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LINEAR MODEL INFERENCE WITH NON-SAMPLE PRIOR INFORMATION

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ABSTRACT

Arguably the most widely used statistical technique is the linear model. Traditionally all classical inferences on the parameters of linear model are based exclusively on the available sample data. Often valuable non-sample prior information on the value of the parameter of interest is available from the expert knowledge or previously conducted studies. Inclusion of such information, in addition to the sample data, is likely to improve the quality of the inference. This paper uses both sample and non-sample information to define estimators of linear model and investigate their statistical properties. It also incorporates the non-sample prior information in defining tests for a subset of parameters when information on the other subset is available. The comparisons of power of the tests are also explored under different conditions.

Keywords: Pretest and shrinkage estimators,bias and mean squared error, pretest test, power and size of test, and non-central bivariate t distribution.

1. Introduction

Classical or frequentist statistics exclusively uses sample information to make inference on population parameters. Incorporation of non-sample prior information with the sample data is likely to improve the quality of inference. However, any such improvement depends on the accuracy of the non-sample prior information. Bayesian approach includes the prior distribution of the parameters of the model along with the sample data to draw inference. The prior distribution is not unique, indeed often subjective, and the posterior distribution depends on the choice of the prior distribution, and that affects the ultimate inference. Nevertheless, non-sample prior information (NSPI) on the value of any parameter from reliable sources can be accurate and lead to correct inference.

As a common practice, classical inferences about population parameters are always drawn from the sample data alone. This applies to methods used in parameter estimation and hypothesis testing. Inferences about population parameters could be improved using non-sample prior information (NSPI) from trusted sources (cf Bancroft, 1944). Such information, which is usually available from previous studies or expert knowledge or experience of the researchers, is un-related to the sample data. It is expected that the inclusion of NSPI in addition to the sample data improves the quality of the estimator and the performance of the test. However, any NSPI on the value of any parameter is likely to be uncertain (or unsure). In this case, the information can be articulated in the form of a null hypothesis. An appropriate statistical test on this null hypothesis is useful to eliminate the uncertainty on the NSPI. Then the outcome of the preliminary test is used in the hypothesis testing or estimation. This approach is likely to improve the quality of the estimator and the performance of the statistical test (see Khan and Saleh 2001; Saleh 2006, p. 1; Yunus 2010; Yunus and Khan, 2011a; and Pratikno 2012).

The NSPI can be classified as (i) unknown (unspecified) if NSPI on the value of the parameter(s) is unavailable, (ii) known (certain or specified) if the exact value of the parameter(s) is available, and (iii) uncertain if the suspected value is unsure (that is, suspected to be a fixed quantity, but not sure). For the three different scenarios, three different estimators, namely the (i) unrestricted estimator (UE), (ii) restricted estimator (RE) and (iii) preliminary test estimator (PTE) are defined in the literature (see,e.g., Judge and Bock, 1978; Saleh, 2006, p. 58). Khan (2003), and Khan and Hoque (2003) provide the UE, RE, and PTE for different linear models. Many authors have contributed to this area to the estimation of parameter(s) in the presence of uncertain NSPI. Bancroft (1944, 1964, 1965) and Han and Bancroft (1968) introduced a preliminary test estimation of parameters. Later, Sclove et al. (1972), Stein (1981), Bhoj and Ahsanullah (1994), Khan (1998, 2003, 2005, 2006a, 2006b, 2008), Khan and Saleh (1995, 1997, 2001, 2005, 2008), Khan et al. (2002a, 2002b, 2005), Khan and Hoque (2003), and Saleh (2006, p. 55) covered various work in the area of improved estimation using NSPI.

For the testing purpose, three different statistical tests, namely the (i) unrestricted test (UT), (ii) restricted test (RT) and (iii) pre-test test (PTT) are defined along the same line as the three different estimators. The UE and UT use the sample data alone but the RE and RT do not use the sample data alone. The PTE and PTT use both the NSPI and the sample data. The PTE is a choice between the UE and RE, whereas the PTT is a choice between the UT and RT. The choice depends on the outcome of the pre-testing on the uncertain NSPI value. Note that by definition the test statistics of the PT and UT are correlated but that of the PT and RT are uncorrelated, indeed independent.

There are a very limited number of studies on the testing of parameters in the presence of uncertain NSPI. Tamura (1965), Saleh and Sen (1978, 1982), Yunus and Khan (2008, 2011a, 2011b), and Yunus (2010) used the NSPI for testing hypothesis using nonparametric methods. Some authors have studied the UE, RE and PTE for parametric cases (for instance Bechhofer (1951), Bozivich et al. (1956), Bancroft (1964), Saleh (2006)), and Hoque et al. (2009) but not the tests. Pratikno (2012) covered the testing after pretest under the parametric framework for a number of linear regression models. The non-parametric approach, namely the M-test method, is used by Yunus (2010) and Yunus and Khan (2011b).

2. The Simple Regression Model

The simple regression model for $[(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)]$ can be represented by

$$\boldsymbol{y} = \theta \boldsymbol{1}_n + \beta \boldsymbol{x} + \boldsymbol{e}, \tag{2.1}$$

where θ and β are the intercept and slope parameters, $\boldsymbol{x}$ is the vector of explanatory variable, $\boldsymbol{y}$ is the vector of the response variable, and the error vector $\boldsymbol{e} \sim N(\mu, \sigma^2 I_n)$ in which I_n is the identity matrix of order n and σ^2 is the spread parameter. Let uncertain NSPI on the value of β be available, and the degree of distrust in the NSPI be $0 \leq d \leq 1$.

In addition to the simple regression model above, the following linear models may be considered to estimate parameters and perform tests: estimate/test (1) the intercept vector of the *multivariate simple regression model* (MSRM) when there is NSPI on the slope vector, (2) a subset of regression parameters of the *multiple regression model* (MRM) when NSPI is available on another subset of the regression parameters, and (3) the equality of the intercepts for $p(\geq 2)$ lines of the *parallel regression model* (PRM) when there is NSPI on the slopes. In this paper we do not include these model. It may be noted that to study the properties (power and size) of the ptetest test of any multivariate model the bivariate noncentral chi-square (cf Yunus and Khan, 2011c) and F distributions are essential. For details on testing after pretest under parametric model, see Pratikno (2012), and Khan and Pratikno (2012, 2013).

3. The Estimation Problem

From the sample data alone the unrestricted estimator (UE) of the parameters are

$$\tilde{\beta} = (\boldsymbol{x}'\boldsymbol{x})^{-1}\boldsymbol{x}'\boldsymbol{y},\ \tilde{\theta} = \bar{y} - \tilde{\beta}\bar{x},\ \text{and}\ S_n^2 = \frac{1}{n-2}(\boldsymbol{y} - \tilde{\boldsymbol{y}})'(\boldsymbol{y} - \tilde{\boldsymbol{y}})\ \text{where}\ \tilde{\boldsymbol{y}} = \tilde{\theta}\mathbf{1}_n + \tilde{\beta}\boldsymbol{x}. \quad (3.2)$$

Note S_n^2 has a scaled χ^2 distribution with d.f. $\nu = (n-2)$. Consider β_0 to be the value of the slope from a credible source. Then this NSPI can be expressed as a null hypothesis $H_0 : \beta = \beta_0$. When the NSPI is correct, the restricted estimator (RE) of β is $\hat{\beta} = \beta_0$, so the RE of θ becomes $\hat{\theta} = \bar{Y} - \beta_0\bar{X}$. If the NSPI is under suspicion, its uncertainty is removed by testing $H_0 : \beta = \beta_0$ against $H_0 : \beta \neq \beta_0$ using the test statistic $\mathcal{L}_\nu = S_n^{-1} S_{xx}^{\frac{1}{2}}(\tilde{\beta} - \hat{\beta}) \sim t_\nu$ with $\nu = (n-2)$ df. This test statistic is used to define the preliminary test estimator (PTE). Note, in general, $t_\nu^2 = F_{1,\nu}$.

Let $0 \leq d \leq 1$ be the coefficient of distrust on the NSPI. The value of d is 0 if there is no distrust in the NSPI. Now the RE, PTE & shrinkage estimator (SE) of the intercept parameter are defined as follows:

$$\begin{aligned}
\text{Restricted estimator (RE)} : \hat{\theta}^{\mathrm{RE}}(d) &= d\tilde{\theta} + (1-d)\hat{\theta}, \quad 0 \leq d \leq 1, \\
\text{Pretest estimator (PTE)} : \hat{\theta}^{\mathrm{PTE}}(d) &= \hat{\theta}^{\mathrm{RE}}(d) I(F < F_\alpha) + \tilde{\theta} I(F \geq F_\alpha) \\
&= \tilde{\theta} + (\hat{\theta} - \tilde{\theta})(1-d) I(F < F_\alpha), \\
\hat{\theta}^{\mathrm{PTE}}(d=0) &= \tilde{\theta} + (\hat{\theta} - \tilde{\theta} I(F < F_\alpha)\ \text{when}\ d = 0, \\
\text{Shrinkage estimator (SE)} : \hat{\theta}^{\mathrm{SE}}(d) &= \tilde{\theta} + (1-d)(\hat{\theta} - \tilde{\theta}) c S_n [\sqrt{S_{xx}} |\tilde{\beta}|]^{-1}, \quad (3.3)
\end{aligned}$$

where c is the shrinkage constant, and $I(\cdot)$ is a binary indicator function.

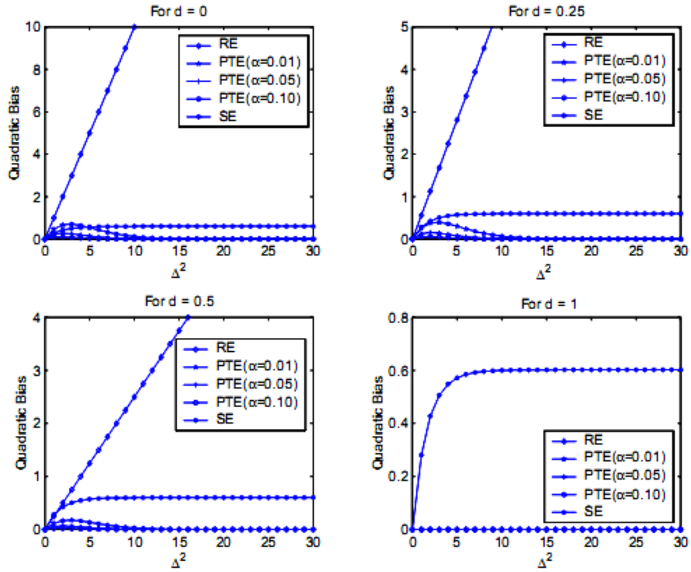


Figure 1: Graph of the quadratic bias of the RE, PTE and SE against Δ^2.

3.1. The Bias of the RE, PTE and SE

The UE is unbiased. The expression for bias of the other estimators are given below.

$$\begin{aligned}
B_2[\hat{\theta}^{\mathrm{RE}}(d)] &= S_{xx}^{-1/2}\bar{x}\sigma(1-d)\Delta, \quad \text{where } \Delta^2 = \sigma^{-2}S_{xx}(\beta-\beta_0)^2.\\
B_3[\hat{\theta}^{\mathrm{PTE}}(d)] &= (1-d)\bar{x}(\beta-\beta_0)G_{3,\nu}\left(3^{-1}F_\alpha;\Delta^2\right),\\
B_4[\hat{\theta}^{\mathrm{SE}}(d)] &= (1-d)S_{xx}^{-1/2}c\bar{x}(\beta-\beta_0)E[S_n]E\left[Z|Z|^{-1}\right],
\end{aligned}$$

where $Z = \sigma^{-1}\sqrt{S_{xx}}(\tilde{\beta}-\beta_0) \sim \mathcal{N}(\Delta, 1)$ and $G_{n_1,n_2}(\cdot;\Delta^2)$ is the c.d.f. of a non-central F-distribution with (n_1, n_2) degrees of freedom and non-centrality parameter Δ^2.

The quadratic bias of the RE, PTE and SE are

$$\begin{aligned}
QB_2[\hat{\theta}^{\mathrm{RE}}(d)] &= S_{xx}^{-1}\bar{x}^2\sigma^2(1-d)^2\Delta^2,\\
QB_3[\hat{\theta}^{\mathrm{PTE}}(d)] &= S_{xx}^{-1}\bar{x}^2\sigma^2(1-d)^2\Delta^2\left\{G_{3,\nu}\left(3^{-1}F_\alpha;\Delta^2\right)\right\}^2,\\
QB_4[\hat{\theta}^{\mathrm{SE}}(d)] &= S_{xx}^{-1}\sigma^2\bar{x}^2K_\nu^2\{2\Phi(\Delta)-1\}^2,
\end{aligned}$$

where $K_\nu = \sqrt{\frac{2}{n-2}}\,\frac{\Gamma(\frac{n-1}{2})}{\Gamma(\frac{n-2}{2})}$.

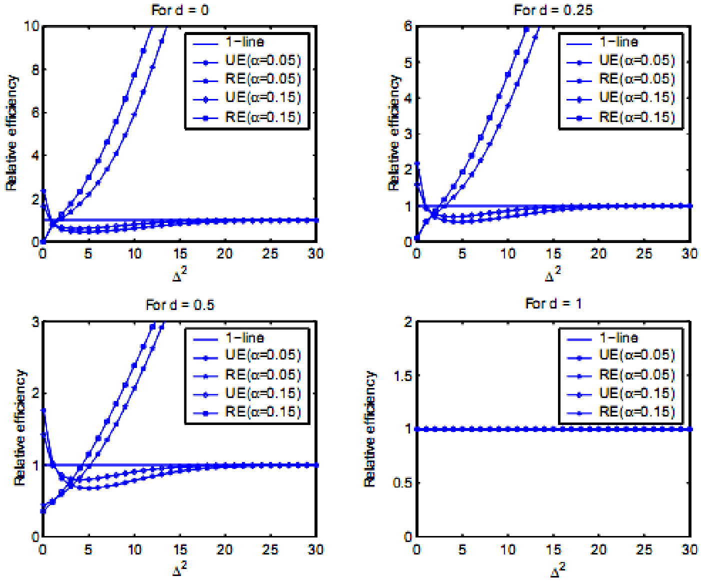


Figure 2: Graph of the relative efficiency of PTE relative to UE and RE against Δ^2.

3.2 The MSE of the RE, PTE and SE

The mean squared error of the estimators are

$$\begin{aligned}
M_2[\hat{\theta}^{\rm RE}(d)] &= \sigma^2\left[d^2H + (1-d)^2 S_{xx}^{-1}\bar{x}^2\Delta^2\right],\\
M_3[\hat{\beta}^{\rm PTE}(d)] &= \sigma^2 H + S_{xx}^{-1}\sigma^2\bar{x}^2\left[\Delta^2\left\{2(1-d)G_{3,v}\left(3^{-1}F_\alpha;\Delta^2\right)\right.\right.\\
&\left.\left.\quad -(1-d^2)G_{5,v}\left(5^{-1}F_\alpha;\Delta^2\right)\right\} - (1-d^2)G_{3,v}\left(3^{-1}F_\alpha;\Delta^2\right)\right],\\
M_4[\hat{\theta}^{\rm SE}(d)] &= \sigma^2\left[n^{-1} + S_{xx}^{-1}\bar{x}^2\left\{1 + 2\pi^{-1}K_\nu^2\left(1-2e^{-\frac{\Delta^2}{2}}\right)\right\}\right].
\end{aligned}$$

4. The Testing Problem

For testing the base load (see Kent 2009) of the energy consumption in a production plant test of intercept is appropriate. The three tests for testing $H_0^*:\theta=0$ are (1) the unrestricted test (UT) if β is unspecified; (2) the restricted test (RT) if β is specified ($\beta=\beta_0$); and (3) the pretest test (PTT), if there is uncertainty on the NSPI, after a preliminary test (PT) on the slope, that is, after testing $H_0^{(1)}:\beta=\beta_0$ to remove any uncertainty.

From now on we only consider tests for unknown σ^2.

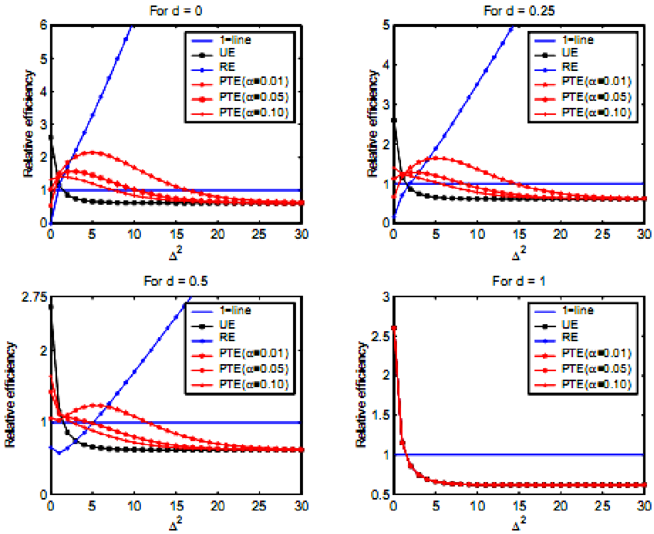


Figure 3: Graph of relative efficiency of SE relative to UE, RE and PTE against Δ^2.

1. $T_1^{(1)} = \sqrt{n}(\tilde{\theta} - \theta_0)[s_*(1 + \frac{n\bar{x}^2}{S_{xx}})^{\frac{1}{2}}]^{-1} = \sqrt{n}(\bar{y} - \tilde{\beta}\bar{x})[s_*(1 + n\bar{x}^2 S_{xx})^{\frac{1}{2}}]^{-1} \sim t_{(n-2)}$.

2. $T_2^{(1)} = \frac{\sqrt{n}(\hat{\theta}-\theta_0)}{s} = \frac{\sqrt{n}\bar{y}}{s} \sim t_{(n-1)}$ under H_0^*, where $s^2 = \frac{\sum_{i=1}^n (y_i-\bar{y})^2}{n-1}$ and $s_*^2 = \frac{\sum_{i=1}^n (y_i-\hat{y}_i)^2}{n-2}$.

3. $T_3^{(1)} = \frac{\tilde{\beta}-\beta_0}{\mathrm{SE}(\tilde{\beta})} = \frac{\sqrt{S_{xx}}\tilde{\beta}-\beta_0}{s_*} \sim t_{n-2}$ under $\mathrm{H}_0^{(1)}$

The associated test functions are defined as (1) $\Omega_1^{(1)} = I\left[T_1^{(1)} > t_{n-2,\alpha_1}\right]$; (2) $\Omega_2^{(1)} = I\left[T_2^{(1)} > t_{n-1,\alpha_2}\right]$; and (3) $\Omega^* = \begin{cases} 1, & \text{if } \{\Psi_1 \text{ or } \Psi_2\} \\ 0, & \text{otherwise,} \end{cases}$
where $\Psi_1 = \left(T_3^{(1)} \le t_{n-2,\alpha_3}, T_2^{(1)} > t_{n-1,\alpha_2}\right)$ and $\Psi_2 = \left(T_3^{(1)} > t_{n-2,\alpha_3}, T_1^{(1)} > t_{n-2,\alpha_1}\right)$.

Consider the local alternative hypothesis

$$K : (\theta, \beta) = (\delta_1/\sqrt{n}, \delta_2/\sqrt{n}),$$

where $\delta_1 = \sqrt{n}\theta, \delta_2 = \sqrt{n}\beta$ are (fixed) real values. For unknown σ^2 (1) $T_1^{(t)} = T_1^{(1)} - \frac{\delta_1}{s_*\sqrt{1+n\bar{x}^2/S_{xx}}} \sim t_{n-2}$; (2) $T_2^{(t)} = T_2^{(1)} - \frac{\delta_1+\delta_2\bar{x}}{s} \sim t_{n-1}$; and (3) $T_3^{(t)} = T_3^{(1)} - \frac{\delta_2\sqrt{S_{xx}}}{\sqrt{n}s_*} \sim t_{n-2}$.
Then

$$\boldsymbol{t} = \begin{pmatrix} T_1^{(t)} \\ T_3^{(t)} \end{pmatrix} = \frac{\boldsymbol{Z}}{\sqrt{\frac{(n-1)s_*^2}{\sigma^2}/(n-2)}}$$

is bivariate t with d.f.$=(n-2)$, location vector $(0,0)'$ and scale matrix $\begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix}$. Here,

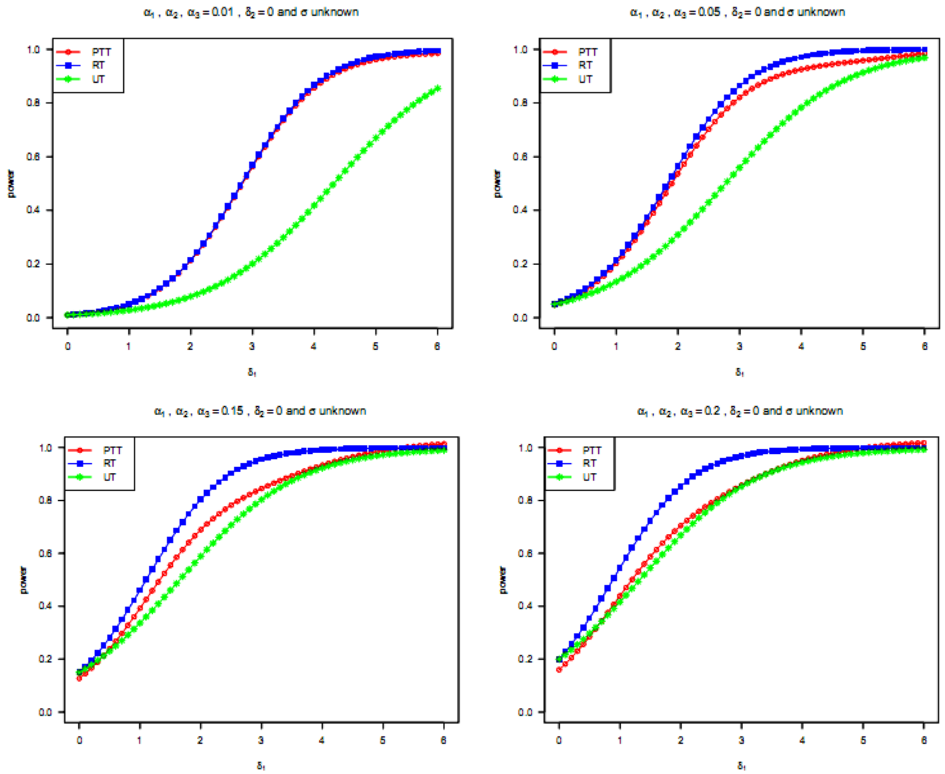


Figure 4: Power function of different tests for different values of δ_1 when $\delta_2 = 0$. $T_2^{(t)}$ and $T_3^{(t)}$ are independent.

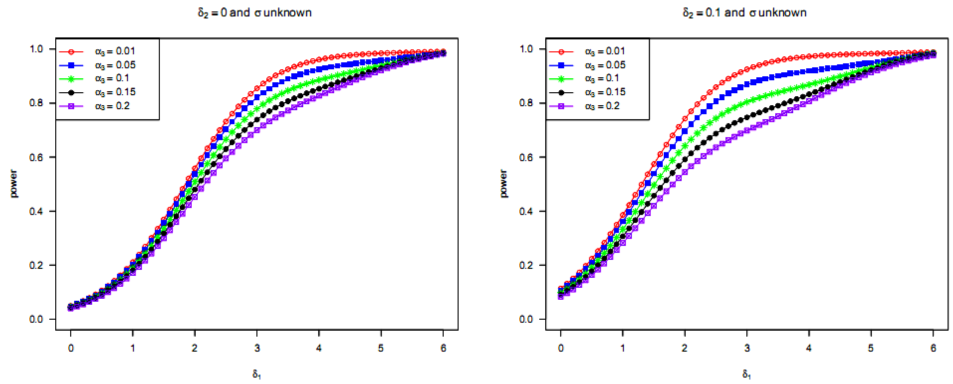
4.1. Properties of the tests - unknown σ^2

The power functions of the three test are given by

- $\Pi_1^{(1)}(\boldsymbol{\delta}) = P\left(T_1^{(t)} > t_{n-2,\alpha_1} - \delta_1[1 + \frac{n\bar{x}^2}{S_{xx}}]^{-1/2}\right)$.
- $\Pi_2^{(1)}(\boldsymbol{\delta}) = P\left(T_2^{(t)} > t_{n-1,\alpha_2} - \delta_1 - \bar{x}\delta_2\right)$, where $\delta_1 = \frac{\lambda_1}{s}(\approx \frac{\lambda_1}{s_*})$ and $\delta_2 = \frac{\lambda_2}{s}(\approx \frac{\lambda_2}{s_*})$.
- $\Pi_3^*(\boldsymbol{\delta}) = P\left(T_3^{(t)} \leq t_{n-2,\alpha_3} - \frac{\delta_2\sqrt{S_{xx}}}{\sqrt{n}}\right) \times P\left(T_2^{(t)} > t_{n-1,\alpha_2} - \delta_1 - \delta_2\bar{x}\right)$
$+ d_{1\rho(a,b,\rho)}\left\{t_{n-2,\alpha_3} - \frac{\delta_2\sqrt{S_{xx}}}{\sqrt{n}}, t_{n-2,\alpha_1} - \delta_1[1 + \frac{n\bar{x}^2}{S_{xx}}]^{-1/2}, -\rho\right\}$

where d_1 is a bivariate Student's t probability integrals defined as

$$d_{1\rho}(a,b,\rho) = \frac{\Gamma\left(\frac{\nu+2}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)n\pi\sqrt{1-\rho^2}} \int_a^{\infty}\int_b^{\infty}\left[1 + \frac{1}{\nu(1-\rho^2)}(x^2 + y^2 - 2\rho xy)\right]^{-\frac{\nu+2}{2}} dxdy, \quad (4.4)$$



in which $-1 < \rho < 1$ is the correlation coefficient between the T^{UT} and T^{PT}, $a = t_{n-2,\alpha_3} - \frac{\delta_2\sqrt{S_{xx}}}{\sqrt{n}}$ and $b = t_{n-2,\alpha_1} - \delta_1[1 + \frac{n\bar{x}^2}{S_{xx}}]^{-1}$. For large sample, $\Omega_1 = \Omega_1^{(1)}$, $\Omega_2 = \Omega_2^{(1)}$ and $\Omega^* = \Omega_1^*$. It follows that for any reasonable small n and for some moderate values of δ_2 and α_3, the size of the UT, RT and PTT will be relatively smaller when σ^2 is unknown than when σ^2 is known because $t_{cric} \geq z_{citc}$.

4.2. Illustrations

To compare the power and size of the tests graphically consider the values of the independent variable to be $1, 2, 3, \ldots\ldots, n$. Then $\bar{x} = \frac{n+1}{2}$, and $S_{xx} = \frac{n(n^2-1)}{12}$. Hence

- $\Pi_1^{(1)}(\boldsymbol{\delta}) = P\left(T_1^{(t)} > t_{n-2,\alpha} - \delta_1\sqrt{\frac{n-1}{2n}}\right)$.
- $\Pi_2^{(1)}(\boldsymbol{\delta}) = P\left(T_2^{(t)} > t_{n-1,\alpha} - \delta_1 - (\frac{n+1}{2})\delta_2\right)$.
- $\Pi_3^*(\boldsymbol{\delta}) = P\left(T_3^{(t)} \leq t_{n-2,\alpha_3} - \delta_2\sqrt{\frac{n^2-1}{12}}\right) P\left(T_2^{(t)} > t_{n-1,\alpha} - \delta_1 - \delta_2(\frac{n+1}{2})\right)$
$+ d_1\left\{t_{n-2,\alpha_3} - \delta_2\sqrt{\frac{n^2-1}{12}}, t_{n-2,\alpha} - \delta_1\sqrt{\frac{n-1}{2n}}, -\frac{\sqrt{3n+3}}{\sqrt{4n+2}}\right\}$.

The power of the tests, for different values of its arguments, are computed from the generated data and plotted in the graphs for comparison.

The size of the PTT is is given in the following table.

4.3. Comparing power and size

For $\bar{x} > 0$: (1) The RT is the best choice for its largest power but the worst choice for its largest size (2) The UT is the best choice for its smallest size but the worst choice for its smallest power (3) The size of the PTT is smaller than that of the RT regardless the value of the slope and the power of the PTT is larger than that of the UT for small and moderate values of the slope. For $\bar{x} = 0$: (1) The size and power of RT, UT and PTT are the same. For $\bar{x} < 0$: (1) The RT is the best choice for its smallest size but the worst choice for its smallest power. (2) The size and power of the UT and PTT are not much different

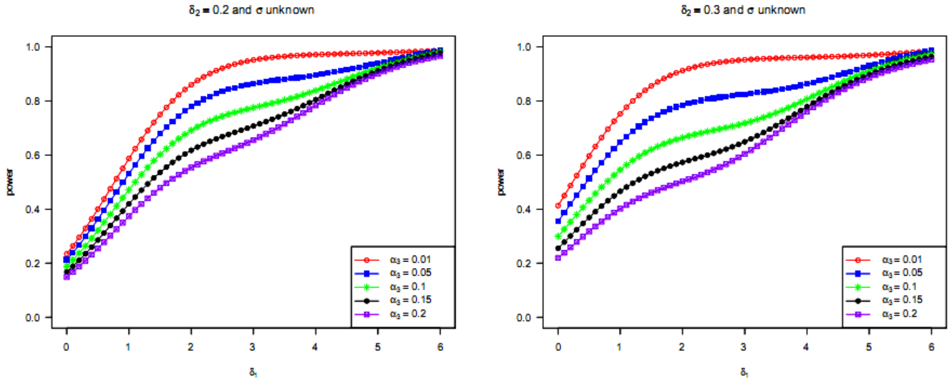
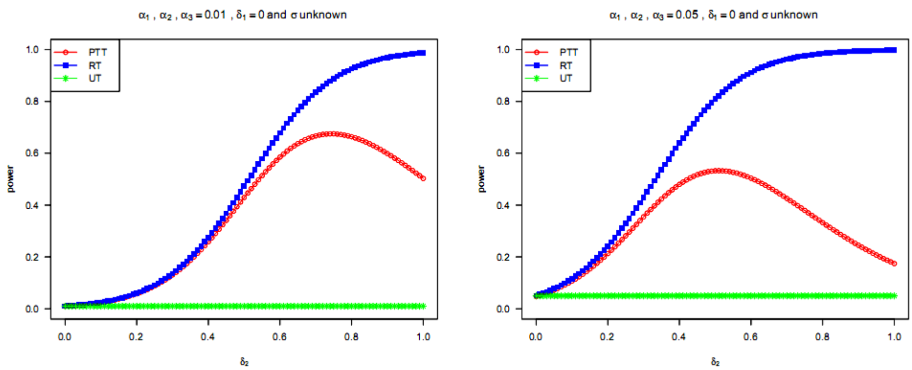


Figure 5: Power function of the PTT for different values of α_3 and varying δ_2.

Table 1: Size of the PTT following PT on slope.

Source of σ	$\delta_2 \backslash \alpha_3$	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90
σ known	0	.04	.04	.04	.03	.03	.02	.02	.01	.01	.01
σ unknown		.04	.04	.04	.03	.03	.02	.02	.01	.01	.01
σ known	.10	.12	.11	.09	.08	.06	.05	.04	.02	.02	.02
σ unknown		.10	.09	.08	.06	.05	.04	.03	.02	.02	.02
σ known	.20	.25	.22	.17	.14	.10	.08	.06	.04	.03	.03
σ unknown		.21	.18	.14	.11	.09	.07	.05	.04	.03	.03
σ known	.30	.39	.33	.24	.18	.13	.09	.07	.05	.04	.04
σ unknown		.35	.29	.21	.16	.12	.09	.06	.05	.04	.04
σ known	.40	.49	.39	.26	.18	.13	.09	.06	.05	.04	.04
σ unknown		.48	.38	.25	.18	.13	.09	.07	.06	.05	.05
σ known	1	.11	.07	.05	.05	.05	.05	.05	.05	.05	.05
σ unknown		.17	.10	.06	.06	.06	.06	.06	.06	.07	.07
σ known	2	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05
σ unknown		.07	.07	.07	.07	.07	.07	.07	.07	.07	.07



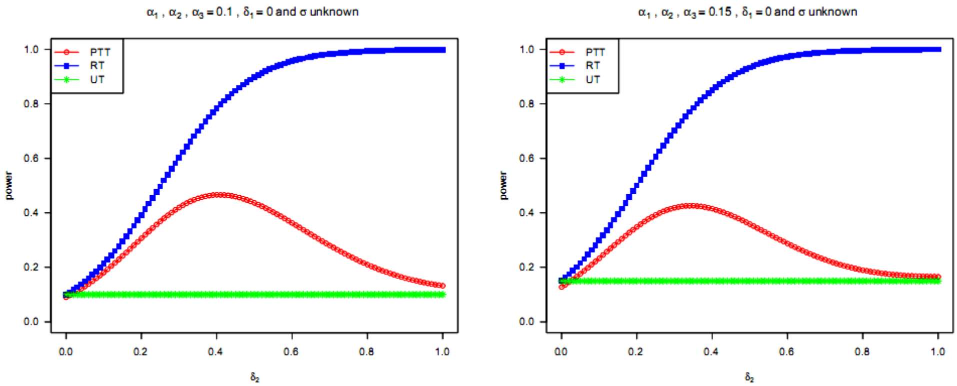


Figure 6: Sizes of UT, RT and PTT for different values of $\alpha_1 = \alpha_2 = \alpha_3$.

for moderate and large values of the slope. The power of the PTT are larger than that of the UT and RT for small values of the slope.

The size and power of the PTT is large when the nominal size of pre-test is very close to 0 especially when the slope (δ_2) is large (& other arguments fixed). This is because it approaches to the size of the RT (which is large when the slope (δ_2) is large).

5. Concluding remarks

Under the unbiasedness criterion, the UE is the best, and RE is the worst if d away from 1. But the PTE is a compromise between the two. The SE is also biased, but it is better than the RT and worse than PTE. For $d = 1$, all estimators are unbiased except the RE.

The UT has the lowest size and lowest power. The RT has the highest power and highest size. The PTT protects against the lowest power of UT and highest size of RT.

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