This does not imply that all nodes are transmitting with the same power.

2.3 Overlapping Wireless Domains

From the fact that we can get close to the Shannon-Hartley bound, it follows that we can use it to estimate system capacity. This is useful in itself, as a simple and effective way to model wireless systems. For example, we can use this principle to model the bandwidth which can be achieved in a configuration of access points and users of the sort depicted in Figure 1.

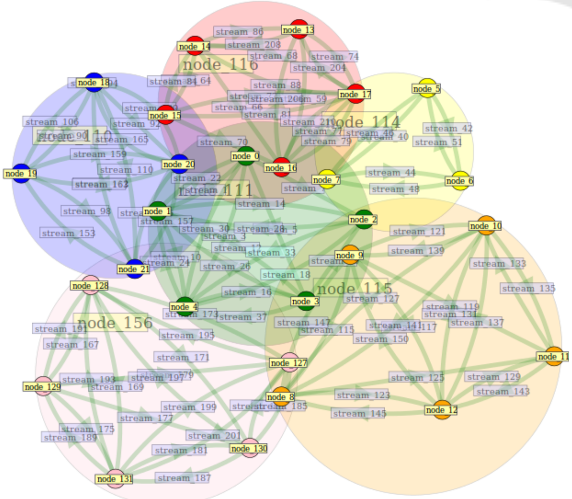

Figure 1: Six wireless networks sharing spectrum.

Currently, the conventional way to model such a system would be to simulate it, for example, using Ns3 (Henderson et al., 2008), Omnet (Varga & Hornig, 2008), or Opnet (Guo et al., 2007). However, setting up such a simulation would be very time consuming and would not necessarily provide useful insight into spectrum sharing.

The study (Alhasnawi et al., 2018) also used the Shannon Hartley bound to model the capacity of wireless communication systems.

3 POWER/EMF CONSTRAINTS

To meet regulations and standards governing wireless communication, all wireless devices must limit the power of their transmissions. This effectively also limits the total EMF generated by these transmissions. In this paper we seek to compare and contrast different approaches to regulating or limiting EMF and/or power.

If shared use of spectrum is mediated by time-segregated use, which is often the case (e.g. as in CSMA/CA), a limit on the power transmitted by any device imposes a constraint on the total electrical field strength (and magnetic field strength), which can occur. Regulations on transmission power are not necessarily imposed for this purpose, however, as the number of devices sharing the same physical and spectral location increases, it may become appropriate, or necessary to view regulation of power in this light, i.e. as a means to limit total electromagnetic field strength.

Suppose there are n transmissions required to take place, as shown in Figure 10, let the transmission power at source k be denoted by P_k, and suppose the maximum power allowed to be transmitted, in order to regulate total EMF, is T_P. More precisely, if there was only one transmitter, in order to acheive the desired limit on EMF, it could not transmit with more power than T_P. We now consider five different approaches to limiting power which vary in the way the aggregate EMF due to all the devices is considered.

Note that the five different approaches to meeting power/EMF constraints that are considered here vary slightly in the way the constraint is *expressed*, but also, and this is the more significant aspect, in the *way* in which the constraint is *enforced*.

These five approaches are:

1. Carrier-sense multiple access (CSMA) method, the modeling of which is presented in Subsection 3.1,
2. Orthogonal Frequency-Division Multiple Access (OFDMA), treated in Subsection 3.2,
3. Electromagnetic Fields (EMF) limited, in Subsection 3.3,

4. Spread Spectrum-Orthogonal Frequency Division Multiplexing (SS-OFDM), in Subsection 3.4, and,
5. mutually interfering (i.e. all transmitters use the entire bandwidth, simultaneously, treating each other as noise), treated in Subsection 3.5.

The purpose of this comparison is not simply to show that one approach has more throughput than another. For example, it will always produce lower throughput when EMF is adopted as the appropriate constraint, rather than transmitted power at each device. The reason for comparing these constraints is that an EMF constraint is more rigorous, and therefore safer. The experiments show that adopting this constraint does not dramatically reduce throughput relative to a constraint on power, and that is the conclusion of interest from these particular experiments.

Likewise, the transmission model adopted is very simple and cannot be used as the basis for designing a communication system. We assume that all communication systems use OFDM with careful channel estimates made dynamically during actual operation. The simple transmission model is being used to compare throughput under the five different configurations which are compared, and is sufficient for that purpose.

3.1 Time-segregated Transmission

If all devices communicate only when others are idle, and when this is the case they use all the available spectrum, the power constraints can be expressed thus:

$$P_n \le T_p, \quad n = 1, \ldots, N \tag{1}$$

These constraints also ensure that at every location, the EMF never exceeds the EMF which would be generated by one device transmitting continuously at the limit power.

3.2 OFDMA

In this case, the power constraints are still expressed by (1). However, because the devices are able to transmit simultaneously, total throughput can be quite different, as shown in the Section 5.

3.3 EMF Constrained

Let

$$G = \begin{pmatrix} g_{11} & \cdots & g_{1n} \\ \vdots & \ddots & \vdots \\ g_{n1} & \cdots & g_{nn} \end{pmatrix}$$

where g_{jk} is the received power at node k due to the transmission from node j, if $j \neq k$, or 1 otherwise. These values can be estimated from Friis transmission formula (Popović & Popović, 2000):

$$g_{jk} = \frac{DA}{4\pi r_{jk}^2} \tag{2}$$

in which D is the directivity of the aerial at node S_j, the source of transmission j, A denotes the relative effective area of the receiving aerial (i.e. the human body) at node S_k, the source of transmission k, and r_{jk} is the distance between the source of transmission j and the source of transmission k. By *relative effective area* of the aerial at node S_k we mean how much less effective a human present at node S_k is, at receiving power from a distant aerial, than they are at receiving power from the source of transmission k. Hence, a simple choice for A is 1.

A constraint on total EMF due to all transmissions, at all the sources, can therefore be expressed in the form:

$$\sum_{j=1}^{n} g_{k,j} P_j \le T_p, \quad k = 1, \ldots, n. \tag{3}$$

3.4 SS-OFDM

Now suppose we use codes, either orthogonal codes or nearly orthogonal ones, in conjunction with OFDM. Thus, codes are used to mediate access rather than frequencies, as in OFDMA. The case where the codes are orthogonal is, in many respects, no different from OFDMA.

Two approaches to limiting power can be distinguished in this case: (a) a simple limit on total power, as in OFDMA, and (b) a limit on total EMF, as in the EMF-limited case. Since the two cases are very similar, we shall confine our investigation in this case to the second of these alternatives.

In this case, the constraints on power are also expressed by (3). If the codes are orthogonal, the throughput will also be the same as in the previous case. A formula for the throughput when the codes are not orthogonal is given in Section 4.5. The only difference is that in this case the power spectral density of the transmitted signal will be different. By judicious use of codes it should be feasible to achieve a virtually flat power spectral density.

However, if the codes are *nearly orthogonal*, as in (Alhasnawi et al., 2018), the throughput of this system will be quite different, and provides an approach intermediate between that of Subsection 3.3 and 3.5.

3.5 Mutually Interfering

In this case, also, the constraints on power are also expressed by (3). Instead of seeking complete independence of different transmissions, by using of time, frequency, or code segregation, in this case we make no attempt to prevent interfence between different transmissions, and simply allow them to proceed simultaneously, with each transmitter treating the others as white noise. We may suppose, for example, that each uses a unique coding which ensures that its signal appears, statistically, as white noise for the others. In a situation where transmitters are far from each other, or where background noise is already of relatavely high power, this approach will be nearly optimal.

4 SUM-RATE OPTIMAL THROUGHPUT

The transmitters sharing the available spectrum are always assumed, when time, frequency, or code resources are shared, to be allocated equal shares. It is therefore possible that higher throughputs than those we obtain below could be attained by unequal allocation of resources. Our intention in this paper is primarily to compare the different sharing strategies rather than to optimize throughput as such. In any case, since focussing on total throughput would often result in some users being allowed no resources at all, it is unlikely that total throughput in this sense is an appropriate objective.

In this section we determine the maximum sum-rate throughput, per Hz, for each of the power allocation and sharing schemes considered in Section 3. We assume that each transmitter has identical access to communication resources – allocation of these resources is not optimized. Rather, it is allocation of power to the resources which is under consideration. Mainly we seek to compare the throughput achieved by the alternative schemes, under different network conditions.

In the first two cases (time segregated, and individual power constraints), the optimal power allocation to devices is obvious. In both these cases, devices simply transmit at their maximum power, while they are active.

In the EMF-constrained case, set out in Subsection 3.3, the vector of power levels is $P = (P_1, \ldots, P_N)'$ where

$$P = T_P G^{-1} u \tag{4}$$

Where u is a vector of $1's$ and $*'s$. If $u_j = *$ we require $P_j = 0$. In other words, we select a subset of sources to transmit at full power and another set of sources that will be idle. One such selection will be optimal.

To work out which ones should be transmitting and which should not, consider a small change to the power of a transmitter, along with the consequential changes to all other transmitters which keep them within their constraint. If this change leads to more throughput, with more power, then this should be one of the transmitters.

The special case where all sources are transmitters will occur frequently because the matrix G will frequently have rather small off-diagonal terms. In this case the vector u, at (4), consists of all 1's.

The total throughput of the system is the same as the *sum rate*, which is the objective of the multiplexing and channel allocation problem considered in this paper. This objective is expressed mathematically in Equation (4) in (Chen & Oien, 2008). In their formulation, the signal from each communication interferes with all others, and appears as white noise of the same power.

4.1 Time-segregated Transmission

In this case each transmitter operates at power $P_n = T_P$ while it is transmitting. The total rate of transmission, in bits/s/Hz, in this case is

$$\sum_{n=1}^{N} \frac{1}{N} \log_2 \left(1 + \frac{P_n G_{n,n}}{\sigma_n^2} \right). \tag{5}$$

4.2 OFDMA

Because the power allocated to the bandwidth assigned to each transmitter is the whole of the allocated power, for this transmitter, i.e. $P_n = T_P$, while the noise is just a $\frac{1}{N}$-th share, and the bandwidth for each transmitter is $\frac{1}{N}$-th of the whole, the total rate of transmission, in bits/s/Hz, in this case is

$$\sum_{n=1}^{N} \frac{1}{N} \log_2 \left(1 + N \frac{P_n G_{n,n}}{\sigma_n^2} \right). \tag{6}$$

4.3 EMF Constrained

The throughput in the EMF-limited case is also given by (6), except that in this case the P_n are given by (4).

4.4 SS-OFDM

In this case, as well as background noise, receiver n experiences user noise, u_n^2, which is given by the formula

$$u_n^2 = \sum_{k \neq n} c P_k G_{k,n} \tag{7}$$

in which c is the correlation between codes, which we assume is the same for all pairs of codes. Naturally $0 \leq c \leq 1$; in the case $c = 0$, we say the codes are orthogonal. Throughput is therefore,

$$\sum_{n=1}^{N} \frac{1}{N} \log_2 \left(1 + \frac{P_n G_{n,n}}{\sigma_n^2 / N + u_n^2} \right). \tag{8}$$

4.5 Mutually Interfering

In this case, as in the previous case, as well as background noise, receiver n experiences user noise, u_n^2, which is now given by the formula

$$u_n^2 = \sum_{k \neq n} P_k G_{k,n} \tag{9}$$

Each transmitter is active all the time, and receivers experience the full background noise, so throughput is

$$\sum_{n=1}^{N} \log_2 \left(1 + \frac{P_n G_{n,n}}{\sigma_n^2 + u_n^2} \right). \tag{10}$$

5 EXPERIMENTS

The throughput model from the previous section has been implemented in Netml (Addie et al., 2011; Addie & Natarajan, 2015) allowing for five different sharing strategies, namely CSMA/CA, OFDMA, EMF-constrained OFDMA, mutually-interfering (i.e. all transmitters use the entire bandwidth, simultaneously, treating each other as noise), and SS-OFDM. This model of sharing which has been implemented in the Netml system is not the same as simulation, and is therefore not available in alternative systems like Opnet, Omnet, or ns-3. Equations (2), (5)–(10) have been used to estimate throughput, instead of simulation. This is much faster and, since it focusses on principles underlying shared use of spectrum, more appropriate in the present context.

The precise values of received power and SNR at each receiver, in (9), depend on the power levels at the transmitters and gain across each pair ($G_{n,n}$), and hence on the geographical layout of the pairs. All these parameters are relatively easy to calculate once the layout has been determined. Using the Netml system, different configurations of communicating pairs can easily be created, the distances between all nodes calculated, the power levels allowed by the constraints for the particular case determined, the gain matrix G calculated by means of (2), and the total throughput calculated.

We have undertaken three experiments, in each of which the geographical configuration of the pairs of communicating devices is arranged somewhat differently. The three cases considered are as follows:

(i) the nodes of each pair are relatively close to each other and the pairs are widely separated. There are 8 pairs of nodes. This case is referred to as *widely separated pairs*, as shown in Figure 2.

(ii) The pairs are closer together than in the previous case, and there is only three pairs, as in Figure 5. This case is referred to as *three close pairs*.

(iii) In this case eight pairs overlap. We refer to this case as *overlapping pairs*, as in Figure 8.

5.1 Widely Separated Pairs

The results, plotted in Figure 3, show that the sum-throughput rate of OFDMA and EMF-constrained OFDMA was equal for all levels of background noise. Total throughput in the CSMA/CA case was always worse than OFDMA and quite significantly so for high levels of noise.

The mutually interfering and SS-OFDM total throughputs were very similar and both were also similar to OFDMA for high noise levels, but a little worse than OFDMA for low noise.

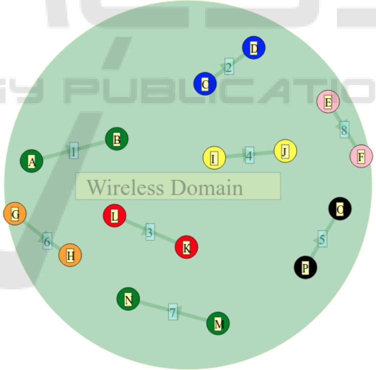

Figure 2: Eight widely separated pairs of nodes.

The power of the signal in each frequency range, when OFDMA is used, has been calculated as well as throughput, and is shown in Figure 4. The power vs frequency distribution will be the same in the EMF-limited case. In the time-segregated case, the SS-OFDM case, or the mutually interfering case, the power vs frequency distribution will be essentially flat.

5.2 Three Close Pairs

The results in this case, plotted in Figure 6 exhibit the same key features: OFDMA and EMF-limited cases

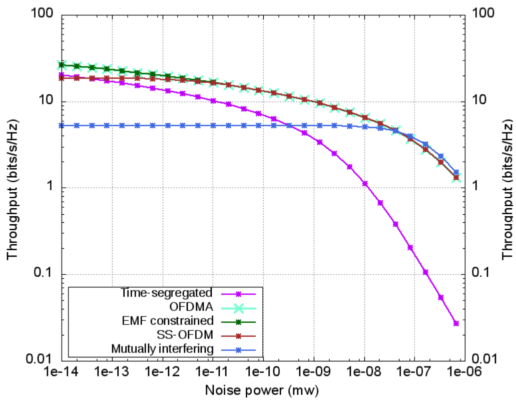

Figure 3: Wireless throughput for widely separated pairs.

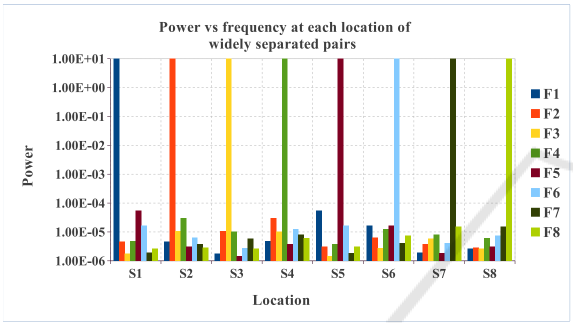

Figure 4: Power vs frequency for widely separated pairs.

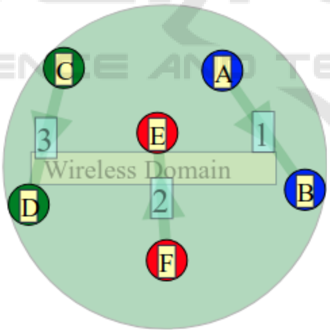

Figure 5: Three pairs of close nodes.

are almost identical and deliver better throughput than all other cases. CSMA/CA is worse, and more significantly so under high noise. The SS-OFDM case is closer to OFDMA but a little worse under low noise. One difference from the previous experiment is that now the mutually interfering case exhibits worse performance than SS-OFDM.

The power of the signal in each frequency range, when OFDMA or the EMF-limited case applies, has been calculated and is shown in Figure 7. In the time-segregated case, the SS-OFDM case, or the mutually interfering case, the power vs frequency distribution will be, as in the first experiment, essentially flat.

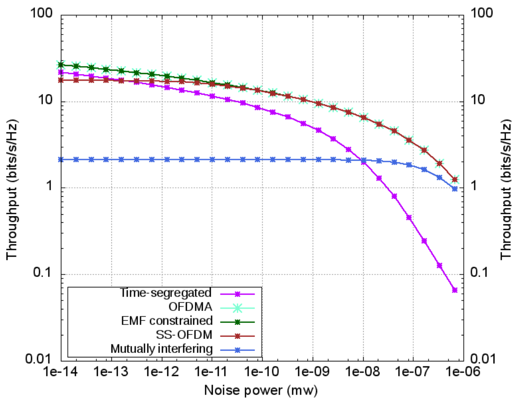

Figure 6: Wireless throughput for three close pairs of nodes.

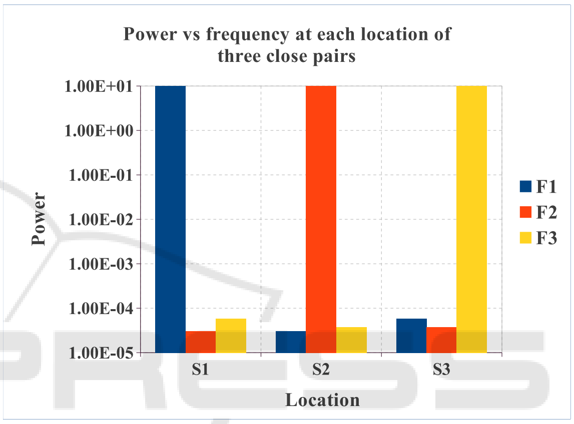

Figure 7: Power vs frequency at each source location of three close pairs.

5.3 Overlapping Pairs

The results, plotted in Figure 9, again show that the OFDMA and EMF-limited cases have higher total throughput than all others. CSMA/CA is again significantly worse for high noise, and SS-OFDM is close to OFDMA, but a little worse for low noise. Also, the mutually interfering case is worse again than SS-OFDM.

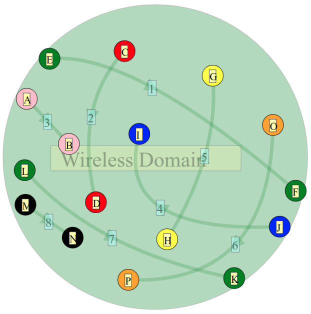

Figure 8: Overlapping communicating pairs.

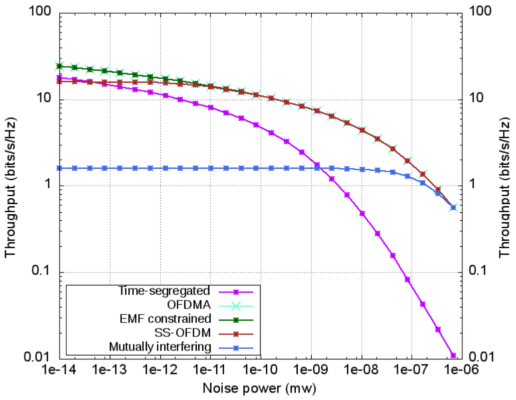

Figure 9: Wireless throughput of overlapping communicating pairs.

The power of the signal in each frequency range, in the OFDMA or EMF-limited cases, has been calculated and is shown in Figure 10. In the time-segregated case, the SS-OFDM case, or the mutually interfering case, the power vs frequency distribution will be, as in the previous experiments, essentially flat.

6 CONCLUDING REMARKS

The experiments all show that OFDMA and the EMF-limited cases are nearly identical. This is because in all the cases considered, the EMF limits on power are not significantly different from simply limiting the transmitted power of each device. If configurations where devices are very close together were considered, this would no longer be the case. Consideration of such cases remains for future work.

Another consistent result was that OFDMA consistently out-performed all other sharing mechanisms. The SS-OFDM case assumed non-orthogonal codes, with correlation at the level 0.1. If orthogonal codes were used, the performormance of SS-OFDM would be identical to OFDMA. Such experiments were conducted, but not shown, because the two performance curves would simply be superimposed.

However, the spectral distribution of SS-OFDM is essentially flat, unlike that of OFDMA. If this is an important consideration, SS-OFDM is therefore the preferred option. It acheives the same throughput as OFDMA, but within a much tighter constrant on the power spectral density.

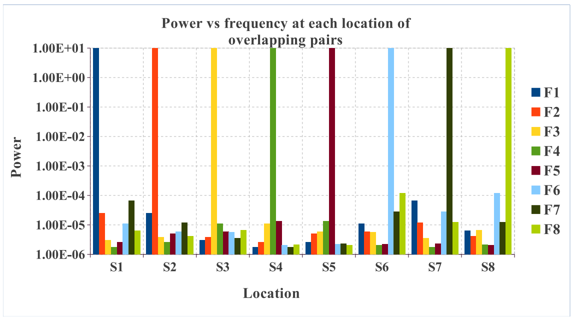

Figure 10: Power vs frequency at each source location of overlapping pairs.

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