

A Simplified Empirical Model for Prediction of Mechanical Properties of Random Short Fiber/Vinylester Composites

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ABSTRACT: This article discusses a simplified approach to analyze mechanical properties of randomly distributed short fiber composites. Mechanical properties of three different randomly oriented short fiber composites, cotton, nylon, and aluminium with vinylester resins, were experimentally investigated. The analytical results were compared with experimental results and a very good correlation was found. Further, the experimental results and the predictions showed that the strength of the composites is less than the strength of the matrix material, for all three composites tested.

KEY WORDS: random short fiber composites, cotton fiber, nylon fiber, aluminium fiber, mathematical modeling, strength, modulus.

INTRODUCTION

THE SHORT FIBER reinforced composite (SFRC) consists of short fibers dispersed into matrix material. The low cost, ease of fabricating complex parts, and isotropic nature are enough to make the short fiber composites the material of choice for large-scale production. Consequently, the SFRCs have successfully established its place in lightly loaded component manufacturing. Three types of short fiber composites are depicted in Figure 1. Aligned SFRC which normally produced by injection molding and extrusion processes have comparatively excellent in-plane mechanical properties, whereas randomly oriented SFRC composite shows quasi-isotropic nature in macroscopic scale. The most widely used SFRC is randomly oriented SFRC composite due to comparatively easy production process [1].

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Figures 2–10 appear in color online: <http://jcm.sagepub.com>

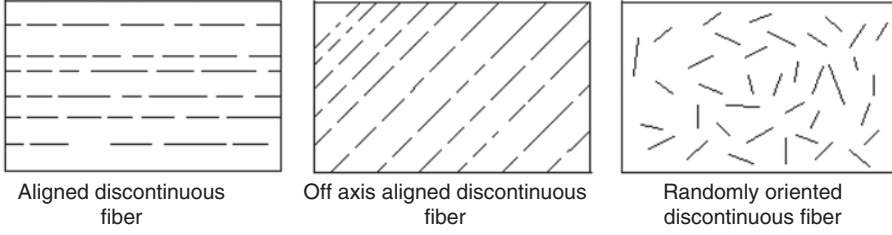


Figure 1. Different types of discontinuous fiber reinforcement.

Unlike continuous fiber composites, the external loads are not directly applied to the fibers in SFRC materials. The load applied to matrix materials is transferred to the fibers via fiber ends and the surfaces of fibers. As a consequence, the properties of SFRC greatly depend on fiber length and the diameter (i.e., fiber aspect ratio = length/diameter of fiber) of the fibers. Further, several factors such as fiber orientation, volume fraction, fiber spacing, fiber packing arrangement, and curing parameters also significantly influence the properties of SFRC materials [2]. There has been a significant development in property prediction models of aligned SFRC based on micro and macro mechanics of composite [3–6]. Most widely used analytical methods, which predict considerably accurate results for aligned short fiber composites, are based on shear lag-analysis, which was originally proposed by Cox [7], Halpin and Kardos [8], Agarwal et al. [4], and Tucker and Liang [5]. It has been shown that the predictions of these models are analogous to the predictions by micromechanical analyses. Direct application of these models in random short fiber property prediction is limited. However, a few empirical relationships are available to be used in conjunction with these analytical models of aligned short fiber composites, for the determination of approximate properties of random short fiber composites [4,9]. These two basic models were developed and modified by various researchers during the past few decades. Following are some of the noticeable models, which have been used for the randomly oriented short fiber composites:

Model proposed by Christensen [10] is:

$$E_C = \frac{v_f E_f}{3} + \frac{(1 - v_f) E_m}{0.3} + \frac{19}{27} E_m \left[\frac{E_f(1 + v_f) + E_m(1 - v_f)}{E_f(1 - v_f) + E_m(1 + v_f)} \right] \quad (1)$$

The modified Halpin–Tsai [8] equation is:

$$n = \frac{3}{8} E_1 + \frac{5}{8} E_2 \quad (2)$$

$$\text{where } E_1 = E_m \left(\frac{1 + \xi \eta_1 v_f}{1 - \eta_1 v_f} \right), \quad E_2 = E_m \left(\frac{1 + \xi \eta_2 v_f}{1 - \eta_2 v_f} \right), \quad \eta_1 = \frac{\left(\frac{E_f}{E_m} \right) - 1}{\left(\frac{E_f}{E_m} \right) + 2(l/d)}$$

$$\text{and } \eta_2 = \frac{\left(\frac{E_f}{E_m} \right) - 1}{\left(\frac{E_f}{E_m} \right) + 2}$$

Both longitudinal and transverse modulus is determined using modified Halpin–Tsai equation.

Model proposed by Puck and Schuermann [11] is:

$$E_C = v_f \left(\frac{16}{45} E_f + 2E_m \right) + \frac{8}{9} E_m \quad (3)$$

While significant progress in modeling of mechanical properties of the aligned SFRC has been achieved, it is found that much of those modeling are not precisely applicable to random SFRC [2]. As such, there is an urgent need for a proper analytical method for randomly oriented short fiber composite materials. This article details an investigation undertaken for the prediction of properties of three random short fiber composites, cotton fiber/vinylester, nylon fiber/vinylester, and a aluminium fiber/vinylester, based on a simplified approach. The results from the proposed algorithm and the results from the experimental properties will be compared. The significance of this work is the use of a natural cotton fiber as reinforcer for the first time, and use of metallic fiber aluminium and synthetic nylon fibers for SFRC.

MODEL FORMULATION

From shear lag model the stiffness can be predicted for aligned fibers as:

$$E_C = \left[v_f E_f \left(1 - \frac{\tanh(ns)}{ns} \right) + (1 - v_f) E_m \right] \quad (4)$$

where s is the aspect ratio (l/d), l is average fiber length, d is diameter of fiber and

$$n = \left[\frac{2E_m}{E_f(1 + v_m) \ln(1/v_f)} \right]^{0.5} \quad (5)$$

When s approaches ∞ Equation (4) becomes rule of mixture

$$E_C = v_f E_f + (1 - v_f) E_m \quad (6)$$

Further inspection of Equations (4) and (5) shows that the governing parameter of modulus in shear lag model is proportional to $(s\sqrt{E_m/E_f})$ [5].

The modified Halpin–Tsai equation for continuous fiber composites are a set of empirical relationships that enable the property of a composite material to be expressed in terms of the properties of the matrix and reinforcing phases together with their proportions and geometry [8]. Halpin and Kardos [8] showed that the property of a composite P_c could be expressed in terms of the corresponding property of the matrix P_m and the reinforcing phase (or fiber) P_f using the following relationships:

$$P_c = P_m \left(\frac{1 + \xi \eta v_f}{1 - \eta v_f} \right) \quad (7)$$

where

$$\eta = \frac{(P_f/P_m) - 1}{(P_f/P_m) + \xi}$$

The factor ξ is used to describe the influence of geometry of the reinforcing phase on a particular property. This factor is good for different properties in the same composite. Halpin and Kardos [8] suggested that $\xi = 2(l/d)$ gives good predictions from

the proposed relationship. Tucker and Liang [5] suggested that the governing parameter of properties is closely proportional to $(v_f(E_m/E_f))^2$ (for fiber volume fraction). Also when ξ approaches ∞ , Equation (4) becomes rule of mixture.

Properties of random short fiber composites can be treated as quasi-isotropic. As a consequence, rule of mixture with a considerable treatment would be appropriate in property prediction as the influence of aspect ratio on randomly oriented short fiber matrix properties are limited or negligible. As such, the following relationship for randomly oriented composite is postulated:

$$E_C = \left(v_f \left(\frac{E_m}{E_f} \right) \right)^2 E_f + \left[1 - \left(v_f \left(\frac{E_m}{E_f} \right) \right)^2 \right] E_m \quad (8)$$

Following the above analysis composite strength σ_c can be formulated as:

$$\sigma_C = \left(v_f \left(\frac{\sigma_m}{\sigma_f} \right) \right)^2 \sigma_f + \left[1 - \left(v_f \left(\frac{\sigma_m}{\sigma_f} \right) \right)^2 \right] \sigma_m \quad (9)$$

However, the empirical model presented in Equation (9) has not included the influences of the stress transfer micromechanisms, which will govern by the geometry and orientation of fibers.

The strength of the composite depends on the stress transfer mechanism between matrix and the fibers [3,4]. Consequently, the analytical methods to predict composite strength become complex due to random nature of fiber orientation in a random SFRC material. To overcome the complexity of an analysis, it is logical to assume that the effective volume fraction, which contains the load carrying fibers i.e., aligned fiber with the loading direction, as the parameter for strength prediction. Therefore Equation (9) is more accurate in the form of:

$$\sigma_C = \left[\left(v_f \left(\frac{\sigma_m}{\sigma_f} \right) \right)^2 \sigma_f + \left[1 - \left(v_f \left(\frac{\sigma_m}{\sigma_f} \right) \right)^2 \right] \sigma_m \right] f(v_f) \quad (10)$$

where $f(v_f)$ is distribution function of volume factor of the matrix which carries fibers aligned with the loading direction.

The distribution of strength of composites can be assumed as Weibull type distribution [3,12,13]. Consequently, the distribution of aligned fibers in load carrying direction can be assumed as a Weibull type distribution as the aligned fibers are directly proportional to the strength of the composite. It is therefore postulated that the volume fraction of the aligned fiber of random fiber composite, in any particular loading direction as single parameter Weibull distribution whose probability density function (PDF) is:

$$f(v_f) = \beta(1 - v_f)^{\beta-1} e^{-(1-v_f)\beta} \quad (11)$$

where $\beta (>0)$ is the shape parameter. As such the empirical relationship for strength σ_c , Equation (10) becomes:

$$\sigma_C = \left[\left(v_f \left(\frac{\sigma_m}{\sigma_f} \right) \right)^2 \sigma_f + \left[1 - \left(v_f \left(\frac{\sigma_m}{\sigma_f} \right) \right)^2 \right] \sigma_m \right] \left(\beta(1 - v_f)^{\beta-1} e^{-(1-v_f)\beta} \right) \quad (12)$$

It should be noted that the shape parameter β is a constant for a particular fiber type i.e., the material and the average fiber geometry.

EXPERIMENTATION

The composite samples were casted in a mold. The mold consisted of a bottom plate attached to detachable pieces to form a rectangular cavity. The arrangement was done in such a way that eight specimens could be prepared at a time at uniform pressure. The top plate of the mold was fixed with individual punches to form punch plate assembly. The entire mold was prepared from wooden material glued with 1 mm thick Formica sheet at contact surfaces to provide smooth surface.

The weighed fibers were uniformly placed in the mold cavity and then a measured volume of vinylester resin was poured into the mold. The fibers have approximate length of 3–7 mm and were distributed randomly in the matrix of vinylester resin. Other parameters were kept constant during the curing of specimens at room temperature; all specimens were allowed to cure in the mold for 24 h. The specimens were manufactured to ASTM Standard D3039 tensile test specimen. For obtaining the same volume fraction, the same weighted short fibers were poured with constant quantity of resin. The same procedure was done for different weighing short fibers for obtaining different volume fraction specimen. Five sets of different volume fraction from each fiber materials were prepared. The static tests were carried out on 2T Mikrotech Tensiometer. The properties of the constituent elements are shown in Table 1.

RESULTS AND DISCUSSION

Figure 2 shows the material of broken samples. The broken samples show the damage plane is nearly 45° to the loading direction, indicating a shear failure. The failure mode can be a fiber-dominated failure due to random fiber orientation or the matrix-dominated failure. Figures 3–8 inclusive show the experimental and predicted values for modulus and strength

Table 1. Properties of the constituent elements of the composites.

Material	Cotton	Nylon	Aluminium	Polyester resin
Modulus of elasticity (N/m ²)	7.9×10^9	3.9×10^9	67.5×10^9	2.285×10^9
Poisson's ratio (ν)	0.30	0.39	0.34	0.37
Density (kg/m ³)	1540	1140	2700	1250
Diameter (μm)	200	100	300	—
Ultimate tensile stress (N/m ²)	3.535×10^8	0.64×10^8	5.75×10^8	0.206×10^8
Diameter of fiber (μm)	200	100	400	—
Length of fiber (mm)	3–7	3–7	3–7	—



Figure 2. Tested samples: (a) cotton, (b) nylon, (c) aluminium.

of the cotton/vinylester, nylon/vinylester, and aluminium/vinylester. Table 2 shows the Weibull parameter β used for proposed model.

Figure 3 shows that the cotton/vinylester modulus predictions from the proposed model correlate well with the experimental results. The predictions by Christensen, Halpin and Kardos, and Puck are considerably higher than experimental results. The error levels of predictions are $<10\%$ for the proposed model while the error levels for the other models are above 13% . Figure 4 shows the experimental and predicted values for cotton/vinylester strength. The proposed model correlates with experimental results by $<2\%$ error levels while the error levels for other models are greater than 13% . The other important observation made here is the strength of the composite is comparatively less than the matrix strength. The predictions in Figure 4 show only a comparatively small increase in strength with the increase of volume fraction.

Figure 5 shows experimental and predicted modulus for nylon/vinylester composite. It shows a good correlation of the proposed model with the experimental results. The error levels are as low as 6% for the proposed models while error level for the other models

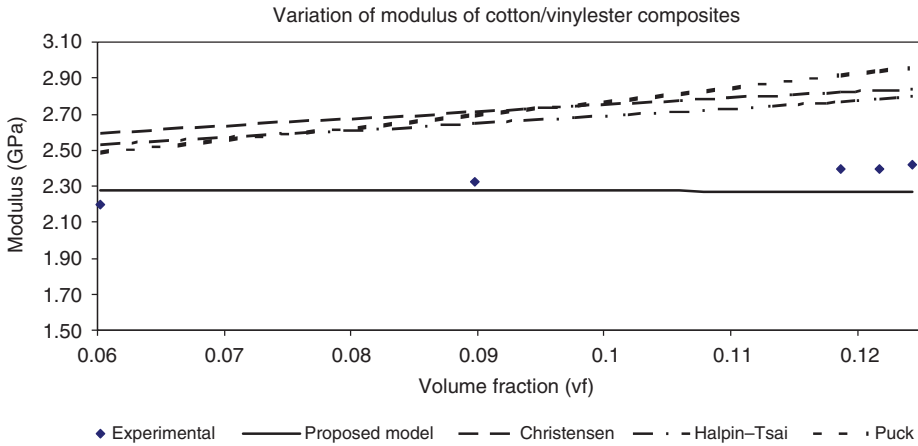


Figure 3. Modulus of cotton/vinylester composites.

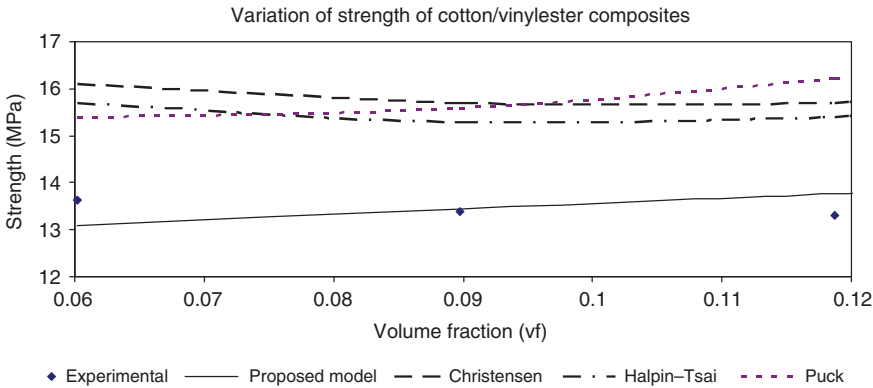


Figure 4. Strength of cotton/vinylester composites.

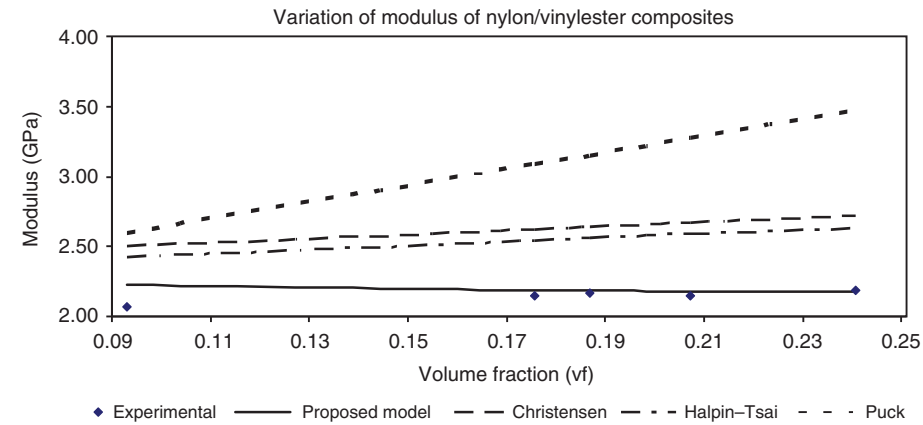


Figure 5. Modulus of nylon/vinylester composites.

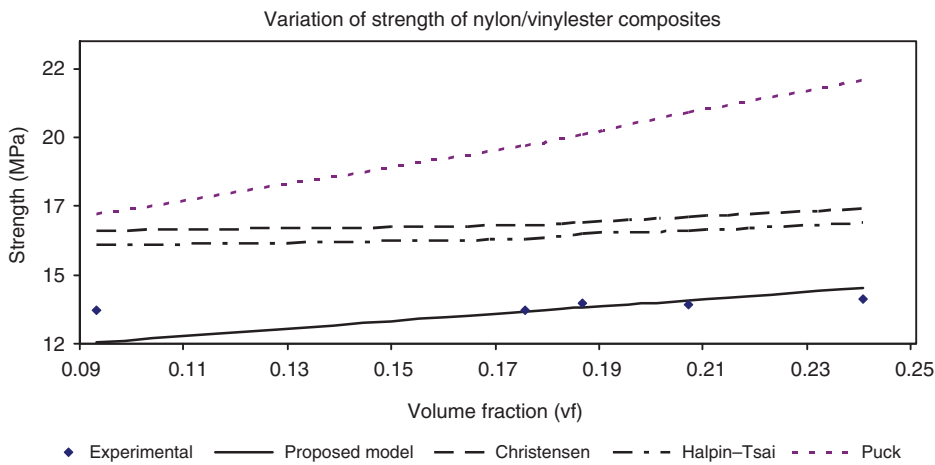


Figure 6. Strength of nylon/vinylester composites.

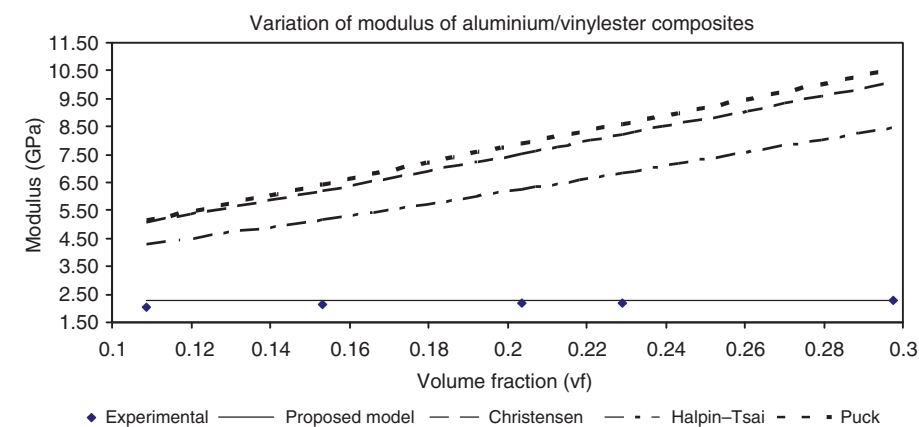


Figure 7. Modulus of aluminium/vinylester composites.

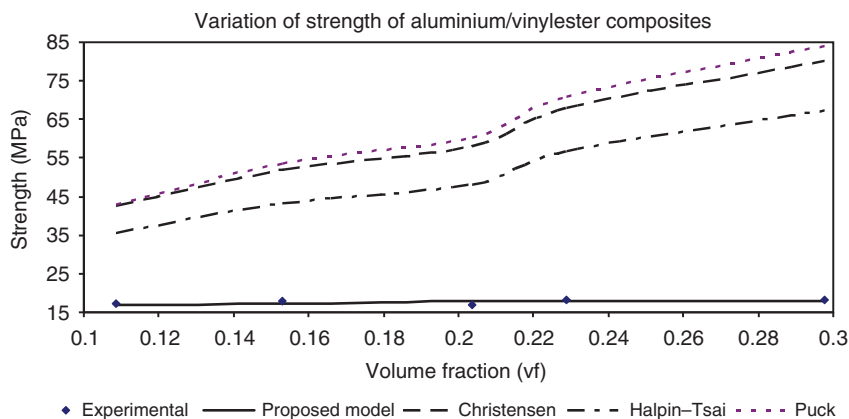


Figure 8. Strength of aluminium/vinylester composites.

Table 2. Parameters used for fibers.

Fiber	Weibull parameter β
Cotton	1.5
Nylon	1.4
Aluminium	2.0

are high as 36%. Figure 6 shows strength of nylon/vinylester composite obtained experimentally and calculated from the models. The proposed model shows a good correlation with the experimental results with error levels of <9%. The predictions from other models are comparatively higher with error levels as high as 36%. The strength of the composite is less than the strength of matrix material. The strength predictions for higher volume fractions (Figure 6) show an increase in strength; however, it stays under matrix strength.

Figure 7 shows experimental and predicted modulus for aluminium/vinylester composite. It shows a good correlation of the proposed model with experimental results. The error levels are <10% for the proposed models while error level for the other models are as high as 75%. Figure 8 shows strength of aluminium/vinylester composite. The proposed model shows a good correlation with the experimental results with error levels of <5%. The predictions from other models are comparatively higher with error levels extremely as high as 78%. The strength of the composite is less than the strength of matrix material. The strength predictions for higher volume fractions (Figure 9) show an increase in strength; however, it starts decreasing after volume fraction around 0.3. Figure 10 shows that closely flat trend of the modulus of aluminium/vinylester composite with the increase of the fiber volume fraction.

Table 2 shows parameter β that is used for this analysis for three different materials. The cotton and nylon, which are nonmetallic materials, share reasonably closed β value whereas β value for aluminium is 2.

The module for cotton/vinylester and aluminium/vinylester composites are reasonably closer to their matrix module. The module for nylon/vinylester is less than the matrix module. Further, the predicted strength of three composites is always less than the matrix strength showing the matrix is weakened in the presence of fibers. This may be caused by the creation of brittle matrix–fiber interface [14,15].

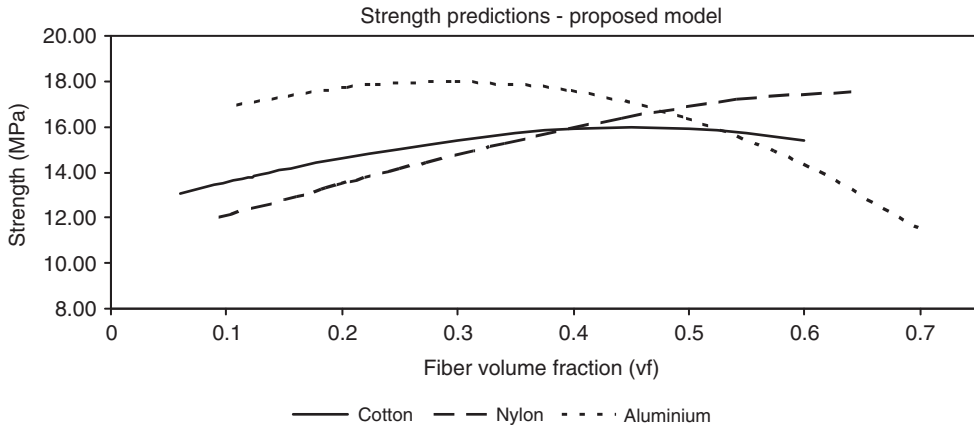


Figure 9. Strength predicted by the proposed model.

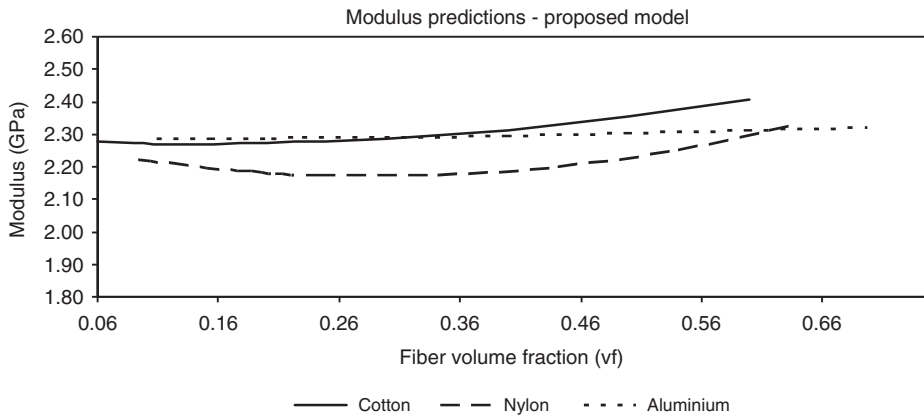


Figure 10. Modulus predicted by the proposed model.

Since the Weibull parameter β was defined as a function of aspect ratio and the material property of the fibers, parameter β is not valid for a wide range of fibers. Next logical step is to include fiber geometry and other properties in Weibull model independent of parameter β make the model suitable for a variety of fibers.

CONCLUSION

Two simplified empirical models were presented for mechanical property calculations of random short fiber composites. The properties of three different random short fiber composites were experimentally determined and compared with the predictions. The model predictions have a very good agreement with the experimental values. The Weibull parameter β has a considerable influence on the strength predictions depending on fiber type as anticipated. Unfortunately, nothing can be concluded about parameter β due to the limited scope of this project.

The surface conditions of the three fiber selected are different, which will highly affect the ultimate interfacial bonding properties of the composites. However, in this study only the

moduli of cotton/vinylester composites was compared with themselves using the model developed in this study and those of others as depicted in Figure 3. Similarly, the same was done for nylon/vinylester and aluminium/vinylester composites as depicted in Figures 5 and 7, respectively. The same argument applies to the strength of the three types of composites as depicted in Figures 4, 6, and 8. In Figures 9 and 10, the trends and not the absolute values of the three types of composites were compared. Therefore, the difference in surface conditions of the composites will not affect the results and trends of this study.

The predictions by Christensen, modified Halpin–Tsai, and Puck models are comparatively higher than experimental values. This over prediction has been reported in the literature regularly. The proposed model predicted that the strength of the composites will not increase above the matrix strength with the increase of volume fraction. Further, the model predictions indicated that cotton and nylon fiber composites have a trend of increase in modulus with the increase of fiber volume. The aluminium fiber composite shows a reasonably constant modulus with the increase of fiber volume.

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