An Extended Bilevel Programming Model and Its Kth-Best Algorithm for Dynamic Decision Making in Emergency Situations

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***Abstract*— Linear bilevel programming has been studied for many years and applied in different domains such as transportation, economics, engineering, environment, and telecommunications. However, there is lack of attention of the impacts on dynamic decision making with abrupt or unusual events caused by unpredictable natural environment or human activities (e.g. Tsunami, earthquake, and malicious or terrorist attacks). In reality these events could happens more often and have more significant impacts on decision making in an increasingly complex and dynamic world. This paper addresses this unique problem by introducing a concept of Virtual Follower (VF). An extended model of bilevel multi-follower programming with a virtual follower (BLMFP-VF) is defined and the kth-best algorithm for solving this problem is proposed. An example is given to illustrate the working of the extended model and approach.**

Keywords*— *Bilevel programming, virtual follower, kth-best approach, decision making.

I. INTRODUCTION

Bilevel programming (BLP) techniques have been developed to solve decentralized optimization problems [1]. In a classical BLP model, there are two hierarchical classes of decision makers. The upper-level is termed as the leader and the lower-level is termed as the follower. The control for the decision variables is shared amongst the players who seek to optimize their individual objective functions. It is assumed that perfect information sharing is available for all the players. The leader attempts to optimize his objective function given the assumption that the leader anticipates all possible responses of its followers. The follower observes the leader's decision and reacts in a way that is to optimize his personal objective function. Because the set of feasible choices available to all players is interdependent, the leader's decision affects the follower's objective function and allowable actions, and vice versa.

Although bilevel programming were initiated by Von Stackelberg [1] back in the early nineteen fifties, it was not drawn much attention until the mid-seventies and the early eighties, which were motivated by the increasingly complex real world problems in the processes of hierarchical decision making and engineering design [2-6]. Since then, bilevel-programming techniques have been applied in different domains such as transportation [7], decentralized resource planning [8], electronic power market [9], and telecommunications [9]. However, even though with the recent advances in bi-level programming, its theoretical foundation remains unsatisfactory and incomplete. Due to the intrinsically complex nature of bilevel programming problems, it is not surprising that the vast majority of theoretic work to date has been depending on the simplified version of the real-world problems [10]. We observed that, in the previous research, there is lack of attention of the impacts on dynamic decision making with abrupt or unusual events caused by unpredictable natural environment or human activities (e.g. Tsunami, earthquake, and malicious or terrorist attacks). In reality these events could happens more often and smart decision making are becoming more critical in an increasingly complex and dynamic world.

One typical example of such case is in telecommunications. In reality, wireless communications and Internet environment have becoming increasingly dynamic and vulnerable. Computer network management in terms of Quality of Service (QoS), reliability and security are crucial for the smooth operations of our modern society. The complex network management can be complicated by unexpected events caused by unpredictable natural environment or human activities (e.g. Tsunami, earthquake and malicious attack). These events or accidents may introduce huge surge of network traffic, bring very negative impacts to user experience, and even make the network paralyze. In these scenarios, the malicious Internet user or a cluster of abrupt Internet users has distinct behavior from ISPs (Internet Service Providers) and any other normal users.

Bilevel programming optimization has been extended to telecommunications and Internet services [15] with the deregulation of Internet service providers and the rapid growth of mobile multimedia applications. In recent years, the Internet traffic has increased exponentially with the increasing proliferation of different kinds of attractive smart phones and handhold mobile devices and their innovative content-rich bandwidth-hungry applications. For examples, Internet-based radio/television broadcasts, video conferencing, video telephony, real-time interactive and collaborative work environments, and video on demand. There is clear limitation of the pure optimization approach in the telecommunications and Internet industries with respect to routing, resource or quality of service allocation and pricing. The management decisions on routing strategy, allocation, or price choices of one Internet service provider (ISP) cannot be made independently without the considerations of the other ISPs and its subscribers. Internet Service Providers (ISP), telecommunication firm, the subscribers and other actors can be all regarded as the players in the networking games. The choices of any player will influence the choices of the others. The networking system requires an equilibrium, or stable operating point, to maintain its reliability and robustness.

This research addresses the above dynamic networking management issue and other similar emergency management issues as a unique bilevel decision-making problem. To tackle the problem, an extended bilevel multi-followers programming (BLMFP) model is defined and a solution for this model together with related theories is presented based on our previous research results [16,18,21,23].

The rest of this paper is organized as follows. In section 2, a general BLMFP model with a virtual follower is defined. In Section 3, an extended Kth-best approach for this problem is presented. Then a numerical example of the extended kth-best approach is illustrated in Section 4. This paper is concluded in Section 5 with future research directions.

II. A MODEL OF BILEVEL MULTI-FOLLOWER PROGRAMMING WITH A VIRTUAL FOLLOWER

A BLMF decision problem has been defined to have two or more followers at the low lever of the bilevel problem. Under this definition, if the followers don't have any shared decision variables, it is called an uncooperative relationship between/among followers.

During a special event or disaster, normally there is abnormal behaviour (e.g. a surge of traffic, service breakdown), which are caused by hidden factor(s) or unexpected player(s). We define the hidden factor or unexpected player as a *virtual follower*. In this case, both the leader and the followers may be affected by the virtual follower's decision information in his/her objective or constraints. We present this model as follows.

For $x \in X \subset R^n$, $y_i \in Y_i \subset R^{m_i}$, $z \in Z \subset R^m$ $F: X \times Y_1 \times \ldots \times Y_K \times Z \to R^1$, and $f_i: X \times Y_1 \times \ldots \times Y_K \times Z \to R^1$, $i = 1,2,\ldots,K$, a linear BLMFP problem, where $K(\geq 2)$ followers are involved and there are *k* shared variables and one partial shared variable, individual objective functions and constraint functions among the followers, is defined as follows:

$$\min_{x \in X} F(x, y_1, \ldots, y_K, z) = cx + \sum_{s=1}^{K} d_s y_s + dz \tag{1a}$$

$$\text{Subject to } Ax + \sum_{s=1}^{K} B_s y_s + Bz \leq b \tag{1b}$$

$$\min_{y_i \in Y_i, z \in Z} f_i(x, y_i, z) = c_i x + e_i y_i + e_i^{'} z \tag{1c}$$

$$\text{subject to } A_i x + C_i y_i + C_i^{'} z \leq b_i, \tag{1d}$$

where $c \in R^n$, $c_i \in R^n$, $d_i \in R^{m_i}$, $d \in R^m$, $e_i \in R^{m_i}$, $e_i^{'} \in R^m$, $b \in R^p$, $b_i \in R^{q_i}$, $A \in R^{p \times n}$, $B_i \in R^{p \times m_i}$, $B \in R^{p \times m}$, $A_i \in R^{q_i \times n}$, $C_i \in R^{q_i \times m_i}$, $C_i^{'} \in R^{q_i \times m}$, $i, s = 1,2,\ldots,K$.

The main idea to deal with linear BLMFP problems with partial sharing variables among the followers is that an assumed third party controls the shared variable z. It means that the i^{th} follower controls the variable y_i ($i = 1,2,\ldots,K$), and a third party called a virtual follower: the $(K+1)^{th}$ follower, controls the variable z.

Definition 1: A topological space is compact if every open cover of the entire space has a finite subcover. For example, $[a,b]$ is compact in R (the Heine-Borel theorem) [23].

III. AN EXTENDED *K*TH-BEST ALGORITHM FOR LINEAR BLMFP WITH A VIRTUAL FOLLOWER

A. Model transformation

Based on *Definition 3.3* of the linear BLP [16], a constraint region for linear BLFMP problems with a virtual follower should refer to all possible combinations of choices that the leader and all the followers may make. That means if there exists an optimal solution for a linear BLMFP problem with a virtual follower, this solution has to satisfy all constraints. The individual constraints can be equally treated no matter the sharing constraints belong to themselves or not,

The i^{th} follower controls the variable y_i ($i = 1,2,\ldots,K$) and the virtual follower, called the $(K+1)^{th}$ follower, controls the variable z. By using this splitting method, (1) can be rewritten as follows:

$$\min_{x \in X} F(x, y_1, \ldots, y_K, z) = cx + \sum_{s=1}^{K} d_s y_s + dz \tag{2a}$$

$$\text{subject to } Ax + \sum_{s=1}^{K} B_s y_s + Bz \leq b \tag{2b}$$

$$\min_{y_i \in Y_i} f_i(x, y_i, z) = c_i x + e_i y_i + e_i^{'} z \tag{2c}$$

$$\min_{z \in Z} f_i(x, y_1, \ldots, y_K, z) = c_i x + e_i y_i + e_i^{'} z \tag{2d}$$

$$\text{subject to } A_i x + C_i y_i + + C_i^{'} z \leq b_i \tag{2e}$$

Actually (2d) is that the K followers share the variable z. By using a weighting method proposed by [1], we can obtain a compromised formulation for (2) as follows:

$$\begin{aligned} &\min_{x \in X} F(x, y_1, \ldots, y_K, z) = cx + \sum_{s=1}^{K} d_s y_s + dz \\ &\text{subject to } Ax + \sum_{s=1}^{K} B_s y_s + Bz \leq b \end{aligned} \tag{3}$$

$$\min_{y_i \in Y_i} f_i(x, y_i, z) = c_i x + e_i y_i + e_i' z$$

$$\min_{z \in Z} \sum_{i=1}^{K} f_i(x, y_1, \ldots, y_K, z) = \sum_{i=1}^{K} (c_i x + e_i y_i + e_i' z)$$

subject to $A_i x + C_i y_i + + C_i' z \le b_i$.

To simply (3), we have

$$\min_{x \in X} F(x, y_1, \ldots, y_K, y_{K+1}) = cx + \sum_{s=1}^{K+1} d_s y_s \qquad \text{(4a)}$$

$$\text{subject to } Ax + \sum_{s=1}^{K+1} B_s y_s \le b \qquad \text{(4b)}$$

$$\min_{y_i \in Y_i} f_i(x, y_1, \ldots, y_K, y_{K+1}) = c_i x + \sum_{s=1}^{K+1} e_{is} y_s \qquad \text{(4c)}$$

$$\text{subject to } A_i x + \sum_{s=1}^{K+1} C_{is} y_s \le b_i , \qquad \text{(4d)}$$

where $i = 1, \ldots, K, K+1$, $y_{K+1} = z$, $d_{K+1} = d$, $B_{K+1} = B$, $e_{ii} = e_i (i = 1, \ldots, K)$, $e_{ij:i,j=1,\ldots,K,i \ne j} = (0)_{R^{m_j}}$, $e_{i(K+1)} = e_i' (i = 1, \ldots, K)$, $c_{K+1} = \sum_{s=1}^{K} c_s$, $e_{(K+1)i} = e_i (i = 1, \ldots, K)$, $e_{(K+1)(K+1)} = \sum_{s=1}^{K} e_s'$, $C_{ii} = C_i (i = 1, \ldots, K)$, $C_{ij:i,j=1,\ldots,K,i \ne j} = (0)_{q_i \times m_j}$, $C_{i(K+1)} = C_i' (i = 1, \ldots, K)$, $A_{K+1} = (0)_{q_{K+1}}$, $C_{(K+1)l} = (0)_{q_{K+1} \times m_l} (l = 1, \ldots, K)$, $b_{K+1} = (0)_{q_{K+1}}$.

This simple transformation has shown that solving the linear BLMFP (1) is equivalent to solving (4). There are K followers that have the shared variable z for the linear BLMFP (1). However, (4) has $K+1$ followers and is the linear BLMFP without shared variables among the followers. We can also find that all the variables of the followers parameterise into the objective functions and constraint functions of the followers.

In order to demonstrate the application for the proposed model transformation technology, the decision problem model for emergency QoS management in computer networks can be established by simplifying it into the following linear BLMF decision model:

Example 1:

$$\min_{x \in X} F(x, y_1, y_2, z) = 3x + 7y_1 + 11y_2 + 8z$$

$$\begin{aligned} \text{subject to } & 5x - y_1 + 6y_2 + 2z \le 40 \\ & 6x + 13y_1 - z \le 15 \\ & x - 7y_2 + z \le 10 \\ & 7y_1 + 4y_2 \le 20 \end{aligned}$$

$$\min_{y_1 \in Y} f_1(x, y_1, z) = 2x - y_1 + z$$

$$\begin{aligned} \text{subject to } & 5x + 7y_1 \le 15 \\ & 25y_1 - 4z \le 3 \end{aligned}$$

$$\min_{y_2 \in Y} f_2(x, y_2, z) = 15x + 80y_2 - z$$

$$\text{subject to } 40x + y_2 \le 5 .$$

where $x \in R^1$, $y_1 \in R^1$, $y_2 \in R^1$, $z \in R^1$ and $X = \{x > 0\}$, $Y = \{y_1 > 0, y_2 > 0\}$, $Z = \{z > 0\}$.

The variable z is controlled by the virtual follower. According to the way of model transformation, (2), (3) and (4), we have as follows:

$$\min_{x \in X} F(x, y_1, y_2, z) = 3x + 7y_1 + 11y_2 + 8z$$

$$\begin{aligned} \text{subject to } & 5x - y_1 + 6y_2 + 2z \le 40 \\ & 6x + 13y_1 - z \le 15 \\ & x - 7y_2 + z \le 10 \\ & 7y_1 + 4y_2 \le 20 \end{aligned}$$

$$\min_{y_1 \in Y} f_1(x, y_1, y_2, z) = 2x - y_1 + z$$

$$\begin{aligned} \text{subject to } & 5x + 7y_1 \le 15 \\ & 25y_1 - 4z \le 3 \end{aligned}$$

$$\min_{y_2 \in Y} f_2(x, y_1, y_2, z) = 15x + 80y_2 - z$$

$$\text{subject to } 40x + y_2 \le 5 .$$

$$\min_{z \in Z} f_3(x, y_1, y_2, z) = 17x - y_1 + 80y_2$$

By using proposed model transformation technology, the model (1) can be transferred into model (4). It is a linear BLMFP and all the variables of the followers parameterise into the objective functions and constraint functions of the followers.

B. Definition of solution

For simplification and convenience, we write model (4) as follows:

$$\min_{x \in X} F(x, y_1, \ldots, y_K) = cx + \sum_{s=1}^{K} d_s y_s \qquad \text{(5a)}$$

$$\text{subject to } Ax + \sum_{s=1}^{K} B_s y_s \le b \qquad \text{(5b)}$$

$$\min_{y_i \in Y_i} f_i(x, y_1, \ldots, y_K) = c_i x + \sum_{s=1}^{K} e_{is} y_s \qquad \text{(5c)}$$

$$\text{subject to } A_i x + \sum_{s=1}^{K} C_{is} y_s \le b_i , \qquad \text{(5d)}$$

where $c \in R^n$, $c_i \in R^n$, $d_i \in R^{m_i}$, $e_{is} \in R^{m_s}$, $b \in R^p$, $b_i \in R^{q_i}$, $A \in R^{p \times n}$, $B_i \in R^{p \times m_i}$, $A_i \in R^{q_i \times n}$, $C_{is} \in R^{q_i \times m_s}$, $i, s = 1, 2, \ldots, K$.

The formulation (5) is the same as (4) except the number of followers. They have the same solution algorithms. Corresponding to (5), we give following basic definition.

Definition 2:

(a) Constraint region:

$$S=\{(x,y_1,\ldots,y_K)\in X\times Y_1\times\ldots\times Y_k, Ax+\sum_{s=1}^{K}B_s y_s\le b,$$

$$A_i x+\sum_{s=1}^{K}C_{is}y_s\le b_i, i=1,2,\ldots,K\}.$$

The constraint region refers to all possible combinations of choices that the leader and followers may make.

(b) Projection of S onto the leader's decision space:

$$S(X)=\{x\in X:\exists y_i\in Y_i, Ax+\sum_{s=1}^{K}B_s y_s\le b, A_i x+\sum_{s=1}^{K}C_{is}y_s\le b_i\}.$$

(c) Feasible set for each follower $\forall x\in S(X)$:

$$S_i(x)=\{y_i\in Y_i:(x,y_1,\ldots,y_K)\in S\}.$$

The feasible region for each follower is affected by the leader's choice of x, and the allowable choices of each follower are the elements of S.

(d) Each follower's rational reaction set for $x\in S(X)$:

$$P_i(x)=\{y_i\in Y_i:y_i\in\arg\min[f_i(x,\hat{y}_i,y_j,j\neq i):\hat{y}_i\in S_i(x)]\},$$

where $i,j=1,2,\ldots,K$,

$$\arg\min[f_i(x,\hat{y}_i,y_j,j=1,2,\ldots,K,j\neq i):\hat{y}_i\in S_i(x)]=$$
$$\{y_i\in S_i(x):f_i(x,y_1,\ldots,y_K)\le f_i(x,\hat{y}_i,y_j,j=1,2,\ldots,K,j\neq i),\hat{y}_i\in S_i(x)\}$$

The followers observe the leader's action and simultaneously react by selecting y_i from their feasible set to minimize their objective functions.

(e) Inducible region:

$$IR=\{(x,y_1,\ldots,y_K):(x,y_1,\ldots,y_K)\in S, y_i\in P_i(x), i=1,2,\ldots,K\}.$$

Thus in terms of the above notations, (5) can be written as

$$\min\{F(x,y_1,\ldots,y_K):(x,y_1,\ldots,y_K)\in IR\} \tag{6}$$

We propose the following theorem to characterize the condition under which there is an optimal solution for (5).

Theorem 1: If S is nonempty and compact, there exists an optimal solution for a linear BLMFP problem.

Proof: Since S is nonempty, there exist a point $(x^*,y_1^*,\ldots,y_K^*)\in S$. Then, we have

$$x^*\in S(X)\neq\varphi,$$

by Definition 2(b). Consequently, we have

$$S_i(x^*)\neq\varphi,\ i=1,2,\ldots,K,$$

by Definition 2(c). Because S is compact and *Definition* 2(d), we have

$$P_i(x^*)=\{y_i\in Y_i:y_i\in\arg\min[f_i(x^*,\hat{y}_i,y_j,j\neq i):\hat{y}_i\in S_i(x^*)]\}$$

$$=\{y_i\in Y_i:y_i\in\{y_i\in S_i(x^*):$$

$$f_i(x^*,y_1,\ldots,y_K)\le f_i(x^*,\hat{y}_i,y_j,j\neq i),\hat{y}_i\in S_i(x^*)\}\}\neq\varphi,$$

where $i=1,2,\ldots,K$. Hence, there exists $y_i^0\in P_i(x^*)$, $i=1,2,\ldots,K$ such that $(x^*,y_1^0,\ldots,y_K^0)\in S$. Therefore, we have

$$IR=\{(x,y_1,\ldots,y_K):(x,y_1,\ldots,y_K)\in S, y_i\in P_i(x), i=1,2,\ldots,K\}\neq\varphi,$$

by *Definition* 2(e). Because we are minimizing a linear function $\min_{x\in X}F(x,y_1,\ldots,y_K)=cx+\sum_{s=1}^{K}d_s y_s$ over IR, which is nonempty and bounded, an optimal solution to the linear BLMFP problem must exist. So the proof is completed.

C. An extended Kth-best Algorithm

Theorem 2: The inducible region can be written equivalently as a piecewise linear equality constraint comprised of supporting hyperplanes of constraint region S.

Corollary 3.1: The problem (5) is equivalent to minimizing F over a feasible region comprised of a piecewise linear equality constraint.

Corollary 3.2: A solution for the linear BLMFP problem occurs at a vertex of IR.

Theorem 3: The solution $(x^*,y_1^*,\ldots,y_K^*)$ of the linear BLMFP problem occurs at a vertex of S.

Corollary 3.3 If x is an extreme point of IR, it is an extreme point of S.

Due to the page limit, proofs of *Theorem 2&3* and *Corollaries 3.1-3.3* are omitted here. Please refer to [27] for the proofs.

Theorem 2 and *Corollary 3.3* have provided theoretical foundation for our new algorithm. It means that by searching extreme points on the constraint region S, we can efficiently find an optimal solution for a linear BLMFP problem. The basic idea of our algorithm is that according to the objective function of the upper level, we arrange all the extreme points in S in descending order, and select the first extreme point to check if it is on the inducible region IR. If yes, the current extreme point is the optimal solution. Otherwise, the next one will be selected and checked. More specifically, let $(x^1,y_1^1,\ldots,y_K^1),\ldots,(x^N,y_1^N,\ldots,y_K^N)$, denote the N ordered extreme points to the linear BLMFP problem

$$\min\{cx+\sum_{s=1}^{K}d_s y_s:(x,y_1,\ldots,y_K)\in S\}, \tag{7}$$

such that $cx^j+\sum_{s=1}^{K}d_s y_s^j\le cx^{j+1}+\sum_{s=1}^{K}d_s y_s^{j+1}, j=1,2,\ldots,N-1.$

Let $(\tilde{y}_1,\tilde{y}_2,\ldots,\tilde{y}_K)$ denote the optimal solution to the following problem

$$\min(f_i(x^j,y_1,\ldots,y_K):y_i\in S_i(x^j),i=1,2,\ldots,K) \tag{8}$$

We only need to find the smallest j, $j = 1,2,\ldots,N$ under which $y_i^j = \tilde{y}_i$, $i = 1,2,\ldots,K$.

Let us write (13) as follows

$\min f_i(x, y_1, \ldots, y_K)$ subject to $y_i \in S(x)$; $x = x^j$, where $i = 1,2,\ldots,K$.

We only need to find the smallest j under which $y_i^j = \tilde{y}_i$, $i = 1,2,\ldots,K$. From *Definition 2.2(b),* we have

$$\min f_i(x, y_1, \ldots, y_K) = c_i x + \sum_{s=1}^{K} e_{is} y_s$$

$$\text{subject to } Ax + \sum_{s=1}^{K} B_s y_s \le b$$

$$A_l x + \sum_{s=1}^{K} C_{ls} y_s \le b_l,\ l = 1,2,\ldots,K \tag{9}$$

$x = x^j$; $y_1 \ge 0, y_2 \ge 0, \ldots, y_K \ge 0$, where $i = 1,2,\ldots,K$.

The solving is equivalent to select one ordered extreme point $(x^j, y_1^j, \ldots, y_K^j)$, then solve (9) to obtain the optimal solution $\tilde{y}_i$. If for all i, $y_i^j = \tilde{y}_i$, then $(x^j, y_1^j, \ldots, y_K^j)$ is the global optimum to (5). Otherwise, check the next extreme point.

IV. A Numeric Example for the *K*th-best Approach

Let us give **Error! Reference source not found.** to show how the *K*th-best approach works. According to the *K*th-best approach, Example 1 can be rewritten as follow in the format of (8),

$$\min_{x \in X} F(x, y_1, y_2, z) = 3x + 7y_1 + 11y_2 + 8z$$

subject to $5x - y_1 + 6y_2 + 2z \le 40$

$6x + 13y_1 - z \le 15$

$x - 7y_2 + z \le 10$

$7y_1 + 4y_2 \le 20$

$5x + 7y_1 \le 15$

$25y_1 - 4z \le 3$

$40x + y_2 \le 5$.

Step 1, set $j = 1$, and solve the above problem with the simplex method to obtain the optimal solution $(x_{[1]}, y_{[1]}, z_{[1]}) = (1.5, 1.83, 2.5)$. Let $W = \{(1.5, 1.83, 2.5)\}$ and $T = \varphi$. Go to Step 2.

Loop 1: Setting $i \leftarrow 1$ and by (11), we have

$\min f_1(x, y) = x + y$ subject to

$-x + 3y \le 4$; $-x + z \le 1$; $x - y \le 0$; $-x - y \le 0$; $x + z \le 4$; $2x - 5z \le 1$; $2x + z \ge 1$; $x = 1.5$; $y \ge 0$; $z \ge 0$.

Using the bounded simplex method, we have $\tilde{y}_j = 1.5$. Because of $\tilde{y}_j \ne y_{[j]}$, we go to Step 3. We have $W_{[j]} = \{(1.5,1.83,2.5),(0,1.33,1),(1.5,1.5,2.5),(0.5,1.5,0),(2,2,2)\}$, $T = \{(1.5,1.83,2.5)\}$ and $W = \{(0,1.33,1),(1.5,1.5,2.5),(0.5,1.5,0),(2,2,2)\}$, then go to Step 4. Update $j = 2$, and choose $(x_{[j]}, y_{[j]}, z_{[j]}) = (1.5,1.5,2.5)$, then go to Step 2.

Loop 2: Setting $i \leftarrow 1$ and by (11), we have

$\min f_1(x, y) = x + y$ subject to

$-x + 3y \le 4$; $-x + z \le 1$; $x - y \le 0$; $-x - y \le 0$; $x + z \le 4$; $2x - 5z \le 1$; $2x + z \ge 1$; $x = 1.5$; $y \ge 0$; $z \ge 0$.

Using the bounded simplex method, we have $\tilde{y}_j = 1.5$ and $\tilde{y}_j = y_{[j]}$. Setting $i \leftarrow i + 1$ and by (11), we have

$\min f_2(x, z) = x + z$ subject to

$-x + 3y \le 4$; $-x + z \le 1$; $x - y \le 0$; $-x - y \le 0$; $x + z \le 4$; $2x - 5z \le 1$; $2x + z \ge 1$; $x = 1.5$; $y \ge 0$; $z \ge 0$.

Using the bounded simplex method, we have $\tilde{z}_j = 0.4$. Because of $\tilde{z}_j \ne z_{[j]}$, we go to Step 3. We have

$W_{[j]} = \{(1.5,1.83,2.5),(0,0,1),(1.5,1.5,2.5),(0.5,0.5,0),(2,2,2)\}$,

$T = \{(1.5,1.83,2.5),(1.5,1.5,2.5)\}$ and

$W = \{(0,1.33,1),(0.5,1.5,0),(2,2,2),(0,0,1),(0.5,0.5,0)\}$, then go to Step 4. update $j = 3$, and choose $(x_{[j]}, y_{[j]}, z_{[j]}) = (2,2,2)$, then go to Step 2.

Loop 3: Setting $i \leftarrow 1$ and by (11), we have

$\min f_1(x, y) = x + y$ subject to $-x + 3y \le 4$; $-x + z \le 1$;

$x - y \le 0$; $-x - y \le 0$; $x + z \le 4$; $2x - 5z \le 1$; $2x + z \ge 1$; $x = 2$; $y \ge 0$; $z \ge 0$.

Using the bounded simplex method, we have $\tilde{y}_j = 2$ and $\tilde{y}_j = y_{[j]}$. Setting $i \leftarrow i + 1$ and by (11), we have

$\min f_2(x, z) = x + z$ subject to

$-x + 3y \le 4$; $-x + z \le 1$; $x - y \le 0$; $-x - y \le 0$; $x + z \le 4$; $2x - 5z \le 1$; $2x + z \ge 1$; $x = 2$; $y \ge 0$; $z \ge 0$.

Using the bounded simplex method, we have $\tilde{z}_j = 0.6$. Because of $\tilde{z}_j \ne z_{[j]}$, we go to Step 3. We have

$W_{[j]} = \{(1.5,1.83,2.5),(1.5,1.5,2.5),(2,2,0.6),(2,2,2)\}$,

$T = \{(1.5,1.83,2.5),(1.5,1.5,2.5),(2,2,2)\}$ and

$W = \{(0,1.33,1),(0.5,1.5,0),(0,0,1),(0.5,0.5,0),(2,2,0.6)\}$, then go to Step 4. Update $j = 3$, and choose $(x_{[j]}, y_{[j]}, z_{[j]}) = (2,2,0.6)$, then go to Step 2.

Loop 4: Setting $i \leftarrow 1$ and by (11), we have

$\min f_1(x,y) = x + y$ subject to

$-x+3y \le 4$; $-x+z \le 1$; $x-y \le 0$; $-x-y \le 0$; $x+z \le 4$; $2x-5z \le 1$; $2x+z \ge 1$; $x=2$; $y \ge 0$; $z \ge 0$

Using the bounded simplex method, we have $\tilde{y}_j = 2$ and $\tilde{y}_j = y_{[j]}$. Setting $i \leftarrow i+1$ and by (11), we have

$\min f_2(x,z) = x + z$ subject to

$-x+3y \le 4$; $-x+z \le 1$; $x-y \le 0$; $-x-y \le 0$; $x+z \le 4$; $2x-5z \le 1$; $2x+z \ge 1$; $x=2$; $y \ge 0$; $z \ge 0$.

Using the bounded simplex method, we have $\tilde{z}_j = 0.6$ and $\tilde{z}_j = z_{[j]}$. Solution $(x_{[j]}, y_{[j]}, z_{[j]}) = (2,2,0.6)$ is the global solution to the example. . Therefore, the optimal solution of the bilevel multi-follower problem occurs at the point $(x^*, y^*, z^*) = (2,2,0.6)$ with the leader's objective value $F^* = -4.4$, and two followers' objective values $f_1^* = 4$ and $f_2^* = 2.6$ respectively.

V. Conclusion and Further Studies

This paper addresses the theoretical properties of a newly defined linear BLMFP-VF problem in which there are not common variables among all the followers. This paper presents the extended *K*th-best approach for linear BLMFP-VP problem and gives a numeric example for using the approach.

This research can be used for making dynamic decisions to detect abnormal events and prevent service from breakdown in emergency situations. Future research work includes developing a web-based dynamic system for government and business to make real-time intelligent decisions in emergency situations and therefore building robust and resilient management systems to deal with potential environmental and human disasters.

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