AN INVESTIGATION INTO THE FERTILIZER PARTICLE DYNAMICS OFF-THE-DISC

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ABSTRACT. *The particle size range specifications for two biosolids-derived organomineral fertilizers (OMF) known as OMF_{10} (10:4:4) and OMF_{15} (15:4:4) were established. Such specifications will enable field application of OMF with spinning disc systems using conventional tramlines spacing. A theoretical model was developed, which predicts the trajectory of individual fertilizer particles off-the-disc. The drag coefficient (C_d) was estimated for small time steps (10^{-6} s) in the trajectory of the particle as a function of the Reynolds number. For the range of initial velocities (20 to 40 m s^{-1}), release angles (0° to 10°) and particle densities (1000 to 2000 kg m^{-3}) investigated, the analysis showed that OMF_{10} and OMF_{15} need to have particle diameters between 1.10 and 5.80 mm, and between 1.05 and 5.50 mm, respectively, to provide similar spreading performance to urea with particle size range of 1.00 to 5.25 mm in diameter. OMF_{10} and OMF_{15} should have 80% (by weight) of particles between 2.65 and 4.30 mm, and between 2.55 and 4.10 mm, respectively. Due to the physical properties of the material, disc designs and settings that enable working at a specified bout width by providing a small upward particle trajectory angle (e.g., 10°) are preferred to high rotational velocities. However, field application of OMF with spinning discs applicators may be restricted to tramlines spaced at a maximum of 24 m; particularly, when some degree of overlapping is required between two adjacent bouts. The performance of granular fertilizers can be predicted based on properties of the material, such as particle density and size range, using the contour plots developed in this study.*

Keywords. *Biosolids granules, Fertilizer particle trajectory, Fertilizer spreading, Landing distance, Organomineral fertilizers (OMF), Particle density, Particle diameter, Urea.*

Uneven spreading of fertilizers affects the overall performance of crops, reduces fertilizer use efficiency and profit margins due to loss of crop yield and quality, and increases the risk of nutrients losses to the environment (Jensen and Pesek, 1962a,b; Dilz and Van Brakel, 1985; Van Meirvenne et al., 1990; Søgaard and Kierkegaard, 1994; Miller et al., 2009). Inaccurate fertilizer spreading can result from improper application rates or non-uniform spreading, which requires that the optimum rate is determined and delivered correctly (Richards and Hobson, 2013). The components of the application system with performance targets relating to delivery rate and uniformity of distribution include the following (after Miller, 1996): **a.** Machine design (Olieslagers et al., 1996), settings, calibration and maintenance (Bull and Crowe, 1985), **b.** Physical and chemical properties of the fertilizer material (Hofstee, 1993), and **c.** Weather conditions during fertilizer spreading, particularly, wind speed, which influences particles' trajectory, and relative air humidity, which influences the behavior of the fertilizer material (Svenssen, 1994).

In the United Kingdom, the most popular fertilizer applicator is the spinning disc type spreaders (about 70% of total) (DEFRA, 2013) whose main advantages are low capital and operating costs, robust construction, and simplicity of operation, and ability to work at relatively wide tramline spacing with a range of fertilizer materials (Davis and Rice, 1973; Aphale et al., 2003; Dampney et al., 2003). Theoretical concepts relating to centrifugal distributors have been studied in detail e.g., Cunningham and Chao (1967), Inns and Reece (1962), Patterson and Reece (1963), Mennel and Reece (1963), Olieslagers et al. (1996), and Dintwa et al. (2004). Due to difficulties commonly encountered in trying to predict accurately the behavior of fertilizers on the surface of the disc, particularly, the effects of contact material-material, much of the practical aspects of design of spinning disc systems are empirical (Dampney et al., 2003). The study of aerodynamic properties of fertilizer materials and the interaction fertilizer-spreader has received considerable attention, e.g., Bilanski et al. (1962), Mennel and Reece (1963), Reints and Yoerger (1967), Grift et al. (1997), and Lawrence and Yule (2007). Research has focused on

theoretical models to study particle trajectories on- and off-the-disc while experimental work has also been conducted using 'ideal' particles or reduced number of granular materials (Aphale et al., 2003). Pitt et al. (1982) derived approximating equations for particle trajectory, which enable estimating their landing point depending on initial velocity and height. A comprehensive review of the early research was conducted by Hofstee and Huisman (1990), and Hofstee (1992, 1994) who investigated physical properties of fertilizers relating to particle dynamics. The basic principle governing the functioning of spinning disc spreaders is that the fertilizer is first discharged onto a spinning plate and it moves outward under the action of centripetal forces until particles reach the vanes (Dampney et al., 2003). Subsequently, particles are displaced along the vanes leaving the edge of the disc with velocity and trajectory that depend on a number of parameters, importantly, rotational speed, disc diameter, and disc and vane geometry (Olieslagers et al., 1996; Grift and Hofstee, 1997; Dampney et al., 2003). The terminal velocity of a particle at the instant at which it leaves the disc includes both radial and tangential velocity components (Patterson and Reece, 1963; Aphale et al., 2003). Patterson and Reece (1963) concluded that in practice, fertilizer particles leave the disc with a wide range of velocities and directions, which result in random variation in the performance of the spreader. For twin discs spreaders, and depending on the factors listed above, this velocity can be in the range of 20 to 40 m s^{-1} given the particles diameters commonly found in mineral fertilizers (Mennel and Reece, 1963; Hofstee, 1993, 1995; Miller, 1996; Grift and Hofstee, 2002; Miller and Parkin, 2005; Parkin et al., 2005). However, higher velocities (e.g., 40 to 70 m s^{-1}) are also reported (Persson, 1996; Grift et al., 1997). The trajectory of a fertilizer particle off-the-disc is dependent on its velocity and direction when leaving the disc, which determines the point of landing of the particle on the ground (Olieslagers et al., 1996). The fertilizer spread pattern may be widened by increasing the diameter of the disc and the length of vanes, by increasing the rotational velocity of the disc (Dampney et al., 2003) or by changing the point at which fertilizer is discharged on the disc e.g., near- or off-center feed (Inns and Reece, 1962; Patterson and Reece, 1963; Persson, 1996; Grift and Kweon, 2006).

Studies (Antille, 2011; Antille et al., 2013c) with biosolids-derived organomineral fertilizers (OMF) indicated the need to determine the suitability of OMF for application with standard fertilizer spreading equipment, such as spinning disc systems. There is also a need to determine whether field application of OMF can be satisfactorily performed using tramline spacing considered to be typical of grain cropping systems in the United Kingdom (e.g., 18 or 24 m), which are compatible with most mineral fertilizers and fertilizer spreading equipment. Since OMF has only been produced in relatively small quantities for experimental purposes (Antille et al., 2013b, 2014a,b,c), it has not been possible to conduct full-scale spreading tests with twin discs spreaders. However, Antille (2011) reported satisfactory results from distribution uniformity and machine calibration tests conducted with

OMF using a pneumatic fertilizer applicator Kuhn 2212 (Kuhn, 2014). Despite that the physical characteristics (particle size and size distribution) of the OMF used in Antille (2011) were relatively poor, the pneumatic applicator performed well (CV =12.4%) when delivering an application rate equivalent to 455 kg ha^{-1} of OMF, which was uniform across the treated swath and along the tramline. Results from Antille (2011) demonstrated the suitability of OMF for application with pneumatic applicators, however further work is required to determine the particle size range specifications that enable satisfactory application with spinning discs systems.

Transverse tray testing (e.g., ISO, 1985; *ASAE Standards*, 1999) are reliable means of determining distribution patterns and the interaction of machine components on fertilizer particle distribution but are difficult and time-consuming to perform in on-farm situations (Miller, 1996; Lawrence and Yule, 2007). Such tests may require the use of indoor facilities to isolate from the influence of environmental conditions, which makes them costly (Grift et al., 1997; Walker et al., 1997). Several studies (e.g., Bull and Crowe, 1985; Miller, 1996; Richards and Hobson, 2013) have indicated that fertilizer spreaders are often used without being calibrated for the material to be applied. Therefore, the ability to determine the landing position of fertilizer particles prior to conducting field operations is an important practical consideration in achieving uniform distribution patterns from spinning disc systems (Dampney et al., 2003). The point of landing of a particle on the ground can be estimated from physical properties of the fertilizer material and the media, which is valuable to parameterize the spreading behavior of such materials with differing diameters and particle densities (Parkin et al., 2005).

OBJECTIVES

The objectives of this work were to: (1) develop a theoretical model to investigate the trajectory of individual fertilizer particles off-the-disc to determine the travel distance when particles are projected from a spinning disc system based on physical properties of the material; and (2) determine the required particle size range for biosolids-derived organomineral fertilizers (OMF) reported in earlier studies (Antille, 2011; Antille et al., 2013c) that may enable field application with spinning disc systems using conventional tramline spacing. An advantage of the proposed method is that it requires a reduced number of readily available input parameters, and that it can be used to pre-assess the behavior of fertilizer materials using the software specially developed, which can be accessed with this article from the ASABE Technical Library (https://elibrary.asabe.org/). Instructions to operate the software are given in the Appendix.

THEORY

NOTATION
α_0 = launch angle (rad);
d = particle diameter (m);

C_d = drag coefficient;
D = drag force [modulus] (N);
g = gravity acceleration (m s^{-2});
m = mass (kg);
p = number of points in the integration interval;
r = particle radius (m);
Re = Reynolds number;
S = frontal projected area (m^2);
t = time (s);
v = velocity of the particle (m s^{-1});
v_0 = initial velocity (m s^{-1});
Δt = time step (s);
ρ_a = air density at 15°C (1.225 kg m^{-3});
ρ_p = particle density (kg m^{-3});
μ = dynamic viscosity of the air; (kg m^{-1} s^{-1}).

Figure 1 shows the trajectory and forces acting on a fertilizer particle launched from a spinning disc system under conditions of still air with friction. These forces are proportional to the characteristics of the particle (particle mass, frontal projected area, and drag coefficient), instantaneous velocity, and air density (Grift et al., 1997). A simplification of the analysis is usually made by regarding fertilizer particles as spherical (symmetrical), which is considered to be a fair assumption for most particle shapes commonly spread with spinning disc systems (Mennel and Reece, 1963). When the particle is launched from a height (h_0) and angle (α_0) immerse in air, it is subjected to the action of gravity (g) and drag force (D) that acts in the direction of velocity (v) and opposite to it.

Newton's momentum equation applied to the particle and projected on the parallel (x) and perpendicular (y) axes to the ground yields:

$$-D\cos\alpha = m\frac{d^2x}{dt^2} = m\,\ddot{x} \qquad (1)$$

$$-D\sin\alpha - g = m\frac{d^2y}{dt^2} = m\,\ddot{y} \qquad (2)$$

The following cinematic equation applies:

$$\frac{dy}{dx} = \tan\alpha = \frac{dy}{dt}\frac{dt}{dx} = \frac{\dot{y}}{\dot{x}} \qquad (3)$$

In aerodynamics, air drag is given by:

$$D = \frac{1}{2}\rho C_d S v^2 = \frac{1}{2}\rho C_d S\left(\dot{x}^2 + \dot{y}^2\right) \qquad (4)$$

Since velocity, as defined by its components, is:

$$v = \sqrt{\left(\dot{x}^2 + \dot{y}^2\right)} \qquad (5)$$

The drag coefficient (C_d) is an empirical number which, to a first order, is a function of the Reynolds number (Re) and the shape of the particle (Eisner, 1930):

$$C_d = f\left\{Re,\ shape\right\} \qquad (6)$$

Since:

$$Re = Re\left(\dot{x}, \dot{y}\right) = Re\left(t\right) \qquad (7)$$

Therefore,

$$C_d = f\left\{Re\left(\dot{x}, \dot{y}\right), shape\right\} \qquad (8)$$

The Reynolds number (Re) is given by:

$$Re = \frac{\rho d v}{\mu} = \frac{\rho d}{\mu}\sqrt{\left(\dot{x}^2 + \dot{y}^2\right)} \qquad (9)$$
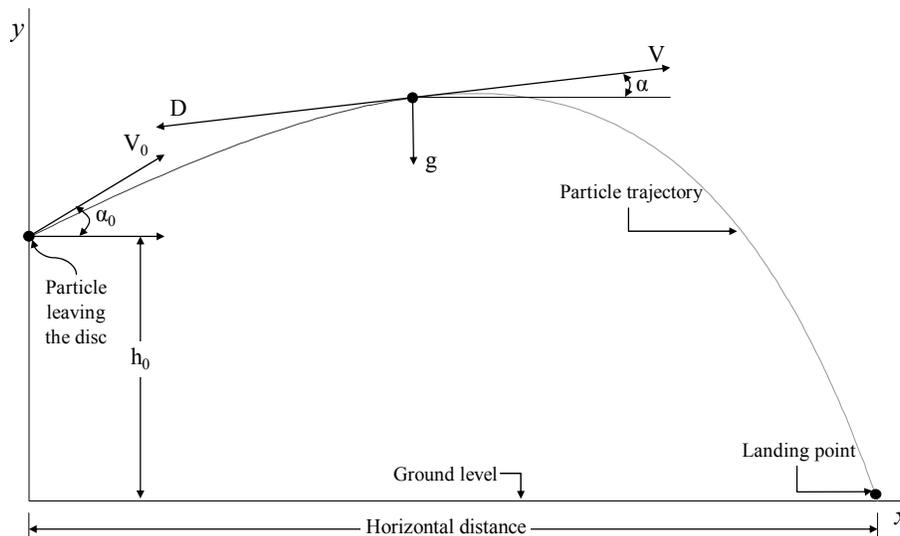
Figure 1. Trajectory and forces acting on a fertilizer particle after leaving a spinner disc.

By replacing 4 in 1 and 2, it results that:

$$\ddot{x} = -\frac{1}{2m}\rho C_d S\left(\dot{x}^2 + \dot{y}^2\right)\cos\left(a\tan\frac{\dot{y}}{\dot{x}}\right) \tag{10}$$

And,

$$\ddot{y} = -\frac{1}{2m}\rho C_d S\left(\dot{x}^2 + \dot{y}^2\right)\sin\left(a\tan\frac{\dot{y}}{\dot{x}}\right) - \frac{g}{m} \tag{11}$$

where

$$C_d\left(\dot{x}, \dot{y}\right) \tag{12}$$

The system given in equations 10 and 11 is non-linear with second order differential equations. By introducing a change of variables as shown in equations 13 and 14, respectively, this can be reduced to a non-linear system of first order differential equations, which is shown in equations 15 and 16, respectively.

Therefore:

$$\frac{dx}{dt} = \xi \tag{13}$$

And,

$$\frac{dy}{dt} = \eta \tag{14}$$

Then,

$$\dot{\xi} = -\frac{1}{2m}\rho C_d S\left(\xi^2 + \eta^2\right)\cos\left(a\tan\frac{\eta}{\xi}\right) \tag{15}$$

And,

$$\dot{\eta} = -\frac{1}{2m}\rho C_d S\left(\xi^2 + \eta^2\right)\sin\left(a\tan\frac{\eta}{\xi}\right) - \frac{g}{m} \tag{16}$$

where

$$C_d\left(\xi, \eta\right) \tag{17}$$

C_d depends on the air flow around the particle and its geometrical characteristics (Mennel and Reece, 1963). The characteristics of this flow and the ratio of the resulting drag force due to inertia and fluid's viscosity are described by Re (Mennel and Reece, 1963). Re, as defined in equation 9, can be expressed in the form shown in equation 18:

$$Re = \frac{2\rho r v}{\mu} = \frac{2\rho r}{\mu}\sqrt{\left(\xi^2 + \eta^2\right)} \tag{18}$$

The relationship between C_d and Re is complex because of the velocity (Parkin et al., 2005). Mennel and Reece

(1963) simplified this relationship to two straight lines for Re between 10 and 10000, regarding $C_d = 0.44$ for turbulent flow ($Re>500$), $C_d = 18.5 \times Re^{-0.6}$ for the transition region from turbulent to laminar flow ($1<Re<500$), and $C_d = 24 \times Re^{-1}$ for laminar flow ($Re<1$). Parkin et al. (2005) used a similar scheme based on Douglas et al. (1995). Grift and Hofstee (2002), and Aphale et al. (2003) used a constant C_d, which was considered to be a fair assumption given the range of Re typically found by fertilizer particles (turbulent flow) and indicated that with non-spherical particles travelling through the air at high velocities, the transition to turbulent flow occurs at relatively low Re numbers. Grift et al. (1997) regarded C_d as constant for small intervals in the trajectory of the particle and calculated it for each of these intervals based on the approach of Von-Zabeltitz (1967) for the transitional region. The approach presented in our study uses a C_d which is calculated for every instant (10^{-6} s) in the trajectory of the particle, as a function of Re, so that it can be computed more accurately. The value of Re is not constant as it depends on the velocity of the particle, which also changes with time in the particle's trajectory (eq. 7). Equations 15 and 16 can be solved by imposing the initial velocity (v_0) and the angle (α_0) at which the particle is launched from the edge of the disc as boundary conditions.

Therefore:

$$\xi(t = 0) = v_0 \cos\alpha_0 \tag{19}$$

And,

$$\eta(t = 0) = v_0 \sin\alpha_0 \tag{20}$$

Equations 15 and 16 are coupled and do not admit primitives in terms of elementary functions. A convenient method to solve this system is by employing the numerical *Euler* scheme, which is convergent and zero-stable for sufficiently small time steps (Δt). This scheme produces the solution for the instant ($n+1$) from the solution in the previous instant (t).

Therefore:

$$t^{n+1} = t^n + \Delta t \tag{21}$$

The general form of the scheme is:

$$u^{n+1} = u^n + \Delta t F^n \tag{22}$$

For a differential equation in the form of:

$$\frac{du}{dt} = F(u, t) \tag{23}$$

For this particular system of equations, the solution in the instant ($n+1$) is given by:

$$\xi^{n+1} =$$
$$\xi^n - \Delta t\left(\frac{1}{2m}\rho C_d^n S\left(\left(\xi^n\right)^2 + \left(\eta^n\right)^2\right)\cos\left(a\tan\frac{\eta^n}{\xi^n}\right)\right) \tag{24}$$

and,

$$\eta^{n+1} =$$

$$\eta^n - \Delta t \left(\frac{1}{2m} \rho C_d^n S \left(\left(\xi^n \right)^2 + \left(\eta^n \right)^2 \right) \sin \left(a \tan \frac{\eta^n}{\xi^n} \right) - \frac{g}{m} \right) \quad (25)$$

Equations 24 and 25 provide the velocity field (ξ, η), which must be integrated to obtain the trajectory of the particle. Due to ξ and η being expressed in a discrete form, it is also necessary to conduct the integration numerically. This can be done by applying the trapezoid rule which, in its general form, is given by:

$$\int_a^b f(x)dx \approx \frac{b-a}{p} \left[\frac{f(a)+f(b)}{2} + \sum_{k=1}^{p-1} f\left(a+k\frac{b-a}{p} \right) \right] \quad (26)$$

Then, when applied to the problem under study, ξ and η are integrated as follows:

$$x = \int_0^t \xi dt \approx \frac{t}{p} \left[\frac{\xi(t=0)+\xi(t=t)}{2} + \sum_{k=1}^{p-1} \xi\left(k\frac{t}{p} \right) \right] \quad (27)$$

And,

$$y = \int_0^t \eta dt \approx \frac{t}{p} \left[\frac{\eta(t=0)+\eta(t=t)}{2} + \sum_{k=1}^{p-1} \eta\left(k\frac{t}{p} \right) \right] \quad (28)$$

MATERIALS AND METHODS

FERTILIZER MATERIALS

The spreading characteristics of granular urea (46:0:0) were compared with two biosolids-derived organomineral fertilizers (Antille, 2011; Antille et al., 2013c) known as OMF_{15} (15:4:4) and OMF_{10} (10:4:4), and biosolids granules (4.5:5.5:0.2). Samples corresponding to the three fertilizer types used in this study are shown in figure 2. Physical and chemical properties of urea, OMF_{15}, OMF_{10}, and biosolids granules, and tests conducted to characterize these materials are described in detail in Antille et al. (2013c). Properties relevant to this study are shown in table 1.

MODEL SOLUTION

The proposed method predicts the horizontal distance travelled by individual fertilizer particles from the edge of a spinning disc to the landing point on the ground. The system of equations given earlier was processed with FORTRAN 90. The first part of the analysis calculated landing distances based on the physical properties of the materials reported in table 1, which included particle density, mean particle diameter, and particle diameters corresponding to values of percentiles D_{10}, D_{16}, D_{50}, D_{84}, and D_{90}. The values of percentiles were required to characterize the fertilizer materials (British Standard, 1995). The analysis was conducted for particles leaving the disc assuming height above the ground ($h_0 = 1$ m), launch angles ($\alpha_0 = 0°$ and $10°$), and initial velocities ($v_0 = 20, 30,$ and 40 m s^{-1}) to investigate differences in spreading performance between fertilizer types in the samples analyzed. Such values of parameters (α_0 and v_0) are available in the literature and are considered to be typical of spinning disc systems (Parkin et al., 2005). Subsequently, based on the work of Miller (1996) and Parkin et al. (2005), the relationships between initial velocity (v_0), launch angle (α_0), particle diameter (d) and particle density (ρ_p) were explored further for fixed height above the ground ($h_0 = 1$ m) so that landing distances of individual fertilizer particles were estimated for a range of values of the above parameters (figs. 3 and 4). From this, and based on the study of Parkin et al. (2005), contour plots were developed, which help to overcome difficulties that arise when trying to estimate the spreading performance of fertilizer materials with different physical properties (fig. 5). The variability commonly encountered in particle size and composition of granular materials is discussed in Smith et al. (2005) who reported significantly different particle size distributions for similar fertilizers materials used in practice. Contour plots allow for rapid interpolation of data to determine likely spreading performance of granular fertilizers based on properties, such as particle density and particle diameter that are relatively straightforward to determine. For a specified tramline spacing and fertilizer applicator of known performance, the particle size range and particle density need to be chosen to match the required spreading width or adjust the spreading equipment to achieve the required spreading width with a given fertilizer material (Parkin et al., 2005). These considerations become particularly important in situations where vehicle wheeling is confined to permanent traffic lanes, namely, controlled traffic farming systems (Antille et al., 2013a).

Figure 2. Samples of fertilizer materials used in the study (after Antille, 2011; Antille et al., 2013c).

PARTICLE SIZE RANGE SPECIFICATIONS FOR ORGANOMINERAL FERTILIZERS

A condition was imposed that OMF particles should match the minimum and maximum landing distances achieved with particles of urea to enable application at conventional tramline spacing. For urea, given velocity and angle at the instant at which particles leave the disc, such distances are determined by its mean particle density (ρ_p =1432 kg m^{-3}), and by the smallest (1 mm) and largest (5.25 mm) particle diameters encountered in the sample. A second condition was that particle diameters corresponding to percentiles D_{10} and D_{90} of urea will determine the range of travelling distances within which 80% (by weight) of OMF particles will fall. These conditions will ensure that OMF has a relatively narrow particle size range, which will minimize unwanted effects of granulometric segregation during handling and spreading (Hoffmeister et al., 1964; Bridle et al., 2004). Modifying density properties of OMF is more difficult than selecting a specific particle size range, which is possible during the granulation process of sludge (Antille, 2011). Therefore, the particle size range specifications for OMF$_{10}$ and OMF$_{15}$ were obtained by calculating landing distances for varying particle diameters (all other parameters being constant) until they matched, approximately, the minimum and maximum landing distances achieved with urea. The same approach was applied to obtain particle diameters equivalent to D_{10} and D_{90}. A 50 mm difference in landing distance calculations was allowed between fertilizer materials to yield particle diameters that were multiple of 0.05 mm, and to avoid particle sizes that may not be possible to produce in practice. Since mean particle densities of OMF are significantly lower (P<0.05) compared with urea (table 1), the required particle size range of OMF will produce slightly larger particles diameters when all other input parameters are set constant.

RESULTS AND DISCUSSION

PROPERTIES OF FERTILIZER MATERIALS INFLUENCING SPREADING PERFORMANCE

The measured physical properties of the fertilizer materials used in this study are summarized in table 1 (Antille et al., 2013c). Samples of OMF and biosolids granules exhibited a wider range of particle sizes compared with urea (P<0.05), which is denoted by the values of percentiles. Particle size distribution showed relatively larger variability between samples in OMF and biosolids granules compared with urea, which is denoted by the corresponding standard deviation (SD) values of percentiles. Across all samples (*n*=4), OMF particles ranged between <0.60 mm (up to 6% by weight) and 25 mm (up to 5% by weight) in diameter (Antille et al., 2013c). Such particle size and size distribution will likely affect fertilizer uniformity of distribution during field spreading due to particle segregation (Jensen and Pesek, 1962c; Antille et al., 2013c). Segregation is produced because smaller particles percolate through the voids of the material and are released in turns during spreading according to their relative sizes (Lance, 1996). As a result, the spreading width may be initially narrow and it may widen up progressively as larger particles reach the disc, which will produce inconsistent particle distribution patterns along the tramline (Jensen and Pesek, 1962c; Bradley and Farnish, 2005; Virk et al., 2013). Severe granulometric segregation can occur when Granulometric Spread Index (GSI) is above 25% (Miserque and Pirard, 2004), value that is largely exceeded in samples of OMF and biosolids granules (GSI >40%) presented in table 1 (Antille et al., 2013c). Urea particles ranged between 1 mm (0.01% by weight) and 5.25 mm (0.11% by weight) in diameter, and the mean particle diameter was significantly (P<0.05) smaller than OMF and biosolids granules (Antille et al., 2013c).

Compression tests showed that unlike urea particles, OMF and biosolids granules did not exhibit a characteristic force that induced the breaking of the granule (Antille et al., 2013c). Instead, OMF and biosolids granules deformed permanently when a relatively small vertical load was applied and behaved in a plastic fashion, which was attributed to the moisture content (range of 11% to 17% by weight) and the organic nature of the materials. As highlighted earlier, the spreading width may be increased by increasing disc diameter or rotational velocity, and length of vanes (Dampney et al., 2003). A disadvantage of increasing rotational velocity is that it can lead to shattering because of greater forces exerted on the particles, which could affect uniformity of distribution (Dampney et al., 2003; Miller and Parkin, 2005). For synthetic nitrogen fertilizers, Miller and Parkin (2005) suggest that the velocity of the particle leaving the disc should not be higher than 40 m s^{-1} to reduce the risk of particle shattering.

Table 1. Measured physical properties of fertilizer materials used in the study (after Antille et al., 2013c).

Parameter	Fertilizer Material			
	Urea	OMF$_{15}$	OMF$_{10}$	Biosolids Granules
D_{10} (mm)[a]	2.43	1.97 ±0.97	1.91 ±0.37	2.22 ±2.58
D_{16} (mm)[a]	2.52	2.50 ±1.0	2.40 ±0.6	2.74 ±3.1
D_{50} (mm)[a]	3.03	4.60 ±2.8	4.45 ±2.8	4.87 ±5.6
D_{84} (mm)[a]	3.73	7.24 ±4.4	6.91 ±5.2	7.08 ±7.5
D_{90} (mm)[a]	3.90	9.97 ±4.98	10.47 ±6.78	8.18 ±9.03
Mean particle diameter (*d*, mm)[b]	3.10[a] ±0.35	5.42[b] ±2.6	5.28[b] ±2.9	4.97[c] ±5.50
Mean particle density (ρ_p, kg m^{-3})[b]	1432[a] ±255	1357[b] ±178	1297[b] ±263	1333[b] ±315
Mean bulk density (ρ_b, kg m^{-3})[b]	748[a] ±5.7	603[b] ±73.1	623[b] ±85.5	578[b] ±87.2

[a] D_{10}, D_{16}, D_{50}, D_{84}, and D_{90} are, respectively, values of percentiles corresponding to particle diameters below which 10%, 16%, 50%, 84%, and 90% (by weight) of material is collected after sieving (British Standard, 1995).

[b] Different letters indicate that mean values are significantly different at a 95% confidence interval. The standard deviation (SD) is shown as ± the mean value, except when not shown (*n*=1).

For OMF, high rotational velocities can produce deformation of particles (change in shape), which will affect their aerodynamic properties, as discussed in Walker et al. (1997). Spreader distribution and metering flow performances are influenced by particle shape as it affects particle motion in the distributor (Miller, 1996). Depending on particle diameter, full compression of OMF and biosolids granules was achieved with vertical loads in the range of 18 to 44 N (Antille et al., 2013c). An important feature is that OMF and biosolids granules exhibited multiple failures during the compression tests conducted but particles did not disintegrate into smaller particles, as it was observed with urea when the breaking force was reached (Antille et al., 2013c). Urea granules exhibited breaking forces greater than 15 N, which is the suggested lower limit to avoid particle fracture during handling and spreading (Hignett, 1985). Data from dimensional analysis indicate that the flow of granular materials through circular orifices depends on density properties (Gregory and Fedler, 1987). For straight nitrogen fertilizers, Miller (1996) showed a linear decrease in flow time with increasing bulk density. Hence, higher flow time will be expected with OMF and biosolids granules compared with urea.

SPREADING PERFORMANCE

Table 2 shows landing distance calculations for particles of urea, OMF and biosolids granules corresponding to mean diameter (d) and values of percentiles (D_{10} to D_{90}), based on the physical properties of the fertilizer materials (table 1) and the specified model parameters. The relatively wider particle size range of OMF and biosolids granules results in wider spreading width compared with urea, however, the likely occurrence of particle segregation will lead to inconsistent fertilizer distribution both longitudinally (direction of travel) and transversally (treated swath).

Figure 3a confirms that the horizontal distance travelled by a fertilizer particle will increase with particle diameter and initial velocity, however, the rate of increase in landing distance decreases with increasing particle diameter. Such relationship is influenced by particle density (fig. 3b); however, the effect on landing distance appears to be relatively smaller compared with particle diameter. For a

particle with $d = 3.0$ mm, and given initial velocity and release angle, the landing distance will be greater with urea ($\rho_p \approx 1400$ kg m^{-3}) than biosolids granules ($\rho_p \approx 1300$ kg m^{-3}). Similarly, Parkin et al. (2005) determined that a 25% to 30% reduction in particle density, as it occurs when swapping from ammonium nitrate to granular urea, resulted in about 15% reduction in landing distance, which agrees closely with the data shown in figure 3b. Further calculations demonstrated that depending on initial velocity (range: 20 to 40 m s^{-1}), an increase in particle diameter from 3 to 4 mm results in approximately 10% to 14% increase in landing distance when α_0 is 0°, and between 16% and 23% increase when α_0 is 10°. An increase in particle density (from 1300 to 1700 kg m^{-3}) results in approximately 9% to 15% increase in landing distance when α_0 is 0°, and between 15% and 20% increase when α_0 is 10°. Landing distance calculations shown in figure 3 are in agreement with those reported in Miller (1996) for the range of particle sizes and densities investigated, despite applying a different approach to estimating the drag coefficient (C_d).

Figure 3a also suggests that some spinning disc mechanisms may not be capable of operating at standard tramline spacing (e.g., 24 m) as particles will fall short, particularly, when a small overlapping is required between adjacent bouts. This effect was previously observed by Miller (1996) who calculated similar landing distances with measured initial velocity of about 25 m s^{-1} using a disc of 600 mm in diameter operating at 750 rpm. However, in practice it is possible to modify machine settings to achieve more convenient distances, for example, increase disc rotational speed, disc height above the ground and angle (Miller, 1996). The feasibility of using higher rotational speeds depends on the fertilizer material (particle strength), which may pose a limitation with OMF as discussed earlier. Relatively small changes in launch angles to the horizontal ($\alpha_0 > 0$) produce significant increases in landing distances, as demonstrated in figure 4 for particles of urea with $d = 3$ mm. Further analyses showed that all fertilizer materials reach maximum landing distances with launch angles of 10° or greater but not exceeding 25°. However, there is an interaction between launch angle and initial velocity, which influences landing distance and it depends

Table 2. Calculated landing distances for particles of urea, OMF$_{15}$, OMF$_{10}$, and biosolids based on physical properties reported in table 1.

Parameter			Landing Distance (m)										
Fertilizer Material			Urea			OMF$_{15}$			OMF$_{10}$			Biosolids Granules		
$h_0 = 1$ m, $\alpha_0 = 0°$	v_0 (m s^{-1}) =	20	30	40	20	30	40	20	30	40	20	30	40
D_{10}[a]		5.63	7.35	8.68	4.96	6.35	7.43	4.78	6.08	7.09	5.24	6.76	7.93
D_{16}[a]		5.72	7.48	8.85	5.58	7.27	8.58	5.38	6.96	8.18	5.78	7.56	8.93
D_{50}[a]		6.16	8.15	9.70	6.90	9.28	11.25	6.76	9.06	10.93	6.95	9.40	11.41
D_{84}[a]		6.61	8.82	10.61	7.54	10.46	12.98	7.42	10.24	12.65	7.48	10.36	12.84
D_{90}[a]		6.70	8.96	10.80	7.88	11.15	14.05	7.89	11.15	14.06	7.66	10.70	13.35
d[b]		6.22	8.22	9.81	7.14	9.73	11.90	7.04	9.54	11.62	6.98	9.45	11.50
$h_0 = 1$ m, $\alpha_0 = 10°$	v_0 (m s^{-1}) =	20	30	40	20	30	40	20	30	40	20	30	40
D_{10}[a]		7.01	9.21	10.85	5.87	7.53	8.77	5.58	7.10	8.26	6.33	8.20	9.58
D_{16}[a]		7.19	9.46	11.16	6.94	9.08	10.68	6.58	8.55	10.00	7.29	9.60	11.33
D_{50}[a]		8.05	10.78	12.83	9.75	13.50	16.42	9.42	12.93	15.64	9.92	13.80	16.82
D_{84}[a]		9.03	12.32	14.83	11.58	16.94	21.36	11.23	16.23	20.31	11.43	16.62	20.90
D_{90}[a]		9.24	12.65	15.27	12.77	19.47	25.27	12.78	19.50	25.31	11.98	17.76	23.00
d[b]		8.16	10.95	13.04	10.44	14.73	18.13	10.15	14.20	17.38	10.00	13.94	17.03

[a] D_{10}, D_{16}, D_{50}, D_{84}, and D_{90} are, respectively, values of percentiles corresponding to particle diameters below which 10%, 16%, 50%, 84%, and 90% (by weight) of material is collected after sieving (British Standard, 1995).
[b] d is mean particle diameter.
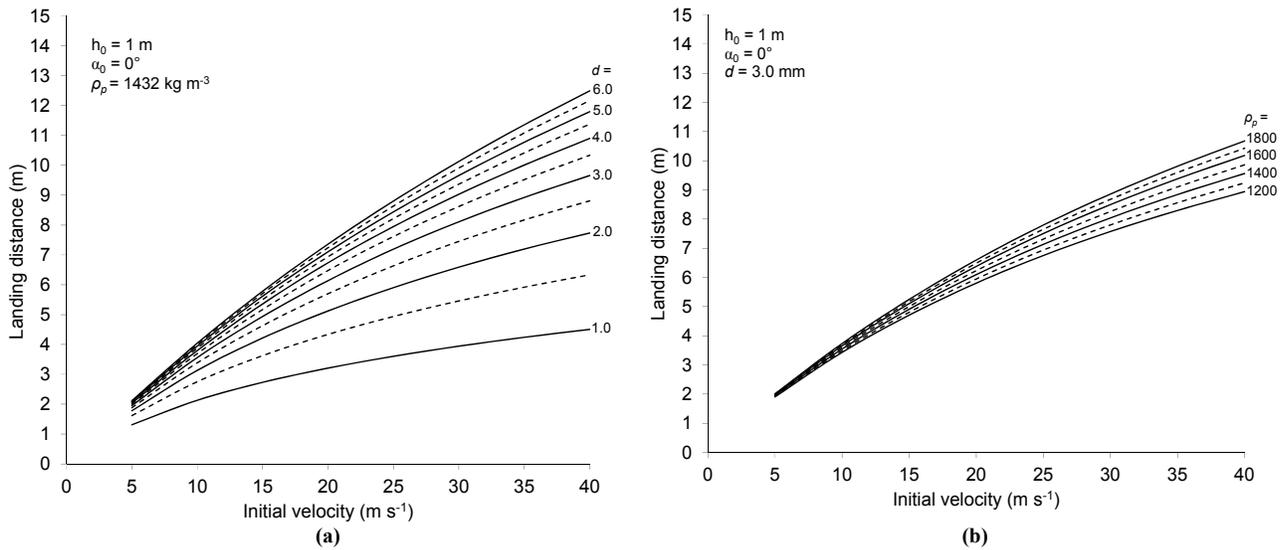
Figure 3. The effects of (a) particle diameter and initial velocity, and (b) particle density and initial velocity on landing distance of individual fertilizer particles. Dashed lines represent responses for intermediate values of particle diameters or particle densities, respectively.

on the fertilizer material and the particle diameter. For instance, OMF and biosolids granules will produce a different set of responses compared to those shown in figure 4 for urea. Similar relationships to those presented in figures 3 and 4 can be obtained to study the effects of height (h_0) and changes in aerodynamic drag characteristics of particles. Miller (1996) demonstrated that a change in air speeds from 0 to 3.3 m s^{-1} in the same direction as the particle's trajectory, will change landing distance by about 15% when d =2 mm, h_0 =0.75 m, v_0 =25 m s^{-1}, α_0 =0°, and ρ_p =1500 kg m^{-3}, which highlights that wind has a significant influence on the resultant fertilizer spread pattern.

Olieslagers et al. (1996) concluded that in order to obtain uniform distribution patterns from spinning disc systems, wide and symmetrical Gaussian-shaped patterns are preferred, with the edges of the pattern producing
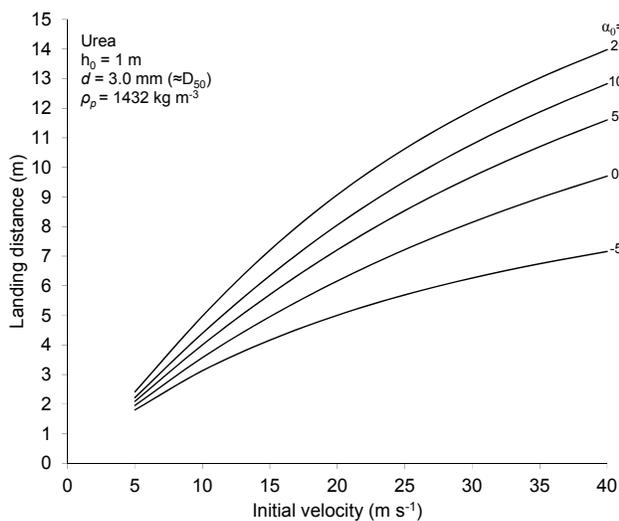
Figure 4. The effect of launch angle and initial velocity on landing distance of individual fertilizer particles. Particle density used in calculations is representative of urea with particle diameter equivalent to D_{50}.

relatively shallow angles to allow for small alterations in the treated width. Miller (1996) emphasizes that such uniformity of distribution can be achieved by appropriately combining input variables relating to spreader design and settings, including application rate (Fulton et al., 2005), and the characteristics of the fertilizer material to be applied. Based on Parkin et al. (2005), contour plots (fig. 5) were constructed, which aid the study of these relationships despite that accurate performance and distribution cannot be predicted from basic properties such as particle diameter and density. However, it is possible to provide broad indication of likely performance. For example, for given swath width and a twin disc spinner system of known performance, the particle diameter required to operate at that width can be predicted from particle density (Parkin et al., 2005). Figure 5b shows that a fertilizer material with particle density of 1200 kg m^{-3} and median particle diameter of 3.5 mm will provide a swath width of 16 m when the spreader (twin discs) is set to release particles at 10° angle and 20 m s^{-1} initial velocity. A similar swath width can be obtained with the same spreader and settings but using a denser material (e.g., 1800 kg m^{-3}) with smaller particle diameter (e.g., 2.5 mm). The particle size range specifications for OMF were established following the procedure shown in the example in figure 3b, which is discussed in the next section. Spreading tests conducted by Parkin et al. (2005) under semi-controlled experimental conditions confirmed that particle diameter has a significant effect on spreading width. They found a significant correlation between spreading width and particle landing distance as derived from trajectory theory, which was confirmed by wind tunnel dispersion tests. Parkin et al. (2005) used a similar approach to determining the particle size range suitable for application of granular urea with spinning disc systems at a 24 m bout width.
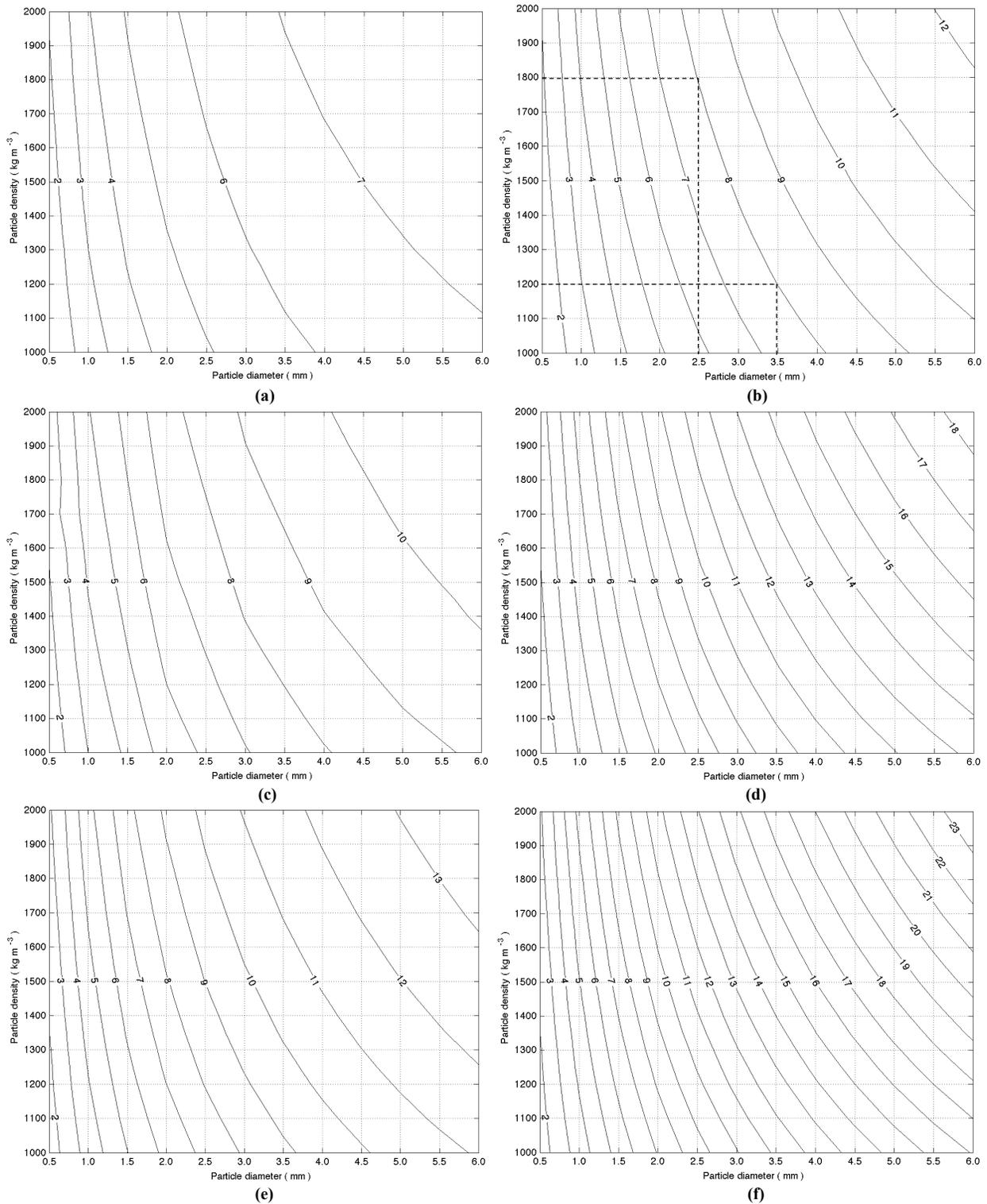
Figure 5. Contour plots used to determine landing distance of individual fertilizer particles based on particle diameter and particle density. Launch conditions are: $h_0 =1$ m, and (a): $\alpha_0 =0°$, $v_0 =20$ m s^{-1}, (b): $\alpha_0 =10°$, $v_0 =20$ m s^{-1}, (c): $\alpha_0 =0°$, $v_0 =30$ m s^{-1}, (d): $\alpha_0 =10°$, $v_0 =30$ m s^{-1}, (e): $\alpha_0 =0°$, $v_0 =40$ m s^{-1}, and (f): $\alpha_0 =10°$, $v_0 =40$ m s^{-1}. In (b), dashed lines illustrate the example given in the text for two fertilizer materials with $d =2.5$ mm and $\rho_p =1800$ kg m^{-3}, and $d =3.5$ mm and $\rho_p =1200$ kg m^{-3}, respectively.

PARTICLE SIZE RANGE SPECIFICATIONS FOR ORGANOMINERAL FERTILIZERS

The particle size range that would be required to achieve a similar spreading performance as urea was derived from the contour plots following the approach given in the example shown in figure 5b. For example, particles of urea with $d =1.00$ mm and $d =5.25$ mm, which represent, respectively, the smallest and largest particle diameters encountered in the sample, released at 20 m s^{-1} and 0° angle, will land at 3.21 and 7.17 m, respectively (fig. 5a).

For OMF_{10}, that distance range (3.21 to 7.17 m) can be achieved with particles with $d = 1.10$ mm and $d = 5.80$ mm, respectively, assuming the same launch conditions, while OMF_{15} will require particles with $d = 1.05$ mm and $d = 5.50$ mm, respectively. The same exercise was repeated for varying initial velocities (range: 20 to 40 m s^{-1}) and release angles (range: 0° to 10°), and it was found that the particle size range of OMF_{10} (1.10 to 5.80 mm) and OMF_{15} (1.05 to 5.50 mm) produced landing distances which were within 50 mm compared with particles of urea (1.00 to 5.25 mm) for the six launch conditions shown in figure 5. The landing distances achieved with urea with particle diameters equivalent to D_{10} and D_{90} (table 2) can be achieved with OMF_{10} with particle diameters equivalent to 2.65 mm ($\approx D_{10}$) and 4.30 mm ($\approx D_{90}$), and with OMF_{15} with particle diameters equivalent to 2.55 mm ($\approx D_{10}$) and 4.10 mm ($\approx D_{90}$). Therefore, it is suggested that OMF_{10} and OMF_{15} have about 80% (by weight) of particles between the ranges of diameters specified above. Fine particles (<1.00 mm) must be maintained to a minimum since this fraction can be responsible for high coefficient of variation (e.g., >10%) during broadcast spreading (Kämpfe et al., 1982).

Given the assumptions made in the analyses, it appears that machine settings such as those shown in figure 5a may not be able to provide satisfactory performances with overlapping spread patterns at swath widths of 18 m or greater, however, this may be overcome by small adjustments such as an increase in release angle (figs. 5b, 5d, and 5f). For OMF, discs settings or designs that achieve a specified landing distance using a small angle (e.g., 10°) may be preferred to increasing rotational velocity, which could result in greater forces being exerted on the particles and induce particle deformation, which could change their aerodynamic behavior. Whilst changing the release angle during field spreading is not an adjustment that most spreaders have, a disc design could be selected that will provide a slightly upward trajectory angle for particle delivery. This upward trajectory can be controlled by the degree of concavity on the outer part of the disc and by the design of the delivery vanes. Subsequently, a satisfactory distribution pattern could be achieved by adjusting the rotational velocity of the discs over a lower range than that needed if flat discs were to be used. When this rotational speed is increased care must be exercised to avoid damaging the particles in contact with the discs and vanes during spreading. Miller and Parkin (2005) suggest a threshold velocity for particles leaving the disc of 40 m s^{-1} for synthetic nitrogen fertilizers, which until further studies are undertaken, is suggested as a reference for OMF.

The influence of air speed on the landing distance of fertilizer particles (Miller, 1996) suggests that the model presented herein may underestimate travelling distance calculations because of the effect of fan that is produced by the discs and vanes rotating at high velocities during spreading, which agrees with observations made by Miller and Parkin (2005). Since distance calculations corresponding to D_{16}, D_{50} and D_{84} for urea were provided, the contour plots can be used to determine those percentiles for OMF to have a more complete characterization of the particle size range required for optimal spreading.

CONCLUSIONS

The main conclusions derived from this research are:

1. The proposed approach can be applied to predict the travelling distance of fertilizer particles when these are projected from a spinning disc system. Contour plots of landing distance versus particle density enable the performance of granular fertilizers to be pre-assessed using a reduced number of readily available input parameters relating to the characteristics of the material and the machine settings.

2. Particle size range specifications for OMF_{10} and OMF_{15} indicate that particles diameters need to be between 1.10 and 5.80 mm, and between 1.05 and 5.50 mm, respectively, to produce a similar spreading performance to urea with particle size range of 1.00 to 5.25 mm. It is also required that 80% (by weight) of particles have diameters between 2.65 and 4.30 mm, and between 2.55 and 4.10 mm for OMF_{10} and OMF_{15}, respectively. The complete characterization of the particle size range required for OMF can be derived from the contour plots developed in this study. Since landing distance is significantly affected by particle diameter, producing the correct particle size and size distribution for the spreading mechanism requires strict quality control. A narrower particle size range is preferable to a wider one to minimize granulometric segregation, which could adversely affect uniformity of distribution during field spreading.

3. This study shows that application of OMF_{10} and OMF_{15} with spinning disc systems may be possible at tramlines spaced at a maximum of 24 m, depending on the degree of overlapping between adjacent bouts. Disc designs and settings that enable working at a specified bout width by providing a small upward particle trajectory angle (e.g., 10°) are preferred to high rotational velocities, which could result in greater forces being exerted on the particles and induce particle deformation, which could change their aerodynamic behavior. A threshold velocity for particles leaving the disc of 40 m s^{-1} is suggested as a reference for OMF.

4. The results derived from the theoretical model reported in this article will benefit from comparisons with data obtained experimentally using the transverse tray testing (ISO, 1985; *ASAE Standards*, 1999).

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APPENDIX

INSTRUCTIONS TO OPERATE THE SOFTWARE

The particle trajectory model presented in this article is available through the ASABE Technical Library (http://elibrary.asabe.org/data/software/3/aeaj2014/30/7/MS%2010729%20Ejecutable.zip) in its electronic version. The following steps are required to run the software and provide a solution to model:

1. Open folder "Ejecutable;"

2. Open file "TiroParabolico.exe;"

3. Define "ambient temperature" (°C) and press "Enter;"

4. Define "launch speed" and press "Enter." This corresponds to the initial velocity (v_0; m s^{-1}) of the particle at the instant it leaves the disc;

5. Define "launch angle" and press "Enter." This corresponds to the angle (α_0, degrees) at which the particle is projected from the edge of the disc;

6. Define "particle radius" and press "Enter." This corresponds to r (mm);

7. Define "particle density" and press "Enter." This corresponds to ρ_p (kg m^{-3});

8. Define "initial height above the ground" and press "Enter." This corresponds to the vertical distance from the ground level to the edge of the disc (m);

9. Define "C_d" as follows:

a. Enter 0 for no drag conditions, and press "Enter"; or

b. Enter 1 for drag conditions and press "Enter".

10. The horizontal distance travelled by a particle will appear on the screen and it is given in meters (m). The file "Results.txt" in the "Ejecutable" folder provides a complete dataset for the particle's trajectory. This file contains the time (s), as well as the x and y velocities and trajectory components. The dataset is summarized at the bottom of the sheet and it contains the following information:

a. Duration of particle's flight (s);

b. Distance travelled by the particle (m);

c. Angle on impact (degrees); and

d. Speed on impact (m s^{-1}).