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Numerical Analysis of Corrugated Tube Flow using RBFNs

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ABSTRACT

This paper reports the application of neural networks for the numerical analysis of steady-state axisymmetric flow through an indefinitely long corrugated tube. Meshless global radial basis function networks (RBFNs) are employed to represent all dependent variables in the governing differential equations. For a better quality of approximation, the networks used here are constructed based on the integration process rather than the usual differentiation process. Multiple spaces of network weights for each variable are converted into the single space of nodal variable values, resulting in the square system of equations with usual size. The governing equations are discretized in the strong form by point collocation and the resultant nonlinear system is solved with trust-region methods. The corrugated tube flow of a Newtonian fluid, power-law fluid and Oldroyd-B fluid are considered. With relatively low numbers of data points, flow resistance predictions obtained are in good agreement with the benchmark solutions.

INTRODUCTION

The principal methods of discretization for the analysis of non-Newtonian flow include finite difference methods (FDM), finite element methods (FEM), finite volume methods (FVM), boundary element methods (BEM) and spectral methods. Although much progress has been made, there still remain great challenges for the achievement of accurate numerical solutions at high values of Weissenberg number (We) and there still exist great difficulties in the numerical modeling process such as the generation of meshes.

The development of numerical methods without using a mesh for the solution of engineering problems has been an active research area recently. The meshless methods do not require any connectivities between data points, resulting in an easy process of numerical modeling.

For the group of meshless methods based on radial basis function networks (RBFNs), it requires only a minimum amount of effort to implement. Furthermore, the governing equations involving high order or complicated differential operators can be discretized in a straightforward manner with RBFNs. The networks can be constructed based on a differentiation process, namely direct RBFNs (DRBFNs) or based on an integration process, namely indirect RBFNs (IRBFNs). Previous findings showed that the indirect approach performs better than the direct approach in terms of both solution accuracy and convergence rate [1].

In the present study, the meshless IRBFN method is developed to simulate the corrugated tube flow of a Newtonian fluid, power-law fluid and Oldroyd-B fluid. Nonlinearities of the discretized system are handled by using trust region methods that retain two best features, namely rapid local convergence of the Newtonian iteration method and strong global convergence of the Cauchy method. The computed results are compared with the benchmark solutions obtained by the full pseudo-spectral (FCC) and the mixed spectral finite difference (PCFD, PSFD) methods [2-4]. Two salient features of the proposed high order IRBFN method are the mesh-free feature and the capability to achieve high accuracy using low numbers of data points.

REVIEW of RBFNs

RBFNs have been proven to have the property of universal approximation. The network allows a conversion of a function from low dimension space (1D-3D) to high dimension space in which the function can be expressed as a linear combination of RBFs. There is a large class of radial basis functions whose design matrices are always invertible provided that the data points are distinct whatever the number of data points and the dimensionality of problem. On the other hand, the Cover theorem, that can be stated as follows: the higher the number of neurons (RBFs) used the more accurate the approximation will be [5], indicates the property of "mesh" convergence of RBFNs. These important theorems can be seen to provide the theoretical basis for the design of RBFNs to the field of numerical solution of PDEs.

The superior accuracy of the IRBFN method over the DRBFN method can be argued as follows. Any inaccuracy (noise) in the assumed RBFN decomposition is badly magnified in the process of differentiation (the slope of the curve). However, the effects of noise can be suppressed by the process of integration (the area under curve). The approximating functions are expected to be much smoother thorough the integration process.

IRBFNs for SOLUTION of PDEs

Each dependent variable and its derivatives in the governing equations are represented by IRBFNs. Prior conversions of the multiple spaces of network weights into the single space of nodal variable values are employed to form the square system of equations of usual size. These closed-form representations are substituted into the governing equations and the obtained system is then discretized by point collocation.

The present method appears to be close to the FCC method [4] in the sense a) they are global high order methods, b) the governing equations are approximated in the strong form by point collocation and c) the resultant matrices are dense.

In contrast to the FCC method, the present method uses only IRBFNs to represent the field variables and their derivatives in both radial and axial directions. Furthermore, collocation points in the IRBFN method can be chosen randomly, while the coordinates of data points in the radial direction in the FCC method should be chosen as the roots of Chebyshev polynomials. It is known that spectral methods are typically employed for "nice geometries".

In the following section, it will be shown that like the FCC method, the present IRBFN method can produce accurate results using relatively coarse densities of data points.

NUMERICAL RESULTS

Consider the fluid flow through an infinitely long corrugated tube. Relevant geometry parameters are defined in Figure 1. Creeping flows of a Newtonian fluid and an Oldroyd-B fluid as well as inertial flows of a Newtonian fluid and a power-law fluid are simulated. To study "mesh" convergence, three densities of 17×17, 21×21 and 25×25 data points are employed. Results for the flow resistance (fRe) obtained by the IRBFN method and other methods are displayed in Tables 1-4 and Figures 2-3. The present results are in good agreement with the benchmark solutions [2-4] for all tested cases. In the case of (a=0.1, N=0.16) (moderate amplitude and moderate wavelength), convergence can be obtained up to high Weissenberg number, at least of about 30 (Figure 2). For the range of We from 0 to 20, the flow resistance is not much different from that obtained at We=0 (Newtonian fluid), which looks feasible when compared to the available results in the literature. However, the present flow resistance is observed to increase quickly when We>20. The reason could be that data densities become too coarse to capture the solution, especially for the stress fields near boundaries.

SUMMARY

This paper reports a numerical method based on universal high order RBFNs for the analysis of a steady-state axisymmetric flow in a corrugated tube with the periodic boundary conditions. Nonlinearities of the discretized system are treated using trust region methods. Like the FCC method, the present method can produce accurate results using low numbers of data points. The IRBFN method is a truly meshless method and can be extended straightforwardly to simulate non-periodic flows.

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Table 1. Creeping flow of a Newtonian fluid, 25×25 points: flow resistance for different tube geometries by the IRBFN method and other methods. Good agreement is achieved.

Method	fRe		
	a=0.2 N=0.1042	a=0.3 N=0.1592	a=0.286 N=0.2333
IRBFN	19.7582	26.4331	26.3814
FCC [4]	19.7655	26.437	26.383
PSFD [2]	19.765	26.436	26.383
PCFD [2]	19.761	26.432	26.377
FEM [2]	19.756	26.385	26.293

Table 2. Inertial flow of a Newtonian fluid, a=0.3, N=0.16: flow resistance for different Re numbers by IRBFNs. Good convergence is achieved for every Re.

Re	fRe (Error %)		
	17×17	21×21	25×25
0	26.45 (0.03)	26.44 (0.00)	26.44
0.012	26.45 (0.03)	26.44 (0.00)	26.44
12	27.19 (0.07)	27.18 (0.01)	27.17
22.6	28.57 (0.09)	28.55 (0.02)	28.55
51	31.75 (0.02)	31.75 (0.01)	31.75
73	33.43 (0.06)	33.45 (0.01)	33.45
132	36.42 (0.33)	36.54 (0.01)	36.54
207.4	38.68 (0.80)	39.02 (0.05)	38.99
264	39.77 (1.31)	40.34 (0.09)	40.30
397.2	41.13 (3.12)	42.57 (0.26)	42.45
783	--------------	46.08 (0.75)	45.74

Table 3. Inertial flow of a Newtonian fluid: flow resistance by the IRBFN method and other methods. GSM-Galerkin spectral method, GFE-Galerkin finite element. The IRBFN and the FCC results are in the most agreement.

Re	fRe			
	IRBFN 25×25	GSM [4]	FCC [4] Nx=16,Nc=33	GFE [4] Nr=40,Nz=40
0	26.44	26.4	26.44	26.41
0.012	26.44	26.4	26.44	26.41
12	27.17	27.1	27.17	27.09
22.6	28.55	28.5	28.55	28.44
51	31.75	31.7	31.74	31.69
73	33.45	33.4	33.44	33.40
132	36.54	36.7	36.52	36.53
207.4	38.99	38.9	38.96	38.93
264	40.30	39.7	40.24	40.15
397.2	42.45	40.6	42.34	42.11
783	45.74	41.2	45.58	45.07

Table 4: Inertial flow of a power-law fluid (n=0.54, k=1), a=0.3, N=0.1592: flow resistance by the IRBFN and the PCFD methods. The IRBFN results agree well with the PCFD results.

Re	fRe	
	IRBFN 25×25	PCFD [3] Nx=16,Np=100
0	9.1268	9.1052
1.528	9.1434	9.1240
12.484	9.8270	9.8508
21.581	10.3788	10.3885
36.912	11.0120	11.0083
50.430	11.4189	11.3988
62.905	11.7202	11.6876
85.934	12.1586	12.1067

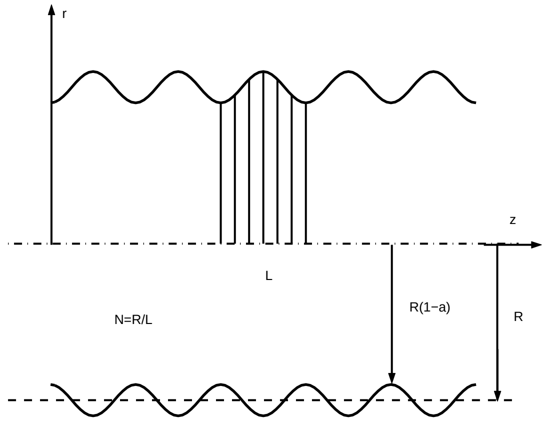

Figure 1. ''Wiggly'' tube problem: geometry. The shaded area represents a unit computation cell.

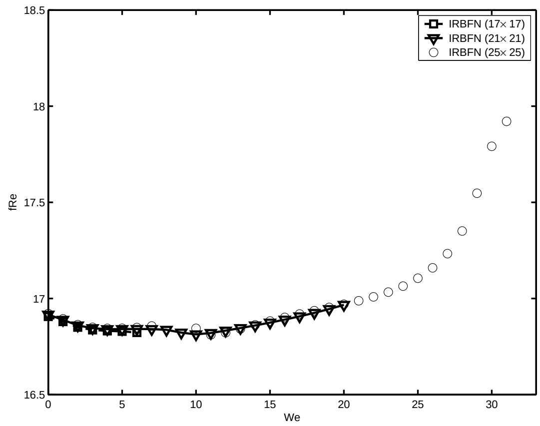

Figure 2. Geometry (a=0.1, N=0.16), Oldroyd-B fluid: the plot of flow resistance (fRe) versus Weissenberg number (We) using three relatively coarse discretizations (17×17, 21×21 and 25×25 data points).

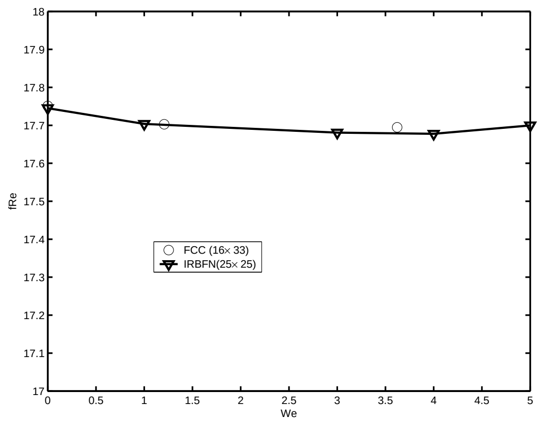

Figure 3. Geometry (a=0.1, N=0.5), Oldroyd-B fluid: Comparison of the flow resistance obtained by the present IRBFN method and that obtained by the FCC method [4]. Good agreement is achieved.