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To cite this article:

Getenet, S. T. (2022). Teachers' knowledge framework for designing numeracy rich tasks across non-mathematics curriculum areas. *International Journal of Education in Mathematics, Science, and Technology (IJEMST), 10*(3), 663-680. https://doi.org/10.46328/ijemst.2137

International Journal of Education in Mathematics, Science and Technology

2022, Vol. 10, No. 3, 663-680 https://doi.org/10.46328/ijemst.2137

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Article Info

Article History

Received:
13 October 2021
Accepted:
28 April 2022

Keywords

Non-mathematics
Curriculum areas
Numeracy
Numeracy across curriculum area
Numeracy rich tasks
Teachers' knowledge

Abstract

There is a consensus that numeracy is important for students to develop logical thinking and reasoning strategies in their everyday activities. As a result, teachers are encouraged to design numeracy rich tasks that incorporate real-life contexts across non-mathematics curriculum areas. However, it is not clear what types of knowledge teachers require to design such tasks. In this study, the authors used the stepwise generalization method and developed a framework for the knowledge required by teachers to design numeracy rich tasks across non-mathematics curriculum areas. The study begins with a brief introduction to numeracy and its definition in various contexts. The nature of knowledge required to design numeracy rich tasks (mathematics content knowledge, a non-mathematics curriculum area knowledge, activity design skills and knowledge of context)) are further described. In the later section of the study, each element of teacher knowledge and the framework to design numeracy rich tasks across non-mathematics curriculum areas are described. The knowledge framework developed in this study can be used to analyze teachers designed numeracy-rich tasks and identify their professional learning requirements.

Introduction

There is a consensus that students need to be numerate to make a positive contribution to society. As a result, developing students' numeracy is a priority in many countries, including Australia (Australian Curriculum and Assessment Reporting Authority [ACARA], 2019), South Africa (Modisaotsile, 2012), the United States of America [USA] (Ford, 2018), the United Kingdom [UK] (Brown et al., 1998) and many European countries (Parveva et al., 2011). Numeracy was first introduced in the UK, and it was defined as the mirror image of literacy but involving quantitative thinking (Goos et al., 2019). Later in 1982, Cockcroft described numeracy as using mathematical skills to cope confidently with the practical demands of everyday life. The definition adopted at the 1997 national numeracy strategy conference became widely accepted and formed the basis of numeracy related research and curriculum development in Australia. The conference contributed to the public debate on key issues in the importance of numeracy and the state of numeracy in Australia and how best to move forward in education. Numeracy was defined as: "to be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life" (Department of Employment, Education, Training and Youth Affairs [DEETYA], 1997, p.15). This broad interpretation of numeracy embraced by educators and policymakers in Australia is similar to the definition provided by the Organization for Economic Co-operation and Development (OECD). OCED (2012) defined numeracy as an individual's capacity to formulate, employ, and interpret mathematics in various contexts. In Australia and this article, numeracy is used to encompass the knowledge, skills, behaviors and dispositions that students need to use mathematics in a wide range of situations in their lives (Forgasz et al., 2017; Young-Loveridge et al., 2012). Hence, the global common theme for numeracy involves understanding and recognizing the importance of mathematics in the world and using mathematical knowledge and skills purposefully in various contexts relevant to students' lives.

Students become numerate as they develop the knowledge and skills to use mathematics confidently across curriculum areas (across curriculum referred in this study to the non-mathematics curriculum areas) at school and in their lives more broadly (ACARA, 2019; Bennison, 2015; Brown et al., 1998; Ford, 2018). As a result, numeracy is approached as a cross-curriculum area in various countries, including the United Kingdom, United States, New Zealand and Australia. Consistent with this, numeracy is included as a general capability in the Australian Curriculum, alongside literacy, information and communication technology, ethical understanding, personal and social capability. While it is recognized that numeracy draws upon mathematical skills, it is not situated within the mathematics curriculum alone. Teachers at all grade levels and in all non-mathematics curriculum areas are expected to support students to develop their numeracy capabilities (ACARA, 2019; Forgasz et al., 2017). When teachers identify numeracy demands and use across the non-mathematics curriculum areas, students have opportunities to transfer their mathematical knowledge and skills to contexts outside the mathematics classroom (Bennison, 2015; Thornton & Hogan, 2004). These opportunities can help students recognize the interconnected nature of mathematics with other non-mathematics curriculum areas and its application in the wider world. Teachers are, therefore, encouraged to design numeracy rich tasks (NRT) and integrate their use across non-mathematics curriculum areas to enhance students' numeracy skills.

In this study, 'task' is defined and uses the same definition provided by Watson et al. (2014) (in numeracy called NRT). Watson et al. (2014, p.9) defined task as: "a wider range of things to do or what students are asked to do that include repetitive exercises, constructing objects, exemplifying definitions, solving single-stage and multi-stage problems, deciding between two possibilities, or carrying out an experiment or investigation". Therefore, a teacher uses a task to demonstrate numeracy, complete activities interactively with students or ask students to solve problems and allows encounters with numeracy concepts, ideas, and strategies across various contexts.

However, the design of NRT across non-mathematics curriculum areas requires teachers to have knowledge in designing and implementing these tasks (e.g., Bennison, 2015; Goos et al., 2013, Geiger et al., 2015). Teachers' knowledge required to design such tasks remains an open issue in research and practice. Researchers have provided models for numeracy tasks (e.g., Goos et al., 2010, 2013) and professional development strategies (e.g., Bennison, 2015; Thornton & Hogan, 2004) to support the development of teachers' competencies to design NRT. For example, Goos et al.'s (2010, 2013) model, describing a numerate person, has been used to provide principles for the design of NRT. However, it is unclear what knowledge teachers require to design NRT in non-mathematics curriculum areas. The need for a more coherent theoretical framework has become rapidly apparent.

In this study, the authors propose a framework for the knowledge required by teachers to design NRT across non – mathematics curriculum areas. The proposed framework helps teachers understand and approach the successful use of mathematics across non-mathematics curriculum areas and develops students' numeracy competencies and skills. The research question guiding the study was: "What type of knowledge do teachers require to design NRT across non-mathematics curriculum areas?" The study is organized as follows. First, the method used to design the framework is described. Second, numeracy across curriculum areas and the knowledge required by teachers to design NRT are explained. This is followed by a discussion of the implications of developing a framework for understanding teacher knowledge for designing NRT.

Method

In this study, the authors used the "stepwise generalization" method suggested by Koskimies and Mössenböck (1995) to design the knowledge framework. Koskimies and Mössenböck suggested that the design of a framework proceeds in three basic phases (see Figure 1) with an alteration process; problem generalization, initial framework design and implementation. As shown in Figure 1, the first phase starts with problem identification and problem generalization. This phase starts from the specification of a representative application of the intended framework and generalizes it in a sequence of steps into the most general (sensible) form. In the current study, the importance of using numeracy across non- mathematics curriculum areas and the need for teachers to have the knowledge to design NRT is described and discussed in the first phase of the knowledge framework design process.

The second phase is called initial framework design. During this phase, the problem generalization levels of the previous phase are considered in reverse order leading to the third phase of implementation. This phase is grounded in the deductive thematic analysis approach (Pearse, 2019), where the specific purpose of creating the framework is to explore the need for teachers' knowledge to design NRT. Using this process, the authors identified the knowledge types needed to design NRT through filtering and adapting existing knowledge and the input of additional contextual and experiential knowledge to co-construct a knowledge or conceptual framework based on agreed categories or themes.

The last phase of a framework design is implementation. It is the process of applying the initial framework in various contexts. The implementation can be achieved by using and testing the framework for the next refinement and generalization of the problem. The second alteration of generalization of problem and redesign of the framework typically requires adding a few specific issues identified during implementation and applying for refining the initial framework. This process uses the fact that the framework is open-ended for further refinement. This approach results in a hierarchy of more refined frameworks, and it is an iterative approach in which the initial framework gets more mature.

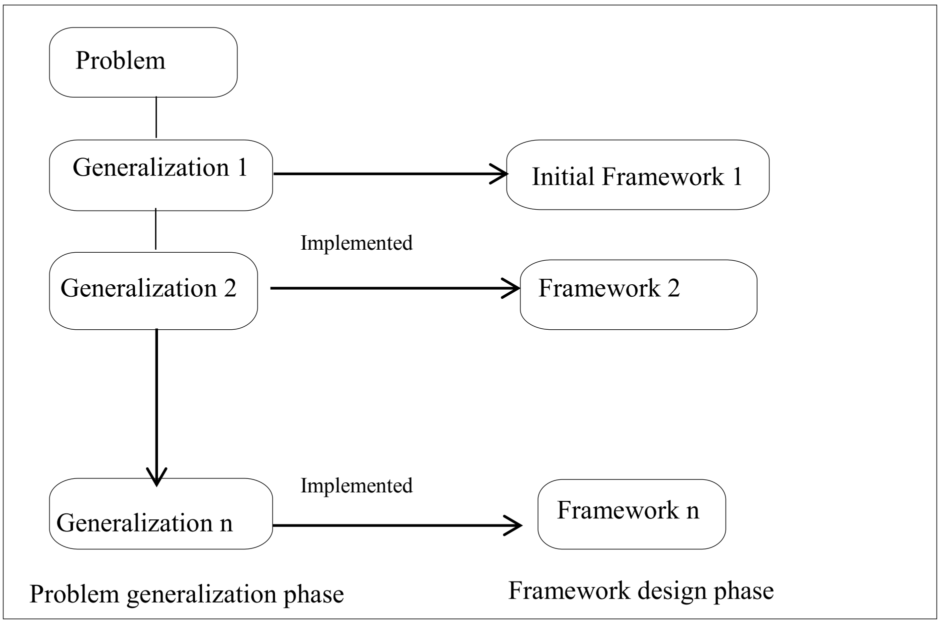

Figure 1. The Development Process of a Framework (adapted from Koskimies & Mössenböck, 1995)

Various studies (e.g., Ben-Abdallah et al., 2004; Yang et al., 1998) suggested and used the first two phases to build an initial version of a framework - problem generalization and design of the initial framework. Accordingly, the current study used the first two phases to design the initial knowledge framework.

In this study, the authors propose an initial version of a framework for teachers' knowledge required to design NRT. The framework is open to be applied to various contexts in the follow-up studies, such as analyzing numeracy tasks and designing professional learnings. The initial framework can later be revised according to the framework's new requirements for further refinement.

According to Yang et al. (1998), one of the limitations of this framework design process is that essential tasks to problem generalization, or framework design are not defined concretely. In addition, this framework design process is used in the science problem domain. However, we believe that the phases can be used to design a framework in mathematics education research. The sections follow the three phases of the framework design process by Koskimies and Mössenböck (1995) to design the knowledge framework required by teachers to design NRT.

Numeracy across Curriculum Areas

Globally, teachers are encouraged to identify and exploit the numeracy learning opportunities that exist in various non-mathematics curriculum areas (e.g., Bennison, 2015; Brown et al., 1998; Ford, 2018). For example, in the USA, numeracy is intended to be taught by all teachers across different curriculum areas (e.g., Ford, 2018). Steen (2001) recommended considering numeracy opportunities across curriculum areas based on the premise that it must be useful to students, suggesting that it must be learned in mathematics and multiple contexts and all school subjects. The Australian government recognized that numeracy development requires experience beyond the mathematics classroom (Council of Australian Governments, 2008). ACARA, which manages the Australian curriculum stresses that connecting mathematics to other non-mathematics curriculum areas and students' everyday lives is an important goal of school mathematics (ACARA, 2019). As a result, ACARA has included numeracy as one of seven general capabilities' areas to create students who are successful, confident, and creative individuals and active and informed citizens.

Further descriptions of numeracy in the Australian Curriculum emphasizing the knowledge and skills related to mathematics content is likely to occur across the non-mathematics curriculum areas. They include examples to illustrate numeracy that teachers can use across the non-mathematics curriculum areas in schools. This approach is seen as the best strategy to develop students' numeracy skills in the primary contexts, including Australian schools (Goos et al., 2013). Bennison (2015) and Ford (2018) also stressed that this approach has more significant potential to empower students than approaches that seek to develop numeracy capabilities solely through mathematics curriculum areas. Consequently, numeracy became formally recognized as an across-¬ the-¬ curriculum commitment in Australian schools, and it is differentiated from the mathematics curriculum area (ACARA, 2019). In addition, the need for teachers' knowledge and teaching strategies is incorporated into the Australian Professional Standards for Teachers (Australian Institute for Teaching and School Leadership, 2019).

Bennison (2015) and Brown et al. (1998) showed that teachers are encouraged to identify and exploit the numeracy learning opportunities in various non-mathematics curriculum areas. According to Bennison (2015), however, teachers are required to complete at least three tasks to teach numeracy across non-mathematics curriculum areas. First, they must recognize numeracy learning opportunities across curriculum areas; second, they need to design NRT; and third, they need to implement the NRT. In this article, the author focuses on the second of these - the design of NRT. The following section describes the knowledge required to design NRT across non-mathematics curriculum areas.

Teachers' Knowledge for Designing Numeracy Rich Tasks

Teachers work with 'tasks"' all the time, in their daily preparation work and during classroom teaching to provide opportunities to learn for students. The tasks are often regarded as influencing and determining students' learning opportunities (e.g., Johnson et al., 2017; Schmidt et al., 1997; Tornroos, 2005). Hence, an approach or a framework is needed that helps teachers design numeracy-rich tasks across non- mathematics curriculum areas, including what teachers know and how they apply what they know in the unique circumstances or contexts within their classrooms.

Callingham et al. (2015) argued that it is unlikely that teachers who focus on various non-mathematics curriculum areas, such as art or history, will feel equipped to develop mathematical ideas disposed towards focusing on numeracy. A challenge is to raise teachers' knowledge and importance of numeracy and encourage their engagement with numeracy aspects outside mathematics curriculum areas (e.g., Beswick & Fraser, 2019; Callingham et al., 2015; Goos et al., 2013). Hence, these studies (e.g., Beswick & Fraser, 2019; Callingham et al., 2015; Geiger et al., 2015) highlighted the importance of mathematical knowledge to design and teach mathematics across non–mathematics curriculum areas. Similarly, studies that highlighted the knowledge and skills teachers require to design NRT included Goos et al. (2010, 2013) and Geiger et al. (2015). They suggested the need for teacher knowledge, such as applying mathematics knowledge and context to integrate mathematics (numeracy) into other non-mathematics curriculum areas.

Geiger et al. (2015) designed a numeracy model to capture the demands of 21st century personal, civic and work life. The model was developed to support teachers to promote their students' numeracy capabilities in the range of non-mathematics curriculum areas. The model consists of four core dimensions: attention to contexts; application of mathematical knowledge; use of physical and digital tools; and promotion of positive dispositions, the use of mathematics to solve real-¬life problems. They further included a fifth dimension— a critical orientation —intertwined through and between the core dimensions. Geiger et al. (2015) argued that the numeracy model assisted teachers in planning and designing learning experiences and creating and structuring numeracy tasks so that students had opportunities to apply mathematics in diverse contexts. The model has been used to develop numeracy-¬ focused activities in a range of non – mathematics curriculum areas— for example, English (Geiger et al., 2013), health and physical education (Peters et al., 2012), civics education (Willis et al., 2012) and environmental education (Cooper et al., 2012). They did not highlight what type of knowledge teachers required to design such numeracy rich activities and did not provide direction for promoting teachers to have such knowledge to design NRT. However, they have highlighted the importance of mathematical and context knowledge to design NRT. In various contexts, other studies emphasized the importance of having at least one non–mathematics curriculum area knowledge (e.g., Beswick & Fraser, 2019; Phillips, 2002) and design skill (Beetham, 2007; Hofer & Harris, 2010) for teachers to design NRT across non- mathematics curriculum areas.

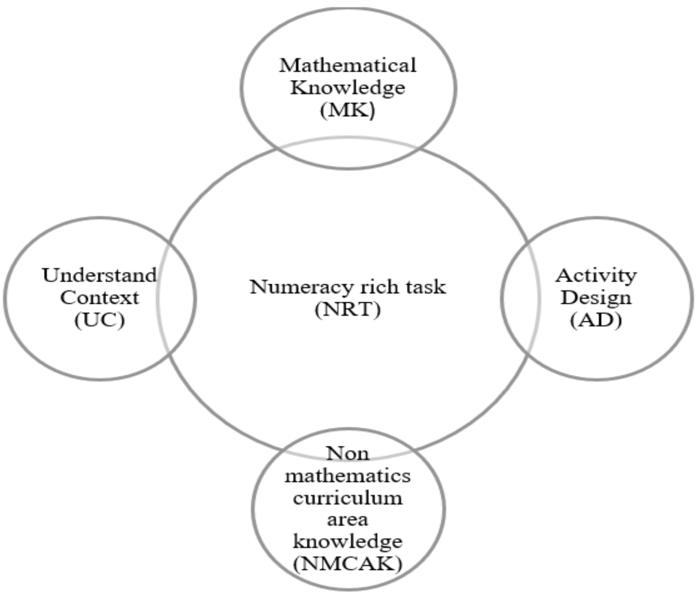

Figure 2. Teachers' Knowledge for Designing NRT across Non- Mathematics Curriculum Areas

Each of them is further described in the following section. Later, indicators that can describe each of these knowledge types are identified. The examples of indicators are highlighted and derived from the description of the knowledge required to design NRT.

Mathematics Content Knowledge (MCK)

Teachers' mathematical knowledge has long been scrutinized in both primary and secondary classrooms contexts. Notably, the scrutiny is more on Australian primary school teachers as they are not required to be subject specialists to teach. However, studies have shown that teachers need to be competent in substantive knowledge regarding mathematics content and pedagogy to be effective teachers (e.g., Ball et al., 2008; Blömeke et al., 2011; Loughran et al., 2008; Schmidt et al., 2008). Ball and colleagues (e.g., Ball et al., 2008) and Park and colleagues (Park et al., 2011) have emphasized the importance of mathematics content knowledge for teachers to use in teaching. Ball et al. (2008) suggested that mathematics content knowledge includes knowledge of concepts, ideas, procedures, how they work, and how one decides that a claim is valid, a solution complete, or a representation accurate. This indicates that mathematics teaching demands teachers to acquire the appropriate mathematical knowledge to use in teaching.

According to Ball et al. (2008), teachers should be competent in four mathematics content areas to design teaching activities and teach mathematics. First, teachers need to be aware of the mathematical knowledge and skills used in settings other than teaching. This type of knowledge is used in various settings, and although essential for teaching mathematics, it is not unique to teaching. Second, teachers need to know the mathematical knowledge that might be taught directly to students to understand. Third, the knowledge of teachers about students and their mathematical thinking and development in the context of learning mathematics is required. Fourth, teachers require the knowledge to design pedagogical approaches for teaching mathematics. Goos et al. (2013) suggested that designing a numeracy task requires using and applying mathematical knowledge. Goos et al. (2013) identified the importance of mathematical knowledge to design NRT, including appropriate concepts and skills and capabilities such as making sensible estimations and applying problem-¬ solving strategies. This shows that to design NRT, teachers need mathematical knowledge and the know-¬ how to apply this knowledge in ways relevant to a specific context. Table 1 presents the description of MCK and its indicators and samples of the literature sources.

Table 1. Sample Literature and Indicators for Mathematics Content Knowledge

Description	Indicators	Examples of Sources
The knowledge of mathematics concepts that are to be learned or taught and mathematics knowledge for teaching Appropriate concepts and skills and applying to a relevant and a specific context	• The knowledge of the concept of mathematics • Uses of relevant materials • Mathematics relevance – Age and prior knowledge	Ball et al. (2008), Callingham et al. (2015); Forgasz et al. (2017), Goos et al. (2013)

A close study of teachers' mathematics knowledge specific to designing NRT across non-mathematics curriculum areas could inform the need to develop teachers' knowledge in, for example, the concepts of mathematics (e.g., its definition), the mathematics to be used in teaching (e.g., help students to understand the concept) and, understanding students' levels of mathematical thinking. These knowledge types can be used to assess and evaluate the quality of teachers designed NRT.

Non-Mathematics Curriculum Areas Knowledge (NMCAK)

Developing numeracy requires that students gain the confidence and experience to use their mathematical knowledge in everyday situations and non-mathematics curriculum areas at school (Goos et al., 2019). The purpose of understanding the numeracy demands of the subject teachers are teaching is to enhance students' learning of this subject, not to teach them more mathematics. Students develop numeracy capability as they learn discipline-specific knowledge about, for example, history, civics and citizenship, technology, and economics and business (ACARA, 2019). Hence, teachers require the knowledge of the contents of these non-mathematics curriculum areas to deliver the content intended (such as history, science, geography, etc.), in addition to the mathematics content knowledge. Shulman (1987) called this content knowledge which encompasses "the knowledge, understanding, skill, and disposition that are to be learned" (p. 8). Beswick and Fraser (2019), focusing on Science, Technology, Engineering and Mathematics (STEM), argued that numeracy used in non – mathematics curriculum areas (such as science, technology and engineering) rely on teachers having the expertise in at least one non-mathematics curriculum area. The description of NMCK and its indicators and sample of the literature are provided in Table 2.

Table 2. Sample of the Literature and the Indicators for the Non-Mathematics Curriculum Areas Knowledge

Description	**Indicators**	**Examples of Sources**
The knowledge of a non-mathematics curriculum area/ concepts that mathematics will be used to facilitate the learning Appropriate concepts, skills, and key ideas that organised the curriculum scope for the non-mathematics curriculum area (English, art, science, etc.)	• Focus on the curriculum area • Mathematics is used as a vehicle to teach a non-mathematics concept • Demonstrate and understand the concept • Structure of the content	ACARA (2019), Beswick and Fraser (2019), Geiger et al. (2013), Philip (2002)

Further to having mathematics content knowledge, Goos et al. (2019) argued teachers should; "identify the specific numeracy demands of their learning area/s, provide learning experiences and opportunities that support the application of students' general mathematical knowledge and skills. In addition, they argued that teachers should be aware of the correct use of mathematical terminology in their learning area/s and use this language in their teaching as appropriate. This knowledge is needed to have sensible conversations with students and for effective planning of relevant activities. For example, in history, students are required to develop numeracy capability as they learn to organize and interpret historical events and developments and analyze numerical data to make meaning of the past. In addition to the mathematics concepts, teachers require the historical content knowledge embedded in these numerical concepts so that mathematics is used to deliver the intended history concept (Blow et al., 2012; Phillips, 2002). The teacher's non-mathematics knowledge included demonstrating knowledge and understanding of the target teaching area's concepts, substance, and structure (e.g., history). In addition, teachers require the knowledge to organize content into an effective learning and teaching sequence. From the above analysis, the key knowledge needed in the target curriculum area's context is the underlying subjects' concepts, structures, and sequencing of the relevant teaching strategies. This includes teachers having a focused knowledge and understanding of the non-mathematics curriculum area, structuring the contents, and using mathematics as a vehicle to teach a non-mathematics curriculum area concept.

Activity Design (AD)

The numeracy tasks with which students engage can shape students' mathematical learning opportunities and their experiences with mathematics in various contexts (Watson & Mason, 2007). Once teachers identify numeracy demands and opportunities, they need to plan for numeracy learning by designing activities that enhance numeracy and non-mathematics curriculum areas. Beetham (2007, p. 28) defined learning activities as a specific interaction of learner (s) with other (s) using specific tools and resources, oriented towards specific outcomes. Watson et al. (2014) defined activity as 'activity is the subsequent mathematical (and other) motives that emerge from the interaction between student, teacher, resources, environment, and so on around the task. There has been considerable research on how to design effective activities for students' learning. Goos et al. (2019, p. 106) proposed the importance of checking on 'how does the activity address curriculum goals in the target learning area and how does the activity develop students' numeracy capabilities' to identify the effectiveness of activities. Studies have supported that good learning activities engage students and significantly improve learning outcomes (e.g., Beetham, 2007; Hofer & Harris, 2010). Beetham (2007) emphasized that good learning activities are pedagogically meaningful and focus on students' effective learning. Beetham (2007) and Hofer and Harris (2010) have suggested that good learning activity design requires applying theoretical principles and criteria for sound learning approaches. Consequently, Beetham (2007) suggested five criteria for the design of practical learning activities and an outline for a learning activity. An overview of a learning activity is shown in Figure 3. Considering learning outcomes, the learners, learning environment and specific roles of others involved in the learning separately seem vital in designing learning activities. These elements are highly relevant to designing learning activities to achieve the intended objectives of classroom teaching.

Activities in a highly authentic context are more productive. Students enjoy activities and examples relevant to their daily lives (e.g., Herrington et al., 2014; Reys et al., 2019). According to Beetham (2007), effective learning depends on authenticity and how the concepts are structured. Carefully organized and sequenced activities help students to learn the focus concept more easily. Good learning activities support students in integrating a concept or skill with their existing beliefs and capabilities in various contexts. In addition, the value of collaboration should be reflected in the learning activities for effective students' learning. Quality learning activities allow students to guide their learning in the classroom. Considering these criteria can help the best students benefit from the learning experience when undertaking the design process. On the other hand, Hofer and Harris (2010) suggested teachers follow a general planning sequence to design learning activities. The planning sequences includes identifying learning goals, understanding classroom context and student learning styles and preferences, sequencing appropriate learning activities, selecting appropriate assessment strategies and using relevant tools and resources. Table 3 presents and the description and the indicators for the knowledge to design numeracy activities and sample literature sources.

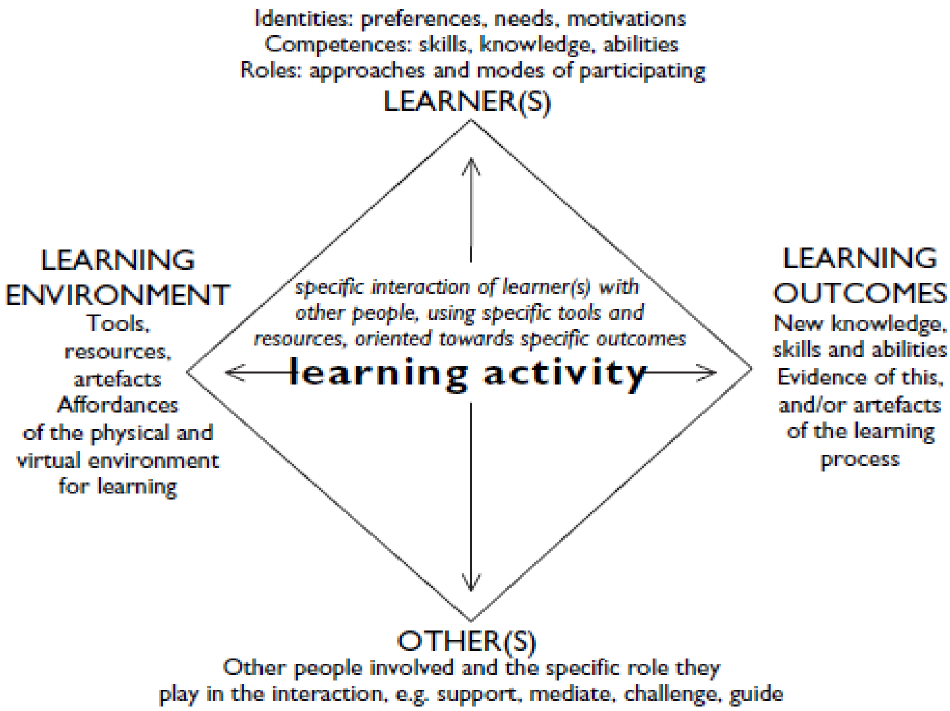

Figure 3. An Outline for a Learning Activity (Beetham, 2007, p. 29)

As shown in Table 3, combining Beetham's (2007) and Hofer and Harris's (2010) suggestions about effective learning activities, the following criteria can be used as indicators to design NRT that lead to significant positive learning outcomes. These emphasize students' engagement, relevance to students' lives, consideration of students' existing capabilities, including options for diverse students, providing opportunities for discussion, use of relevant tools, and inclusion of assessment strategies.

Table 3. Sample of the Literature and the Indicators for the Knowledge to Design Numeracy Activities

Description	Sample indicators	Examples of Sources
Knowledge of specific interaction of student(s) with others using specific tools and resources, orientated towards specific outcomes and pedagogically meaningful and focused on students' effective learning	• Student-focused • Relevance to students' lives • Consideration of students existing capabilities, including options for diverse students • Provision of opportunities for discussion • Use of relevant tools • Inclusion of assessment strategies	Beetham (2007), Herrington et al. (2014), Hofer and Harris (2010), Reys et al. (2019)

Understand the Context (UC)

Context is a complex, multifaceted, perspective dependent concept extending from the specific characteristics of the learning and teaching environment to disciplinary and personal issues (Benson & Samarawickrema, 2007). According to Morrison et al. (2004) and Borko (2004), context is a collection of factors that can influence the design of learning activities. Context plays an important role in providing rich data for designing real examples, practical content and scenarios (Morrison et al., 2004; Tessmer & Richey, 1997). Callingham et al. (2015) argued that understanding a context helps teachers recognize the appropriate mathematical ideas used in non-mathematics curriculum areas and suggest ways to acknowledge and develop these ideas. Understanding context can help the designer to suggest context-relevant teaching activities (Borko, 2004). Therefore, teachers need to have mathematics content, a non-mathematics curriculum area, and selecting and designing learning activities. They also require the knowledge of classroom contexts (such as diversity of students' learning styles and preferences) to design NRT. Understanding the context offers teachers opportunities to plan for effective learning experiences (Hofer & Harris, 2010). Table 4 presents samples of the literature, description and the indicators for the knowledge for context.

Table 4. Description and Sample of the Literature and the Indicators for the Knowledge for Context

Description	Sample indicators	Examples of sources
The knowledge of how classroom contexts and the diversity of students' learning styles and preferences offer opportunities to plan for effective learning experiences Use mathematics in a manner specific to the features of a particular problem	• Students' prior knowledge • Age appropriateness • Relevance to existing curricula • Available potential tools • The characteristics of the school system	Benson and Samarawickrema (2007), Callingham et al. (2015), Goos et al. (2013), Hofer and Harris (2010), Morrison et al. (2004)

Context includes understanding both classroom context and the students' context (Hailikari et al., 2008; Smith & Ragan, 2005). Classroom context covers matters such as the availability of potential learning resources such as technology, the characteristics of the school system and relevance to existing curricula (Goos et al., 2013). The students' context involves students' prior knowledge, age, beliefs about the subject matter and preferences, attitudes and interests (Benson & Samarawickrema, 2007; Smith & Ragan, 2005). There is also scope for content to be taught using specific local contexts that align with students' own lives and interests. Barbosa and de Oliveira (2013) argued that the extents to which tasks are set in a realistic context to maximize engagement of students and, on the other hand, whether the context detracts from the potential of the task to achieve the intended learning. As shown in Figure 2 and described in Tables 1-4, teachers' effective design of NRT across non-mathematics curriculum areas lies in teachers' knowledge of mathematics content, a non –mathematics curriculum area, design activities and the context of the classrooms and students.

In addition to the need for these knowledge types, some non – mathematics curriculum areas might be difficult relative to other curriculum areas for teachers to integrate numeracy. For example, teachers often have difficulty using numeracy across English or literacy because of the perceived lack of materials recommended and readily available to teachers (Koellner et al., 2009). Similarly, Geiger et al. (2013) argued that English curricula provided little opportunity for teachers to identify numeracy demands in its content. On the other hand, Gough (2007) argued that science, for example, is a very common non-mathematics curriculum area to integrate numeracy concepts such as generating numerical data, making and reading data tables and graphs.

Discussion

In this study, the author argued that teachers need to have four knowledge types for designing numeracy-rich tasks across non-mathematics curriculum areas. These are knowledge of mathematics content, non-mathematics curriculum area, activity designs and context to design effective NRT (see Figure 2 and Tables1-4).

Teachers require appropriate mathematical knowledge to use in various contexts, including across non–mathematics curriculum areas. These include considering and using the mathematics concepts relevant to students' existing knowledge and year levels in their designed NRT. While using relevant mathematics, studies highlighted the importance of using appropriate resources such as teaching patterns using manipulatives in teaching mathematics (e.g., Koellner et al., 2009; Herrington et al., 2014). Effective NRT require the desire to embed the related mathematics concepts within numeracy tasks and appropriate resources that best support the implementation of the NRT (Forgasz et al., 2017; Young-Loveridge et al., 2012). The inclusion of relevant resources is considered a factor for meaningful pedagogy and students' effective learning.

Teachers require the content knowledge of non-mathematics curriculum areas and the mathematics content to be used and then deliver the intended content. In addition, teachers who are usually non-specialists require knowledge of the content of curriculum areas in addition to the mathematics content to use and deliver the learning intended in those non-mathematics curriculum areas. Studies argued that mathematics should be a tool for curriculum area understanding (e.g., Phillips, 2002). Phillips (2002) also stressed that mathematics should be seen as a tool to aid understanding and enrich students' understandings of the non-mathematics curriculum area concepts. Beswick and Fraser (2019) emphasized the importance of teachers having at least one non-mathematics curriculum area (in the context of science, technology and engineering) knowledge to use numeracy in those curriculum areas. However, regardless of the curriculum area knowledge, teachers might show low confidence in integrating numeracy across selected curriculum areas such as English/literacy (Geiger et al., 2013; Koellner et al., 2009). Similarly, Koellner et al.'s (2009) study results showed that teachers were less confident integrating numeracy across English because of the perceived lack of recommended and readily available materials. Geiger et al. (2013) attributed this to the nature of the Australian Curriculum - English, which has little opportunity for teachers to identify numeracy demands in its contents.

When designing engaging activities, studies have recommended that effective learning activities support students to integrate a concept or skill with their existing beliefs and capabilities in various contexts (Beetham, 2007; Hofer & Harris, 2010). A good design of learning activities can engage students in their learning and lead to significant positive learning outcomes (e.g., Herrington et al., 2010; Reys et al., 2019). However, studies such as Beetham (2007) and Hofer and Harris (2010) recommended that effective learning activities support students in integrating a concept or skill with their existing beliefs and capabilities in various contexts.

Knowledge of classroom contexts and the diversity of students' learning styles and preferences offer opportunities to plan for learning experiences. Teachers who lack the expertise to address or consider contexts in their design of NRT could make the task design process challenging to provide students with the learning experiences necessary to adapt the knowledge they learn in school to the outside world. Steen (2001) also argued that for students to become numerate, they must engage with tasks that demand using mathematics in multiple contexts.

The design of NRT by teachers in Australia is becoming a policy imperative; however, requiring teachers to develop rich numeracy tasks might be another 'ask' of already over-burdened teachers. Several studies have reported positively involving teachers as partners in task design (e.g., Askew & Canty, 2013; Knott et al., 2013; Jones & Pepin, 2016) for effective design of tasks. Jones and Pepin (2016) stated that various research and development teams have, over the years, undertaken collaborative efforts aimed at developing and testing task designs for improving the teaching and learning of mathematics. For example, Askew and Canty (2013) examined how teachers developed classroom tasks as springboards to develop students' mathematics reasoning. They concluded that introducing a framework for working on and with the tasks and treating this as a joint endeavor promoted teacher inquiry and collegiality. They found that their way of working supported teachers who could independently create rich tasks that moved their students towards justifying and generalizing. Hence, a collaborative approach can be an effective strategy to design NRT.

Limitations and Future Directions

There were at least two limitations to the findings of this study. In our attempt to develop the examples indicators used to unpack the various elements of teachers' knowledge, there were overlaps between the different aspects of the framework, such as mathematics content and non-mathematics curriculum area content knowledge. This overlap suggests a further refinement of this initial framework could be beneficial. For example, the indicators of each knowledge type are illustrative only and are not meant to be a definitive list. Further studies could support generating more and quality indicators through the implementation of the initial framework. Despite these limitations, this study can help explore the gaps and options for integrating numeracy across non-mathematics curriculum areas and suggest relevant professional learning strategies for teachers. However, the framework is likely to be revised as teachers use it in various contexts and provide feedback on their experiences.

Conclusion

The purpose of this study was to develop a knowledge framework required by teachers to design effective Numeracy rich tasks (NRT) across non-mathematics curriculum areas. This study can have various implications. First, it highlighted that for school teachers to contribute to students' numeracy competence, they need mathematics knowledge, non-mathematics curriculum area knowledge and skills of activity design and relevant contexts, and strategies to support teachers to develop this knowledge and skills. Reasonably, influential knowledge frameworks for teachers to integrate numeracy across non-mathematics curriculum areas could help teachers to build these knowledge types. The author believes that the knowledge framework suggested in this study is the starting point. Second, the framework included the essential knowledge elements required to design NRT. The knowledge framework and approach adopted in this study can be used to analyse teachers' designed NRT and identify their professional learning requirements. The knowledge framework can also be used to analyse the knowledge and skill requirements for teaching numeracy across non-mathematics curriculum areas and make an explicit criterion for each knowledge type. Third, the framework proposed adds dimensions to existing numeracy models such as Goos et al. (2013). Goos and colleagues emphasized the importance of having mathematics knowledge and context to design numeracy tasks. This study highlighted the importance of having additional knowledge: a non-mathematics curriculum area and activity design. Future studies could include using this framework to evaluate a numeracy rich task.

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