Stability Analysis of the Model-Based Networked Control System with Unreliable Links

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Abstract. This paper studies the system stability of a Model-Based Networked Control System (MB-NCS). In the MS-NCS, the sensors send information through the network to update the model state. The system estimation error is reset when the packets are received. We define the maximum interval between the received packets as the maximum update time (MUT) while assuming the update frequency is constant. In practice, packet drops randomly. In this work, we assume that intervals between the received packets follow Poisson Distribution. The result shows that the system is stable if the expected interval is less than MUT. This result is verified in simulations.

Keywords: Packet Loss Distribution, Model-Based Networked Control System, System Stability, Poisson Packet Drop

1 Introduction

In the past decade, networked control systems (NCS) have gained great attention in control system theory and design. The term NCS is used to describe the combined system of controllers, actuators, sensors and the communication network that connects them together. Compared with traditional feedback control systems, NCS reduces the system wiring, make the system easy to operate, maintain and diagnose in case of malfunctioning. In spite of the great advantages that the networked control architecture brings, inserting a communication network between the plant and the controller introduces many problems as well. Constrains have been brought in as the information must be exchanged according to the rules and dynamics of the network. Network induced delays are unavoidable because of the scheduling schemes. Communication link failures cause the information flow between the controller and the plant to be disrupted. Packets may also be lost due to insufficient process power, bus capacity in the end machines or by congestion in routers on the link. Time delays and packet drops deteriorate the networked control system performance. In [1-2], system stability has been studied while network time delays are considered. Gupta et al [3] investigated the system performance with packet drops, and concluded that packet drops degrade a system's performance and possibly cause system instability.

Yook et al [4] used state estimator techniques to reduce the communication volume in a networked control system.

It is important to develop the understanding of how much loss the control system can tolerate before the system becomes unstable. Spencer et al [5] stated that by experiments the assumption of Poisson statistics for the distribution of packet loss is a good approximation. Packet loss should be kept to less than certain rates to avoid loss of synchronization. Montestruque [6] proposed a Model-based NCS, and provided the necessary and sufficient conditions for stability in terms of the update time and the parameters of the plant and its model, assuming that the frequency at which the network updates the state in the controller is constant. Teng et al [7] modeled the unreliable nature of the network links as a stochastic process, and assume that this stochastic process is independent of the system initial condition and the plant model state is updated with the plant state at the time when packet arrives. A model for the model-based NCS is built up and a new system matrix is obtained regarding the intervals between the arrived packets following random distributions. The result shows that the system is stable as long as the system error is reset within the maximum update time. Apparently, it is very conservative.

If the statistical description of the link failure process is given a priori, a problem of interest is to determine the optimal control and estimation policies under the link failure constrains. In the authors' best knowledge, the packet drop distribution has not been fully investigated. In NCS, we consider the communication between the sensor and the controller or estimator is subject to unpredictable packet loss. We assume that packet drops obey Poisson distribution. This work studied how the packet drops affect on the system stability in terms of random distribution.

This paper is organized as follows. In section 2, system stability is analyzed in the cases where packet drops follow Poisson distribution. In section 3, example is provided to verify our conclusion. Conclusion is drawn in section 4.

2 Paper Preparation

A model-based control system in Fig. 1 is considered. The system dynamics are given by:

Plant:

$$x(n+1) = Ax(n) + Bu(n) . \tag{1}$$

Model:

$$\hat{x}(n+1) = \hat{A}\hat{x}(n) + \hat{B}u(n) . \tag{2}$$

Controller:

$$u(n) = L\hat{x}(n) . \tag{3}$$

where x(n) is the plant state vector, A and B are system parameter matrices, $\hat{x}(n)$ is the estimate of the plant state, $\hat{A}$ and $\hat{B}$ are the model matrices, L is the controller feedback gain matrix. We define the modelling error matrices $\tilde{A} = A - \hat{A}$, $\tilde{B} = B - \hat{B}$.

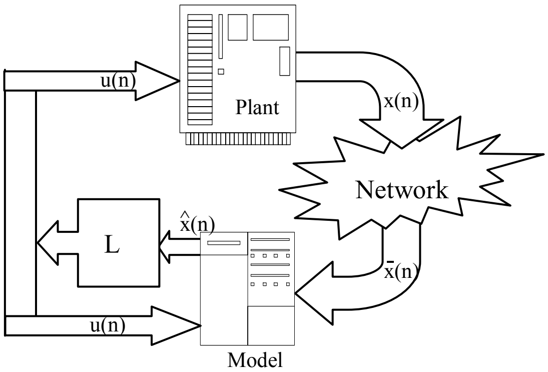

Fig. 1 Model-Based Networked Control System

The stochastic process $\{\gamma_n\}$ models the unreliable nature of the network links. $\gamma_n = 0$, when the packet is not received. $\gamma_n = 1$, otherwise. γ_n takes value 0 with small probability α, and γ_n takes value 1 with big probability $1-\alpha$. α is a known constant. We assume that γ_n is independent of the initial condition, x(0). The vector $\bar{x}(n)$ is current state x(n) if a packet is received. $\bar{x}(n) = 0$, otherwise. That gives us the following equation:

$$\bar{x}(n) = \gamma_n x(n). \tag{4}$$

We define the state error as:

$$e(n) = \bar{x}(n) - \hat{x}(n). \tag{5}$$

The frequency at which the network updates the state is not constant. We assume that the intervals obey Poisson distribution. The plant model state $\hat{x}(n)$ be updated with plant's state $\bar{x}(n)$, at every n_k, where $n_k - n_{k-1} = h_k$, h_k is the interval between the received packets, k=0, 1, 2, ... Then, $e(n_k) = 0$.

Now we can write the evolution of the closed loop NCS,

$$\begin{pmatrix} x(n+1) \\ e(n+1) \end{pmatrix} = A(\gamma_n) \begin{pmatrix} x(n) \\ e(n) \end{pmatrix}. \tag{6}$$

where

$$A(\gamma_n) = \begin{cases} A_0 = \begin{pmatrix} A & -BL \\ 0 & \hat{A} + \hat{B}L \end{pmatrix}, & \gamma_n = 0 \\ A_1 = \begin{pmatrix} A + BL & -BL \\ \tilde{A} + \tilde{B}L & \hat{A} - \tilde{B}L \end{pmatrix}, & \gamma_n = 1 \end{cases}. \quad (7)$$

We modeled the system as a set of linear systems, in which the system jumps from one mode representing by A_0 to another representing by A_1. We define matrix Λ as the function of A_0, A_1 and α:

$$\Lambda = \alpha A_0 + (1 - \alpha) A_1 . \quad (8)$$

We define $z(n) = \begin{pmatrix} x(n) \\ e(n) \end{pmatrix}$, (6) can be represented by

$$z(n+1) = \Lambda z(n) . \quad (9)$$

Theorem 1: The system described by (8) is globally exponentially stable around the solution, if the eigenvalues of $\begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \Lambda^{\tau} \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$ are inside the unit circle. τ represents the expected interval between the received packets.

3 Simulation

To verify our conclusion, a simple control system is used to estimate the system response and test the system stability in case of Poisson packet drops. A full state feedback is given by:

$$x(n+1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} x(n) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(n) .$$

$$u(n) = \begin{pmatrix} -1 & -2 \end{pmatrix} \hat{x}(n) .$$

$$\hat{x}(n+1) = \begin{pmatrix} 1.3626 & 1.6636 \\ -0.2410 & 1.0056 \end{pmatrix} \hat{x}(n) + \begin{pmatrix} 0.4189 \\ 0.8578 \end{pmatrix} u(n) .$$

We have two matrices,

$$A_0 = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0.9437 & 0.8258 \\ 0 & 0 & -1.0988 & -0.7100 \end{pmatrix}$$

and

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$$A_1 = \begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & -1 & 1 & 2 \\ 0.0563 & 0.1742 & 0.9437 & 0.8258 \\ 0.0988 & -0.2900 & -0.0988 & 1.2900 \end{pmatrix}.$$

Based on [6-7], assuming that there is no packet dropout and the frequency at which the network updates the state is constant, the magnitude of the maximum eigenvalues of $\begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \Lambda^h \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$ against update time h as shown in Figure 2. We define the maximum interval between the received packets as maximum update time (MUT). From Fig. 2 it can be seen that MUT is 4.

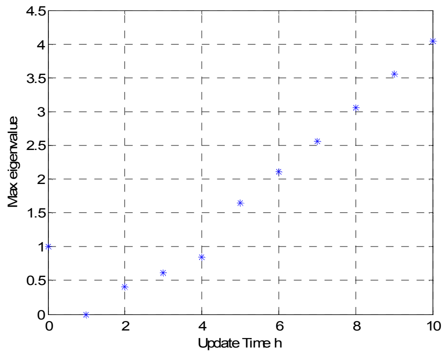

Fig. 2 The Plot of Magnitude of the Maximum eigenvalues of the Test Matrix

In this work, we use the same system and assume that packet drops obey Poisson distribution. The system jumps from one mode with h≥MUT, representing by A_0 to another mode with h<MUT, representing by A_1.

Fig. 3 shows the plots of the system responses with initial condition $z(0) = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ in case the packet intervals are attributed to Poisson Distribution with mean values of $\tau < MUT$. Using Matlab function poissrnd to generate Poisson random numbers with mean $\tau = 3$ as follows: 4, 3, 4, 3, 5, 1, 4, 4, 3, 4, 4, 3, 4, 4, 2, 2, 3, 2, 6, 2, 3, 7, 7, 4, 5, 5, 2, 3, 1, 2. From the graphics in Figure 3, it can be seen the system is stable.

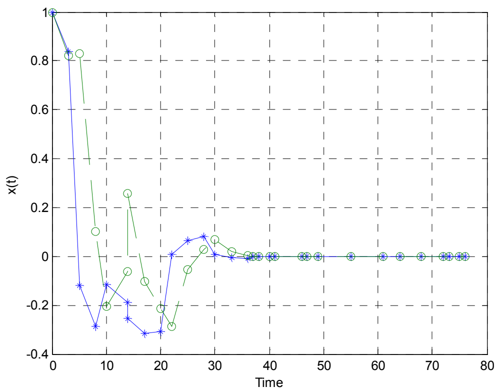

Fig. 3 The System Response with Bernoulli Distribution Packet Intervals

4. Conclusion

In this paper, the stability problem in Model-Based Networked Control System (MB-NCS) with unpredictable packet drops has been investigated. In MB-NCS, the sensors send information through the network to update the model state. The system estimation error is reset when the packets are received. We define the maximum interval between the received packets as the maximum update time (MUT) while assuming the update frequency of the model is constant. If the frequency at which the network updates the model state is constant, and the update time is less than MUT, the system is stable. In practice, packet drops randomly. We modeled the unreliable nature of network links as a stochastic process. In our previous work, the system is stable if the model state is updated within MUT. In this work, we assume that packet losses follow Poisson Distribution. The result shows that the system is stable if the expected interval between the received packets is under MUT. This conclusion is demonstrated in examples at the end.

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