

## Climate modelling for the classroom: Applying physics and mathematics in a new context

by Joachim Ribbe

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Emerging scientific disciplines such as climate system and earth system science pose additional challenges to the traditional enabling disciplines physics, mathematics, biology, and chemistry, and their teaching. Yet these new disciplines rely on students being trained in the enabling sciences. In this article I introduce the art of climate modelling which, as a system science, applies traditional physical and mathematical concepts in new contexts. I use a simple climate model to develop several exercises that can be applied to current environmental issues, including the debate on climate change. Students still use the tools of the enabling sciences; however their application is problem-based and linked to crucial environmental science issues.

### Introduction

Over the last twenty years, a new science has emerged that blends mathematics, the traditional, enabling sciences—physics, biology, chemistry and geology—with 20<sup>th</sup> century sciences like oceanography and meteorology into what is often now referred to as climate system science or earth system science (Steffen et al, 2005).

In Australia, the federal government has recognised these developments by funding the Australian Earth System Science Network, a consortium of many Australian universities and research institutions (see web links below). The emergence of earth system science has been driven by the rapid environmental changes that have accompanied human population growth and development that has in turn led to unsustainable use of natural resources. Scientists, often trained in the traditional fields, have nonetheless recognised that a new approach—system science—is

essential to understand the bases of environmental change.

Climate system and earth system science recognise that the fundamental nature of our living environment can only be understood from a systems perspective. That is, what is required is a holistic framework of thought that takes into account the complex interaction between the natural and human systems that traditionally have been studied in isolation by individual disciplines.

The emergence of these system sciences has occurred against the background of a declining student interest in the traditional science fields as taught in Australian high schools and universities. Strong competition from new emerging areas of scientific endeavour has compounded falling student enrolments in the enabling sciences (AUTC 2005). A dwindling student interest in the traditional sciences is further compounded by a shortage of suitably qualified science teachers who could stimulate senior

students' interest in science and deliver the rapidly evolving content of the modern sciences (Harris et al. 2005).

The emerging climate and earth system sciences require the skills and understandings of the traditional science disciplines, in particular of physics and mathematics. What is new is that these skills are applied in contexts that are central to, and drive the discussion of, current political and societal issues pertaining to global climate change and variability. Much of what we know about climate change is based upon the application of climate models. This paper is intended to help secondary teachers of science to develop modules that will introduce their students to the art of climate modelling. It employs traditional physical principles, mathematical concepts and modern technologies such as the internet to engage students in this art (Ribbe 2002).

## The concept of energy balance modelling

One of the most complex tasks that the scientific research community has undertaken during the last 30–40 years has been to model the global climate system. Complexity is an intrinsic feature of a climate system that is itself composed of a series of sub-systems that include the ocean, atmosphere, land, biosphere, and cryosphere. Consequently, constructing models of our climate system requires expert knowledge drawn from a wide range of scientific disciplines. An excellent series of articles on the state of climate and climate modelling can be found in *National Geographic* (September, 2004).

When we describe climate, we often refer to a temperature since temperature is an easy-to-measure environmental indicator of climate. It was this climate variable that was used to describe climate when scientists started to model climate in the early 1960s. In a sense this is still valid; we refer to climate change as global warming.

In its simplest form we refer to climate modelling as Energy Balance Modelling (Hartmann 1994). If our climate doesn't change—that is, if, on average, we detect an unchanged pattern in the annual cycle of Earth's mean surface temperature—then the heat or thermal energy of our planet must be in a steady-state. That is, the amount of radiation the earth receives from the Sun must equal the amount re-radiated back into space otherwise our planet would continue to warm. Energy balance modelling is based on this assumption.

Energy balance modelling may appear a rather simplistic approach to understanding climate, yet it illustrates some basic behaviours of climate. Energy balance modelling can be used to discuss phenomena that are currently discussed in media reports that include phenomena such as global dimming, global warming, the ice ages, and 'Snow Ball' Earth. We can discuss with our students such questions as: How does climate respond to changes in solar energy output? Could current global warming be caused by changes in solar energy output? How does climate respond to changes in albedo or reflectivity

of Earth? How does climate respond to changes in the composition of the atmosphere? But before we use this simple climate model to explore our climate system, we need to learn a little bit about the underlying physical and mathematical concepts required to provide us with answers to some of our questions.

## Physical laws and mathematical requirements

Energy balance modelling therefore relies on the assumption that our climate is in steady-state. The globally averaged surface temperature of Earth is about +15 °C. This temperature has been constant for many thousands of years; it makes the planet habitable and water available in liquid form. To maintain this equilibrium temperature, the amount of incoming solar radiation (heat or thermal energy gain) that drives our climate and life on Earth, needs to be equivalent to the amount of outgoing radiation (heat or thermal energy loss). Any changes in this balance would either lead to global cooling or global warming.

This amount is referred to as the Solar Constant ( $S$ ) and is about  $1370 \text{ Wm}^{-2}$ . (The physical unit [ $\text{W} = \text{Watt}$ ] is equivalent to the amount of energy [ $\text{J} = \text{Joule}$ ] per second [ $\text{s}$ ]). In a period of one year, only a fraction of Earth's surface actually captures the full amount of solar radiation since it is only the 'Shadow Area' of the planet that intercepts the beam of solar radiation coming from the sun (Figure 1).

In mathematical terms, we can describe the amount of captured (i.e. absorbed) energy through the following relationship:

**Equation 1:**

$$E_{\alpha} = S \cdot \pi \cdot r^2$$

Here  $\pi r^2$  is the shadow area and  $r$  is the radius of Earth ( $r = 6370 \text{ km}$ ). Yet, we know that in addition to capturing energy, the planet also reflects energy. It is not a perfect mirror, but at least a fraction of the solar radiation is sent back directly into space without being available to heat the climate system. We can express this through the following relationship:

**Equation 2:**

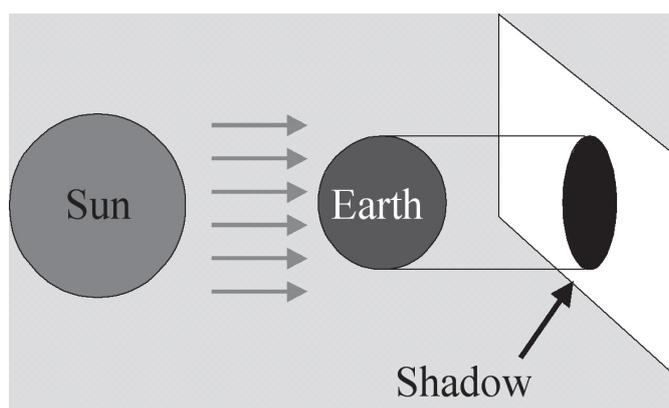
$$E_r = -S \cdot \alpha \cdot \pi \cdot r^2$$

Here,  $\alpha$  is referred to as the reflectivity or albedo of the earth. For example, if the surface is very white as for example in the case of snow and ice, the albedo is large ( $\alpha = 0.9$ ) and most of the incoming energy is reflected. By contrast, for a very dark surface such the ocean,  $\alpha = 0.1$ . In these circumstances most of the incoming solar radiation or energy is absorbed. The minus sign in Equation 2 indicates that the reflected energy is removed from the climate system. Earth's albedo results from the combined reflectivity of clouds, ocean, land, and snow and ice, and is at present about  $\alpha = 0.3$ , meaning the earth reflects 30 % of incoming radiation. The sum of Equation 1 and 2 now provides us with the total amount of energy  $E_{in}$  that Earth receives during a year:

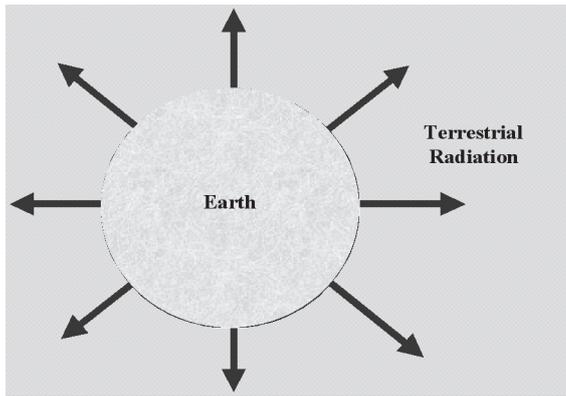
**Equation 3:**

$$E_{in} = E_u + E_r + S \cdot \pi \cdot r^2 - S \cdot \alpha \cdot \pi \cdot r^2 = S \cdot \pi \cdot r^2 \cdot (1 - \alpha)$$

Next, we need to calculate the amount of thermal energy that is radiated by the earth in order to maintain the required energy equilibrium (Figure 2).



**Figure 1. Solar radiation as received by Earth is captured by an area referred to as the shadow area**



**Figure 2. Radiation emitted by Earth is maintaining our globally averaged mean temperature at about +16 °C**

The amount of energy emitted follows from a simple physical law that links the amount of heat or thermal energy leaving the planet to the surface temperature of the planet. This law is referred to as the Stefan Boltzmann's Law and is expressed as:

**Equation 4:**

$$E_{out} = \sigma \cdot T^4 \cdot 4 \cdot \pi \cdot r^2$$

Here, T [K] is the surface temperature of the planet and the term  $4\pi r^2$  relates to the surface area of the sphere (i.e. Earth) that emits the radiation. The variable  $\sigma = 5.67 \cdot 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$  is a universal physical constant referred to as the Boltzmann constant. A unique feature of Earth is the existence of an atmosphere that traps a fraction  $\epsilon$  of the radiation emitted from the surface of the earth. The trapping of Earth's radiation is called the Greenhouse Effect and Equation 4 is modified to include this effect:

**Equation 5:**

$$E_{out} = \epsilon \cdot \sigma \cdot T^4 \cdot 4 \cdot \pi \cdot r^2$$

The current natural composition of the atmosphere (and in particular, the greenhouse gases, water vapour and carbon dioxide) traps about 40 % of the outgoing radiation. That is,  $\epsilon = 0.6$  since 60 % of the outgoing radiation balance the incoming solar radiation. This variable is often referred to as the transparency of the atmosphere.

We now have both an estimate for the incoming radiation that warms the climate system (Equation 3) and an estimate for the amount of outgoing radiation that cools the climate system (Equation 4). In order to maintain equilibrium, both amounts of thermal energy need to be equal:

**Equation 6:**

$$S \cdot \pi \cdot r^2 \cdot (1 - \alpha) = \epsilon \cdot \sigma \cdot T^4 \cdot 4 \cdot \pi \cdot r^2$$

This equation can be rearranged to yield an equation that allows us to calculate the equilibrium surface temperature of the climate system:

**Equation 7:**

$$T = \sqrt[4]{\frac{S \cdot (1 - \alpha)}{4 \cdot \epsilon \cdot \sigma}}$$

T [K] is the surface temperature of the Earth. All other variables in this equation are known. This energy balance model is based upon the balance between incoming solar and outgoing planetary radiation. We can now use the equation to calculate the surface temperature of Earth and test the response of the climate system – that is Earth's mean temperature – to changes in the radiation balance. These changes may result from changes in albedo  $\alpha$ , Solar constant S or composition, i.e. transparency  $\epsilon$  of the atmosphere.

**Exercise 1: Global equilibrium temperature**

In this first exercise, we calculate the mean surface temperature of Earth by substituting all variables in Equation 6 with the known quantities:

$$T = \sqrt[4]{\frac{S \cdot (1 - \alpha)}{4 \cdot \epsilon \cdot \sigma}}$$

$$T = \sqrt[4]{\frac{1370 \cdot (1 - 0.3)}{4 \cdot 0.6 \cdot 5.67 \cdot 10^{-8}}}$$

$$T = 289K$$

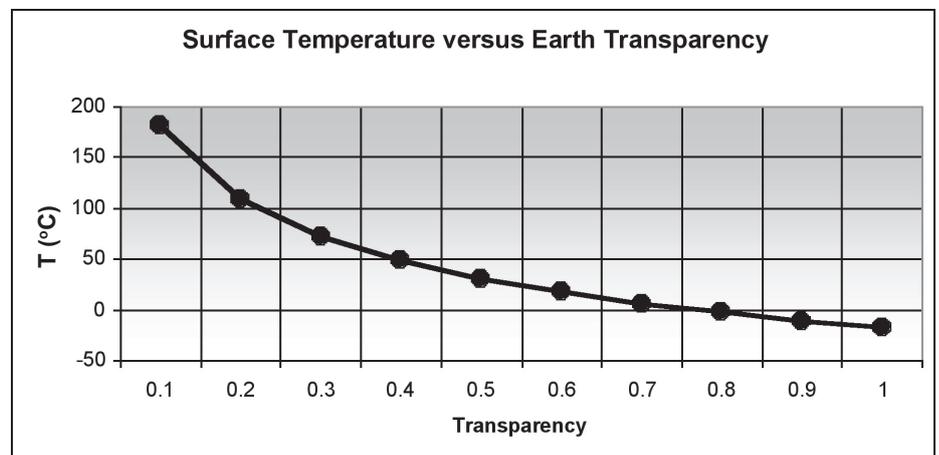
Note that temperature is given as absolute temperature [K] and can be converted into [°C]:

$$T = 289 - 273 = 16^\circ\text{C}$$

Interpreting this result leads to the following statement: under current conditions, which include the composition of the atmosphere (transparency  $\epsilon$ ), the distribution of land, ocean and ice (albedo  $\alpha$ ), and the amount of solar radiation arriving at Earth (S), the global mean surface temperature of the planet is about 16 °C. This value is very close to the observed value of 15 °C. It follows that any changes to the above properties will result in changes to the global mean surface temperature of the earth.

**Exercise 2: Sensitivity to atmospheric composition**

This exercise uses a simple climate model to discuss what would happen if we change the composition of the atmosphere. The net effect of the atmosphere, expressed through the variable  $\epsilon$ , is referred to as transparency. The smaller  $\epsilon$ , the weaker the greenhouse effect and less outgoing energy is trapped. Assume we change the transparency of the atmosphere, for example, by altering the amount of carbon dioxide or other greenhouse gases in the atmosphere. This would lead to a change in the transparency  $\epsilon$ . Students can investigate the response of the climate system to changes in  $\epsilon$ , by substituting a range of values into Equation 7, essentially determining T as a function of  $\epsilon$ , that is T( $\epsilon$ ). All other data are kept constant. The data can be entered into a Microsoft



**Figure 3. Presentation of the surface temperature as a function of the transparency  $\epsilon$ . The albedo is varied between 0.1 and 1 representing an atmosphere that is fully transparent for emitted radiation (i.e. no Greenhouse effect) to an atmosphere that is trapping most of the outgoing radiation**

Excel spreadsheet and is graphically presented in Figure 3.

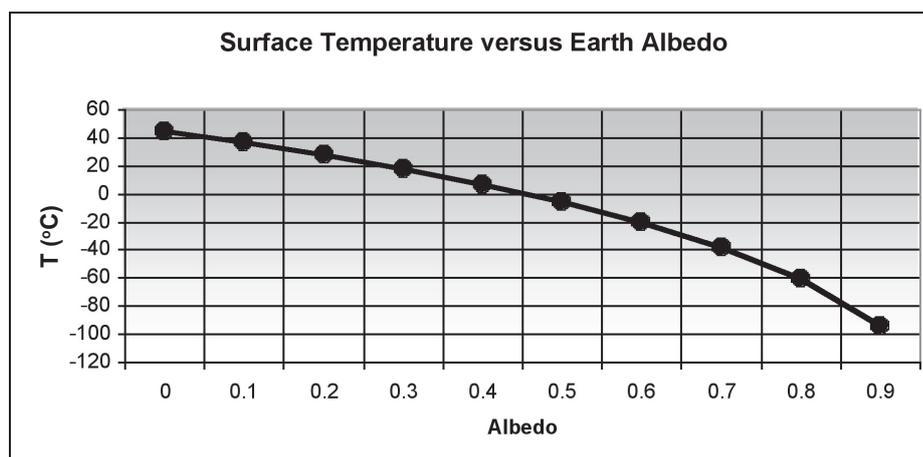
The physical interpretation of this experiment allows students to conclude that the Greenhouse Effect is a direct consequence of the atmospheric composition. The more intense the Greenhouse effect, the higher the surface temperature of Earth will climb, reaching a value of almost 200 °C with an opaque, non-transparent atmosphere. On the other hand, it enables students to state that with a fully transparent atmosphere ( $\epsilon = 1$ ) no energy is trapped in the atmosphere. The simple energy balance model of Earth dictates that global mean temperature would be about  $T = -18$  °C. All water on the planet would be frozen and life as we know it could have not developed.

While this is a very simple approach to climate modelling, it allows us to link our result to the current public debate of possible future climatic changes due to changes in the composition of the atmosphere. Humans are actively altering the transparency of the atmosphere by altering its composition.

### Exercise 3: Sensitivity to albedo

This exercise explores the response of the climate system to changes in reflectivity. Changes in reflectivity could occur as a result of changes in the amount of ice and snow accumulating on the surface of Earth. For example, during the ice ages more ice and snow amassed on the surface of the planet. Earth was 'whiter', therefore the albedo was higher and more energy was reflected back into space. This cooled the planet further. Other possible increases of the albedo could result from more intense global winds that would result in more waves and white caps covering the global ocean surface, or from more white clouds in the upper layers of the atmosphere. Students can test the Earth's climate response to a larger or smaller albedo by varying the variable  $\alpha$  between 0 and 0.9. The data can be entered onto a spreadsheet and results are shown in Figure 4.

An interpretation of these data is that with a larger value of reflectivity ( $\alpha = 0.9$ ), more than 90% of incoming solar radiation is reflected directly



**Figure 4. Presentation of the surface temperature as a function of the reflectivity  $\alpha$ . The albedo is varied between 0 and 0.9 representing a surface that ranges from dark (high absorption) to white surface (high reflection)**

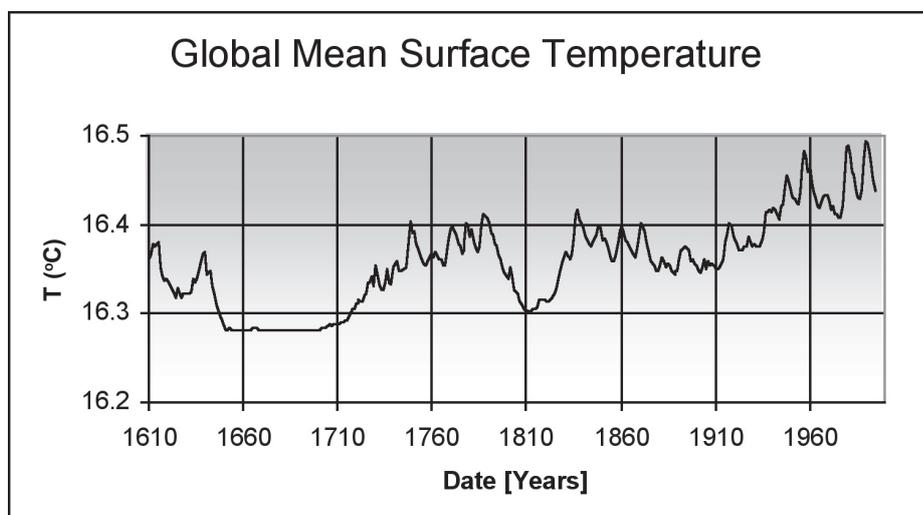
back into space and is not available to warm the planet. As a consequence, global cooling sets in and the mean temperature drops to below the freezing point. Again, water would be frozen and life as we know it could not be sustained. The phenomenon is referred to as 'Snowball' Earth and there is some evidence that it has occurred in the past. On the other hand, a darker planet leads to a low albedo and temperature would increase to as much as 45 °C.

### Exercise 4: Sensitivity to solar variability

In this final exercise students access a palaeoclimatology webpage (see link) and download information about

solar radiation changes that occurred during the last 500 years. These data can be entered into an Excel spreadsheet and with the help of the simple climate model, they can directly calculate how much global temperature has varied as a consequence of small changes in solar radiation.

An interpretation of the results will lead students to the inference that, during at least the last 500 years, the global mean temperature has varied only marginally. Due to natural forces such as changes in the solar constant, global temperature has only varied by about 0.1–0.2 °C. This is a much smaller value than the warming the world currently experiences, which is in the order of about 0.6 °C.



**Figure 5. Changes in solar radiation by about 3-4  $Wm^{-2}$  during the last five hundred years were accompanied by only small changes (about 0.1-0.2 °C) in the mean global temperature**

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### Summary

In this article I have outlined a series of exercises that will increase students' competency in the enabling sciences, physics and mathematics, and that can easily be adopted wholly, or partly, as classroom activities for senior science students. Finally, these exercises provide a basis to discuss pressing environmental problems by enabling our students to apply their acquired knowledge in the context of earth system science. For help with adapting the exercises above as classroom activities, I can be contacted by e-mail: [Joachim.Ribbe@usq.edu.au](mailto:Joachim.Ribbe@usq.edu.au)

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### Web Links

Australian Earth System Science Network: <http://www.es.mq.edu.au/physgeog/staff/ap/ACSN>

NOAA Paleoclimate Webpage: <http://www.ngdc.noaa.gov/paleo/paleo.html>