Parametric Space for Elastic Waves in Porous Media with Complex Rheology

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***Abstract*—We evaluate the influence of mechanical parameters on the decay rate of elastic waves in fluid-saturated media with bubbles. The parameters describe the complex rheological scheme adopted as a model of the grainy porous medium. Areas of validity of the model in the space of mechanical parameters are determined.**

***Index Terms*—elastic waves, rheology, decay rate, parameters**

I. Introduction

Wave propagation in fluid-saturated porous media attracts constant attention of researches and engineers due to the many applications in geomechanics, oil extraction industry and because of theoretical significance [1]-[7]. Recently we used [8] an extended rheological model relative to the earlier model of Nikolaevskiy [9, 10], to study the influence of the bubbles on the linear Frenkel–Biot waves of P1 type. The rheological model for the wave with the bubbles, shown in Fig. 1, consists of three segments representing the solid continuum, fluid continuum and a bubble surrounded by the fluid. Based on the rheological model, we derived, using the procedure similar to [10], the one-dimensional partial differential equation for the nonlinear longitudinal seismic wave,

$$C_1\frac{\partial v}{\partial \tau} - C_2\frac{\partial^2 v}{\partial \xi^2} + \varepsilon C_3\frac{\partial^3 v}{\partial \xi^3} - \varepsilon^2 C_4\frac{\partial^4 v}{\partial \xi^4} - \varepsilon^4 C_6\frac{\partial^6 v}{\partial \xi^6} - C_7\frac{\partial vv}{\partial \xi} = 0, \quad (1)$$

where v is the velocity of the solid matrix, ε is the small parameter responsible for slow behaviour of the wave in space and time, and the coefficients C_i, $i=1, 2, 3, 4, 6, 7$ are constants linked to the mechanical parameters of the system. From (1) follows the dispersion relation

$$\lambda(k) = -\frac{C_2}{C_1}k^2 + \varepsilon^2\frac{C_4}{C_1}k^4 - \varepsilon^4\frac{C_6}{C_1}k^6, \quad (2)$$

Describing the behavior of the wave in time, $v \sim \exp(\lambda t)$; k is the wave number. We discovered [8] that the increase of the radius of the bubbles, R_0, leads to faster decay, while the increase of the number of the bubbles, n_0, leads to slower decay of the wave as shown in Fig 2.

The aim of this paper is to evaluate the ranges of the parameters of the rheological model that lead to negative values of λ, which describe a *decaying* wave and are, therefore, physically acceptable (valid), and to positive λ which describe a *growing* wave, and are, therefore, physically not acceptable (invalid). In this paper we use k = 0.25 1/m, ε = 0.01 [8].

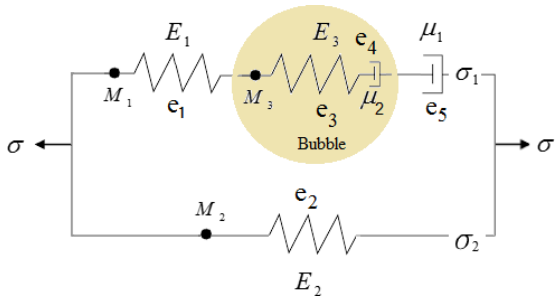

Figure 1. Rheological scheme including a gas bubble [8].

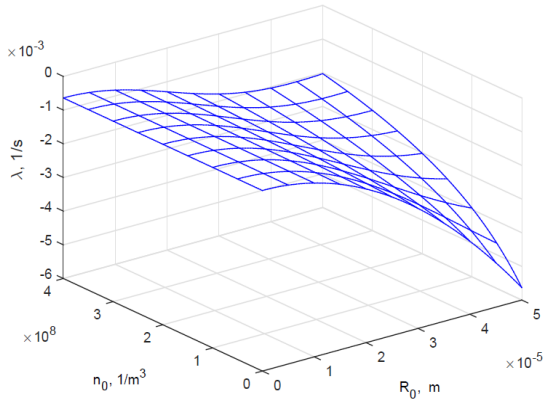

Figure 2. The decay rate λ versus R_0 and n_0, from [8].

Manuscript received July 7, 2019; revised March 11, 2020.

doi: 10.18178/ijmerr.9.6.825-829

II. Rheological Parameters

In this section we focus on the parameters of the rheological model shown in Fig. 1. The model includes two friction elements with viscosities μ_1 and μ_2, three elastic springs with the elastic moduli E_1, E_2, and E_3, and three oscillating masses M_1, M_2, and M_3. Further, σ is the total stress; e with respective subscript denotes the displacement of the element of the model. Overall we have 8 independent rheological parameters, E_1, E_2, E_3, M_1, M_2, M_3, μ_1 and μ_2. The magnitudes of the viscosities μ_1 and μ_2 are well known from literature [11, 12] while the parameters E_1, E_2, E_3, M_1, M_2 and M_3 are known quite poorly and only be roughly estimated by the order of magnitude. We aim to determine a possibly wider range of the values of the parameters making the decay rate negative (valid).

As for the values of E_i and M_i, they are roughly suggested in [10, 13, 14, 15], and we let them vary from these reference levels by a few orders of magnitude. Tables I and II show the values for E_i and M_i which we use in the present study.

TABLE I. The Values of E_1, E_2 and E_3

	$E_{1,\text{Pa}}$	$E_{2,\text{Pa}}$	$E_{3,\text{Pa}}$
1	10^4	10^5	10^2
2	10^5	10^6	5×10^2
3	10^6	10^7	10^3

TABLE II. The Values Of M_1, M_2 and M_3

	$M_{1,\text{Kg/m}}$	$M_{2,\text{Kg/m}}$	$M_{3,\text{Kg/m}}$
1	10^{-2}	5×10^{-2}	5×10^{-5}
2	10^{-3}	5×10^{-3}	5×10^{-6}
3	10^{-4}	5×10^{-4}	5×10^{-7}

Tables I and II give $3^3\times3^3=27\times27$ possible combinations of the parameter values. In order to reduce the amount of calculations, for each of the 27 combinations of E_i from Table I we consider only one combination of M_i from Table II, namely one with the same row and column number, see Table III. For each row from Table III, we plot the decay rate against the radius of the bubbles, R_0, and their number (concentration), n_0. In total Table III gives 27 possible graphs, but some of these graphs are very close as we comment in the figure captions. The results are presented in Figs. 3-5.

We found the following. When the values of E_1 and E_3 are simultaneously increased to 10^{12} Pa the decay rate λ remains negative (valid). When they are simultaneously decreased down to 10^2 Pa the decay rate λ gets less negative. With the increase of the value of E_2 to 6×10^9 Pa the decay rate λ becomes positive (invalid), see Figs. 6 and 7. This indicates that our model is not applicable to such extreme values of E_2.

TABLE III. The Values of E_1, E_2, E_3, M_1, M_2 and M_3 Considered in This Paper

	$E_{1,\text{Pa}}$	$E_{2,\text{Pa}}$	$E_{3,\text{Pa}}$	$M_{1,\text{Kg/m}}$	$M_{2,\text{Kg/m}}$	$M_{3,\text{Kg/m}}$
1	10^4	10^5	10^2	10^{-2}	5×10^{-2}	5×10^{-5}
2	10^4	10^5	5×10^2	10^{-2}	5×10^{-2}	5×10^{-6}
3	10^4	10^5	10^3	10^{-2}	5×10^{-2}	5×10^{-7}
4	10^4	10^6	10^2	10^{-2}	5×10^{-3}	5×10^{-5}
5	10^4	10^6	5×10^2	10^{-2}	5×10^{-3}	5×10^{-6}
6	10^4	10^6	10^3	10^{-2}	5×10^{-3}	5×10^{-7}
7	10^4	10^7	10^2	10^{-2}	5×10^{-4}	5×10^{-5}
8	10^4	10^7	5×10^2	10^{-2}	5×10^{-4}	5×10^{-6}
9	10^4	10^7	10^3	10^{-2}	5×10^{-4}	5×10^{-7}
10	10^5	10^5	10^2	10^{-3}	5×10^{-2}	5×10^{-5}
11	10^5	10^5	5×10^2	10^{-3}	5×10^{-2}	5×10^{-6}
12	10^5	10^5	10^3	10^{-3}	5×10^{-2}	5×10^{-7}
13	10^5	10^6	10^2	10^{-3}	5×10^{-3}	5×10^{-5}
14	10^5	10^6	5×10^2	10^{-3}	5×10^{-3}	5×10^{-6}
15	10^5	10^6	10^3	10^{-3}	5×10^{-3}	5×10^{-7}
16	10^5	10^7	10^2	10^{-3}	5×10^{-4}	5×10^{-5}
17	10^5	10^7	5×10^2	10^{-3}	5×10^{-4}	5×10^{-6}
18	10^5	10^7	10^3	10^{-3}	5×10^{-4}	5×10^{-7}
19	10^6	10^5	10^2	10^{-4}	5×10^{-2}	5×10^{-5}
20	10^6	10^5	5×10^2	10^{-4}	5×10^{-2}	5×10^{-6}
21	10^6	10^5	10^3	10^{-4}	5×10^{-2}	5×10^{-7}
22	10^6	10^6	10^2	10^{-4}	5×10^{-3}	5×10^{-5}
23	10^6	10^6	5×10^2	10^{-4}	5×10^{-3}	5×10^{-6}
24	10^6	10^6	10^3	10^{-4}	5×10^{-3}	5×10^{-7}
25	10^6	10^7	10^2	10^{-4}	5×10^{-4}	5×10^{-5}
26	10^6	10^7	5×10^2	10^{-4}	5×10^{-4}	5×10^{-6}
27	10^6	10^7	10^3	10^{-4}	5×10^{-4}	5×10^{-7}

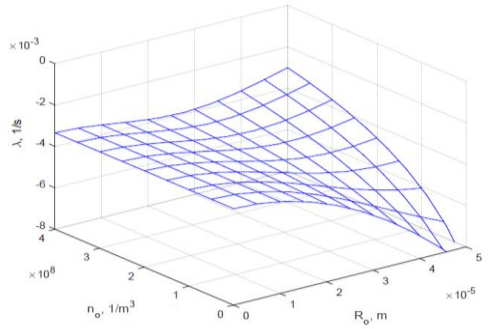

Figure 3. The decay rate by formula (2) for row 1 of Table III.

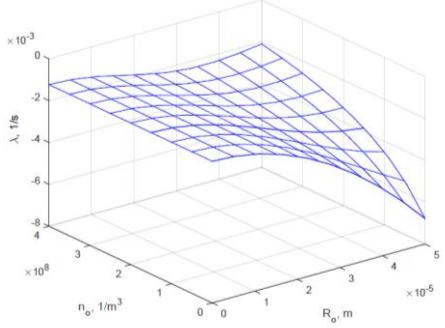

Figure 4. The decay rate by formula (2) for row 11 of Table III.

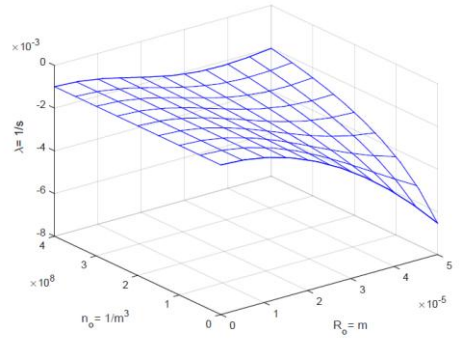

Figure 5. The decay rate by formula (2) for row 27 of Table III.

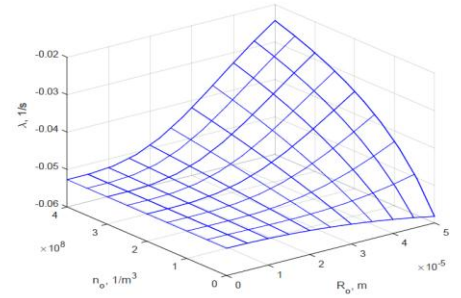

Figure 6. The decay rate by formula (2) for E_1, E_3=10^2 Pa.

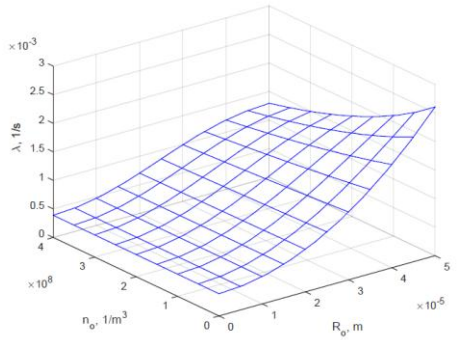

Figure 7. The decay rate by formula (2) for E_2=6×10^9 Pa.

All the presented calculations are carried out for the number of bubbles $n_{0\,=}$ 10^8 1/m^3 and the radius of bubbles R_0 = 5×10^{-5} m [8, 16]. Similar results are obtained when possible values of E_i are extended as shown in Table IV. Figs. 8-10 show the results for the parameter values given in Table IV.

TABLE IV. THE VALUES OF E_1, E_2 AND E_3

	$E_{1,\,\mathrm{Pa}}$	$E_{2,\,\mathrm{Pa}}$	$E_{3,\,\mathrm{Pa}}$
1	10^9	5×10^7	10^6
2	10^9	5×10^7	5×10^6
3	10^9	5×10^7	8×10^7
4	10^9	10^8	10^6
5	10^9	10^8	5×10^6
6	10^9	10^8	8×10^7
7	10^9	4×10^9	10^6
8	10^9	4×10^9	5×10^6
9	10^9	4×10^9	8×10^7
10	5×10^9	5×10^7	10^6
11	5×10^9	5×10^7	5×10^6
12	5×10^9	5×10^7	8×10^7
13	5×10^9	10^8	10^6
14	5×10^9	10^8	5×10^6
15	5×10^9	10^8	8×10^7
16	5×10^9	4×10^9	10^6
17	5×10^9	4×10^9	5×10^6
18	5×10^9	4×10^9	8×10^7
19	10^{10}	5×10^7	10^6
20	10^{10}	5×10^7	5×10^6
21	10^{10}	5×10^7	8×10^7
22	10^{10}	10^8	10^6
23	10^{10}	10^8	5×10^6
24	10^{10}	10^8	8×10^7
25	10^{10}	4×10^9	10^6
26	10^{10}	4×10^9	5×10^6
27	10^{10}	4×10^9	8×10^7

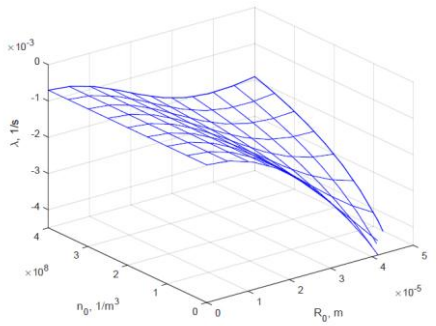

Figure 8. The decay rate by formula (2) for rows 5 of Tables III and IV.

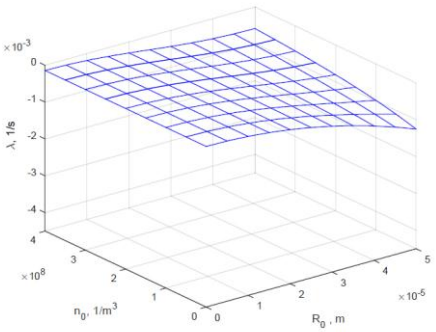

Figure 9. The decay rate by formula (2) for rows 17 of Table III and IV.

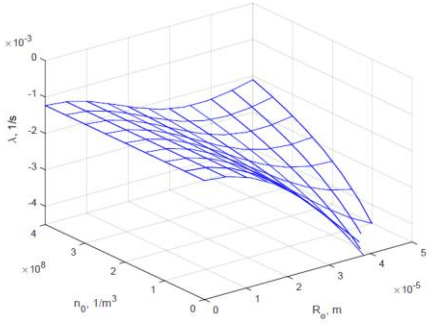

Figure 10. The decay rate by formula (2) for rows 25 of Table III and IV.

III. Validity Space

In this section we investigate the parametric space in terms of negative/positive λ (valid/invalid model). To present results in the form of 3D plots we choose two options: (a) vary E_i with M_i being fixed, (b) vary M_i with E_i being fixed. The points where λ is negative are displayed as 'o', the points where λ is positive are displayed as '*'. The results are presented in Figs. 11-14.

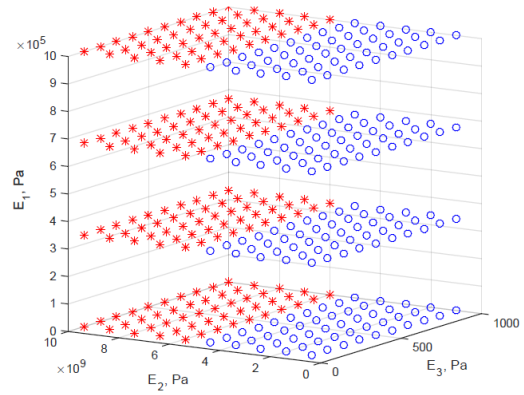

Figure 11. Valid and invalid points in the space of E_i.

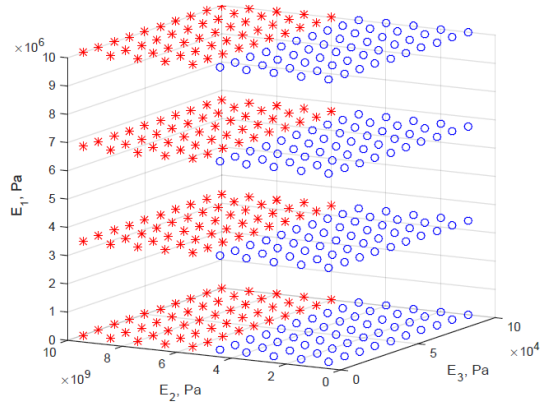

Figure 12. Valid and invalid points in the space of E_i.

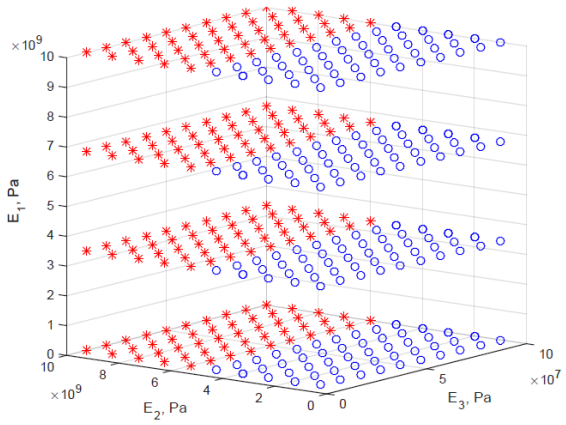

Figure 13. Valid and invalid points in the space of E_i.

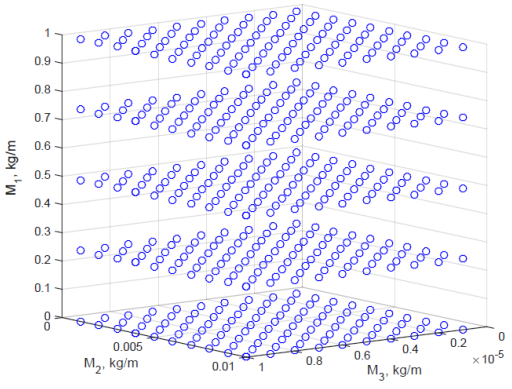

Figure 14. The validity area in the space of M_i.

See from Fig. 14 that all the points are valid points. Therefore, for different values of M_i the elastic wave is a *decaying* wave according to our model, which is a physically acceptable outcome.

IV. Conclusions

We investigated the influence of the values of the rheological parameters on linear dynamics of elastic waves in porous media with fluid including gas bubbles. We evaluated the areas of validity of the decay rate of the wave in the parametric space.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Conceptualization, A.A and D.S.; Formal analysis, A. A.; Investigation, A.A and D.S.; Methodology, A.A.; Visualization, A.A. and D.S.; Writing–original draft, A.A.; Writing – review and editing, A.A and D.S.; all authors had approved the final version.

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