Performance-Based Optimization: A Review

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ABSTRACT: The performance-based optimization (PBO) method has recently been developed for topology design of continuum structures with stress, displacement and mean compliance constraints. The PBO method incorporates the finite element analysis, modern structural optimization theory and performance-based design concepts into a single scheme to automatically generate optimal designs of continuum structures. Performance indices are used to monitor the performance optimization history of topologies while performance-based optimality criteria are employed to identify the optimum from the optimization process. The performance characteristics of a structure in an optimization process are fully captured. The PBO technique allows the designer to tailor the design to a specific performance level required by the owner. This paper reviews the state-of-the-art development of automated performance-based optimal design techniques for topology design of continuum structures. Several practical design examples are provided to demonstrate the effectiveness and validity of the PBO technology as an advanced design tool.

1. INTRODUCTION

Performance-based design is a developing technology, which allows the designer to tailor a structure to a specific performance level required by the owner. In this design methodology, structural design criteria are expressed in terms of achieving performance objectives. There are usually multiple performance objectives that must be considered by the structural designer when designing a structure. The main performance objectives are functionality, serviceability, strength and economy. The cost performance (economy) of a structure, which is of great importance to the owner, has a significant effect on the final design of the structure. The cost of a structure including the initial cost and cost of maintenance is significantly influenced by the structural form, materials used and construction methods. One of the challenges in performance-based design is to develop optimal design tools that can be used by structural designers to achieve cost-effective and high performance structures. The performance-based optimal design of frame structures has been reported by researchers (Ganzerli et al. 2000; Foley and Schinler 2003; Gong et al. 2005; Zou and Chan 2005; Liu et al. 2005; Xu et al. 2006; Fragiadakis et al. 2006). It is recognized that topology optimization of structures offers more material savings than shape and sizing optimization.

Topology optimization of continuum structures has gained popularity in structural optimization in recent years. Many innovative optimization methods and algorithms have been developed and reported in the literature. Haftka and Grandhi (1986) reviewed methods for structural shape optimization. Rozvany et al. (1995) presented a comprehensive survey on the layout optimization of structures. Recently, Eschenauer and Olhoff (2001) provided a review on topology optimization of continuum structures. The publications of several advanced textbooks on topology optimization of continuum structures indicate the mature of

these topology optimization techniques (Bendsøe 1995; Xie and Steven 1997; Seireg and Rodriguez 1997; Mattheck 1998; Hassani and Hinton 1999; Bendsøe and Sigmund 2003; Liang 2005).

In 1988, Bendsøe and Kikuchi (1988) developed a homogenization-based optimization (HBO) method for topology design of continuum structures. Since then, extensive studies on the HBO method have been reported in the literature (Suzuki and Kikuchi 1991; Díaz and Bendsøe 1992; Tenek and Hagiwara 1993; Bendsøe et al. 1995; Hassani and Hinton 1999; Krog and Olhoff 1999; Bendsøe and Sigmund 2003). The HBO method treats topology optimization of a continuum structure as a material redistribution problem in the structure made of composite material with microstructures. The effective material properties of the composite material are computed using the homogenization theory. The material volume is used in the HBO method as a constraint.

The density-based optimization (DBO) method has been developed as an alternative to the topology optimization of continuum structures (Bendsøe 1989; Mlejnek and Schirrmacher 1993; Yang and Chuang 1994; Ramm et al. 1994; Yang 1997; Sigmund 2001). The DBO method assumes that material properties are constant within each finite element whose relative densities are treated as design variables. The effective material properties are computed by the relative material density raised to some power times the material properties of the solid material. The power law approach must be combined with perimeter constraints, gradient constraints or filtering techniques to ensure the existence of solutions (Sigmund 2001). Gea (1996) presented a microstructure-based design domain method, which employs a closed-form expression for the effective Young's modulus and shear modulus in terms of phase properties and volume fractions.

In the absence of effective topology optimization techniques, structural systems are traditionally developed based on the designer's experience and intuition that often leads to oversized designs. To improve the design, underutilized material should be removed from the structure. The material removal concept has been employed in several topology optimization techniques for continuum structures (Rodriguez and Seireg 1985; Atrek 1989; Rozvany et al. 1992; Xie and Steven 1993). Without element elimination, Mattheck and Burkhardt (1990) and Baumgartner et al. (1992) proposed a soft kill optimization (SKO) method that uses the Young's modulus to represent the effective stress of elements in the structure. The optimization technique developed by Xie and Steven (1993) is called evolutionary structural optimization (ESO). In the ESO method, inefficient finite elements identified by the sensitivity numbers are slowly removed from the structures to produce optimal designs. Since 1993, extensive studies on ESO have been published in the literature (Xie and Steven 1994; Hinton and Sienz 1995; Chu et al. 1996; Xie and Steven 1997; Sienz and Hinton 1997; Guan et al. 1999; Kim et al. 2000; Rong et al. 2000; Li et al. 2001a; Steven et al. 2002; Li et al. 2003). The ESO technique was extended to a bi-directional evolutionary structural optimization (BESO) method, which allows for elements to be removed from the structure as well as to be added to the structure (Querin et al. 1998; Yang et al. 1999).

The challenge in topology optimization of continuum structures is to incorporate an appropriate termination criterion in optimization algorithms to determine the optimum. Although the material volume or iteration number can be used as a termination criterion in topology optimization of continuum structures, the final solution may vary with the material volume fraction or the iteration number. Zhou and Rozvany (2001) reported that continuum topology optimization procedures will give wrong results if no extended optimality criteria such as performance-based optimality criteria are used in the optimization algorithms.

Recently, the performance-based optimization (PBO) method has been developed for topology design of continuum structures by Liang et al. (1999a, 2000a), Liang (2001a) and Liang and Steven (2002). Performance indices and performance-based optimality criteria were proposed and incorporated in PBO algorithms to identify the optimum from an optimization process. The PBO method allows the designer to tailor a design to a specific performance level while practical design requirements are taken into account. This paper presents the state-of-the-art review of recent advances in the theory and applications of the PBO method. The PBO technique for continuum structures with stress, displacement and mean compliance constraints is reviewed. The element removal criteria, checkerboard patterns, performance indices, performance-based optimization criteria, performance optimization procedure and practical applications are described. Examples are provided to demonstrate the practical applications of the PBO technique to structural design problems.

2. PERFORMANCE OBJECTIVES

In performance-based optimal design, the design of a structure must satisfy the strength, serviceability and cost performance requirements. The strength and serviceability design criteria require that the stress within the structure and its deformation must be within acceptable performance levels, which are measured by limiting values specified in design codes. In the PBO method, the weight of a structure is used as the performance objective and stresses, displacements and mean compliance are treated as performance-based constraints. The goal of the performance-based optimization is to minimize the weight of a structure while maintaining its stress or displacements or mean compliance within limiting values. The mean compliance (strain energy) of a structure is usually used as an inverse measure of the overall

stiffness of the structure. The optimization problems can be stated in mathematical forms as follows:

$$\text{minimize} \quad W = \sum_{e=1}^{N} w_e(t) \tag{1}$$

$$\text{subject to} \quad \sigma_{max} \leq \sigma^* \quad \text{or} \tag{2}$$

$$u_j \leq u_j^* \quad (j = 1,...,m) \quad \text{or} \tag{3}$$

$$C \leq C^* \tag{4}$$

$$t^L \leq t \leq t^U \tag{5}$$

where W is the total weight of the structure, w_e is the weight of the eth element, t is the thickness of all elements, t^L is the lower bound on the element thickness, t^U is upper bound on the element thickness, N is the total number of elements, σ_{max} is the maximum von Mises stress of an element in the structure under applied loads, σ^* is the maximum allowable stress, u_j is the absolute value of the jth constrained displacement, u_j^* is the prescribed limit of u_j, m is the total number of displacement constraints, C is the absolute value of the mean compliance of the structure, C^* is the prescribed limit of C. It should be noted that the thickness of a continuum structure has a significant effect on the weight of the structure and must be specified by the designer in practice. As a result, the element thickness is treated as one of the design variables. However, only uniform sizing of the element thickness is considered here to simplify the optimization problem.

3. ELEMENT REMOVAL CRITERIA

3.1 Stress Constraints

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The nature of material distribution in an optimized structure depends on the type of constraint. A different type of constraint leads to different optimal topology (Liang et al. 1999a). The stress constraint considered in the PBO method is to maintain the maximum von Mises stress in the optimal design within an acceptable stress limit. The finite element analysis of a continuum structure indicates that the stress distribution within the structure is not uniform and some of the elements are not effective in transferring loads. These lowly stressed elements should be removed from the design domain to improve its efficiency. Element removal criteria can be expressed by (Liang et al. 1999a, 1999b).

$$\sigma_{i,e} < R_j^d \sigma_{i,\max} \tag{6}$$

where $\sigma_{i,e}$ is the von Mises stress of the eth element at the ith iteration, $\sigma_{i,\max}$ is the maximum von Mises stress of an element in the structure at the ith iteration and R_j^d is the deletion ratio of elements at the jth steady state. All elements that satisfy Eq. (6) are removed from the structure. The cycle of the finite element analysis and the element removal is repeated by using the same R_j^d until no more elements can be removed from the structure at the current state. In order to continue the optimization process, the element removal ratio R_j^d is increased by an incremental removal ratio (R_i^d). The element deletion ratio can be expressed by

$$R_j^d = R_0^d + (j-1)R_i^d \qquad (j = 1, 2, 3, 4, \ldots) \tag{7}$$

where R_0^d is the initial deletion ratio of elements. The optimal topology of a continuum structure under one load case can be iteratively generated by using element removal criteria described in Eq. (6). For structures subject to multiple load cases, only those elements that

satisfy Eq. (6) for all load cases are removed from the design domain at each iteration. This criterion leads to an optimal design that can perform the required function under all load cases.

3.2 Displacement Constraints

In order to measure the effect of element elimination on the performance of a structure, virtual unit loads are applied to the degree of freedom of constrained displacements. The sensitivity analysis indicates that the change in the constrained displacement due to the elimination of the *e*th element can be calculated approximately by the virtual strain energy of the *e*th element (Liang et al. 2000a). Considering two elements with the same virtual strain energy, eliminating the element with a larger weight will result in a lighter design while changes in specific displacements are the same. Therefore, in order to obtain the most efficient design, the virtual strain energy per unit weight of an element, which is defined as the virtual strain energy density (VSED) of the element, should be used as element removal criteria. The virtual strain energy density of the *e*th element is denoted as (Xie and Steven 1997)

$$\psi_e = \{u_{ej}\}^{\mathrm{T}}[k_e]\{u_e\}/w_e \tag{8}$$

where $\{u_{ej}\}$ is the nodal displacement vector of the *e*th element under the virtual unit loads, $\{u_e\}$ is the displacement vector of the *e*th element under real loads and $[k_e]$ is the stiffness matrix of the *e*th element. The virtual strain energy densities of elements can be calculated at the element level from the results of the finite element analysis. To achieve the performance objective, it is obvious that elements with the lowest virtual strain energy densities should be systematically eliminated from a continuum design domain being optimized. For a structure

subject to multiple displacement constraints under multiple load cases, the virtual strain energy density of the eth element can be evaluated by using the weighted average approach as

$$\psi_e^m = \sum_{l=1}^{p}\sum_{j=1}^{m}\beta_j\psi_e \tag{9}$$

where the weighting parameter β_j is defined as $u_j^l / u_j^{l^*}$, which is the ratio of the jth constrained displacement to the prescribed limit under the lth load case, and p is the total number of load cases. It is noted that the absolute values of displacements are used in Eq. (9).

3.3 Mean Compliance Constraints

Sensitivity analysis on structures with mean compliance constraints indicates that the change in the strain energy of a continuum structure due to the removal of the eth element can be approximately evaluated by the strain energy of the eth element (Liang and Steven 2002). As a result, the strain energy density of an element can be used as a measure of element contribution to the overall stiffness performance of a structure, and is denoted as

$$\gamma_e = \frac{1}{2}\{u_e\}^{\mathrm{T}}[k_e]\{u_e\}/w_e \tag{10}$$

To achieve the maximum stiffness designs, it is obvious that a small number of elements with the lowest strain energy densities should be systematically removed from a structure. In the PBO algorithms, a loop is used to count elements with the lowest strain energy densities until they made up the specified amount that is the element removal ratio times the total number of elements in the initial design domain. The element removal ratio (R) for each iteration is

defined as the ratio of the number of elements to be removed to the total number of elements in the initial structure. The element removal ratio is not changed in the whole optimization process. Tested examples indicated that the element removal ratio of 1-2% can be used to obtain smooth solutions.

4. CHECKERBOARD PATTERNS

Checkerboard patterns commonly appear in the optimized topologies of continuum structures produced by numerical optimization techniques. Checkerboard patterns are not optimal material distribution patterns, but are numerical anomalies, which are caused by the errors in the displacement-based finite element formulation (Díaz and Sigmund 1995; Jog and Haber 1996; Sigmund and Petersson 1998). The presence of checkerboard patterns leads to difficulty in interpreting and manufacturing optimal structures. It is desirable to suppress the formation of checkerboard patterns in continuum topology optimization.

Several methods for preventing the formation of checkerboard patterns have been suggested by researchers. Jog and Haber (1996) reported that either using higher order finite elements or modifying the functional of a finite element model could suppress the formation of checkerboard patterns in optimal topologies of continuum structures. However, the use of higher order elements will significantly increase the computational cost because of the increase in the degrees of freedom of the structure. Beckers (1999) used the perimeter constraint imposed on the boundary of a continuum structure to prevent checkerboard patterns from formation in optimal topologies. Sigmund and Petersson (1998) proposed a filtering technique based on an image process to eliminate checkerboard patterns and mesh dependency in continuum topology optimization. The filtering technique treats the discretized

continuum structure as a digital image and the weighted average of strain energies over neighboring elements is used to produce a checkerboard free image. The density redistribution method suggested by Youn and Park (1997) is shown to be effective in eliminating checkerboard patterns from optimal designs. Fujii and Kikuchi (2000) incorporated a gravity control scheme into the HBO method to generate checkerboard pattern free designs. Zhou et al. (2001) proposed a density slop control algorithm for checkerboard patterns and direct member size control. A simple smoothing scheme in terms of the surrounding elements' reference factors for eliminating checkerboard patterns in ESO was presented by Li et al. (2001b).

Checkerboard patterns are also observed in optimal topologies and shapes generated by the PBO techniques when four-node finite elements are used in the finite element analysis. This is mainly due to the instability of four-node elements. The use of higher order elements such as eight-node elements in the PBO method can effectively suppress the formation of checkerboard patterns. However, the computational cost will significantly increase especially for practical structures, which are simulated using very fine finite elements. A simple checkerboard suppression algorithm has been implemented in the PBO method (Liang and Steven 2002). For continuum structures subject to mean compliance constraints, the nodal strain energy densities of an element are calculated by averaging the strain energy densities of neighboring elements as follows

$$\zeta_{nd} = \frac{1}{M}\sum_{e=1}^{M}\gamma_e \tag{11}$$

in which ζ_{nd} is the nodal strain energy density and M is the number of elements that connect to that node. The strain energy density of each element can be recalculated from the nodal strain energy densities at the nodes of the element as

$$\zeta_e = \frac{1}{Q}\sum_{nd=1}^{Q}\zeta_{nd} \qquad (12)$$

where ζ_e is the recalculated strain energy density of the eth element and Q is the number of nodes in the element. Element removal criteria are now based on the recalculated strain energy densities of elements. It has been observed that the strain energy density redistribution scheme can effectively suppress the formation of checkerboard patterns in performance-based topology optimization. For structures subject to displacement constraints, the virtual strain energy densities of elements are used in Eqs. (11) and (12) (Liang 2005).

5. PERFORMANCE INDICES

The strength and stiffness of a two-dimensional continuum structure depends on its thickness assigned to the initial design domain. This implies that satisfying the stress or displacement constraints does not guarantee an optimum. In the PBO method, inefficient elements are gradually eliminated from a continuum structure to improve topology performance. In order to evaluate the performance of resulting topologies, performance indices are needed. Performance indices were used to assist the selection of materials, sectional shapes and the layouts of truss structures (Ashby 1992; Burgess 1998). The scaling design concept was used in structural optimization to obtain the best feasible constrained design for trusses (Kirsch 1982). The advantages of scaling the design are that it can monitor the history of the weight

reduction after each iteration and pick the most active constraints. This method can be applied to structural optimization when the stiffness matrix of a structure is a linear function of design variables. The scaling design concept has been utilized to develop a set of performance indices for evaluating the performance of continuum structures by Liang et al. (1999a, 2000a), Liang et al. (2001a) and Liang and Steven (2002). The continuum structure is scaled with respect to the most active constraint at each iteration in an optimization process by changing its thickness. These performance indices have been incorporated in the PBO algorithms to monitor the performance optimization history of resulting topologies in the optimization process.

For continuum structures with stress constraints, the stress-based performance index is defined by (Liang et al. 1999a)

$$PI_s = \frac{\sigma_{0,\max} W_0}{\sigma_{i,\max} W_i} \tag{13}$$

where $\sigma_{0,\max}$ is the maximum von Mises stress of an element in the initial structure under the applied loads, $\sigma_{i,\max}$ is the maximum von Mises stress of an element in the current structure at the ith iteration, W_0 is the actual weight of the initial structure, and W_i is the actual weight of the current structure at the ith iteration. It can be seen from Eq. (13) that the stress-based performance index is a dimensionless number, which measures the performance of a structural topology in terms of material efficiency (the weight of the structure) and structural response (the maximum stress within the structure). The performance index formula is independent of the scale of the loads and the structure.

For plane stress continuum structures, the displacement-based performance index is expressed by (Liang et al. 2000a)

$$PI_{ds} = \frac{u_{0j}W_0}{u_{ij}W_i} \qquad (14)$$

where u_{0j} is the absolute value of the most critical constrained displacement in the initial structure under applied loads, u_{ij} is the absolute value of the most critical constrained displacement in the current structure at the ith iteration under applied loads. The displacement-based performance index for a plate in bending is determined by (Liang et al. 2001a)

$$PI_{dp} = \left(\frac{u_{0j}}{u_{ij}}\right)^{1/3}\frac{W_0}{W_i} \qquad (15)$$

It can be seen from Eqs. (14) and (15) that displacement-based performance indices measure the performance of a structural topology in terms of material efficiency in resisting deformations.

For plane stress continuum structures with mean compliance constraints, the energy-based performance index can be expressed by (Liang and Steven 2002)

$$PI_{es} = \frac{C_0W_0}{C_iW_i} \qquad (16)$$

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where C_0 is the absolute value of the strain energy of the initial structure under applied loads, and C_i is the absolute value of the strain energy of the current structure under applied loads at the ith iteration. The energy-based performance index for plates in bending is expressed by (Liang and Steven 2002)

$$PI_{ep} = \left(\frac{C_0}{C_i}\right)^{1/3} \frac{W_0}{W_i} \tag{17}$$

It can be seen from Eqs. (16) and (17) that energy-based performance indices measure the performance of a structural topology in terms of material efficiency and the overall stiffness of the structure. For structures subject to multiple load cases, the performance index of a structure at each iteration can be calculated by using the response parameter of the structure under the most critical load case in the optimization process as suggested by Liang and Steven (2002).

Performance indices are useful tools in topology optimization of continuum structures. They can be used to evaluate the performance of optimized designs for continuum structures. In addition, they can be used to monitor the performance optimization history of topologies generated in an optimization process. Moreover, they can be used to assist the selection of topologies in structural design when esthetic and construction requirements are taken into account (Liang and Steven 2002). Furthermore, they can be used to compare the performance of optimal topologies produced by any continuum topology optimization methods (Liang et al. 1999a; Liang et al. 2000a).

6. PERFORMANCE-BASED OPTIMALITY CRITERIA

By gradually eliminating lowly stressed or lowly strained elements from a continuum design domain, the distribution of element stresses or strain energy densities within the resulting structure will consequently become more and more uniform. The fully stressed design and the uniform strain energy density distribution have been used as optimality conditions in structural optimization. However, the uniform condition of stresses/strain energy densities in a continuum structure may not be achieved even if the constraint has been violated. This means that a minimum-weight design with acceptable performance levels is not necessarily a structure in which the distribution of element stresses/strain energy densities is absolutely uniform. Therefore, the uniformity of element stresses/strain energy densities cannot be incorporated in continuum topology optimization methods as a termination condition to identify the optimum. Performance-based optimality criteria have been proposed for identifying the optimum from an optimization process by Liang et al. (2000a, 2000b) and Liang (2001a) and Liang and Steven (2002).

Performance-based optimality criteria (PBOC) are the maximization of the performance index:

$$\text{maximize } PI = \alpha^n \left(\frac{W_0}{W_i} \right) \qquad (18)$$

where α is the ratio of the response parameter (stress/displacement/strain energy) in the initial structure to the response parameter in the current structure and n is the exponent, which is equal to 1.0 for plane stress continuum structures and $\frac{1}{3}$ for plates in bending. The PBOC mean that the optimal topology or shape of a continuum structure under applied loads is achieved when the product of its associated structural response parameter and material

consumption is a minimum. The optimal topology obtained represents an efficient load-carrying mechanism within the design domain. The performance index can be employed to monitor the optimization process so that the optimum can be identified from the performance index history. Rozvany et al. (2002) realized that traditional optimality criteria are not adequate for continuum structures so that they used extended optimality criteria in continuum topology optimization. As reported by Setoodeh et al. (2005), the extended optimality criteria are similar to the PBOC proposed by Liang et al. (2000a) and Liang and Steven (2002).

7. PERFORMANCE OPTIMIZATION PROCEDURE

The PBO technique utilizes the finite element analysis (FEA) method as a modeling and computational tool. The stresses or strain energy densities of elements can be calculated from the results of finite element analyses. Lowly stressed/strained elements are identified as inefficient elements for elimination. The performance of a structure can be improved by gradually eliminating inefficient elements from the structure. The process of FEA, performance evaluation and element removal is repeated until the performance of the structure is maximized. The flowchart of the performance optimization procedure is depicted in Figure 1. The optimization procedure is also summarized as follows:

(1) Model the initial continuum structure with fine finite elements.

(2) Perform the finite element analysis on the structure.

(3) Evaluate the performance of the resulting structure using performance indices.

(4) Calculate the stresses/strain energy densities of elements under each load case.

(5) Remove a small number of inefficient elements from the design regions.

(6) Check the continuity of the resulting structure.

(7) Check the symmetry of the resulting structure

(8) Repeat step (2) to (7) until the performance index is less than unity.

(9) Plot the performance index history and select the optimal design.

In the PBO method, the elimination of a small number of inefficient elements from a structure at each iteration may result in singular elements which are disconnected to the continuum design domain. A continuity scheme has been implemented in PBO algorithms to avoid the discontinuity of the design domain. This scheme assumes that two elements are connected together if they have at least one common edge. Any element that is not connected with other elements is considered as a singular element, which is removed from the model.

Numerical errors may occur in the calculation of element strain energy densities due to approximate concepts adopted in the formulation. This may lead to an unsymmetrical structure even if the initial structure has a symmetrical geometry, loading and support condition. A scheme for checking the symmetry of resulting structures has been implemented in PBO algorithms. Extra elements are removed from the structure to maintain the symmetry of the resulting structure under an initially symmetrical condition.

8. PRACTICAL APPLICATIONS

The ultimate goal of the development of optimization techniques is to provide practicing design engineers with advanced design tools that can be used in the design of engineering structures in practice. The PBO technique has the capacity not only to generate the global optimum but also to monitor the performance optimization history of resulting topologies in an optimization process. This allows the designer to tailor the design to a specific performance level which satisfies the requirements of structural performance, esthetic and

construction (Liang 2005). The PBO technique can be used to solve a wide range of structural design problems. It can be used to generate optimal topology designs for bracing systems in multistory steel framed buildings (Liang et al. 2000c) and strut-and-tie models in structural concrete (Liang et al. 2000b, 2001b, 2002).

Strut-and-tie models are rational tools for the design and detailing of structural concrete (Marti 1985; Schlaich et al. 1987). However, traditional methods for developing strut-and-tie models in structural concrete involve a trial-and-error process. The strut-and-tie model developed using these methods is not unique and will vary with the designer's experience. To overcome this problem, the PBO techniques have been developed for automatically generating optimal strut-and-tie models for the design and detailing of reinforced and prestressed concrete structures (Liang et al. 1999c, 1999d, 2000b, 2001b, 2002; Liang 2006). Worked examples on the optimization and detailed design of strut-and-tie models in concrete structures were provided by Liang (2005). The development of strut-and-tie models using topology optimization techniques has attracted the attentions of other researchers (Ramm et al. 1997; Ali and White 2001; Biondini et al. 2001; Guan 2005; Leu et al. 2006; Kwak and Noh 2006).

9. DESIGN EXAMPLES

9.1   Topology Design of Bridges

The PBO technique was used to find the optimal topology design for a bridge subject to uniformly distributed traffic loads as depicted in Figure 2 (Liang and Steven 2002). The bottom supports of the bridge were fixed. The design domain was simulated using 90 × 30

four-node plane stress finite elements. The bridge deck was modeled by two rows of elements below the loading level and was treated as a non-design domain. The uniformly distributed loading was simulated by applying a 500 kN point load per node. The Young's modulus of material $E = 200$ GPa, Poisson's ratio $\nu = 0.3$ and the thickness of elements $t = 300$ mm were specified in the finite element analysis. The element deletion ratio of 1 per cent was used in the optimization process.

The performance characteristic curve for the bridge is presented in Figure 3. The figure shows that when inefficient elements were gradually removed from the design domain, the weight of the bridge structure was reduced with an increase in its mean compliance. The performance index history of the bridge is shown in Figure 4. It can be seen from Figure 4 that when underutilized elements were eliminated from the design domain, the performance index of the bridge increased from 1.0 to the maximum value of 1.4. After iteration 64, the performance index dropped sharply and this indicated that the load transfer mechanism was destroyed by further element removal. Therefore, any topology obtained after iteration 64 is not recommended as the final design.

The topology optimization history of the bridge is presented in Figure 5. The optimal topology generated by the PBO technique shown in Figure 5(c) indicates a well-known tie-arch bridge structural system that has commonly been used in bridge construction. The esthetic issue is often an important consideration in the design of large-scale bridges. The performance index can be used to assist the selection of a bridge form that not only has a good looking but also possesses high structural performance. The performance index of the optimal topology is 1.4 while the topology shown in Figure 5(d) is 1.38. However, the bridge form

shown in Figure 5(d) looks better than that of the optimum. Therefore, the topology presented in Figure 5(d) shall be used as the final form for the bridge.

9.2   Strut-and-tie models in concrete structures

A simply supported concrete deep beam shown in Figure 6 is to be designed to support a concentrated design load of 3000 kN (Liang 2005). The width of the beam was 400 mm. The automated PBO technique was employed to develop a strut-and-tie model for the design and detailing of this concrete deep beam. The deep beam was modeled using 66×30 four-node plane stress elements. The compressive cylinder strength of concrete $f_c^{'}$ = 32 MPa, Young's modulus of concrete E_c = 28600 MPa, and Poisson's ratio v = 0.15 were specified in the finite element analysis. The mean compliance constraint was considered. The element removal ratio of 2 per cent was used in the optimization process.

Figure 7 depicts the performance characteristic curve for the concrete deep beam in the optimization process. The figure shows that the mean compliance of the concrete deep beam increased when lowly strained elements were eliminated from the deep beam. The performance of the deep beam in the optimization process was monitored by the performance index shown in Figure 8. The maximum performance index was 1.57. After iteration 32, however, the performance index decreased sharply because further element elimination resulted in the breakdown of the load transfer mechanism in the concrete deep beam.

The optimization history of strut-and-tie model in the concrete deep beam is shown in Figure 9. It appears from the figure that the load transfer mechanism in the concrete deep beam gradually became clear when inefficient elements were systematically eliminated from the

model. The optimal topology depicted in Figure 9(d) occurred at iteration 32. The optimal structure represents the most efficient load transfer mechanism (strut-and-tie model) in the concrete deep beam at the ultimate limit state. The inclined tension tie shown in Figure 9(d) is needed to make the forces equilibrium at points B and D as depicted in Figure 10. If steel reinforcement is not provided to resist tension force in this tie, the deep beam will crack in this region. The optimal topology was transformed to a discrete strut-and-tie model illustrated in Figure 10, where the solid lines represent tension ties and the dashed lines represent compression struts. A detailed design of this concrete deep beam using the optimal strut-and-tie model generated by the PBO technique can be found in Liang (2005).

9.3   Bracing systems for multistory steel frames

Figure 11 depicts a 3-bay, 12-story rigid steel frame under revisal lateral loads. The frame structure was fixed at points A, B, C and D as shown in Figure 11. The automated PBO technique was employed to generate an optimal bracing system for this steel frame. Gravity loads were not considered in the optimization of the lateral bracing system. The beam and column were modeled using 15 and 9 beam elements respectively with a total of 684 elements for the whole frame. The Young's modulus $E = 200$ GPa, shear modulus $G = 7690$ MPa, and the material density $\rho = 7850$ kg / m^3 were specified for steel sections. A linear elastic finite element analysis was undertaken on the unbraced frame structure. The BHP hot rolled standard steel sections were then selected from databases to size the members of the frame based on strength performance criteria. Beams were grouped together as having a common section for each floor whilst columns were grouped for every two stories. Steel sections selected for frame members are given in Liang et al. (2000c).

The maximum lateral displacement of the unbraced frame obtained was 618 mm, which exceeds the drift limit of $H/400$, where H is the total height of the frame. Therefore, a continuum structure with a uniform thickness of 25 mm was used to fully brace the frame to reduce lateral deflections. The continuum structure was treated as a design domain, which was divided into 45×108 four-node plane stress elements. The steel frame and the continuum structure were connected together at the locations of beams and columns with compatible degrees of freedoms. The Young's modulus $E = 200$ GPa and Poisson's ratio $v = 0.3$ were used for the continuum design domain. The element removal ratio of 2 per cent was adopted in the optimization process.

The performance characteristic curve and the performance index curve for the braced steel frame are presented in Figures 12 and 13 respectively. In these curves, the weight of the continuum structure was used while the mean compliance of the braced frame was used. It appears from Figures 12 and 13 that the element removal resulted in a significant reduction in the weight of the continuum structure and the increase in the mean compliance of the braced frame. The optimal topology of the bracing system for the frame is shown in Figure 14(a), which provides the structural designer with very useful information on which member of the steel frame should be stiffened by resizing. The optimal topology of the bracing system for the steel frame can be transformed to a large-scale bracing system depicted in Figure 14(b). Since the mean compliance constraint in terms of the lateral drift has not been reached the actual limit at the optimum, sizing techniques can be employed to further optimize the braced frame using available standard steel sections. This example shows that continuum topology optimization is an effective tool for use in the conceptual design of bracing systems for multistory steel frameworks (Mijar et al. 1998). Advanced analysis techniques can be used to

analyze the bracing steel frame to carry out a final check for its strength and stiffness performance (Xu et al. 2006; Gong et al. 2006).

10. CONCLUSIONS

A state-of-the art review of the performance-based optimization techniques for topology design of continuum structures subject to stress, displacement and mean compliance constraints has been presented in this paper. Element removal criteria for continuum structures with performance-based constraints used in the PBO method were described. Techniques for suppressing the formation of checkerboard patterns in continuum topology optimization were reviewed. The importance of performance indices for monitoring the performance optimization history of plane stress continuum structures and plates in bending was highlighted. Performance-based optimality criteria have been generalized as the maximization of the performance index in an optimization process. A general performance optimization procedure was presented. Three design examples were presented to demonstrate the efficiency of the PBO technology as a practical performance-based design tool.

The PBO technique incorporates performance indices and performance-based optimality criteria into optimization algorithms to monitor the performance optimization history and to identify the optimum from an optimization process. The optimization technique allows the designer to tailor the design to a specific performance level required by the owner. The PBO technique can be used to solve a wide range of structural design problems. It can be used to automatically generate optimal strut-and-tie models for the design and detailing of structural concrete and optimal bracing systems for multistory steel and composite frames. Developing strut-and-tie models in structural concrete members using trial-and-error methods is a time-

consuming and challenging task for structural designers. The automated PBO technique developed overcomes the limitations of trial-and-error methods for developing strut-and-tie models and is an efficient design tools for structural designers. It can be used in the performance-based design of engineering structures.

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Figures
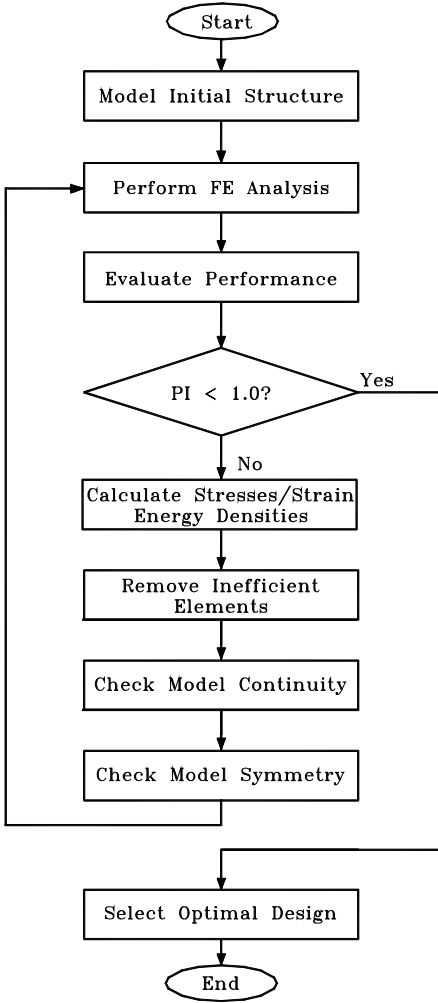
Figure 1. Flowchart of performance optimization procedure
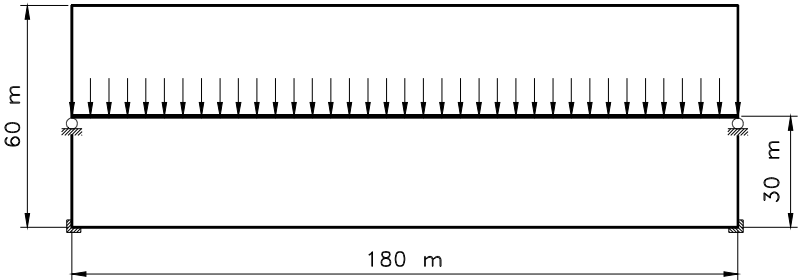
Figure 2. Design domain for a bridge (Liang and Steven 2002)
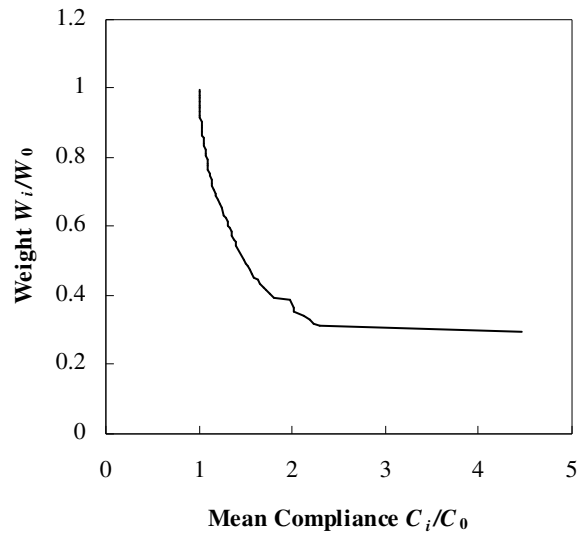
Figure 3. Performance characteristic curve for the bridge (Liang and Steven 2002)
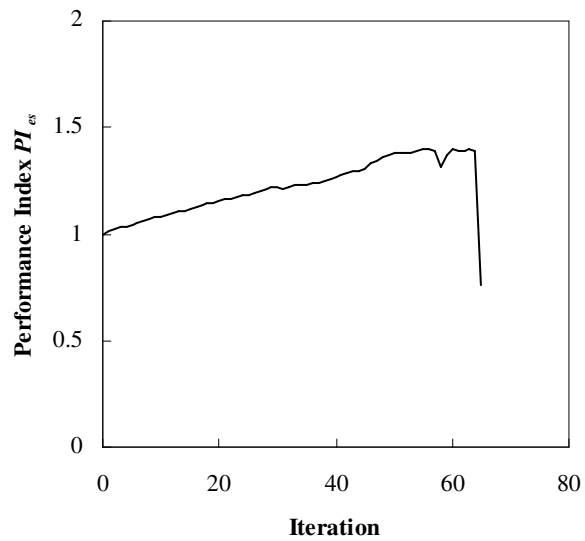
Figure 4. Performance index history of the bridge (Liang and Steven 2002)
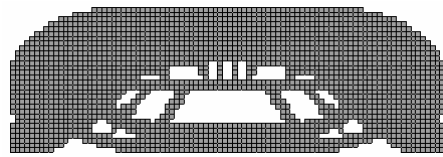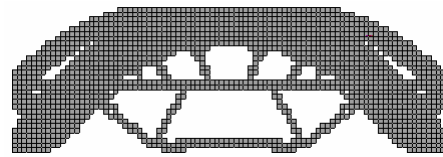
(a) Topology at iteration 20   (b) Topology at iteration 40
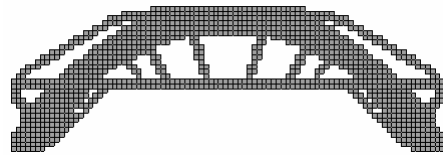
(c) Optimal topology at iteration 56   (d) Topology at iteration 64
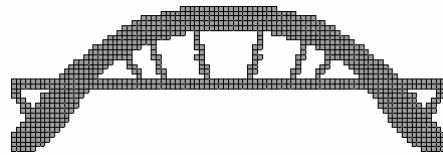
Figure 5. Topology optimization history of a tie-arch bridge (Liang and Steven 2002)
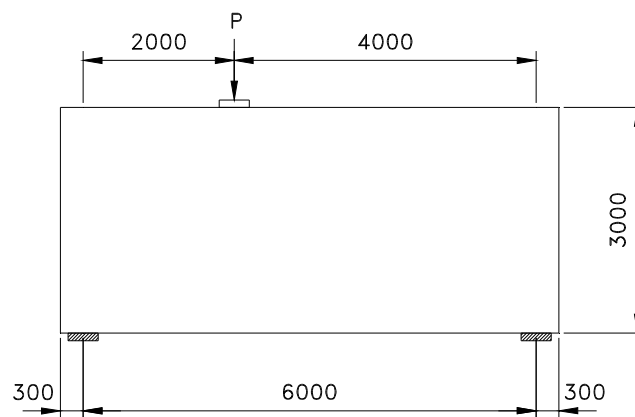
Figure 6. Simply supported concrete deep beam (Liang 2005)
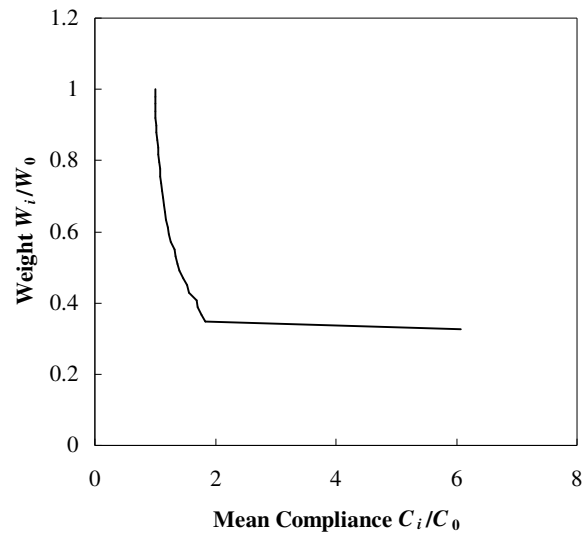
Figure 7. Performance characteristic curve for the concrete deep beam
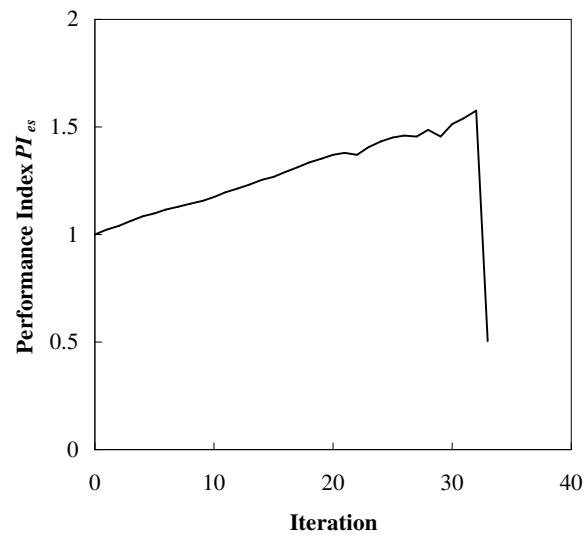
Figure 8. Performance index history of the concrete deep beam
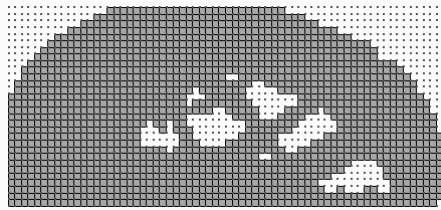
(a) Topology at iteration 10   (b) Topology at iteration 20
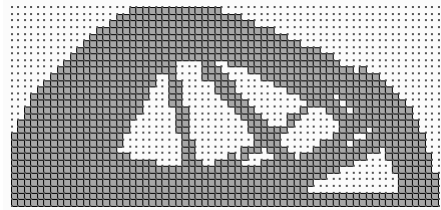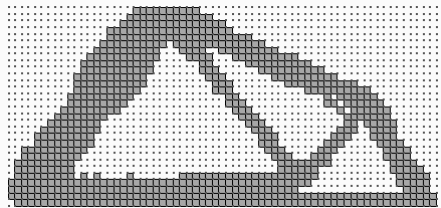
(c) Topology at iteration 30   (d) Optimum at iteration 32
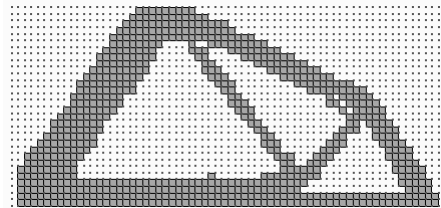
Figure 9. Optimization of strut-and-tie model in the concrete deep beam
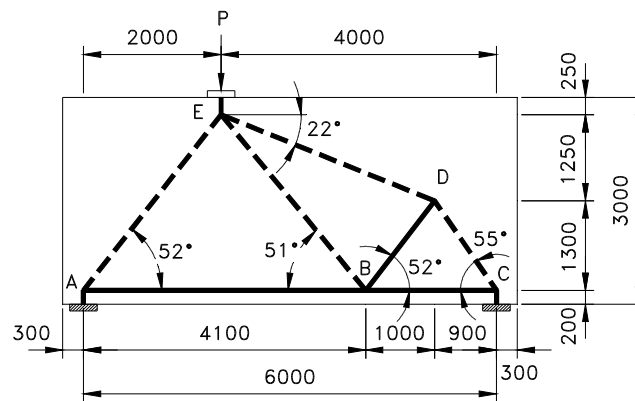
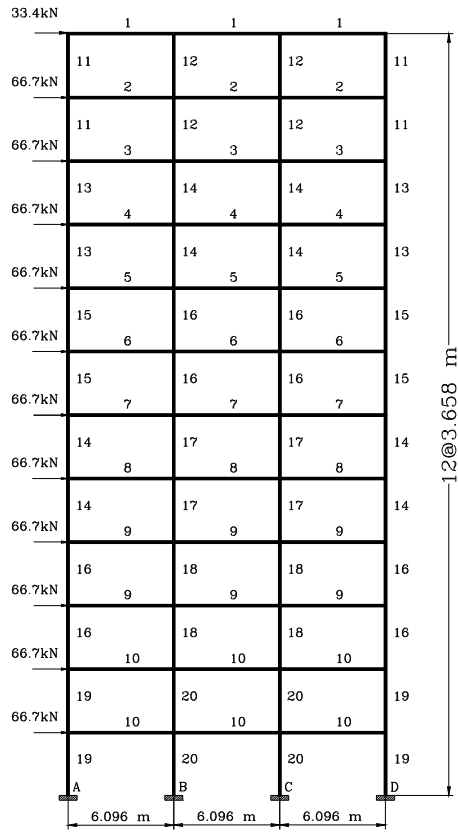**Figure 10.** Discrete strut-and-tie model in the concrete deep beam (Liang 2005)

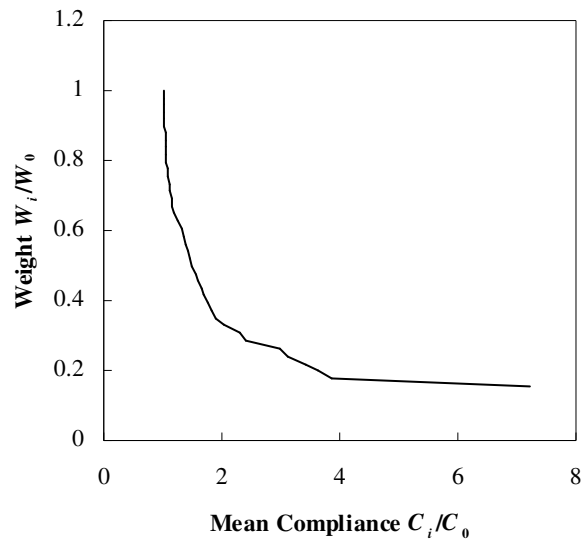**Figure 11.** A 3-bay, 12-story steel frame (Liang et al. 2000c)

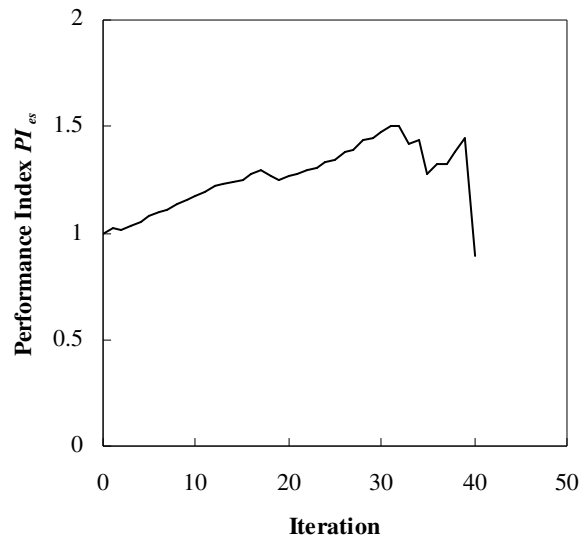**Figure 12.** Performance characteristics of bracing system

Figure 13. Performance index history of bracing system (Liang et al. 2000c)

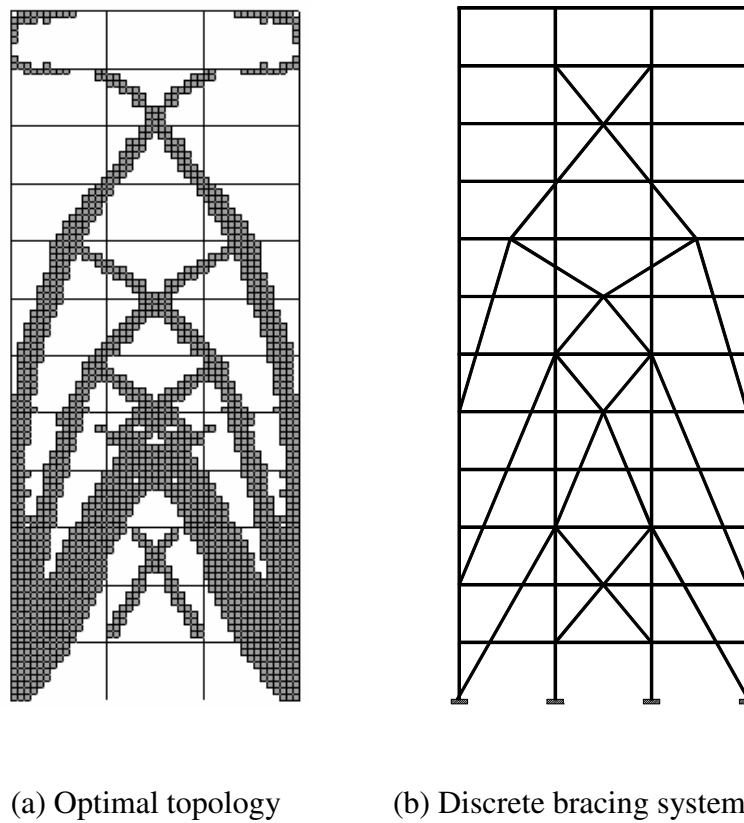(a) Optimal topology   (b) Discrete bracing system

Figure 14. Optimal bracing system for the 12-story steel frame (Liang et al. 2000c)