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ORIGINAL ARTICLE

Designing pentapartitioned neutrosophic cubic set aggregation operator-based air pollution decision-making model

Yi-ming Li[1] · Majid Khan[2] · Adnan Khurshid[3] · Muhammad Gulistan[2,4] · Ateeq Ur Rehman[2] · Mumtaz Ali[5] · Shahab Abdulla[5]· Aitazaz A. Farooque[6]

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Abstract

Environmental pollution is a global concern that has economic and health implications. Therefore, proper estimation using precise modeling can help in decision-making to address this externality. In science and engineering, there are a lot of different theories to help deal with the complex frame of the environment. The prime objective of these theories is to impart a plan of action to handle fuzzy data more precisely. Furthermore, humans need a platform that can correctly assign a value to optimize credence in a belief system. The indeterminacy is further classified into contradiction, ignorance, and unknown by a pentapartitioned neutrosophic set. On the other hand, a cubic set characterizes both the combined and the crisp value. The study introduces pentapartitioned neutrosophic cubic set, as it illustrates all of these attributes, allowing credence to be appropriately handled. The study also explained its operational laws and aggregation operators. Finally, this technique is used to develop and evaluate the air pollution models in major Pakistani cities like Karachi, Lahore, Islamabad, and Peshawar. It will help the legislators to reevaluate current policies to mitigate this externality.

Keywords Neutrosophic set (NS) · Neutrosophic cubic set (NCS) · Pentapartitioned neutrosophic cubic set (PNCS) · Aggregation · Air pollution

Introduction

Uncertainty is a complex phenomenon that occurs in the real world. Since uncertainty is inevitably involved in problems, it occurs in different domains of life such that traditional methods have broken out to manage such problems. The considerable task is to deal with fuzzy information more effectively. Many theories have been established to assimilate uncertainty into the interpretation of the system. Zadeh instigate fuzzy set (FS) [1]. Henceforth, it is applied in different areas of sciences like information sciences, artificial intelligence, decision-making theory, medical sciences and

✉ Adnan Khurshid
adnankhurshid83@gmail.com

✉ Muhammad Gulistan
gulistanmath@hu.edu.pk; mgulista@ualberta.ca

Yi-ming Li
li-yiming@zjnu.edu.cn

Majid Khan
majidmaths@hu.edu.pk

Ateeq Ur Rehman
atteqmaths@hu.edu.pk

Mumtaz Ali
Mumtaz.Ali@usq.edu.au

Shahab Abdulla
Shahab.Abdulla@usq.edu.au

Aitazaz A. Farooque
afarooque@upei.ca

1 School of Economics and Management, Zhejiang Normal University, Jinhua, China

2 Department of Mathematics and Statistics, Hazara University, Mansehra 21130, Pakistan

3 College of Economics and Management, Zhejiang Normal University, Jinhua 321004, China

4 Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB, Canada

5 UniSQ College, University of Southern Queensland, Darling Heights, QLD 4350, Australia

6 Canadian Center for Climate Change and Adaptation University of Prince Edward Island, St Peters Bay, PE, Canada

much more. Due to its aptness in science and daily life problem, the fuzzy set has been generalized into interval valued fuzzy sets (IVFS) [2, 3], intuitionistic fuzzy set (IFS) [4], interval valued intuitionistic fuzzy set (IVIFS) [5] and cubic set [6]. IFS comprises of membership and non-membership, whereas the hesitant component is considered under the condition that the sum of these components is one. Indeterminacies are an imperative part of daily life. The collection and evaluation of data always contain inconsistent, vague and indeterminant components. Smarandache introduces a neutrosophic set (NS) [7] to discuss the indeterminacies. NS, which provides a more general platform to elongate the ideas of classic and fuzzy set theory. The NS expresses truth, indeterminacy and falsity components independently. NS is the generalization of IFS [8]. For engineering problems and science Wang et al., presented a single valued neutrosophic set (SVNS) [9], while Wang et al., introduced the interval neutrosophic set (INS) [10]. Jun et al., characterized INS and NS to establish neutrosophic cubic set (NCS) [11] which empowers us to consider both interval value and a single value of membership, indeterminacy and falsehood components, hence presenting a more general platform for vague and uncertain data. Smarandache characterized uncertainty in functions of unknown membership function, conflicting membership function, and ignorant membership function to design a five-symbol neutrosophical logic (FSVNL) [12]. Later, the use of the FSVNL concept of a pentapartitioned neutrosophic set (PNS) was introduced by Mallick et al. [13] which further extended into interval valued pentapartitioned neutrosophic set (IVPNS) [14]. The air pollution model was discussed by Khan et al. in [15]. See also [16]. But in the practical scenario of dealing with human insecurity, there are many cases where the values of membership (truth, contradiction, ignorance, unknown and untruth) fall within a certain interval and a single value both the same time. Therefore, to overcome this type of scenario, we are developing PNCS.

Motivation To deal the vague and inconsistent data has always been a challenging task. Many conjectures are presented to deal with this challenge. One of these is a neutrosophic set that deals with the indeterminate part that is not covered in its predecessors. However, the issue remains that indeterminant components can be further classified into parts. The pentapartitioned set classifies indeterminancy further into three parts contradiction, ignorance and unknown. On the other hand, the cubic set has the characteristic to produce the selection in the form of interval and single value. This motivates the author to define the pentapartitioned neutrosophic cubic set which will generalize all its predecessors.

Contribution The key contribution of this paper is the following.

- Introduction of the pentapartitioned neutrosophic cubic set (PNCS).
- Defining operational laws on PNCS.
- Defining score and accuracy function for comparison of two PNC values.
- Defining of pentapartitioned neutrosophic arithmetic (PNCA) aggregation operators.
- Development of PNCA aggregation operator.
- PNCS is solicited to determine the air pollution in major cities of Pakistan.
- The effects of pollution on health and precautionary measures needed are discussed.
- Comparative analysis among these cities is given.

Organization The paper is sorted as follow. “Preliminaries” consists of preliminaries definition and results. “Diagrammatic approach to pentapartitioned neutrosophic set” presents Venn’s diagram interpretation of the pentapartitioned set. “Pentapartitioned neutrosophic cubic sets” presents definitions and operational laws. “Aggregation operator on pentapartitioned” presents pentaneutrosophic cubic arithmetic aggregation operators. “Model formulation of air pollution” presents of air pollution model as an application along with figures.

Preliminaries

This section comprises of some definitions and results which provide the foundation of the work.

Definition 2.1 [7] A NS $N = \{(T_N(u), I_N(u), F_N(u))|u \in U\}$ consist of $T_N(u)$ is truth, $I_N(u)$ is indeterminancy and $F_N(u)$ is falsity function. Where $\{T_N(u), I_N(u), F_N(u) \in [0^-, 1^+]\}$.

Definition 2.2 [9] A SVNS $N = \{(T_N(u), I_N(u), F_N(u))|u \in U\}$ consist of $T_N(u)$ is truth $I_N(u)$ is indeterminancy and $F_N(u)$ is falsity function. Where $\{T_N(u), I_N(u), F_N(u) \in [0, 1]\}[0, 1]$ and $0 \le T_N + I_N + F_N \le 3$, simply denoted by $N = (T_N, I_N, F_N)$.

Definition 2.3 [10] An INS $N = \left\{\left(\tilde{T}_N(u), \tilde{I}_N(u), \tilde{F}_N(u)\right)|u \in U\right\}$ consist of $\tilde{T}_N(u)$ is truth, $\tilde{I}_N(u)$ is indeterminancy and is $\tilde{F}_N(u)$ falsity function. Where $\left\{\tilde{T}_N(u), \tilde{I}_N(u), \tilde{F}_N(u) \in D[0, 1]\right\}$ and $[0, 0] \le \tilde{T}_N + \tilde{I}_N + \tilde{F}_N \le [3, 3]$, simply denoted by $N = \left(\tilde{T}_N, \tilde{I}_N, \tilde{F}_N\right) = ([T_N^L, T_N^U], [I_N^L, I_N^U], [F_N^L, F_N^U])$.

Definition 2.4 [11] A structure $N = \left\{\left(u, \tilde{T}_N(u), \tilde{I}_N(u), \tilde{F}_N(u), T_N(u), I_N(u), F_N(u)\right)|u \in U\right\}$ is neutrosophic cubic set (NCS) in U in which

$\left(\tilde{T}_N = [T_N^L, T_N^U], \tilde{I}_N = [I_N^L, I_N^U], \tilde{F}_N = [F_N^L, F_N^U]\right)$ is an interval neutrosophic set and (T_N, I_N, F_N) is neutrosophic set in U, where

$$N = \left(\tilde{T}_N, \tilde{I}_N, \tilde{F}_N, T_N, I_N, F_N\right),\ \tilde{0} \le \tilde{T}_N + \tilde{I}_N + \tilde{F}_N \le \tilde{3} \text{ and } 0 \le T_N + I_N + F_N \le 3.$$

Definition 2.5 [13] A PNS α over U designate each element u in U by a truth membership function $T_\alpha(u)$, a contradiction membership function $C_\alpha(u)$, an ignorance membership function $G_\alpha(u)$, an unknown membership function $U_\alpha(u)$ and a falsity membership function $F_\alpha(u)$ such that for each $u \in U$, $T_\alpha(u)$, $C_\alpha(u)$, $G_\alpha(u)$, $U_\alpha(u)$, $F_\alpha(u) \in [0, 1]$ and $0 \le T_\alpha(u) + C_\alpha(u) + G_\alpha(u) + U_\alpha(u) + F_\alpha(x) \le 5$.

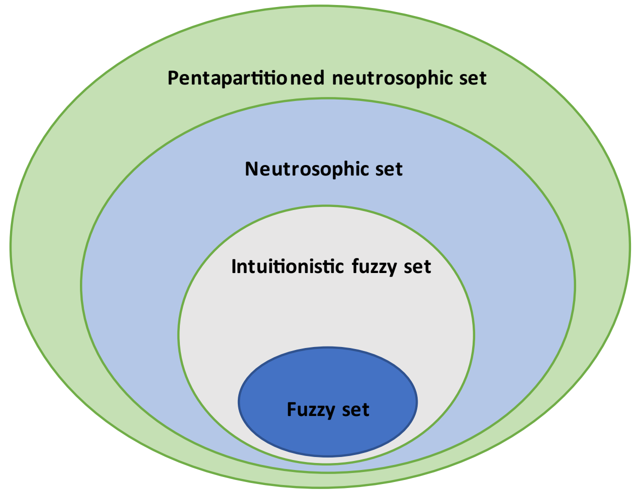
In particular case of the refined neutrosophic set with T is refined in $T1, T2, \ldots I$ is refined into $I1, I2, \ldots F$ is refined into F1, $F2, \ldots$ If one takes unrefined T, unrefined F, and refined "I" into three subcomponents $I1 = C, I2 = G, I3 = U$, into the Refined neutrosophic set, then it is reduced into the pentapartitioned neutrosophic set [9].

Definition 2.6 [14] A IVPNS $\tilde{\alpha}$ on the universe of discourse U is defined as $\tilde{\alpha} = \left\{\left(u, \tilde{T}_\alpha(u), \tilde{C}_\alpha(u), \tilde{G}_\alpha(u), \tilde{U}_\alpha(u), \tilde{F}_\alpha(u)\right) : u \in U\right\}$, where $\tilde{T}_\alpha(u)$, $\tilde{C}_\alpha(u)$, $\tilde{G}_\alpha(u)$, $\tilde{U}_\alpha(u)$, $\tilde{F}_\alpha(u) \in D[0, 1]$ are functions satisfying the condition: $\forall u \in U$, $\tilde{0} \le \sup \tilde{T}_\alpha(u) + \sup \tilde{C}_\alpha(u) + \sup \tilde{G}_\alpha(u) + \sup \tilde{U}_\alpha(u) + \sup \tilde{F}_\alpha(u) \le \tilde{5}$.

Here $\tilde{T}_\alpha(u)$, $\tilde{C}_\alpha(u)$, $\tilde{G}_\alpha(u)$, $\tilde{U}_\alpha(u)$, $\tilde{F}_\alpha(u)$ represent the interval truth membership function, interval contradiction membership function, interval ignorance membership function, interval unknown membership function and interval falsity membership function, respectively.

Diagrammatic approach to pentapartitioned neutrosophic set

FS theory is the generalization of classic set theory. FS is practical plate form to deal uncertain information. IFS is an extension of FS consisting of non-membership function addition to membership. NS consists of three independent components truth, indeterminancy and falsehood. PNS consist of five independent components namely truth membership function, contradiction membership function, ignorance membership function, unknown membership function and falsity membership function.



Pentapartitioned neutrosophic cubic sets

This section comprises of pentapartitioned neutrosophic cubic sets and study its basic properties.

Definition 4.1 A pentapartitioned neutrosophic cubic set (PNCS) is defined as $A = \{(\tilde{\alpha}(u), \alpha(u)) : u \in U\}$ where $\tilde{\alpha} = \left\{\left(u, \tilde{T}_\alpha(u), \tilde{C}_\alpha(u), \tilde{G}_\alpha(u), \tilde{U}_\alpha(u), \tilde{F}_\alpha(u)\right) : u \in U\right\}$ and $\alpha = \{(u, T_\alpha(u), C_\alpha(u), G_\alpha(u), U_\alpha(u), F_\alpha(u)) : u \in U\}$ are IVPNS and PNS, respectively. $\tilde{T}_\alpha(u)$, $\tilde{C}_\alpha(u)$, $\tilde{G}_\alpha(u)$, $\tilde{U}_\alpha(u)$, $\tilde{F}_\alpha(u) \in D[0, 1]$ and $T_\alpha(u)$, $C_\alpha(u)$, $G_\alpha(u)$, $U_\alpha(u)$, $F_\alpha(u) \in [0, 1]$ are functions satisfying the conditions: $\forall u \in U$, $\tilde{0} \le \sup \tilde{T}_\alpha(u) + \sup \tilde{C}_\alpha(u) + \sup \tilde{G}_\alpha(u) + \sup \tilde{U}_\alpha(u) + \sup \tilde{F}_\alpha(u) \le \tilde{5}$ and $0 \le T_\alpha(u) + C_\alpha(u) + G_\alpha(u) + U_\alpha(u) + F_\alpha(x) \le 5$. Here, $\tilde{T}_\alpha(u)$, $\tilde{C}_\alpha(u)$, $\tilde{G}_\alpha(u)$, $\tilde{U}_\alpha(u)$, $\tilde{F}_\alpha(u)$ represent interval truth membership function, an interval contradiction membership function, an interval ignorance membership function, an interval unknown membership function and an interval falsity and $T_\alpha(u)$, $C_\alpha(u)$, $G_\alpha(u)$, $U_\alpha(u)$, $F_\alpha(u)$ represent the truth membership function, contradiction membership function, ignorance membership function, unknown membership function and falsity membership function, respectively.

Definition 4.2 A PNCS is categorized as:

Truth-internal (briefly, T-internal) if $(\forall u \in U)\left(T_\alpha^L(u) \le T_\alpha(u) \le T_\alpha^U(u)\right)$.

Contradiction-internal (briefly, C-internal) if $(\forall u \in U)$ $\left(C_\alpha^L(u) \le C_\alpha(u) \le C_\alpha^U(u)\right)$.

Ignorance-internal (briefly, G-internal) if $(\forall u \in U)$ $\left(G_\alpha^L(u) \le G_\alpha(u) \le G_\alpha^U(u)\right)$.

Unknown-internal (briefly, G-internal) if $(\forall u \in U)$ $\left(U_\alpha^L(u) \le U_\alpha(u) \le U_\alpha^U(u)\right)$.

Falsity-internal (briefly, F-internal) if $(\forall u \in U)$ $\left(F_\alpha^L(u) \le F_\alpha(u) \le F_\alpha^U(u)\right)$.

Example 4.3 $\alpha = (([0.2, 0.8], [0.3, 0.7], [0.3, 0.6], [0.6, 0.9], [0.1, 0.4]), (0.4, 0.5, 0.4, 0.8, 0.3))$ is internal PNCS.

Definition 4.4 A PNCS is categorized as:

Truth-external (briefly, T-external) if $(\forall u \in U)$ $\left(T_\alpha(u) \notin \left(T_\alpha^L(u), T_\alpha^U(u)\right)\right)$.

Contradiction-external (briefly, C-external) if $(\forall u \in U)$ $\left(C_\alpha(u) \notin \left(C_\alpha^L(u), C_\alpha^U(u)\right)\right)$.

Ignorance-external (briefly, G-external) if $(\forall u \in U)$ $\left(G_\alpha(u) \notin \left(G_\alpha^L(u), G_\alpha^U(u)\right)\right)$.

Unknown-external (briefly, U-external) if $(\forall u \in U)\left(U_\alpha(u) \notin \left(U_\alpha^L(u), U_\alpha^U(u)\right)\right)$.

Falsity-external (briefly, F-external) if $(\forall u \in U)$ $\left(F_\alpha(u) \notin \left(F_\alpha^L(u), F_\alpha^U(u)\right)\right)$.

Example 4.5 $A = (([0.2, 0.4], [0.5, 0.7], [0.3, 0.5], [0.5, 0.7], [0.2, 0.4]), (0.8, 0.9, 0.6, 0.8, 0.1))$ is external PNCS.

Definition 4.6 Let $A = (\tilde{\alpha}, \alpha)$ and $B = \left(\tilde{\beta}, \beta\right)$ be two PNCS then.

$$A = B \Leftrightarrow \tilde{\alpha} = \tilde{\beta} \text{ and } \alpha = \beta \quad \text{(equality)}$$

$$A \subseteq_P B \Leftrightarrow \tilde{\alpha} \subseteq \tilde{\beta} \text{ and } \alpha \le \beta \quad \text{(P - order)}$$

$$A \subseteq_R B \Leftrightarrow \tilde{\alpha} \subseteq \tilde{\beta} \text{ and } \alpha \ge \beta \quad \text{(R - order)}$$

Definition 4.7 Let $A_i = (\tilde{\alpha}_i, \alpha_i)$ be PNCS then P-union, P-intersection, R-union, and R-intersection are defined as

$$\underset{i\in j}{\cup_P} A_i = \left(\underset{i\in j}{\cup} \tilde{\alpha}_i, \underset{i\in j}{\vee} \alpha_i\right) \quad \text{(P - union)}$$

$$\underset{i\in j}{\cap_P} A_i = \left(\underset{i\in j}{\cap} \tilde{\alpha}_i, \underset{i\in j}{\wedge} \alpha_i\right) \quad \text{(P - intersection)}$$

$$\underset{i\in j}{\cup_R} A_i = \left(\underset{i\in j}{\cup} \tilde{\alpha}_i, \underset{i\in j}{\wedge} \alpha_i\right) \quad \text{(R - union)}$$

$$\underset{i\in j}{\cap_R} A_i = \left(\underset{i\in j}{\cap} \tilde{\alpha}_i, \underset{i\in j}{\vee} \alpha_i\right) \quad \text{(R - intersection)}$$

Note that the union and intersection of pentapartitioned neutrosophic set and interval valued pentapartitioned neutrosophic set are as defined in preliminaries.

Operations on pentapartitioned neutrosophic cubic sets

This subsection comprise of some basic operations and results defined over PNCS.

Definition 4.1.1 Let $A = \{(\tilde{\alpha}(u), \alpha(u)) : u \in U\}$ and $B = \left\{\left(\tilde{\beta}(u), \beta(u)\right) : u \in U\right\}$ where $\tilde{\alpha} = \left\{\left(u, \tilde{T}_\alpha(u), \tilde{C}_\alpha(u), \tilde{G}_\alpha(u), \tilde{U}_\alpha(u), \tilde{F}_\alpha(u)\right) : u \in U\right\}$ $\tilde{\beta} = \left\{\left(u, \tilde{T}_\beta(u), \tilde{C}_\beta(u), \tilde{G}_\beta(u), \tilde{U}_\beta(u), \tilde{F}_\beta(u)\right) : u \in U\right\}$ are IVPNS and $\alpha = \{(u, T_\alpha(u), C_\alpha(u), G_\alpha(u), U_\alpha(u), F_\alpha(u)) : u \in \boldsymbol{U}\}$,$\beta = \left\{\left(u, T_\beta(u), C_\beta(u), G_\beta(u), U_\beta(u), F_\beta(u)\right) : u \in U\right\}$ be PNS. Where $\tilde{T}_\alpha(u) = \left[T_\alpha^L(u), T_\alpha^U(u)\right]$, $\tilde{C}_\alpha(u) = \left[C_\alpha^L(u), C_\alpha^U(u)\right]$, $\tilde{G}_\alpha(u) = \left[G_\alpha^L(u), G_\alpha^U(u)\right]$, $\tilde{U}_\alpha(u) = \left[U_\alpha^L(u), U_\alpha^U(u)\right]$, $\tilde{F}_\alpha(u) = \left[F_\alpha^L(u), F_\alpha^U(u)\right]$ and $\tilde{T}_\beta(u) = \left[T_\beta^L(u), T_\beta^U(u)\right]$, $\tilde{C}_\beta(u) = \left[C_\beta^L(u), C_\beta^U(u)\right]$, $\tilde{G}_\beta(u) = \left[G_\beta^L(u), G_\beta^U(u)\right]$, $\tilde{U}_\beta(u) = \left[U_\beta^L(u), U_\beta^U(u)\right]$, $\tilde{F}_\beta(u) = \left[F_\beta^L(u), F_\beta^U(u)\right]$.

- A is contained in B if and only if

$$T_\alpha^L(u) \le T_\beta^L(u),\ T_\alpha^U(u) \le T_\beta^U(u)$$

,

$$C_\alpha^L(u) \le C_\beta^L(u),\ C_\alpha^U(u) \le C_\beta^U(u),$$

$$G_\alpha^L(u) \ge G_\beta^L(u),\ G_\alpha^U(u) \ge G_\beta^U(u),$$

$$U_\alpha^L(u) \ge U_\beta^L(u),\ U_\alpha^U(u) \ge U_\beta^U(u),$$

$$F_\alpha^L(u) \ge F_\beta^L(u),\ F_\alpha^U(u) \ge F_\beta^U(u),$$

$$T_\alpha(u) \ge T_\beta(u),\ C_\alpha(u) \ge C_\beta(u),$$

$$G_\alpha(u) \le G_\beta(u),\ U_\alpha(u) \le U_\beta(u),\ F_\alpha(u) \le F_\beta(u),$$

- The union of A and B is a PNCS, defined by

$$A \cup B = \left\{ u, \begin{pmatrix} \left[\max\left\{T_\alpha^L(u),\, T_\beta^L(u)\right\},\, \max\left\{T_\alpha^U(u),\, T_\beta^U(u)\right\}\right], \\ \left[\max\left\{C_\alpha^L(u),\, C_\beta^L(u)\right\},\, \max\left\{C_\alpha^U(u),\, C_\beta^U(u)\right\}\right], \\ \left[\min\left\{G_\alpha^L(u),\, G_\beta^L(u)\right\},\, \min\left\{G_\alpha^U(u),\, G_\beta^U(u)\right\}\right], \\ \left[\min\left\{U_\alpha^L(u),\, U_\beta^L(u)\right\},\, \min\left\{U_\alpha^U(u),\, U_\beta^U(u)\right\}\right], \\ \left[\min\left\{F_\alpha^L(u),\, F_\beta^L(u)\right\},\, \min\left\{F_\alpha^U(u),\, F_\beta^U(u)\right\}\right], \\ \min\left\{T_\alpha(u),\, T_\beta(u)\right\},\, \min\left\{C_\alpha(u),\, C_\beta(u)\right\}, \\ \max\left\{G_\alpha(u),\, G_\beta(u)\right\},\, \max\left\{U_\alpha(u),\, U_\beta(u)\right\},\, \max\left\{F_\alpha(u),\, F_\beta(u)\right\} \end{pmatrix} : u \in U \right\}$$

- The intersection of A and B is a PNCS, defined by

$$A \cap B = \left\{ u, \begin{pmatrix} \left[\min\left\{T_\alpha^L(u),\, T_\beta^L(u)\right\},\, \min\left\{T_\alpha^U(u),\, T_\beta^U(u)\right\}\right], \\ \left[\min\left\{C_\alpha^L(u),\, C_\beta^L(u)\right\},\, \min\left\{C_\alpha^U(u),\, C_\beta^U(u)\right\}\right], \\ \left[\max\left\{G_\alpha^L(u),\, G_\beta^L(u)\right\},\, \max\left\{G_\alpha^U(u),\, G_\beta^U(u)\right\}\right], \\ \left[\max\left\{U_\alpha^L(u),\, U_\beta^L(u)\right\},\, \max\left\{U_\alpha^U(u),\, U_\beta^U(u)\right\}\right], \\ \left[\max\left\{F_\alpha^L(u),\, F_\beta^L(u)\right\},\, \max\left\{F_\alpha^U(u),\, F_\beta^U(u)\right\}\right], \\ \max\left\{T_\alpha(u),\, T_\beta(u)\right\},\, \max\left\{C_\alpha(u),\, C_\beta(u)\right\}, \\ \min\left\{G_\alpha(u),\, G_\beta(u)\right\},\, \min\left\{U_\alpha(u),\, U_\beta(u)\right\},\, \min\left\{F_\alpha(u),\, F_\beta(u)\right\} \end{pmatrix} : u \in U \right\}$$

- The complement of A is a PNCS A^c, defined by $\alpha^c = \left\{\left(u,\, \tilde{T}_{\alpha^c}(u),\, \tilde{C}_{\alpha^c}(u),\, \tilde{G}_{\alpha^c}(u),\, \tilde{U}_{\alpha^c}(u),\, \tilde{F}_{\alpha^c}(u),\, T_{\alpha^c}(u),\, C_{\alpha^c}(u),\, G_{\alpha^c}(u),\, U_{\alpha^c}(u),\, F_{\alpha^c}(u)\right) : u \in U\right\}$ where $\tilde{T}_{\alpha^c}(u) = \tilde{F}_\alpha(u),\, \tilde{C}_{\alpha^c}(u) = \tilde{U}_\alpha(u)$,

$$G_{\alpha^c}^L(u) = 1 - G_\alpha^U(u),\;\; G_{\alpha^c}^U(u) = 1 - G_\alpha^L(u),$$

$$\tilde{U}_{\alpha^c}(u) = \tilde{C}_\alpha(u),\;\; \tilde{F}_{\alpha^c}(u) = \tilde{T}_\alpha(u),$$

$$T_{\alpha^c}(u) = F_\alpha(u),\;\; C_{\alpha^c}(u) = U_\alpha(u),$$

$$G_{\alpha^c}(u) = 1 - G_\alpha(u),\;\; U_{\alpha^c}(u) = C_\alpha(u),\;\; F_{\alpha^c}(u) = T_\alpha(u),$$

Example 4.1.2 Consider two IVPNSs defined over W as

$$A = \left\{ \begin{array}{c} (u_1,\, ([0.2,\, 0.4],\, [0.3,\, 0.5],\, [0.6,\, 0.9],\, [0.2,\, 0.3], \\ [0.5,\, 0.6],\, 0.4,\, 0.5,\, 0.2,\, 0.6,\, 0.7)), \\ (u_2,\, ([0.7,\, 0.8],\, [0.5,\, 0.8],\, [0.1,\, 0.2],\, [0.3,\, 0.4], \\ [0.1,\, 0.3],\, 0.7,\, 0.2,\, 0.9,\, 0.3,\, 0.5)) \end{array} \right\}$$

$$B = \left\{ \begin{array}{c} (u_1,\, ([0.4,\, 0.6],\, [0.1,\, 0.2],\, [0.5,\, 0.7],\, [0.2,\, 0.4], \\ [0.3,\, 0.6],\, 0.3,\, 0.7,\, 0.8,\, 0.2,\, 0.1)) \\ (u_2,\, ([0.1,\, 0.3],\, [0.4,\, 0.5],\, [0.5,\, 0.7],\, [0.4,\, 0.6], \\ [0.2,\, 0.3],\, 0.2,\, 0.6,\, 0.4,\, 0.9,\, 0.5)) \end{array} \right\}$$

then

$$A \cup B = \left\{ \begin{array}{c} (u_1,\, ([0.4,\, 0.6],\, [0.3,\, 0.8],\, [0.5,\, 0.7],\, [0.2,\, 0.3], \\ [0.3,\, 0.6],\, 0.3,\, 0.5,\, 0.8,\, 0.6,\, 0.7)), \\ (u_2,\, ([0.7,\, 0.8],\, [0.5,\, 0.8],\, [0.1,\, 0.2],\, [0.3,\, 0.4], \\ [0.1,\, 0.3],\, 0.2,\, 0.2,\, 0.9,\, 0.9,\, 0.5)) \end{array} \right\}$$

$$A \cap B = \left\{ \begin{array}{c} (u_1,\, ([0.2,\, 0.4],\, [0.1,\, 0.2],\, [0.6,\, 0.9],\, [0.2,\, 0.4], \\ [0.5,\, 0.6],\, 0.4,\, 0.7,\, 0.2,\, 0.2,\, 0.1)), \\ (u_2,\, ([0.1,\, 0.3],\, [0.4,\, 0.5],\, [0.5,\, 0.7],\, [0.4,\, 0.6], \\ [0.2,\, 0.3],\, 0.7,\, 0.6,\, 0.4,\, 0.3,\, 0.5)) \end{array} \right\}$$

$$A^C = \left\{ \begin{array}{c} (u_1,\, ([0.5,\, 0.6],\, [0.2,\, 0.3],\, [0.7,\, 0.9],\, [0.2,\, 0.3], \\ [0.2,\, 0.4],\, 0.7,\, 0.6,\, 0.8,\, 0.5,\, 0.4)), \\ (u_2,\, ([0.1,\, 0.3],\, [0.3,\, 0.4],\, [0.8,\, 0.9],\, [0.5,\, 0.8], \\ [0.1,\, 0.3],\, 0.5,\, 0.3,\, 0.1,\, 0.2,\, 0.7)) \end{array} \right\}$$

Theorem 4.1.3 *For any three IVPNSs* α, β*and* γ.

$$\left.\begin{array}{l}\alpha\cup\alpha=\alpha\\ \alpha\cap\alpha=\alpha\end{array}\right\}\text{ (Idempotent Law)}$$

$$\left.\begin{array}{l}\alpha\cup\beta=\beta\cup\alpha\\ \alpha\cap\beta=\beta\cap\alpha\end{array}\right\}\text{ (Commutative Law)}$$

$$\left.\begin{array}{l}(\alpha\cup\beta)\cup\gamma=\alpha\cup(\beta\cup\gamma)\\ (\alpha\cap\beta)\cap\gamma=\alpha\cap(\beta\cap\gamma)\end{array}\right\}\text{ (Associative Law)}$$

$$\left.\begin{array}{l}\alpha\cup(\beta\cap\gamma)=(\alpha\cup\beta)\cap(\alpha\cup\gamma)\\ \alpha\cap(\beta\cup\gamma)=(\alpha\cap\beta)\cup(\alpha\cap\gamma)\end{array}\right\}\text{ (Distributive Law)}$$

$$\left.\begin{array}{l}(\alpha\cup\beta)^c=\alpha^c\cap\beta^c\\ (\alpha\cap\beta)^c=\alpha^c\cup\beta^c\end{array}\right\}\text{ (De Morgan's Law)}$$

$$\left.\begin{array}{l}\alpha\cup(\alpha\cap\beta)=\alpha\\ \alpha\cap(\alpha\cup\beta)=\alpha\end{array}\right\}\text{ (Absorption Law)}$$

$$\left(\alpha^c\right)^c=\alpha\quad\text{(Involution Law)}$$

Proof Proofs are straight forward. □

The algebraic addition, multiplication, and scalar multiplication on PNCS are introduced.

Definition 4.1.4 Let $A=\{(\tilde{\alpha}(u),\alpha(u)):u\in U\}$ and $B=\left\{\left(\tilde{\beta}(u),\beta(u)\right):u\in U\right\}$ where $\tilde{\alpha}=\left\{\left(u,\tilde{T}_\alpha(u),\tilde{C}_\alpha(u),\tilde{G}_\alpha(u),\tilde{U}_\alpha(u),\tilde{F}_\alpha(u)\right):u\in U\right\}$ $\tilde{\beta}=\left\{\left(u,\tilde{T}_\beta(u),\tilde{C}_\beta(u),\tilde{G}_\beta(u),\tilde{U}_\beta(u),\tilde{F}_\beta(u)\right):u\in U\right\}$ are IVPNS and $\alpha=\left\{(u,T_\alpha(u),C_\alpha(u),G_\alpha(u),U_\alpha(u),F_\alpha(u)):u\in U\right\}$ $\beta=\left\{\left(u,T_\beta(u),C_\beta(u),G_\beta(u),U_\beta(u),F_\beta(u)\right):u\in U\right\}$ be PNS. Where $\tilde{T}_\alpha(u)=\left[T_\alpha^L(u),T_\alpha^U(u)\right]$, $\tilde{C}_\alpha(u)=\left[C_\alpha^L(u),C_\alpha^U(u)\right]$, $\tilde{G}_\alpha(u)=\left[G_\alpha^L(u),G_\alpha^U(u)\right]$, $\tilde{U}_\alpha(u)=\left[U_\alpha^L(u),U_\alpha^U(u)\right]$, $\tilde{F}_\alpha(u)=\left[F_\alpha^L(u),F_\alpha^U(u)\right]$ and $\tilde{T}_\beta(u)=\left[T_\beta^L(u),T_\beta^U(u)\right]$, $\tilde{C}_\beta(u)=\left[C_\beta^L(u),C_\beta^U(u)\right]$, $\tilde{G}_\beta(u)=\left[G_\beta^L(u),G_\beta^U(u)\right]$, $\tilde{U}_\beta(u)=\left[U_\beta^L(u),U_\beta^U(u)\right]$, $\tilde{F}_\beta(u)=\left[F_\beta^L(u),F_\beta^U(u)\right]$ be two PNCS then,

- The sum of A and B is a PNCS, defined by

$$A\oplus B=\left(\begin{array}{c}\left[T_A^L(u)+T_B^L(u)-T_A^L(u)T_B^L(u),\ T_A^U(u)+T_B^U(u)-T_A^U(u)T_B^U(u)\right],\\ \left[C_A^L(u)+C_B^L(u)-C_A^L(u)C_B^L(u),\ C_A^U(u)+C_B^U(u)-C_A^U(u)C_B^U(u)\right],\\ \left[G_A^L(u)G_B^L(u),\ G_A^U(u)G_B^U(u)\right],\left[U_A^L(u)U_B^L(u),\ U_A^U(u)U_B^U(u)\right],\left[F_A^L(u)F_B^L(u),\ F_A^U(u)F_B^U(u)\right],\\ T_A(u)T_B(u),\ C_A(u)C_B(u),\ G_A(u)+G_B(u)-G_A(u)G_B(u),\ U_A(u)+U_B(u)-U_A(u)U_B(u),\\ F_A(u)+F(u)_B-F_A(u)F(u)_B\end{array}\right)$$

- The sum of A and B is a PNCS, defined by

$$A\otimes B=\left(\begin{array}{c}\left[T_A^L(u)T_B^L(u),\ T_A^U(u)T_B^U(u)\right],\left[C_A^L(u)C_B^L(u),\ C_A^U(u)C_B^U(u)\right],\\ \left[G_A^L(u)+G_B^L(u)-G_A^L(u)G_B^L(u),\ G_A^U(u)G_B^U(u)\right],\left[U_A^L(u)+U_B^L(u)-U_A^L(u)U_B^L(u),\ U_A^U(u)U_B^U(u)\right],\\ \left[F_A^L(u)+F_B^L(u)-F_A^L(u)F_B^L(u),\ F_A^U(u)+F_B^U(u)-F_A^U(u)F_B^U(u)\right],\\ T_A(u)+T_B(u)-T_A(u)T_B(u),\ C_A(u)+C_B(u)-C_A(u)C_B(u),\\ G_A(u)G_B(u),\ U_A(u)U_B(u),\ F_A(u)F(u)_B\end{array}\right)$$

- Scalar multiplication for scalar k is defined as

$$kA = \begin{pmatrix} \left[1-\left(1-T_A^L(u)\right)^k,\ 1-\left(1-T_A^U(u)\right)^k\right],\ \left[1-\left(1-C_A^L(u)\right)^k,\ 1-\left(1-C_A^U(u)\right)^k\right], \\ \left[\left(G_A^L(u)\right)^k,\ \left(G_A^U(u)\right)^k\right],\ \left[\left(U_A^L(u)\right)^k,\ \left(U_A^U(u)\right)^k\right],\ \left[\left(F_A^L(u)\right)^k,\ \left(F_A^U(u)\right)^k\right], \\ (T_A(u))^k,\ (C_A(u))^k,\ 1-(1-G_A(u))^k,\ 1-(1-U_A(u))^k,\ 1-(1-F_A(u))^k \end{pmatrix}$$

Aggregation operator on pentaapartitioned neutrosophic cubic set

To define aggregation operators on pentapartitioned neutrosophic cubic set, some basic definition like score accuracy are required to compare PNC values.

Definition 5.1 Let $A = \{(\tilde{\alpha}(u), \alpha(u)) : u \in U\}$ where $\tilde{\alpha} = \left\{\left(u, \tilde{T}_\alpha(u), \tilde{C}_\alpha(u), \tilde{G}_\alpha(u), \tilde{U}_\alpha(u), \tilde{F}_\alpha(u)\right) : u \in U\right\}$ is IVPNS and $\alpha = \{(u, T_\alpha(u), C_\alpha(u), G_\alpha(u), U_\alpha(u), F_\alpha(u)) : u \in U\}$ be PNS. Where $\tilde{T}_\alpha(u) = \left[T_\alpha^L(u), T_\alpha^U(u)\right]$, $\widetilde{C}_\alpha(u) = \left[C_\alpha^L(u), C_\alpha^U(u)\right]$, $\widetilde{G}_\alpha(u) = \left[G_\alpha^L(u), G_\alpha^U(u)\right]$, $\widetilde{U}_\alpha(u) = \left[U_\alpha^L(u), U_\alpha^U(u)\right]$, $\tilde{F}_\alpha(u) = \left[F_\alpha^L(u), F_\alpha^U(u)\right]$ be a PNC value, we define the score function as

$$S(A) = \left[T_A^L(u) - F_A^L(u) + T_A^U(u) - F_A^U(u) + T_A(u) - F_A(u)\right].$$

Definition 5.1 Sometimes, the situation arises that the score of two neutrosophic cubic values are equal. In such a situation, a comparison is made on the basis of an accuracy function.

Definition 5.2 Let $A = \{(\tilde{\alpha}(u), \alpha(u)) : u \in U\}$ where $\tilde{\alpha} = \left\{\left(u, \tilde{T}_\alpha(u), \tilde{C}_\alpha(u), \tilde{G}_\alpha(u), \tilde{U}_\alpha(u), \tilde{F}_\alpha(u)\right) : u \in U\right\}$ is IVPNS and $\alpha = \{(u, T_\alpha(u), C_\alpha(u), G_\alpha(u), U_\alpha(u), F_\alpha(u)) : u \in U\}$ be PNS. Where $\tilde{T}_\alpha(u) = \left[T_\alpha^L(u), T_\alpha^U(u)\right]$, $\widetilde{C}_\alpha(u) = \left[C_\alpha^L(u), C_\alpha^U(u)\right]$, $\widetilde{G}_\alpha(u) = \left[G_\alpha^L(u), G_\alpha^U(u)\right]$, $\widetilde{U}_\alpha(u) = \left[U_\alpha^L(u), U_\alpha^U(u)\right]$, $\tilde{F}_\alpha(u) = \left[F_\alpha^L(u), F_\alpha^U(u)\right]$, the accuracy function is defined as

$$H(u) = \frac{1}{15}\begin{Bmatrix} T_N^L(u) + C_N^L(u) + G_N^L(u) + U_N^L(u) + F_N^L(u) + T_N^U(u) + C_N^U(u) + G_N^U(u) + U_N^U(u) + F_N^U(u) \\ +T_N(u) + C_N(u) + G_N(u) + U_N(u) + F_N(u) \end{Bmatrix}$$

The following definition is accomplished for the comparison relation of the neutrosophic cubic values.

Definition 5.3 Let A_1 and A_2 be two neutrosophic cubic values, where S_{N_1} S_{A_1} and S_{A_2}, are scores and H_{A_1} and H_{A_2} are accuracy functions of A_1 and A_2, respectively, if

$$S_{A_1} > S_{A_2} \Rightarrow\ A_1 > A_2$$

$$S_{A_1} = S_{A_2} \text{ and } H_{A_1} > H_{A_2} \Rightarrow\ A_1 > A_2$$

$$\text{If } H_{A_1} = H_{A_2} \Rightarrow\ A_1 = A_2$$

Neutrosophic cubic weighted averaging aggregation operator

This section consists of some fundamental definitions of pentapartitioned neutrosophic cubic weighted averaging (PNCWA), pentapartitioned neutrosophic cubic ordered weighted averaging (PNCOWA) and pentapartitioned neutrosophic cubic Einstein hybrid averaging (PNCEHA) aggregation operator, which are defined as follows.

Definition 5.1.1 The neutrosophic cubic weighted averaging is a function, $PNCWA : R^n \to R$ defined by $\text{NCWA}_w(A_1, A_2, \ldots, A_n) = \sum_{k=1}^{n} w_k A_k$, where $W = (w_1, w_2, \ldots, w_n)^T$ of $A_k (k = 1, 2, 3, \ldots, n)$, be the weight such that $w_k \in [0, 1]$ and $\sum_{k=1}^{n} w_k = 1$.

Note that in PNCWA, the PNC values are weighted first and then aggregated.

Definition 5.1.2 The pentapartitioned neutrosophic cubic ordered weighted averaging is a function, PNCOWA: $R^n \to R$ defined by $\text{PNCOWA}_w(A_1, A_2, \ldots, A_n) = \sum_{k=1}^{n} w_k S_k$, where S_k denotes the ordered position of PNC values whereby the PNC values are ordered in descending order, $W = (w_1, w_2, \ldots, w_n)^T$ of $A_k(k = 1, 2, 3, \ldots, n)$, be the weight such that $w_k \in [0, 1]$ and $\sum_{k=1}^{n} w_k = 1$.

Note that in PNCOWA, the PNC values are first ordered and then aggregated. The basic concept of PNCOWA is to rearrange the PNC values in descending order and then aggregate them.

Theorem 5.1.3 *Let* $A_k = \{(\tilde{\alpha}_k(u), \alpha_k(u)) : u \in U\}(k = 1, 2, \ldots, \eta)$ *where* $\tilde{\alpha}_k = \left\{\left(u, \tilde{T}_{\alpha_k}(u), \tilde{C}_{\alpha_k}(u), \tilde{G}_{\alpha_k}(u), \tilde{U}_{\alpha_k}(u), \tilde{F}_{\alpha_k}(u)\right) : u \in U\right\}$ *is IVPNS and* $\alpha_k = \{(u, T_{\alpha_k}(u), C_{\alpha_k}(u), G_{\alpha_k}(u), U_{\alpha_k}(u), F_{\alpha_k}(u)) : u \in U\}$ *be PNS. Where* $\tilde{T}_{\alpha_k} = [T^L_{\alpha_k}, T^U_{\alpha_k}]$, $\tilde{C}_{\alpha_k} = [C^L_{\alpha_k}, C^U_{\alpha_k}]$, $\tilde{G}_{\alpha_k} = [G^L_{\alpha_k}, G^U_{\alpha_k}]$, $\tilde{U}_{\alpha_k} = [U^L_{\alpha_k}, U^U_{\alpha_k}]$, $\tilde{F}_{\alpha_k} = [F^L_{\alpha_k}, F^U_{\alpha_k}]$ $(k = 1, 2, \ldots, \eta)$ *be a collection of PNC values, then the pentapartitioned neutrosophic cubic weighted average operator (PNCWA) operator of* A_k *is also a neutrosophic cubic value and*

$$\text{PNCWA}(A_k) = \begin{pmatrix} \left[1 - \prod_{k=1}^{\eta}(1 - T^L_{A_k})^{w_k}, 1 - \prod_{k=1}^{\eta}(1 - T^U_{A_k})^{w_k}\right], \left[1 - \prod_{k=1}^{\eta}(1 - C^L_{A_k})^{w_k}, 1 - \prod_{k=1}^{\eta}(1 - C^U_{A_k})^{w_k}\right], \\ \left[\prod_{k=1}^{\eta}\left(G^L_{A_k}\right)^{w_k}, \prod_{k=1}^{\eta}\left(G^U_{A_k}\right)^{w_k}\right], \left[\prod_{k=1}^{\eta}\left(U^L_{A_k}\right)^{w_k}, \prod_{k=1}^{\eta}\left(U^U_{A_k}\right)^{w_k}\right], \left[\prod_{k=1}^{\eta}\left(F^L_{A_k}\right)^{w_k}, \prod_{k=1}^{\eta}\left(F^U_{A_k}\right)^{w_k}\right], \\ \prod_{k=1}^{\eta}\left(T_{A_k}\right)^{w_k}, \prod_{k=1}^{\eta}\left(I_{A_k}\right)^{w_k}, 1 - \prod_{k=1}^{\eta}\left(1 - \left(F_{A_k}\right)\right)^{w_k} \end{pmatrix}$$

where $W = (w_1, w_2, \ldots, w_\eta)^T$ *of* $N_k(k = 1, 2, 3, \ldots, \eta)$, *be the weight such that* $w_k \in [0, 1]$ *and* $\sum_{k=1}^{\eta} w_k = 1$.

Proof By mathematical induction for $\eta = 2$,

$$w_1 A_1 \oplus w_2 A_2 = \begin{pmatrix} \left[1 - \prod_{k=1}^{2}(1 - T^L_{A_k})^{w_k}, 1 - \prod_{k=1}^{2}(1 - T^U_{A_k})^{w_k}\right], \left[1 - \prod_{k=1}^{2}(1 - C^L_{A_k})^{w_k}, 1 - \prod_{k=1}^{2}(1 - C^U_{A_k})^{w_k}\right], \\ \left[\prod_{k=1}^{2}\left(G^L_{N_k}\right)^{w_k}, \prod_{k=1}^{2}\left(G^U_{N_k}\right)^{w_k}\right], \left[\prod_{k=1}^{2}\left(U^L_{N_k}\right)^{w_k}, \prod_{k=1}^{2}\left(U^U_{N_k}\right)^{w_k}\right], \left[\prod_{k=1}^{2}\left(F^L_{N_k}\right)^{w_k}, \prod_{k=1}^{2}\left(F^U_{N_k}\right)^{w_k}\right], \\ \prod_{k=1}^{2}\left(T_{N_k}\right)^{w_k}, \prod_{k=1}^{2}\left(C_{N_k}\right)^{w_k}, 1 - \prod_{k=1}^{2}\left(1 - \left(G_{N_k}\right)\right)^{w_k}, 1 - \prod_{k=1}^{2}\left(1 - \left(U_{N_k}\right)\right)^{w_k}, 1 - \prod_{k=1}^{2}\left(1 - \left(F_{N_k}\right)\right)^{w_k} \end{pmatrix}$$

Assume that, the result holds for $\eta = m$. That is

$$\sum_{k=1}^{m} w_k A_k = \begin{pmatrix} \left[1 - \prod_{k=1}^{m}(1 - T^L_{A_k})^{w_k}, 1 - \prod_{k=1}^{m}(1 - T^U_{A_k})^{w_k}\right], \left[1 - \prod_{k=1}^{m}(1 - C^L_{A_k})^{w_k}, 1 - \prod_{k=1}^{m}(1 - C^U_{A_k})^{w_k}\right], \\ \left[\prod_{k=1}^{m}\left(G^L_{N_k}\right)^{w_k}, \prod_{k=1}^{m}\left(G^U_{N_k}\right)^{w_k}\right], \left[\prod_{k=1}^{m}\left(U^L_{N_k}\right)^{w_k}, \prod_{k=1}^{m}\left(U^U_{N_k}\right)^{w_k}\right], \left[\prod_{k=1}^{m}\left(F^L_{N_k}\right)^{w_k}, \prod_{k=1}^{m}\left(F^U_{N_k}\right)^{w_k}\right], \\ \prod_{k=1}^{m}\left(T_{N_k}\right)^{w_k}, \prod_{k=1}^{m}\left(C_{N_k}\right)^{w_k}, 1 - \prod_{k=1}^{m}\left(1 - \left(G_{N_k}\right)\right)^{w_k}, 1 - \prod_{k=1}^{m}\left(1 - \left(U_{N_k}\right)\right)^{w_k}, 1 - \prod_{k=1}^{m}\left(1 - \left(F_{N_k}\right)\right)^{w_k} \end{pmatrix}$$

Consider $\eta = m + 1$, the following result will be proven.

$$\sum_{k=1}^{m} w_k A_k \oplus w_{k+1} A_{k+1} = \left\{ \begin{pmatrix} \left[1 - \prod_{k=1}^{m}(1 - T_{A_k}^{L})^{w_k},\ 1 - \prod_{k=1}^{m}(1 - T_{A_k}^{U})^{w_k}\right], \left[1 - \prod_{k=1}^{m}(1 - C_{A_k}^{L})^{w_k},\ 1 - \prod_{k=1}^{m}(1 - C_{A_k}^{U})^{w_k}\right], \\ \left[\prod_{k=1}^{m}\left(G_{N_k}^{L}\right)^{w_k}, \prod_{k=1}^{m}\left(G_{N_k}^{U}\right)^{w_k}\right], \left[\prod_{k=1}^{m}\left(U_{N_k}^{L}\right)^{w_k}, \prod_{k=1}^{m}\left(U_{N_k}^{U}\right)^{w_k}\right], \left[\prod_{k=1}^{m}\left(F_{N_k}^{L}\right)^{w_k}, \prod_{k=1}^{m}\left(F_{N_k}^{U}\right)^{w_k}\right], \\ \prod_{k=1}^{m}(T_{N_k})^{w_k}, \prod_{k=1}^{m}(C_{N_k})^{w_k}, 1 - \prod_{k=1}^{m}(1 - (G_{N_k}))^{w_k}, 1 - \prod_{k=1}^{m}(1 - (U_{N_k}))^{w_k} 1 - \prod_{k=1}^{m}(1 - (F_{N_k}))^{w_k} \end{pmatrix} \\ \oplus \begin{pmatrix} \left[1 - (1 - T_{A_k}^{L})^{w_k},\ 1 - (1 - T_{A_k}^{U})^{w_k}\right], \left[1 - (1 - C_{A_k}^{L})^{w_k},\ 1 - (1 - C_{A_k}^{U})^{w_k}\right], \\ \left[\left(G_{N_k}^{L}\right)^{w_k}, \left(G_{N_k}^{U}\right)^{w_k}\right], \left[\left(U_{N_k}^{L}\right)^{w_k}, \left(U_{N_k}^{U}\right)^{w_k}\right], \left[\left(F_{N_k}^{L}\right)^{w_k}, \left(F_{N_k}^{U}\right)^{w_k}\right], \\ \left(T_{N_k}\right)^{w_k}, \left(C_{N_k}\right)^{w_k}, 1 - \left(1 - \left(G_{N_k}\right)\right)^{w_k}, 1 - \left(1 - \left(U_{N_k}\right)\right)^{w_k}, 1 - \left(1 - \left(F_{N_k}\right)\right)^{w_k} \end{pmatrix} \right\}$$

$$= \begin{pmatrix} \left[1 - \prod_{k=1}^{m+1}(1 - T_{A_k}^{L})^{w_k},\ 1 - \prod_{k=1}^{m+1}(1 - T_{A_k}^{U})^{w_k}\right], \left[1 - \prod_{k=1}^{m+1}(1 - C_{A_k}^{L})^{w_k},\ 1 - \prod_{k=1}^{m+1}(1 - C_{A_k}^{U})^{w_k}\right], \\ \left[\prod_{k=1}^{m+1}\left(G_{N_k}^{L}\right)^{w_k}, \prod_{k=1}^{m+1}\left(G_{N_k}^{U}\right)^{w_k}\right], \left[\prod_{k=1}^{m+1}\left(U_{N_k}^{L}\right)^{w_k}, \prod_{k=1}^{m+1}\left(U_{N_k}^{U}\right)^{w_k}\right], \left[\prod_{k=1}^{m+1}\left(F_{N_k}^{L}\right)^{w_k}, \prod_{k=1}^{m+1}\left(F_{N_k}^{U}\right)^{w_k}\right], \\ \prod_{k=1}^{m+1}\left(T_{N_k}\right)^{w_k}, \prod_{k=1}^{m+1}\left(C_{N_k}\right)^{w_k}, 1 - \prod_{k=1}^{m+1}\left(1 - \left(G_{N_k}\right)\right)^{w_k}, 1 - \prod_{k=1}^{m+1}\left(1 - \left(U_{N_k}\right)\right)^{w_k} 1 - \prod_{k=1}^{m+1}\left(1 - \left(F_{N_k}\right)\right)^{w_k} \end{pmatrix}$$

Hence proved. □

Example 5.1.4 The NCWA operator is applied on the data as stated in "Diagrammatic Approach to Pentapartitioned Neutrosophic Set" with corresponding weight, $w = (0.21, 0.14, 0.25, 0.29, 0.11)^T$. This weight calculated by Xu and Yager [16] is an essential part of aggregation operators and will be used throughout this paper.

The value of NCWA = ([0.7871, 0.9171], [0.5311, 0.7292], [0.2912, 0.5075], 0.5216, 0.6071, 0.3995).

Theorem 5.1.5 *Let* $A_k = \{(\tilde{\alpha}_k(u), \alpha_k(u)) : u \in U\}(k = 1, 2, \ldots, \eta)$ *where* $\tilde{\alpha}_k = \left\{\left(u, \tilde{T}_{\alpha_k}(u), \tilde{C}_{\alpha_k}(u), \tilde{G}_{\alpha_k}(u), \tilde{U}_{\alpha_k}(u), \tilde{F}_{\alpha_k}(u)\right) : u \in U\right\}$ *is IVPNS and* $\alpha_k = \left\{\left(u, T_{\alpha_k}(u), C_{\alpha_k}(u), G_{\alpha_k}(u), U_{\alpha_k}(u), F_{\alpha_k}(u)\right) : u \in U\right\}$ *be PNS. Where* $\tilde{T}_{\alpha_k} = \left[T_{\alpha_k}^{L}, T_{\alpha_k}^{U}\right]$, $\widetilde{C}_{\alpha_k} = \left[C_{\alpha_k}^{L}, C_{\alpha_k}^{U}\right]$, $\widetilde{G}_{\alpha_k} = \left[G_{\alpha_k}^{L}, G_{\alpha_k}^{U}\right]$, $\widetilde{U}_{\alpha_k} = \left[U_{\alpha_k}^{L}, U_{\alpha_k}^{U}\right]$, $\tilde{F}_{\alpha_k} = \left[F_{\alpha_k}^{L}, F_{\alpha_k}^{U}\right]$ $(k = 1, 2, \ldots, \eta)$ *be a collection of PNC values, with weight* $W = (w_1, w_2, \ldots, w_\eta)^T$ *of* $A_k(k = 1, 2, 3, \ldots, \eta)$, *such that* $w_k \in [0, 1]$ and $\sum_{k=1}^{\eta} w_k = 1$. *The following properties are true.*

1. *Idempotence: Let* $A_k = \{(\tilde{\alpha}_k(u), \alpha_k(u)) : u \in U\}(k = 1, 2, \ldots, \eta)$ *where* $\tilde{\alpha}_k = \left\{\left(u, \tilde{T}_{\alpha_k}(u), \tilde{C}_{\alpha_k}(u), \tilde{G}_{\alpha_k}(u), \tilde{U}_{\alpha_k}(u), \tilde{F}_{\alpha_k}(u)\right) : u \in U\right\}$ *is IVPNS and* $\alpha_k = \left\{\left(u, T_{\alpha_k}(u), C_{\alpha_k}(u), G_{\alpha_k}(u), U_{\alpha_k}(u), F_{\alpha_k}(u)\right) : u \in U\right\}$ *be PNS. Where* $\tilde{T}_{\alpha_k} = \left[T_{\alpha_k}^{L}, T_{\alpha_k}^{U}\right]$, $\widetilde{C}_{\alpha_k} = \left[C_{\alpha_k}^{L}, C_{\alpha_k}^{U}\right]$, $\widetilde{G}_{\alpha_k} = \left[G_{\alpha_k}^{L}, G_{\alpha_k}^{U}\right]$, $\widetilde{U}_{\alpha_k} = \left[U_{\alpha_k}^{L}, U_{\alpha_k}^{U}\right]$, $\tilde{F}_{\alpha_k} = \left[F_{\alpha_k}^{L}, F_{\alpha_k}^{U}\right]$ $(k = 1, 2, ..., \eta)$ *are equal, i.e.* $A_k = A$ *for allk, then PNCWA* $_w(A_1, A_2, \ldots, A_\eta) = A$
2. *Monotonicity: Let* $B_k = \left\{\left(\tilde{\beta}_k(u), \beta_k(u)\right) : u \in U\right\}(k = 1, 2, \ldots, \eta)$ *where* $\tilde{\beta}_k = \left\{\left(u, \tilde{T}_{\beta_k}(u), \tilde{C}_{\beta_k}(u), \tilde{G}_{\beta_k}(u), \tilde{U}_{\beta_k}(u), \tilde{F}_{\beta_k}(u)\right) : u \in U\right\}$ *is IVPNS and* $\beta_k = \left\{\left(u, T_{\beta_k}(u), C_{\beta_k}(u), G_{\beta_k}(u), U_{\beta_k}(u),\right.\right.$

$F_{\beta_k}(u)) : u \in U\}$ *be PNS. Where* $\tilde{T}_{\beta_k} = \left[T^L_{\beta_k}, T^U_{\beta_k}\right]$, $\widetilde{C}_{\beta_k} = \left[C^L_{\beta_k}, C^U_{\beta_k}\right]$, $\widetilde{G}_{\beta_k} = \left[G^L_{\beta_k}, G^U_{\beta_k}\right]$, $\widetilde{U}_{\beta_k} = \left[U^L_{\beta_k}, U^U_{\beta_k}\right]$, $\tilde{F}_{\beta_k} = \left[F^L_{\beta_k}, F^U_{\beta_k}\right]$ $(k = 1, 2, ..., \eta)$ *be the collection of neutrosophic cubic values. If* $S_B(u) \geq S_A(u)$ *and* $B_k(u) \geq A_k(u)$, *where* $u \in U$, *then* $PNCWA_w\left(A_1, A_2, ..., A_\eta\right) \leq PNCWA_w\left(B_1, B_2, ..., B_\eta\right)$

3. *Boundary:* $A^- \leq NCWA_w\{(A_1), (A_2), ..., (A_n)\} \leq A^+$, *where*

$$A^- = \left\{\begin{matrix} \min_k T^L_{N_k}, \min_k C^L_{N_k}, 1 - \max_k G^L_{N_k}, 1 - \max_k U^L_{N_k}, 1 - \max_k F^L_{N_k}, \\ \min_k T_{N_k}, \min_k C_{N_k}, 1 - \max_k G_{N_k}, 1 - \max_k U_{N_k}, 1 - \max_k F_{N_k} \end{matrix}\right\}$$

$$A^+ = \left\{\begin{matrix} \min_k T^U_{N_k}, \min_k C^U_{N_k}, 1 - \max_k G^U_{N_k}, 1 - \max_k U^U_{N_k}, 1 - \max_k F^U_{N_k}, \\ \min_k T_{N_k}, \min_k C_{N_k}, 1 - \max_k G_{N_k}, 1 - \max_k U_{N_k}, 1 - \max_k F_{N_k} \end{matrix}\right\}$$

Proof

1. *Idempotence* Since $A_k = A$ so

$$\text{NCWA}(A_k) = \text{NCWA}(A)$$

$$= \begin{pmatrix} \left[1 - \prod_{k=1}^{\eta}(1 - T^L_{A_k})^{w_k}, 1 - \prod_{k=1}^{\eta}(1 - T^U_{A_k})^{w_k}\right], \left[1 - \prod_{k=1}^{\eta}(1 - C^L_{A_k})^{w_k}, 1 - \prod_{k=1}^{\eta}(1 - C^U_{A_k})^{w_k}\right], \\ \left[\prod_{k=1}^{\eta}\left(G^L_{A_k}\right)^{w_k}, \prod_{k=1}^{\eta}\left(G^U_{A_k}\right)^{w_k}\right], \left[\prod_{k=1}^{\eta}\left(U^L_{A_k}\right)^{w_k}, \prod_{k=1}^{\eta}\left(U^U_{A_k}\right)^{w_k}\right], \left[\prod_{k=1}^{\eta}\left(F^L_{A_k}\right)^{w_k}, \prod_{k=1}^{\eta}\left(F^U_{A_k}\right)^{w_k}\right], \\ \prod_{k=1}^{\eta}\left(T_{A_k}\right)^{w_k}, \prod_{k=1}^{\eta}\left(C_{A_k}\right)^{w_k}, 1 - \prod_{k=1}^{\eta}\left(1 - \left(G_{A_k}\right)\right)^{w_k}, 1 - \prod_{k=1}^{\eta}\left(1 - \left(U_{A_k}\right)\right)^{w_k} 1 - \prod_{k=1}^{\eta}\left(1 - \left(F_{A_k}\right)\right)^{w_k} \end{pmatrix}$$

$$A = (\tilde{\alpha}_k, \alpha_k)$$

2. *Monotonicity* Since pentapartitioned neutrosophic cubic ordered weighted average operator (PNCOWA) is strictly monotone function, hence the proof is trivial.
3. *Boundary* Let $u = \min A^-$ and $y = \max A^+$, then by the idempotent law, we have $u \leq \text{PNCOWA}(A_k) \leq y \Rightarrow A^- \leq \text{PNCOWA}(A_k) \leq A^+$. □

Theorem 5.1.6 *Let* $A_k = \{(\tilde{\alpha}_k(u), \alpha_k(u)) : u \in U\}(k = 1, 2, \ldots, \eta)$ *where* $\tilde{\alpha}_k = \left\{\left(u, \tilde{T}_{\alpha_k}(u), \tilde{C}_{\alpha_k}(u), \tilde{G}_{\alpha_k}(u), \tilde{U}_{\alpha_k}(u), \tilde{F}_{\alpha_k}(u)\right) : u \in U\right\}$ *is IVPNS and* $\alpha_k = \left\{\left(u, T_{\alpha_k}(u), C_{\alpha_k}(u), G_{\alpha_k}(u), U_{\alpha_k}(u), F_{\alpha_k}(u)\right) : u \in U\right\}$ *be PNS. Where* $\tilde{T}_{\alpha_k} = \left[T^L_{\alpha_k}, T^U_{\alpha_k}\right]$, $\widetilde{C}_{\alpha_k} = \left[C^L_{\alpha_k}, C^U_{\alpha_k}\right]$, $\widetilde{G}_{\alpha_k} = \left[G^L_{\alpha_k}, G^U_{\alpha_k}\right]$, $\widetilde{U}_{\alpha_k} = \left[U^L_{\alpha_k}, U^U_{\alpha_k}\right]$, $\tilde{F}_{\alpha_k} = \left[F^L_{\alpha_k}, F^U_{\alpha_k}\right]$ $(k = 1, 2, \ldots, \eta)$ *be a collection of PNC values, with weight* $W = (w_1, w_2, \ldots, w_\eta)^T$ *of* $A_k(k = 1, 2, 3, ..., \eta)$, *such that* $w_k \in [0, 1]$ and $\sum_{k=1}^{\eta} w_k = 1$. *The following properties will hold.*

1. If $W = (1, 0, \ldots, 0)^T$, then PNCOWA $(A_1, A_2, \ldots, A_\eta) = \max A_k$
2. If $W = (0, 0, \ldots, 1)^T$, then PNCOWA $(A_1, A_2, \ldots, A_\eta) = \min A_k$
3. If $w_k = 1$, $w_l = 0$, *and* $k \neq l$, then PNCOWA $(A_1, A_2, \ldots, A_\eta) = A_k$, where

A_k *is the largest kthof* $(A_1, A_2, \ldots, A_\eta)$.

Proof Since in PNCOWA, the pentapartitioned neutrosophic values are ordered in descending order, hence PNCWA operator aggregates the weighted values. On the other hand, PNCOWA weights only the ordering positions.□

The idea of pentapartitioned neutrosophic cubic hybrid aggregation operators (PNCHA) is developed to not only weigh the values but also weigh their ordering position as well.

Definition 5.1.7 PNCHA : $\Omega^\eta \rightarrow \Omega$ is a mapping of n-dimension, which has associated weight $W = (w_1, w_2, \ldots, w_n)^T$, where $w_k \in [0, 1]$ and $\sum_{k=1}^{\eta} w_k = 1$, such that $\text{PNCHA}_w(A_1, A_2, \ldots, A_\eta) = w_1 A^{\varpi}_{\sigma(1)} \oplus w_2 A^{\varpi}_{\sigma(2)} \oplus \cdots \oplus w_n A^{\varpi}_{\sigma(\eta)}$ where $A^{\sim}_k$ is the largest kth of the weighted neutrosophic cubic values A^{ϖ}_k. The A^{ϖ}_k can be calculated by the following formula $A^{\varpi}_k = n w_k A_k$, $(k = 1, 2, 3, \ldots, \eta)$, $W = (w_1, w_2, \ldots, w_\eta)^T$, $w_k \in [0, 1]$ and $\sum_{k=1}^{\eta} w_k = 1$, where η is the balancing coefficient.

Theorem 5.1.8 *Let* $A_k = \left\{\left(\tilde{\alpha}^{\varpi}_{\sigma(k)}(u), \alpha^{\varpi}_{\sigma(k)}(u)\right) : u \in U\right\}$ $(k = 1, 2, \ldots, \eta)$ *be a collection of pentapartitioned neutrosophic cubic values. Then,the aggregated value by PNCHA is also a neutrosophic cubic value and*

$$\text{PNCHA}_w(A_k) = \begin{pmatrix} \left[1-\prod_{k=1}^{\eta}(1-T_{A_k}^L)^{w_k},\ 1-\prod_{k=1}^{\eta}(1-T_{A_k}^U)^{w_k}\right], \left[1-\prod_{k=1}^{\eta}(1-C_{A_k}^L)^{w_k},\ 1-\prod_{k=1}^{\eta}(1-C_{A_k}^U)^{w_k}\right], \\ \left[\prod_{k=1}^{\eta}\left(G_{A_k}^L\right)^{w_k}, \prod_{k=1}^{\eta}\left(G_{A_k}^U\right)^{w_k}\right], \left[\prod_{k=1}^{\eta}\left(U_{A_k}^L\right)^{w_k}, \prod_{k=1}^{\eta}\left(U_{A_k}^U\right)^{w_k}\right], \left[\prod_{k=1}^{\eta}\left(F_{A_k}^L\right)^{w_k}, \prod_{k=1}^{\eta}\left(F_{A_k}^U\right)^{w_k}\right], \\ \prod_{k=1}^{\eta}\left(T_{A_k}\right)^{w_k}, \prod_{k=1}^{\eta}\left(C_{A_k}\right)^{w_k}, 1-\prod_{k=1}^{\eta}\left(1-\left(G_{A_k}\right)\right)^{w_k}, 1-\prod_{k=1}^{\eta}\left(1-\left(U_{A_k}\right)\right)^{w_k}, 1-\prod_{k=1}^{\eta}\left(1-\left(F_{A_k}\right)\right)^{w_k} \end{pmatrix}$$

where the weight $W = (w_1, w_2, \ldots, w_\eta)^T$ *is such that* $w_k \in [0, 1]$ and $\sum_{k=1}^{\eta} w_k = 1$.

Proof The proof is directly concluded by Theorem 4.3. □

Theorem 5.1.9 *The PNCWA operator is a special case of PNCHA operator when all the components of w are equal, i.e.* $w_1 = w_2 = \cdots = w_\eta$.

Proof Let $W = \left(\frac{1}{\eta}, \frac{1}{\eta}, \ldots, \frac{1}{\eta}\right)^T$.

Then PNCHA $_w w(A_1, A_2, \ldots, A_\eta)$

$$= w_1 A_{\sigma(1)}^{\varpi} \oplus w_2 A_{\sigma(2)}^{\varpi} \oplus \cdots \oplus w_n A_{\sigma(\eta)}^{\varpi}$$

$$= \frac{1}{\eta}\left(A_{\sigma(1)}^{\varpi} \oplus A_{\sigma(2)}^{\varpi} \oplus \cdots \oplus A_{\sigma(\eta)}^{\varpi}\right)$$

$$= \frac{1}{n}(A_1, A_2, \ldots, A_\eta)$$

$$= w_1 A_1, w_2 A_2, \ldots, w_\eta A_\eta$$

$$= \text{NCWA}(A_1, A_2, \ldots, A_\eta).$$ □

Theorem 5.1.10 *The PNCOWA is a special case of PNCHA when all the components of w are equal, i.e.* $w_1 = w_2 = \cdots = w_n$.

Proof Let $W = \left(\frac{1}{\eta}, \frac{1}{\eta}, \ldots, \frac{1}{\eta}\right)^T$.

Then PNCHA $_w w(A_1, A_2, \ldots, A_\eta)$

$$= w_1 A_{\sigma(1)}^{\varpi} \oplus w_2 A_{\sigma(2)}^{\varpi} \oplus \cdots \oplus w_n A_{\sigma(\eta)}^{\varpi}$$

$$= \frac{1}{\eta}\left(A_{\sigma(1)}^{\varpi} \oplus A_{\sigma(2)}^{\varpi} \oplus \cdots \oplus A_{\sigma(\eta)}^{\varpi}\right)$$

$$= \frac{1}{n}(A_1, A_2, \ldots, A_\eta)$$

$$= w_1 A_1, w_2 A_2, \ldots, w_\eta A_\eta$$

$$= \text{PNCOWA}(A_1, A_2, \ldots, A_n).$$ □

Model formulation of air pollution

Air pollution is a serious threat to the environment. It causes various diseases in humans. Inhalation of polluted air can cause many diseases, including lung cancer and other respiratory diseases. According to the World Health Organization, air pollution caused 7 million premature deaths worldwide in 2016. In the region alone, 4.2 million people have died and household air pollution from cooking with polluted fuel and technology has killed 3.8 million people in the same year. More than 90% of deaths due to air pollution occur in low- and middle-income countries, especially in Africa and Asia. The incidence of cancer and lung disease in Pakistan has increased in recent years. This type of disease is caused by PM. PM sizes are classified as PM25, PM10 and PM2.5. This study focuses on PM2.5.

What is PM2.5

Particulate matter (PM2.5) is an air pollutant that poses a risk to human health in high concentrations in the air. PM2.5 are small particles in the air that reduce visibility and become cloudy when the level rises.

PM2.5 refers to environmental particles PM less than 2.5 µm in diameter, which is about 3% of the hair thickness. Fragments in this group, commonly referred to as PM2.5, are so small that they can only be seen under a microscope. It is less than PM10, which is 10 µm or less, also called fine dust (Fig. 1).

Where do $PM_{2.5}$ come from

Optimism comes from a variety of sources. These include plants, Cars Aircraft; Forest fires; Forest fires; Farm fires; these include volcanoes and dust storms. Some are released directly into the air, while others are formed when gases and atmosphere particles interact.

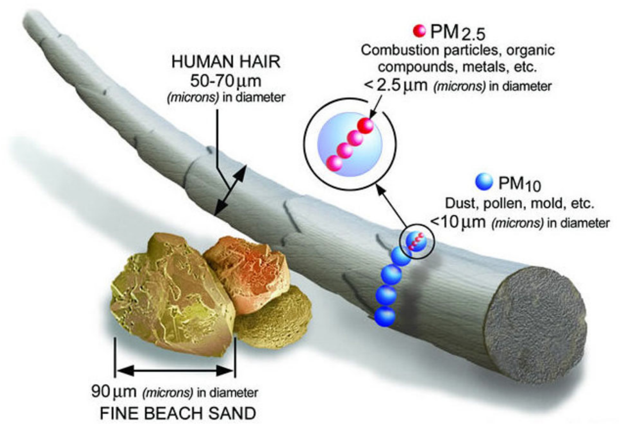


Fig. 1 Comparison of particulate matter

Why are $PM_{2.5}$ dangerous

Because they are so small and light, small particles live in the air longer than larger particles. Increases the ability of humans and animals to breathe in their own bodies. Due to its small size, particles smaller than 2.5 µm can pass through the nose and throat and enter the deep intestine and some into the circulatory system.

In this article, the Moderate, sensitive group, unhealthy and hazard are considered.

The collection of correct and appropriate data is always been a hard task. The inskilled and inappropriate method of data collection causes problem to getting the appropriate results. To minimize these obstacles pentapartitioned neutrosophic cubic set is better choice due to its interesting features of components to determine the indeterminancy. In this article, a problem regarding PM2.5 in major cities of Pakistan, Peshawar, Lahore, Karachi and Islamabad for the month of October 2021 is collected using PNC environment. Then, the data collected is scored using accuracy function then plotted subject to the indicators moderate, sensitive, unhealthy and hazard described in Table 1. After this, the data collected is aggregated using PNCWA and PNCWG aggregation operators to get a collective value for the month of October 2021.

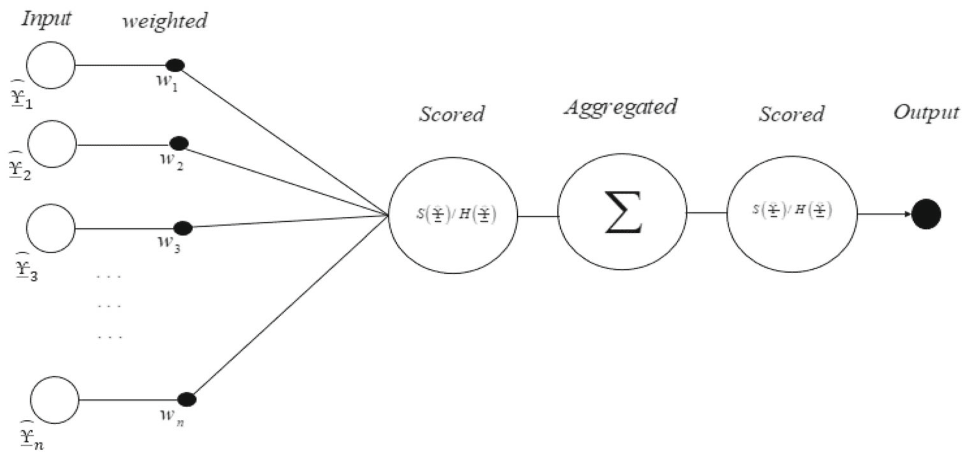
The data collected in the month of October 2021 is in the form of $\widehat{\underline{\Upsilon}}_{\gamma}$ where γ represents the date consequently data collected.

Flowchart

The flowchart presents the methodology used to evaluate the result.

Table 1 24-h PM2.5 levels (µg/m^3)

PM2.5	Air Quality Indexing	$PM_{2.5}$ health repercussion	Precautionary actions
0.0–12.0	Fine 0.00–50.00	Least dangerous	None
1.1–35.4	Moderate 51.0–100.0	Respiratory symptoms may occur in people with abnormal sensitivity	Unusually sensitive people should refrain their prolonged activities
35.5–55.4	Deleterious for Sensitive Groups 101.0–150.0	Increased respiratory symptoms, risk of heart attack or stroke, and untimely death in adults with coronary heart disease	People with heart or respiratory disease, the aged and children should refrain their prolonged activities
55.5–150.4	Unhealthy 151.0–200.0	Worsen of heart, lung disease and untimely deaths in elders and cardiopulmonary disease persons; increase respiratory disease in general inhabitant	People with heart or respiratory disease, the aged and children should refrain their prolonged exertion; everybody else should limit prolonged activities
150.5–250.4	Very Unhealthy 201.0–300.0	Worsen of heart, lung disease and untimely deaths in elders and cardiopulmonary disease persons; notable increase in respiratory disease in general inhabitant	People with heart or respiratory disease, the aged and children should refrain their outdoor activity; everybody else should refrain prolonged activities
250.5–500.4	Hazardous 301.0–500.0	Worsen of heart, lung disease and untimely deaths in elders and cardiopulmonary disease persons; major risk of respiratory disease in general inhabitant	Everybody should refrain any outdoor activities; people with heart or respiratory disease, the aged and children should remain indoors



Peshawar

The data collected in the form of PNCS in the city of Peshawar, Pakistan on the following dates.

$\underline{\widehat{\yen}}_1 = \langle[0.78, 0.84], [0.76, 0.85], [0.77, 0.85], [0.15, 0.24]$
$[0.16, 0.22], 0.79, 0.81, 0.80, 0.29, 0.18\rangle$

$\underline{\widehat{\yen}}_2 = \langle[0.80, 0.84], [0.81, 0.83], [0.81, 0.84], [0.17, 0.19],$
$[0.17, 0.20], 0.82, 0.82, 0.82, 0.28, 0.19\rangle$

$\underline{\widehat{\yen}}_4 = \langle[0.58, 0.64], [0.59, 0.63], [0.59, 0.64], [0.37, 0.41],$
$[0.35, 0.41], 0.60, 0.62, 0.61, 0.48, 0.38\rangle$

$\underline{\widehat{\yen}}_5 = \langle[0.62, 0.65], [0.60, 0.64], [0.61, 0.65], [0.36, 0.40],$
$[0.35, 0.37], 0.64, 0.62, 0.63, 0.48, 0.36\rangle$

$\underline{\widehat{\yen}}_6 = \langle[0.59, 0.63], [0.60, 0.62], [0.60, 0.63], [0.38, 0.40],$
$[0.36, 0.40], 0.61, 0.61, 0.61, 0.49, 0.38\rangle$

$\underline{\widehat{\yen}}_7 = \langle[0.73, 0.78], [0.70, 0.79], [0.72, 0.79], [0.21, 0.30],$
$[0.22, 0.27], 0.75, 0.77, 0.76, 0.33, 0.25\rangle$

$\underline{\widehat{\yen}}_8 = \langle[0.78, 0.82], [0.76, 0.84], [0.77, 0.83], [0.16, 0.24],$
$[0.19, 0.21], 0.80, 0.79, 0.80, 0.31, 0.20\rangle$

$\underline{\widehat{\yen}}_9 = \langle[0.77, 0.80], [0.79, 0.82], [0.78, 0.81], [0.18, 0.21],$
$[0.19, 0.22], 0.78, 0.83, 0.81, 0.27, 0.18\rangle$

$\underline{\widehat{\yen}}_{10} = \langle[0.80, 0.84], [0.79, 0.82], [0.80, 0.83], [0.18, 0.21],$
$[0.17, 0.15], 0.85, 0.79, 0.82, 0.31, 0.16\rangle$

$\underline{\widehat{\yen}}_{11} = \langle[0.76, 0.79], [0.77, 0.80], [0.77, 0.80], [0.20, 0.23],$
$[0.25, 0.20], 0.75, 0.78, 0.77, 0.32, 0.21\rangle$

$\underline{\widehat{\yen}}_{12} = \langle[0.68, 0.70], [0.67, 0.72], [0.68, 0.71], [0.28, 0.33],$
$[0.30, 0.31], 0.71, 0.68, 0.70, 0.42, 0.29\rangle$

$\underline{\widehat{\yen}}_{13} = \langle[0.67, 0.71], [0.68, 0.72], [0.68, 0.72], [0.28, 0.32],$
$[0.28, 0.32], 0.70, 0.67, 0.69, 0.43, 0.33\rangle$

$\underline{\widehat{\yen}}_{14} = \langle[0.76, 0.80], [0.76, 0.82], [0.76, 0.81], [0.18, 0.24],$
$[0.18, 0.26], 0.75, 0.79, 0.77, 0.31, 0.20\rangle$

$\underline{\widehat{\yen}}_{15} = \langle[0.75, 0.78], [0.76, 0.80], [0.76, 0.79], [0.20, 0.24],$
$[0.21, 0.24], 0.76, 0.81, 0.79, 0.29, 0.23\rangle$

$\underline{\widehat{\yen}}_{16} = \langle[0.76, 0.79], [0.75, 0.81], [0.76, 0.80], [0.19, 0.25],$
$[0.21, 0.23], 0.75, 0.76, 0.76, 0.34, 0.22\rangle$

$\underline{\widehat{\yen}}_{17} = \langle[0.71, 0.75], [0.70, 0.76], [0.71, 0.76], [0.24, 0.30],$
$[0.24, 0.29], 0.72, 0.74, 0.73, 0.36, 0.28\rangle$

$\underline{\widehat{\yen}}_{20} = \langle[0.85, 0.87], [0.83, 0.89], [0.84, 0.88], [0.11, 0.17],$
$[0.12, 0.14], 0.86, 0.85, 0.86, 0.25, 0.11\rangle$

$\underline{\widehat{\yen}}_{21} = \langle[0.80, 0.84], [0.81, 0.83], [0.81, 0.84], [0.17, 0.19],$
$[0.15, 0.20], 0.79, 0.82, 0.81, 0.28, 0.17\rangle$

$\underline{\widehat{\yen}}_{22} = \langle[0.86, 0.90], [0.87, 0.91], [0.87, 0.91], [0.09, 0.13],$
$[0.11, 0.15], 0.91, 0.89, 0.90, 0.21, 0.14\rangle$

$\underline{\widehat{\yen}}_{23} = \langle[0.79, 0.82], [0.78, 0.83], [0.79, 0.83], [0.17, 0.22],$
$[0.17, 0.21], 0.81, 0.79, 0.80, 0.31, 0.20\rangle$

$\underline{\widehat{\yen}}_{25} = \langle[0.53, 0.58], [0.54, 0.57], [0.54, 0.58], [0.43, 0.46],$
$[0.41, 0.46], 0.57, 0.53, 0.55, 0.57, 0.47\rangle$

$\widehat{\underline{\Upsilon}}_{26} = \langle[0.66, 0.70], [0.67, 0.71], [0.67, 0.71], [0.29, 0.33], [0.29, 0.35], 0.65, 0.70, 0.68, 0.40, 0.36\rangle$

$\widehat{\underline{\Upsilon}}_{27} = \langle[0.82, 0.85], [0.81, 0.84], [0.82, 0.85], [0.16, 0.19], [0.15, 0.17], 0.81, 0.85, 0.83, 0.25, 0.18\rangle$

$\widehat{\underline{\Upsilon}}_{28} = \langle[0.85, 0.90], [0.84, 0.91], [0.85, 0.91], [0.09, 0.16], [0.09, 0.15], 0.87, 0.86, 0.87, 0.24, 0.10\rangle$

$\widehat{\underline{\Upsilon}}_{29} = \langle[0.72, 0.76], [0.73, 0.77], [0.73, 0.77], [0.23, 0.27], [0.23, 0.28], 0.75, 0.76, 0.76, 0.34, 0.22\rangle$

Lahore

The data collected in the form of PNCS in the city of Lahore, Pakistan on the following dates.

$\widehat{\underline{\Upsilon}}_{1} = \langle[0.41, 0.44], [0.42, 0.45], [0.42, 0.45], [0.55, 0.58], [0.58, 0.60], 0.42, 0.43, 0.43, 0.67, 0.57\rangle$

$\widehat{\underline{\Upsilon}}_{2} = \langle[0.52, 0.55], [0.53, 0.56], [0.53, 0.56], [0.44, 0.47], [0.45, 0.47], 0.53, 0.55, 0.54, 0.55, 0.46\rangle$

$\widehat{\underline{\Upsilon}}_{3} = \langle[0.38, 0.41], [0.37, 0.42], [0.38, 0.42], [0.58, 0.63], [0.59, 0.61], 0.39, 0.39, 0.39, 0.71, 0.60\rangle$

$\widehat{\underline{\Upsilon}}_{4} = \langle[0.37, 0.40], [0.36, 0.42], [0.37, 0.41], [0.58, 0.64], [0.61, 0.63], 0.36, 0.40, 0.38, 0.70, 0.64\rangle$

$\widehat{\underline{\Upsilon}}_{5} = \langle[0.40, 0.42], [0.39, 0.43], [0.40, 0.43], [0.57, 0.61], [0.57, 0.59], 0.39, 0.44, 0.42, 0.66, 0.58\rangle$

$\widehat{\underline{\Upsilon}}_{6} = \langle[0.40, 0.44], [0.39, 0.45], [0.40, 0.45], [0.55, 0.61], [0.56, 0.61], 0.42, 0.34, 0.38, 0.76, 0.60\rangle$

$\widehat{\underline{\Upsilon}}_{7} = \langle[0.56, 0.59], [0.55, 0.60], [0.56, 0.60], [0.40, 0.45], [0.41, 0.44], 0.58, 0.58, 0.58, 0.52, 0.42\rangle$

$\widehat{\underline{\Upsilon}}_{8} = \langle[0.50, 0.54], [0.51, 0.53], [0.51, 0.54], [0.47, 0.49], [0.45, 0.50], 0.52, 0.50, 0.51, 0.60, 0.51\rangle$

$\widehat{\underline{\Upsilon}}_{9} = \langle[0.52, 0.57], [0.51, 0.58], [0.52, 0.58], [0.42, 0.49], [0.43, 0.48], 0.51, 0.59, 0.55, 0.51, 0.50\rangle$

$\widehat{\underline{\Upsilon}}_{10} = \langle[0.56, 0.60], [0.57.0.61], [0.57, 0.61], [0.39, 0.43], [0.39, 0.44], 0.54, 0.56, 0.55, 0.54, 0.38\rangle$

$\widehat{\underline{\Upsilon}}_{11} = \langle[0.48, 0.52], [0.47, 0.53], [0.48, 0.53], [0.47, 0.53], [0.48, 0.53], 0.47, 0.49, 0.48, 0.61, 0.54\rangle$

$\widehat{\underline{\Upsilon}}_{12} = \langle[0.47, 0.49], [0.46, 0.5], [0.47, 0.50], [0.50, 0.54], [0.51, 0.53], 0.46, 0.51, 0.49, 0.59, 0.52\rangle$

$\widehat{\underline{\Upsilon}}_{13} = \langle[0.56, 0.59], [0.54, 0.60], [0.55, 0.60], [0.40, 0.46], [0.40, 0.44], 0.57, 0.61, 0.59, 0.49, 0.39\rangle$

$\widehat{\underline{\Upsilon}}_{14} = \langle[0.66, 0.70], [0.65, 0.71], [0.66, 0.71], [0.29, 0.35], [0.30, 0.34], 0.68, 0.70, 0.69, 0.40, 0.32\rangle$

$\widehat{\underline{\Upsilon}}_{15} = \langle[0.65, 0.70], [0.66, 0.71], [0.66, 0.71], [0.29, 0.34], [0.31, 0.35], 0.67, 0.70, 0.69, 0.40, 0.34\rangle$

$\widehat{\underline{\Upsilon}}_{16} = \langle[0.63, 0.69], [0.62, 0.70], [0.63, 0.70], [0.30, 0.38], [0.31, 0.37], 0.62, 0.68, 0.65, 0.42, 0.35\rangle$

$\widehat{\underline{\Upsilon}}_{17} = \langle[0.64, 0.68], [0.63, 0.69], [0.64, 0.69], [0.31, 0.37], [0.32, 0.36], 0.65, 0.65, 0.65, 0.45, 0.34\rangle$

$\widehat{\underline{\Upsilon}}_{18} = \langle[0.76, 0.79], [0.75, 0.80], [0.76, 0.80], [0.20, 0.25], [0.20, 0.24], 0.78, 0.79, 0.79, 0.31, 0.22\rangle$

$\widehat{\underline{\Upsilon}}_{19} = \langle[0.55, 0.60], [0.50, 0.59], [0.53, 0.60], [0.41, 0.50], [0.39, 0.45], 0.54, 0.55, 0.55, 0.55, 0.40\rangle$

$\widehat{\underline{\Upsilon}}_{20} = \langle[0.60, 0.65], [0.61, 0.66], [0.61, 0.66], [0.34, 0.39], [0.35, 0.40], 0.63, 0.65, 0.64, 0.45, 0.42\rangle$

$\widehat{\underline{\Upsilon}}_{21} = \langle[0.55, 0.65], [0.58, 0.63], [0.57, 0.64], [0.37, 0.42], [0.35, 0.44], 0.54, 0.64, 0.59, 0.46, 0.41\rangle$

$\widehat{\underline{\Upsilon}}_{22} = \langle[0.90, 0.95], [0.89, 0.94], [0.90, 0.95], [0.06, 0.11], [0.05, 0.10], 0.91, 0.88, 0.90, 0.22, 0.08\rangle$

Karachi

The data collected in the form of PNCS in the city of Lahore, Pakistan on the following dates.

$\widehat{\underline{\Upsilon}}_{1} = \langle[0.31, 0.39], [0.29, 0.35], [0.20, 0.37], [0.75, 0.81], [0.61, 0.69], 0.35, 0.32, 0.37, 0.50, 0.67\rangle$

$\widehat{\underline{\Upsilon}}_{2} = \langle[0.21, 0.29], [0.23, 0.30], [0.15, 0.30], [0.80, 0.87], [0.71, 0.79], 0.25, 0.27, 0.36, 0.51, 0.75\rangle$

$\widehat{\underline{\Upsilon}}_{3} = \langle[0.20, 0.26], [0.22, 0.27], [0.14, 0.27], [0.83, 0.88], [0.72, 0.78], 0.24, 0.24, 0.35, 0.50, 0.76\rangle$

$\widehat{\underline{\Upsilon}}_{4} = \langle[0.22, 0.30], [0.25, 0.32], [0.16, 0.31], [0.78, 0.85], [0.70, 0.78], 0.19, 0.34, 0.34, 0.51, 0.68\rangle$

$\widehat{\underline{\Upsilon}}_{5} = \langle[0.25, 0.30], [0.23, 0.28], [0.16, 0.29], [0.82, 0.87], [0.72, 0.77], 0.27, 0.25, 0.36, 0.51, 0.76\rangle$

$\widehat{\underline{\Upsilon}}_{6} = \langle [0.28, 0.35], [0.25, 0.32], [0.18, 0.34], [0.78, 0.85], [0.65, 0.72], 0.32, 0.28, 0.35, 0.48, 0.68 \rangle$

$\widehat{\underline{\Upsilon}}_{7} = \langle [0.41, 0.48], [0.44, 0.51], [0.28, 0.50], [0.59, 0.66], [0.53, 0.60], 0.49, 0.52, 0.44, 0.57, 0.61 \rangle$

$\widehat{\underline{\Upsilon}}_{8} = \langle [0.55, 0.62], [0.52, 0.58], [0.36, 0.60], [0.52, 0.58], [0.38, 0.45], 0.59, 0.55, 0.41, 0.49, 0.42 \rangle$

$\widehat{\underline{\Upsilon}}_{9} = \langle [0.40, 0.45], [0.42, 0.47], [0.27, 0.46], [0.63, 0.68], [0.57, 0.62], 0.48, 0.50, 0.44, 0.58, 0.65 \rangle$

$\widehat{\underline{\Upsilon}}_{10} = \langle [0.38, 0.45], [0.40, 0.48], [0.26, 0.47], [0.62, 0.70], [0.55, 0.62], 0.40, 0.42, 0.38, 0.50, 0.57 \rangle$

$\widehat{\underline{\Upsilon}}_{11} = \langle [0.30, 0.38], [0.28, 0.36], [0.19, 0.37], [0.74, 0.82], [0.62, 0.70], 0.35, 0.32, 0.36, 0.49, 0.65 \rangle$

$\widehat{\underline{\Upsilon}}_{12} = \langle [0.42, 0.47], [0.40, 0.45], [0.27, 0.46], [0.65, 0.70], [0.50, 0.55], 0.50, 0.48, 0.42, 0.53, 0.58 \rangle$

$\widehat{\underline{\Upsilon}}_{13} = \langle [0.68, 0.72], [0.70, 0.74], [0.46, 0.73], [0.36, 0.40], [0.28, 0.32], 0.70, 0.72, 0.45, 0.52, 0.31 \rangle$

$\widehat{\underline{\Upsilon}}_{14} = \langle [0.70, 0.75], [0.72, 0.78], [0.47, 0.77], [0.32, 0.38], [0.32, 0.27], 0.76, 0.79, 0.48, 0.54, 0.29 \rangle$

$\widehat{\underline{\Upsilon}}_{15} = \langle [0.60, 0.68], [0.62, 0.65], [0.41, 0.67], [0.45, 0.48], [0.32, 0.40], 0.58, 0.60, 0.39, 0.45, 0.30 \rangle$

$\widehat{\underline{\Upsilon}}_{16} = \langle [0.57, 0.63], [0.60, 0.66], [0.39, 0.65], [0.44, 0.50], [0.39, 0.45], 0.55, 0.63, 0.43, 0.53, 0.42 \rangle$

$\widehat{\underline{\Upsilon}}_{17} = \langle [0.42, 0.46], [0.45, 0.48], [0.29, 0.47], [0.62, 0.65], [0.54, 0.58], 0.40, 0.43, 0.37, 0.48, 0.52 \rangle$

$\widehat{\underline{\Upsilon}}_{18} = \langle [0.60, 0.65], [0.65, 0.70], [0.42, 0.68], [0.40, 0.45], [0.37, 0.42], 0.62, 0.69, 0.45, 0.54, 0.39 \rangle$

$\widehat{\underline{\Upsilon}}_{19} = \langle [0.35, 0.45], [0.38, 0.48], [0.24, 0.47], [0.62, 0.72], [0.55, 0.65], 0.38, 0.41, 0.37, 0.50, 0.58 \rangle$

$\widehat{\underline{\Upsilon}}_{20} = \langle [0.30, 0.37], [0.32, 0.40], [0.21, 0.39], [0.70, 0.78], [0.65, 0.72], 0.28, 0.30, 0.33, 0.46, 0.62 \rangle$

$\widehat{\underline{\Upsilon}}_{21} = \langle [0.28, 0.35], [0.30, 0.37], [0.19, 0.36], [0.73, 0.80], [0.65, 0.72], 0.31, 0.35, 0.38, 0.53, 0.70 \rangle$

$\widehat{\underline{\Upsilon}}_{22} = \langle [0.25, 0.32], [0.28, 0.35], [0.18, 0.34], [0.75, 0.82], [0.71, 0.78], 0.28, 0.30, 0.37, 0.52, 0.73 \rangle$

$\widehat{\underline{\Upsilon}}_{23} = \langle [0.24, 0.30], [0.25, 0.31], [0.16, 0.31], [0.79, 0.85], [0.68, 0.74], 0.26, 0.27, 0.34, 0.49, 0.70 \rangle$

$\widehat{\underline{\Upsilon}}_{24} = \langle [0.67, 0.73], [0.70, 0.75], [0.46, 0.74], [0.35, 0.40], [0.27, 0.33], 0.65, 0.68, 0.41, 0.47, 0.25 \rangle$

$\widehat{\underline{\Upsilon}}_{25} = \langle [0.70, 0.75], [0.75, 0.80], [0.48, 0.78], [0.30, 0.35], [0.28, 0.33], 0.72, 0.77, 0.47, 0.54, 0.3 \rangle$

$\widehat{\underline{\Upsilon}}_{26} = \langle [0.65, 0.71], [0.68, 0.72], [0.44, 0.72], [0.38, 0.42], [0.25, 0.31], 0.62, 0.66, 0.39, 0.45, 0.23 \rangle$

$\widehat{\underline{\Upsilon}}_{27} = \langle [0.69, 0.75], [0.73, 0.76], [0.47, 0.76], [0.34, 0.37], [0.29, 0.35], 0.66, 0.70, 0.42, 0.49, 0.27 \rangle$

$\widehat{\underline{\Upsilon}}_{28} = \langle [0.70, 0.75], [0.65, 0.70], [0.45, 0.73], [0.40, 0.45], [0.28, 0.33], 0.72, 0.68, 0.44, 0.50, 0.31 \rangle$

$\widehat{\underline{\Upsilon}}_{29} = \langle [0.75, 0.85], [0.70, 0.80], [0.48, 0.63], [0.30, 0.40], [0.15, 0.25], 0.80, 0.75, 0.45, 0.48, 0.20 \rangle$

$\widehat{\underline{\Upsilon}}_{30} = \langle [0.62, 0.68], [0.60, 0.65], [0.41, 0.60], [0.45, 0.50], [0.32, 0.38], 0.66, 0.58, 0.40, 0.44, 0.30 \rangle$

$\widehat{\underline{\Upsilon}}_{31} = \langle [0.70, 0.75], [0.68, 0.73], [0.46, 0.74], [0.37, 0.42], [0.28, 0.33], 0.71, 0.70, 0.45, 0.51, 0.31 \rangle$

Islamabad

The data collected in the form of PNCS in the city of Lahore, Pakistan on the following dates.

$\widehat{\underline{\Upsilon}}_{1} = \langle [0.70, 0.75], [0.72, 0.78], [0.47, 0.77], [0.11, 0.14], [0.25, 0.30], 0.72, 0.76, 0.40, 0.25, 0.28 \rangle$

$\widehat{\underline{\Upsilon}}_{2} = \langle [0.52, 0.58], [0.55, 0.61], [0.36, 0.60], [0.20, 0.23], [0.40, 0.46], 0.60, 0.62, 0.42, 0.39, 0.48 \rangle$

$\widehat{\underline{\Upsilon}}_{3} = \langle [0.25, 0.32], [0.24, 0.30], [0.16, 0.31], [0.35, 0.38], [0.68, 0.75], 0.27, 0.26, 0.40, 0.75, 0.70 \rangle$

$\widehat{\underline{\Upsilon}}_{4} = \langle [0.24, 0.31], [0.23, 0.29], [0.16, 0.30], [0.36, 0.39], [0.67, 0.74], 0.25, 0.24, 0.39, 0.77, 0.69 \rangle$

$\widehat{\underline{\Upsilon}}_{5} = \langle [0.35, 0.40], [0.32, 0.37], [0.22, 0.39], [0.32, 0.34], [0.60, 0.65], 0.30, 0.27, 0.37, 0.74, 0.55 \rangle$

$\widehat{\underline{\Upsilon}}_{6} = \langle [0.32, 0.40], [0.35, 0.43], [0.22, 0.42], [0.29, 0.33], [0.62, 0.70], 0.42, 0.45, 0.43, 0.56, 0.72 \rangle$

$\widehat{\underline{\Upsilon}}_{7} = \langle [0.40, 0.48], [0.42, 0.45], [0.27, 0.47], [0.25, 0.29], [0.55, 0.63], 0.39, 0.41, 0.39, 0.60, 0.54 \rangle$

$\widehat{\underline{\Upsilon}}_{8} = \langle [0.41, 0.47], [0.40, 0.43], [0.27, 0.45], [0.25, 0.29],$

$[0.53, 0.59], 0.45, 0.42, 0.40, 0.59, 0.55\rangle$

$$\widehat{\Upsilon}_9 = \langle [0.50, 0.55], [0.55, 0.60], [0.35, 0.58], [0.20, 0.23], [0.45, 0.50], 0.52, 0.57, 0.40, 0.44, 0.47\rangle$$

$$\widehat{\Upsilon}_{10} = \langle [0.50, 0.60], [0.52, 0.58], [0.34, 0.59], [0.21, 0.24], [0.38, 0.48], 0.51, 0.53, 0.38, 0.48, 0.39\rangle$$

$$\widehat{\Upsilon}_{11} = \langle [0.40, 0.47], [0.42, 0.48], [0.27, 0.48], [0.26, 0.29], [0.53, 0.60], 0.38, 0.40, 0.38, 0.61, 0.52\rangle$$

$$\widehat{\Upsilon}_{12} = \langle [0.30, 0.40], [0.35, 0.40], [0.22, 0.40], [0.30, 0.33], [0.58, 0.68], 0.35, 0.38, 0.40, 0.63, 0.62\rangle$$

$$\widehat{\Upsilon}_{13} = \langle [0.40, 0.47], [0.42, 0.46], [0.27, 0.47], [0.27, 0.29], [0.53, 0.60], 0.42, 0.45, 0.40, 0.56, 0.55\rangle$$

$$\widehat{\Upsilon}_{14} = \langle [0.45, 0.50], [0.42, 0.47], [0.29, 0.49], [0.27, 0.29], [0.45, 0.50], 0.43, 0.41, 0.38, 0.60, 0.44\rangle$$

$$\widehat{\Upsilon}_{15} = \langle [0.45, 0.48], [0.42, 0.50], [0.29, 0.49], [0.25, 0.29], [0.52, 0.55], 0.46, 0.43, 0.40, 0.58, 0.53\rangle$$

$$\widehat{\Upsilon}_{16} = \langle [0.35, 0.40], [0.38, 0.43], [0.24, 0.42], [0.29, 0.31], [0.58, 0.63], 0.32, 0.35, 0.39, 0.66, 0.60\rangle$$

$$\widehat{\Upsilon}_{17} = \langle [0.50, 0.57], [0.52, 0.55], [0.34, 0.56], [0.23, 0.27], [0.45, 0.52], 0.55, 0.53, 0.41, 0.48, 0.51\rangle$$

$$\widehat{\Upsilon}_{18} = \langle [0.85, 0.90], [0.82, 0.87], [0.56, 0.89], [0.07, 0.09], [0.10, 0.15], 0.87, 0.84, 0.40, 0.17, 0.12\rangle$$

$$\widehat{\Upsilon}_{19} = \langle [0.25, 0.35], [0.25, 0.30], [0.17, 0.33], [0.35, 0.38], [0.65, 0.75], 0.38, 0.33, 0.43, 0.68, 0.78\rangle$$

$$\widehat{\Upsilon}_{20} = \langle [0.40, 0.44], [0.39, 0.45], [0.26, 0.45], [0.28, 0.31], [0.55, 0.59], 0.42, 0.38, 0.38, 0.63, 0.46\rangle$$

$$\widehat{\Upsilon}_{21} = \langle [0.55, 0.59], [0.53, 0.57], [0.36, 0.58], [0.22, 0.24], [0.40, 0.45], 0.52, 0.51, 0.38, 0.50, 0.38\rangle$$

$$\widehat{\Upsilon}_{22} = \langle [0.53, 0.57], [0.55, 0.60], [0.36, 0.59], [0.20, 0.23], [0.43, 0.47], 0.52, 0.56, 0.43, 0.45, 0.61\rangle$$

$$\widehat{\Upsilon}_{23} = \langle [0.51, 0.57], [0.50, 0.56], [0.34, 0.57], [0.22, 0.25], [0.45, 0.47], 0.52, 0.53, 0.40, 0.48, 0.47\rangle$$

$$\widehat{\Upsilon}_{24} = \langle [0.35, 0.40], [0.32, 0.38], [0.22, 0.39], [0.31, 0.34], [0.60, 0.65], 0.40, 0.38, 0.40, 0.63, 0.60\rangle$$

$$\widehat{\Upsilon}_{25} = \langle [0.23, 0.29], [0.25, 0.30], [0.16, 0.30], [0.35, 0.38], [0.75, 0.81], 0.25, 0.27, 0.41, 0.74, 0.78\rangle$$

$$\widehat{\Upsilon}_{26} = \langle [0.37, 0.40], [0.35, 0.42], [0.24, 0.41], [0.29, 0.33], [0.62, 0.65], 0.38, 0.36, 0.36, 0.65, 0.43\rangle$$

$$\widehat{\Upsilon}_{27} = \langle [0.41, 0.45], [0.40, 0.47], [0.27, 0.46], [0.27, 0.30], [0.50, 0.55], 0.42, 0.46, 0.39, 0.55, 0.52\rangle$$

$$\widehat{\Upsilon}_{28} = \langle [0.40, 0.50], [0.45, 0.55], [0.28, 0.53], [0.23, 0.28], [0.40, 0.50], 0.45, 0.50, 0.38, 0.51, 0.45\rangle$$

Results and discussion

The accuracy for each PNS in each city is measured using score function. These values are compared with moderate, sensitive group, unhealthy and hazard. For better understanding, graphical presentation is given for each city.

From Fig. 2, it is clearly observed that the index of PM2.5 index in air lies in unhealthy part, above sensitive mark for most of days in the month of October 2021. The linear index of PM2.5 also indicates that the air of Peshawar city is harmful for sensitive individuals, elders and children. Precautionary measure like people with respiratory or heart disease, the elderly and children should limit prolonged exertion.

From Fig. 3, it is clearly observed that the index of PM2.5 index in air lies in unhealthy part, above sensitive mark for most of days in the month of October 2021. The linear index of PM2.5 also indicates that the air of Lahore city is harmful for sensitive individuals, aged and children. Precautionary measure like people with heart, respiratory disease, the aged and children should avoid prolonged outdoor activities.

From Fig. 4, it is clearly observed that the index of PM2.5 index in air lies in unhealthy part, above sensitive mark for most of days in the month of October 2021. The linear index of PM2.5 also indicates that the air of Karachi city is harmful

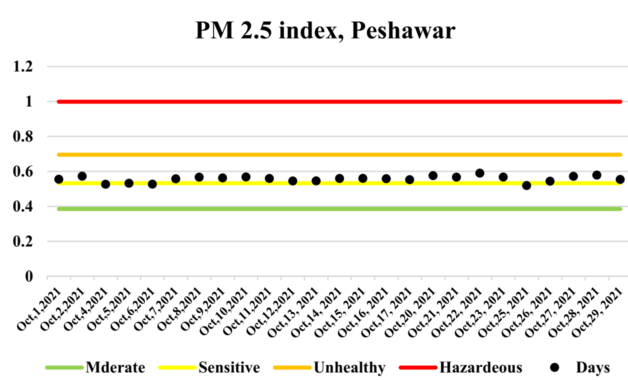


Fig. 2 The graphical interpretation of PM2.5 index in Peshawar

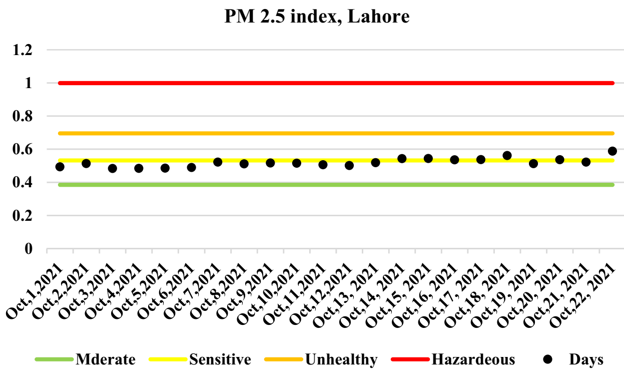


Fig. 3 The graphical interpretation of PM2.5 index in Lahore

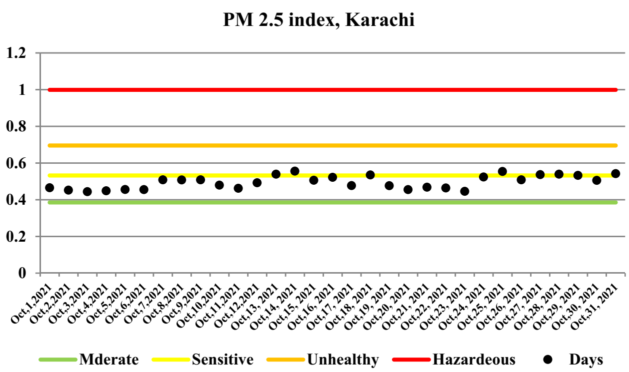


Fig. 4 The graphical interpretation of PM2.5 index in Karachi

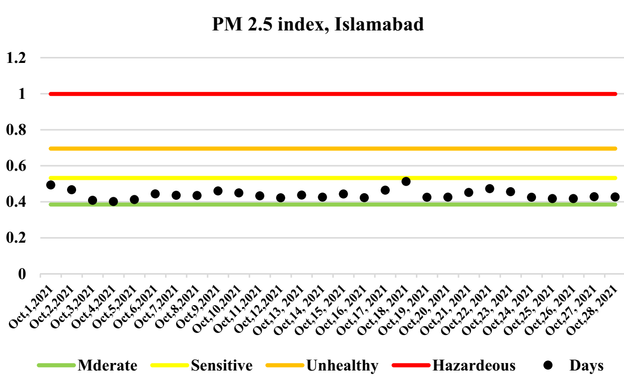


Fig. 5 The graphical interpretation of PM2.5 index in Islamabad

for sensitive individuals. Precautionary measure like people with heart, respiratory disease, the aged and children should avoid prolonged outdoor activities.

From Fig. 5, it is clearly observed that the index of PM2.5 index in air lies in unhealthy part, above sensitive mark for most of days in the month of October 2021. The linear index of PM2.5 also indicates that the air of Islamabad city is harmful for sensitive individuals, elders and children. Precautionary measure like people with heart, respiratory disease, the aged and children should avoid prolonged outdoor activities.

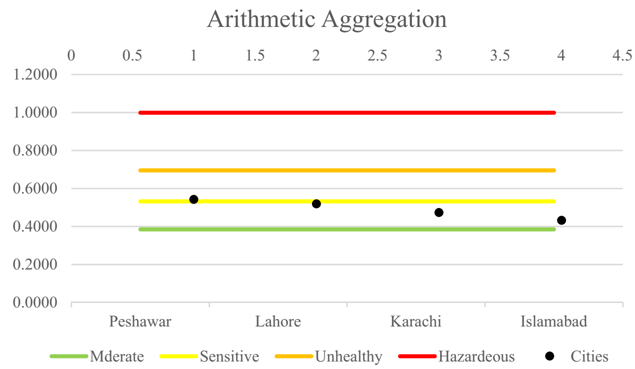


Fig. 6 The graphical interpretation of arithmetic aggregated values of PM2.5 index in all cities

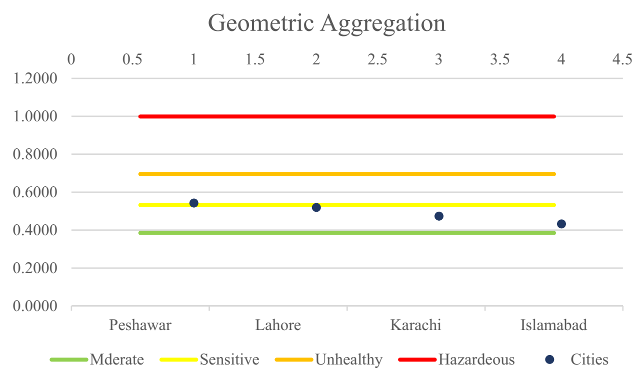


Fig. 7 The graphical interpretation of geometric aggregated values of PM2.5 index in all cities

The PNC values of each city is aggregated using PNCWA and PNCWG aggregation operator.

The arithmetic aggregated value is presented in Fig. 6.

The geometric aggregated value is presented in Fig. 7.

Both the Fig. 6 represent the aggregated values (PNCWA and PNCWG) for the data collected in the respective city in mentioned dates. The aggregated values are considered the aggregated value for the month of October 2021. Both figures indicate the result almost similar which indicates that aggregation operators evaluated the value in better manner. The aggregated value of Peshawar and Lahore indicates that the PM 2.5 index is above the sensitive mark that is the air of city is highly polluted with PM 2.5 which is harmful for natives. In this range the air pollution may cause increase of heart or lung disease and untimely deaths in aged and persons with cardiopulmonary disease; increased respiratory disease in general inhabitant. Precautionary measures like people with heart or respiratory disease, the aged and children should avoid prolonged outdoor activities; everybody else should limit prolonged outdoor activities. The aggregated value of Karachi indicates that the PM 2.5 index is below the sensitive mark, which is harmful for natives having allergic, heart and lung disease. In this range, the air pollution may cause increase of heart or lung disease. Precautionary measures of this group of persons with respiratory or heart disease, should avoid prolonged outdoor activities. The aggregated value of Islamabad indicates that the PM 2.5 index is above the moderate mark and lies in sensitive group, which is harmful for natives having allergic, heart and lung disease. In this range the air pollution may cause increase of heart or lung disease. Precautionary measures of this group of people with respiratory or heart disease, should avoid prolonged outdoor activities.

Comparison in cities

Figures 5 and 6 represent the PNCWA and PNCWG values of Peshawar, Lahore, Karachi and Islamabad, indicates that in the month of October 2021, the PM2,5 index in air of Peshawar city is on border of unhealthy group which alarming. Whereas in Lahore, the situation is like that of Peshawar with marginal limit. Both Peshawar and Lahore air pollution are alarming and need precautionary measure to be taken. The Karachi city air is also polluted but sensitive group of people may be affected. Islamabad has a better aggregated value as compared to other cities, but value is above the moderate.

Conclusion

This paper proposed PNCS along with some operational laws. The score functions are defined to compare PNC values. PNCWA and PNCWG aggregation operators are defined. The PM 2.5 is considered in city of Peshawar, Lahore, Karachi and Islamabad in the month of October 2021. These values are evaluated using score function and then plotted subjected to indicators moderate, sensitive, unhealthy and hazard. The effects on health and precautionary measure needed are discussed for each city. Then, these values are aggregated to get a single value for each city. The comparison among the cities is discussed.

Author contributions All authors contributed equally.

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Code availability Not applicable.

Declarations

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