Problem set 10

The purpose of the section is to supply teachers and students with a selection of interesting problems. In this issue we invite readers to deal with polynomials and their properties. Questions that relate to polynomials have enormous and amazing legacy looking as far back as to Euler and the earlier times of Tartaglia and Cardano when a dispute about the solution of a cubic equation took place. Historically, the solution of polynomial equations in many circumstances served as a starting point for research into mathematical problems and a source for constructing new mathematical theories. The first three problems below are from Gelca and Andreescu (2007). Despite the simple statements they have their solution methods are non-trivial. The other two problems come from by Barbeau (1989). Together they represent the depth of diversity and beauty of polynomials' properties.

1. Find a polynomial with integer coefficients that has the zero $\sqrt{2}+\sqrt[3]{3}$.

2. Find all polynomials satisfying the functional equation
$$(x+1)P(x) = (x-10)P(x+1)$$

3. The zeros of the polynomial $P(x) = x^3 - 10x + 11$ are u, v and w. Determine the value of $\tan^{-1}u + \tan^{-1}v + \tan^{-1}w$.

4. How many roots of the equation $z^6 + 6^z + 10 = 0$ lie in each quadrant of the complex plane?

5. Show that $4x^{16}(x+1)^{64} - 3x^9(x+1)^{27} + 2x^4(x+1)^8 - 1 = 0$ has at most 14 positive roots.

References

Barbeau, E. J. (1989). *Polynomials*. New York: Springer-Verlag.

Gelca, R. & Andreescu, T. (2007). *Putnam and beyond*. New York: Springer.

Solutions to this set of problems for publication should be submitted to:
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Solutions to this set will be made available on the AAMT website (www.aamt.edu.au) after 1 May 2013.