CONCEPTUALISING TECHNOLOGY INTEGRATED MATHEMATICS TEACHING: THE STAMP KNOWLEDGE FRAMEWORK

Seyum Tekeher Getenet[1], Kim Beswick[2] and Rosemary Callingham[2]

[1]University of Southern Queensland, [2]University of Tasmania

The Technological Pedagogical Content Knowledge (TPACK) framework is increasingly in use by educational technology researchers. The framework provides a generic description of the knowledge requirements for teachers using technology in all subjects. This paper describes the development of a mathematics specific version of the TPACK framework. We show how a particular conception of the knowledge required to teach mathematics can be integrated with the TPACK framework as the basis for understanding technology integrated mathematics teaching. The resulting framework provides a sharper lens than the generic TPACK framework alone and a better understanding of the knowledge required to use technology in teaching mathematics.

INTRODUCTION

Over the decades in which mathematics education researchers have worked to conceptualise Shulman's (1987) Pedagogical Content Knowledge (PCK) (e.g., Ball, Thames & Phelps, 2008; Chick, Baker, Pham, & Cheng, 2006; Rowland & Turner, 2007), Information and Communication Technology (ICT) has assumed an increasingly prominent place in students' learning. As a result, the knowledge required for ICT integration into teaching has received considerable attention from researchers who have stressed the importance of technical ICT knowledge, content knowledge, and pedagogical knowledge in teaching (e.g., Chee, Horani, & Daniel, 2005). In response, a conceptual framework that integrates technology, content, and pedagogical knowledge was proposed by Mishra and Koehler (2006), which they called "Technological Pedagogical Content Knowledge," (TPACK). The framework describes the knowledge required by teachers to use technology in teaching in effective ways. TPACK stems from the notion that technology integration benefits from an alignment of content, pedagogy, and technology knowledge. To integrate technology in their teaching practice, teachers need to be competent in all three domains. Only a few studies, however, have related the TPACK framework to particular subject matter content contexts (e.g., Guerrero, 2010; Jang & Chen, 2010).

In this paper, we explore the subject specific use of the TPACK framework in teaching technology integrated mathematics teaching to provide a model for further subject specific definitions and understandings of TPACK. The re-conceptualisation relies on the combination of two frameworks: TPACK and a mathematics specific conceptualisation of PCK, underpinned by the identification of complementary aspects of the two frameworks. The mathematics PCK framework used is Ball et al.'s

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(2008) description of Mathematical Knowledge for Teaching (MKT), chosen because of its influence in mathematics education, and the fact that it elaborates Shulman's (1987) concept of PCK adapted for the context of mathematics. The combined framework extends MKT by adding understandings arising from Mishra and Koehler's (2006) TPACK framework. At the same time, it adapts the TPACK framework by replacing content and pedagogical knowledge with specialised, mathematics specific conceptualisations of these aspects. The result is a new mathematics specific perspective. The paper is organised as follows. First, the development of the two existing frameworks, MKT and TPACK, are discussed separately. This is followed by a discussion of the implications of each framework for, and the development of, a conceptual framework for understanding a mathematics specific TPACK framework.

TEACHERS' KNOWLEDGE FOR TEACHING MATHEMATICS

In mathematics, PCK is regarded as essential (Ball et al., 2008; Park et al., 2011). Some have argued, however, that PCK detracts from the importance of teachers' knowledge of and about mathematics for being effective mathematics teachers (Ball et al., 2008). Thus, the description of PCK in a specialised subject of study, rather than as a generic concept, represents an important contribution to understanding the requirements for effective mathematics teaching (e.g., Ball et al., 2008; Chick, Baker, Pham, & Cheng, 2006; Rowland & Turner, 2007).

Rowland and Turner (2007), for example, identified four different knowledge categories required for teaching mathematics that they described as the knowledge quartet - namely foundation, transformation, connection and contingency. Theirs is a dynamic framework focused on the ways in which teachers use knowledge in the practice of mathematics teaching. Chick et al., (2006) considered the interaction of content and pedagogical knowledge for mathematics teaching in terms of a continuum.

Ball et al.'s (2008) notion of MKT emphasises the role of mathematical content knowledge. Their model comprises two major knowledge types: subject matter knowledge and PCK. Within subject matter knowledge they distinguish Common Content Knowledge (CCK), Specialised Content Knowledge (SCK) and Horizon Content Knowledge (HCK). CCK is necessary but not sufficient for teaching mathematics. It is the mathematical knowledge and skills used in settings other than teaching and used in a wide variety of situations. SCK is the mathematical knowledge and skill unique to teaching and not typically needed for purposes other than teaching mathematics. For example, recognising a wrong answer and being able to carry out a mathematical procedure is part of CCK, whereas recognising the mathematical steps that resulted in a student's error requires SCK in that it is mathematical knowledge not required by people other than teachers. Ball et al. (2008) divided Shulman's notion of PCK into Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT) and Knowledge of Content and Curriculum (KCC).

Familiarity with common errors students are likely to make is an aspect of KCS. Addressing students' errors by selecting appropriate instructional methods is illustrative of KCT. Ball et al. provisionally placed KCC and HCK as third categories of pedagogical content and subject matter knowledge respectively. They described HCK as an awareness of how mathematical topics are related over the span of the mathematics curriculum and KCC as knowledge of the materials and programs that teachers use in their everyday work. Ball and colleagues' conceptualisation of MKT has been criticised, for example, for failing to distinguish clearly between content knowledge and PCK despite purporting to do so; content knowledge is included in each aspect of PCK even though subject matter knowledge is presented as a separate domain (Chick, 2011). Nevertheless, its emphasis on mathematical knowledge suited the purpose of developing a mathematics specific framework for technology integrated teaching.

TEACHERS' KNOWLEDGE FOR TECHNOLOGY INTEGRATED TEACHING

It has been argued that teachers' knowledge of ICT is not the only criterion for effectively using ICT in teaching; sound pedagogical and content knowledge are also critical to success (Chee et al., 2005). Shulman's (1987) notion of PCK is thus relevant but requires the additional dimension of technological knowledge. The incorporation of a technology component into PCK resulted in the development of the notion of "Technological Pedagogical Content Knowledge" (TPACK) (Mishra & Koehler, 2006). According to the TPACK framework, the combination of technology, pedagogy and content knowledge can reinforce each other to realise advantages afforded by technology in the teaching and learning process. The combinations of technology (TK), pedagogy (PK) and content (CK) result in four additional composite knowledge types, namely: Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK), Pedagogical Content Knowledge (PCK) and TPACK. The TPACK framework is presented in Figure 1 followed a definition of each knowledge type.

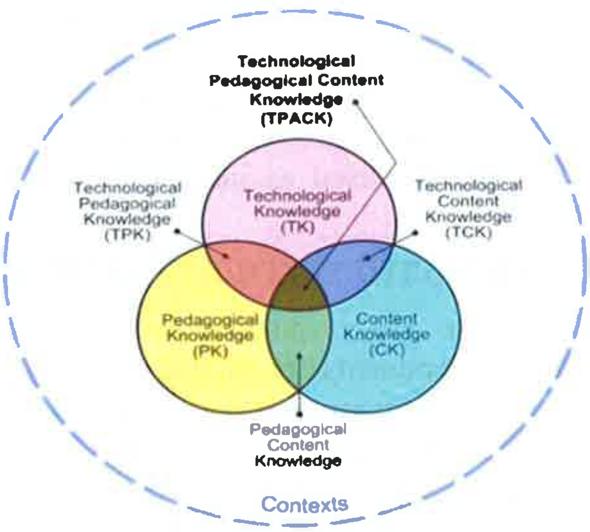


Figure 1: TPACK framework (source: Koehler & Mishra, 2009, p.63)

TK: Skills of teachers to use a particular technology. This could be using a particular software program and installing or removing it.

PK: Knowledge about process and practices of teaching. It includes, for example, students' learning styles, classroom management, students' evaluations and lesson planning.

CK: Knowledge of a subject matter to be taught. It demands understanding core principles, facts, theories, procedures and concepts of a particular subject matter.

PCK: Knowledge of how particular pedagogical approaches are suited to teaching particular content and vice versa.

TCK: Knowledge of how technology and content interact in effective teaching. It includes teachers' understanding of how subject matter can be changed by the use of technology.

TPK: Knowledge of how to use various technologies with different pedagogical approaches. It involves recognising and making use of the affordances of technologies and choosing pedagogical approaches that fit particular technologies and vice versa.

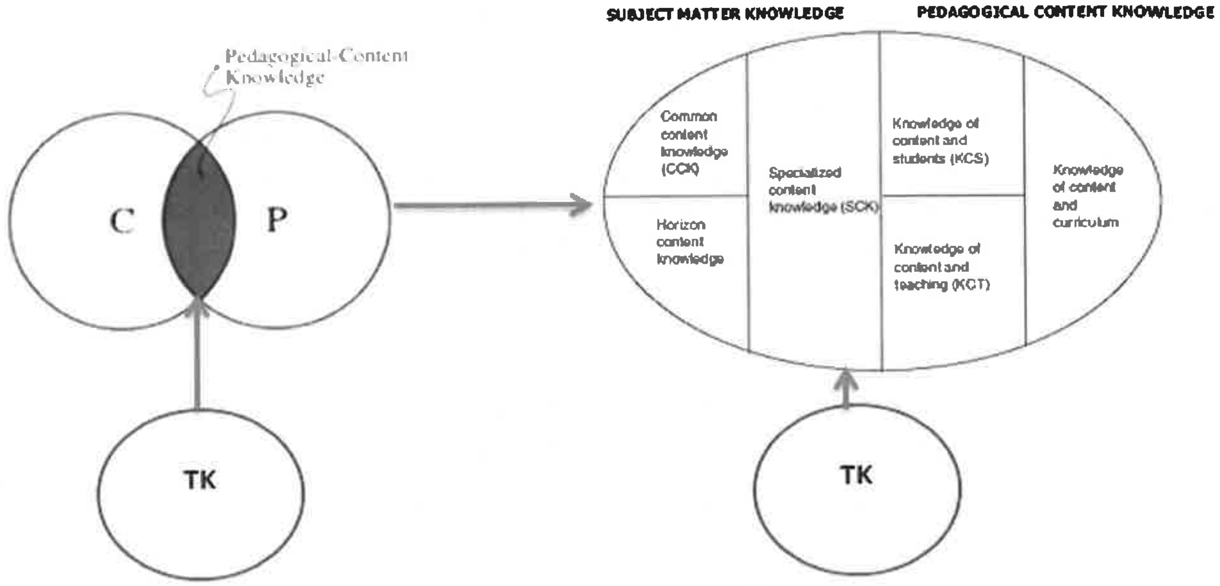
TPACK: The basis of effective teaching with the application of technology and requires an understanding of pedagogical techniques that use technologies in constructive ways to assist students to overcome difficulties and to learn content effectively.

Mishra and Koehler's (2006) knowledge types are based on a generic definition of content and hence of pedagogical knowledge and PCK. The re-conceptualisation of the TPACK framework for mathematics teaching that is presented in this paper is based on a definition of the mathematics content knowledge that teachers need that is more than simple knowledge of mathematics. Rather it is the SCK defined by Ball et al. (2008). This notion in turn influences the conceptualisation of mathematical pedagogical knowledge; it is not simply generic pedagogical knowledge applied to mathematics teaching but rather the knowledge of pedagogy needed to use specialised (mathematical) content knowledge effectively in teaching. This idea in turn has implications for PCK conceptualised as incorporating KCS and KCT as defined by Ball et al. (2008).

TECHNOLOGY INTEGRATED MATHEMATICS TEACHING

Adapting the TPACK framework to apply specifically to mathematics teaching requires understanding the three components (technology, pedagogy and content) from the perspective of mathematics teaching and the knowledge required to teach mathematics. The advantage of using Ball et al. (2008), rather than other conceptualisations of mathematics teacher knowledge, relates to its grounding in Shulman's (1987) notion of PCK which also informed the TPACK framework. The focus of this paper is on bringing together the TPACK framework and of Ball et al.'s (2008) MKT.

While the TPACK framework starts from the definition of PCK of Shulman (1987), we begin from the more detailed conceptualisation of MKT (Ball et al., 2008). In both cases, TK is seen as a component to be added. This is illustrated in Figure 2 with



the left side of the figure showing the addition of TK to Shulman's notion of PCK, the right side showing its addition to Ball et al.'s (2008) MKT framework.

Figure 2. Explaining TPACK in mathematics teaching

Developing Specialised Technological and Mathematics Pedagogical (STAMP) Knowledge Framework

In the STAMP knowledge framework, Mishra and Koehler's (2006) Content Knowledge (CK) is redefined as Specialised Mathematics Knowledge (SMK) and Pedagogical Knowledge (PK) as Specialised Pedagogical Knowledge (SPK). Technological Knowledge (TK) remains as defined by Mishra and Koehler's (2006). SMK and SPK are defined together with each of the knowledge types that result from the intersection of the two frameworks. How each element of the STAMPK framework differs from the corresponding component of the TPACK framework is explained in the following section. The framework is illustrated in Figure 3.

Specialised Mathematics Knowledge (SMK)

SCK, as defined by Ball et al. (2008), is knowledge needed by teachers of mathematics in addition to CCK and HCK, but not by the general population or teachers of other subjects. For example, rather than simply knowing how to perform fraction calculations, teachers need to understand the multiple and subtly different meanings of fractions (e.g., as division, as parts of a whole, as points on a number line).

Specialised Pedagogical Knowledge (SPK)

Ball et al. (2008) did not define pedagogical knowledge but described PCK for mathematics teaching in terms of KCS, KCT, and KCC. Nevertheless, there are parallels with the PK used by Mishra and Koehler (2006) with generic assumptions. Mishra and Koehler (2006, p.1026) defined PK as "knowledge about techniques or methods to be used in the classroom; the nature of the target audience; and strategies for evaluating student understanding. It is about how students construct knowledge, acquire skills, and develop habits of mind and positive dispositions toward learning". From this definition one can see the importance of teaching methods (analogous to KCT), knowledge of students (analogous to KCS), and knowledge of the curriculum (analogous to KCC) which are defined by Ball et al.'s model pertinent to mathematics teaching and redefined in the STAMP knowledge framework by combing all KCT, KCS and KCC as SPK.

Technological Knowledge (TK)

This refers to skills required of teachers to use a particular technology. This could involve using a particular software program and installing or removing it.

Specialised Pedagogical Mathematics Knowledge (SPMK)

The intersection of SPK and SMK takes the place of PCK in Mishra and Koehler's (2006) TPACK framework. SPMK can be understood as the mathematics specific and specialised (as opposed to generic or everyday) knowledge for teaching mathematics. It includes knowledge of ways in which mathematical concepts can be represented, the affordances of particular mathematical problems, resources and the specific difficulties that students are likely to encounter in relation to particular mathematical concepts. For example, knowing the affordances of and appropriate uses of various representations of fractions (e.g., as areas, parts of collections, or points on number lines).

Specialised Technological Mathematics Knowledge (STMK)

The intersection of TK and SMK results in STMK. This is the knowledge required by teachers of mathematics in which the application of technology influences mathematical content. Teachers' selection of technology should fit with the special type of mathematics knowledge needed in teaching. STMK allows teachers to identify and use technology appropriately to facilitate the teaching of mathematical concept effectively. For example, the use of spreadsheets could transform the task of explaining the difference between a square and a rectangle to one of creating, changing and checking the properties of many figures that fit the definition of a rectangle and identifying that some of these are square.

Specialised Technological Pedagogical Knowledge (STPK)

The intersection of TK and SPK (with the definition given earlier) gives rise to STPK. This is the knowledge required by mathematics teachers in which teachers' mathematics specific pedagogical knowledge is influenced by the application of technology. For example, knowing that using dynamic graphing software to remove the tedium of creating scatter plots can enable students to access more sophisticated ideas about the relationships between variables than would be possible in the absence of technology.

Specialised Technological and Mathematics Pedagogical Knowledge (STAMPK)

Finally, the interplay of TK, SMK, and SPK gives rise to STAMPK. This is the unique knowledge for teaching mathematics with the application of technology. It is the integration of these three knowledge types that enables teachers to incorporate technology effectively into mathematics specific pedagogies in such a way that students are assisted to make meaning of the targeted mathematical ideas. It also includes understanding the instructional advantage of different instructional methods, specialised mathematics knowledge and technologies and combining these knowledge types in the classroom for effective learning of mathematics. The TPACK framework (Figure 1) is thus reconceptualised as the STAMPK framework for teaching mathematics with the application of technology as shown in Figure 3.

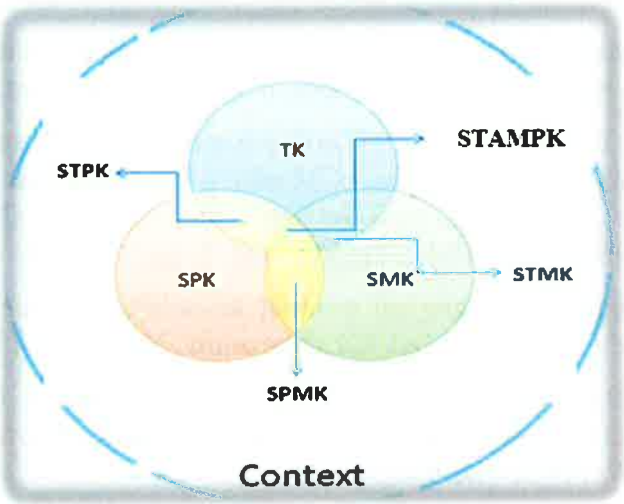


Figure 3. Knowledge required in teaching mathematics with technology

Teaching mathematics successfully with technology requires each component of knowledge described in the STAMPK framework as well as the knowledge types arising from their constructive combination. Their application will depend upon the particular context in which they are employed with such things as the availability of technologies, time, the nature of students, and course assessments.

CONCLUSION

The framework proposed provides an approach to specifying the TPACK framework for teaching mathematics. The resulting STAMPK framework interprets the TPACK framework in terms of Ball et al.'s (2008) influential model of Mathematics Knowledge for Teaching. The work was predicated on the belief that subject specific knowledge frameworks are of greater use than generic frameworks to both researchers and practitioners with interest in a specific subject. Ball et al.'s (2008) model was used because of its emphasis on mathematical knowledge and its origins in the seminal work of Shulman (1987), as well as its status in the field, however, other models of mathematics teachers' knowledge such as those of Chick et al. (2006) and Rowland and Turner (2007) might also have been used. These would likely have yielded different but similarly useful insights into the knowledge demands of technology integrated mathematics teaching. Much work is needed to explore such options, weigh their relative merits, and tackle the ongoing challenge of operationalising conceptions of teacher knowledge within a technology rich environment. The STAMPK framework provides a starting point for one such line of inquiry.

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