



The trends of using the Knowledge Quartet Framework in mathematics education research since 2005: systematic literature review

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Abstract

Numerous frameworks have been developed to describe and assess the pedagogical content knowledge of mathematics teachers. The Knowledge Quartet Framework (KQ), with its focus on professional knowledge for mathematics teaching, has received extensive attention and acknowledgment in various studies. This systematic literature review assessed the growth, use, and application of the KQ in mathematics education research. An analysis of 36 peer-reviewed articles showed that the KQ, initially rooted in UK primary mathematics novice teaching practices, has since gained global prominence. Though originally designed for primary education, applications of KQ span from primary to higher education settings, shedding light on relationships between teacher proficiency, content knowledge, and student performance. Furthermore, while the KQ was primarily associated with mathematics teaching, recent trends have seen its application in disciplines ranging from science to physical education. Key areas of KQ application include reflection on practice, coding and observing classroom practices, understanding the transition of pre-service and novice teachers to experienced classroom teachers, lesson planning, and alignment with other educational frameworks. The review highlights the KQ's role in enhancing teaching practices across various educational levels, its application in diverse research methodologies, and its expanding scope beyond mathematics education.

Keywords Knowledge Quartet framework · Systematic literature review · Teacher professional knowledge · Mathematics education · Observation

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1 Introduction

The field of mathematics education has witnessed the emergence of various theoretical frameworks aimed at enhancing teachers' professional knowledge and promoting effective classroom teaching practices. Various frameworks have been established to articulate and assess teachers' Pedagogical Content Knowledge (PCK) in various regions, such as the United States (e.g., Ball et al.'s (2008) Mathematical Content Knowledge), Germany (Baumert et al.'s (2010) Mathematics Teachers' Professional Competence), the UK (Rowland et al.'s (2005, 2009) Knowledge Quartet), and Australia (Chick et al.'s (2006) Profound Understanding of Fundamental Mathematics). These frameworks were based on early work by Shulman (1986, 1987) on PCK as a fundamental requirement for effective teaching and utilised surveys and classroom observations to examine mathematics teachers' teaching practices. The Knowledge Quartet Framework (KQ) (Rowland et al., 2005, 2009) has received noticeable acknowledgment in academic circles and was selected for this systematic literature review. For example, Rowland et al.'s (2005) comprehensive journal article on the KQ, which builds on their earlier conference paperwork, has been cited in 1069 outputs according to Google Scholar (search conducted on December 01, 2025). In addition, while other frameworks exist, the KQ focuses on teachers' in-the-moment decision-making processes, offering a more tailored approach than general teaching frameworks. This specificity is crucial for understanding and improving mathematics teaching practices. Moreover, our familiarity and prior work with the KQ further informed our decision. Given its significance, this systematic literature review on KQ aims to investigate trends in its use in educational studies since 2005 (when a comprehensive study on KQ was published) and to analyse its application across different educational contexts.

The KQ provides a comprehensive conceptualisation of the knowledge enacted by mathematics teachers as played out (Rowland et al., 2003) or "mobilised" in their teaching (Gumiero & Pazuch, 2021). The framework consists of four key dimensions: foundational, transformative, connective, and contingent. The foundational aspect pertains to the mathematical knowledge and comprehension that trainee teachers acquire during their training, equipping them for effective classroom practice. In contrast, the transformative dimension highlights the teacher's skill in presenting complex or unfamiliar concepts in ways that are accessible and understandable to students. As Rowland et al. (2005) noted — drawing on Shulman (1987) — this involves "the capacity of a teacher to transform the content knowledge he possesses into forms that are pedagogically powerful." This includes the deliberate selection of modes of representation, the crafting of examples, and the structuring that aligns with students' current understanding. The connection dimension highlights the importance of teachers being responsive and flexible in their teaching, adjusting their approach to meet the unique learning needs of their students (Rowland et al., 2005, 2009). The contingent dimension refers to a teacher's ability to respond spontaneously and adaptively to unexpected events during mathematics lessons, such as addressing learners' questions, misconceptions, or new ideas as they arise. These in-the-moment decisions often require improvisation and draw on the teacher's subject knowledge and pedagogical reasoning (Rowland et al., 2005, 2009).

This study aims to analyse existing research on the application of the KQ since 2005 through a systematic literature review. The review employed systematic search strategies to identify relevant studies published from the inception of the framework to July 2023. These

studies encompassed diverse educational contexts, including primary and secondary education, and covered various subject areas within mathematics education.

The research questions guiding this systematic literature review are as follows:

1. How has the application of the KQ changed from 2005 to 2023 in various educational settings, contexts, and disciplines?

In answering this research question, we seek to identify the school year levels and mathematical concepts in which the KQ has been utilised in research.

2. In what ways has the Knowledge Quartet been used methodologically in research studies—either as a standalone framework or in combination with other theoretical or analytical models?

This research question aims to reveal how the KQ is integrated and adapted, providing insights that could inform its future application and enhance research methodologies in the field.

This study's findings will highlight the use of the KQ since 2005, identifying areas of consistent application and those needing further exploration. By examining its use over the years, we gain valuable insights into the professional knowledge of mathematics teachers and the framework's strengths and limitations. This review serves as a foundation for future research, guiding scholars and practitioners in exploring the KQ's effectiveness in enhancing mathematics teaching practices and informing future research on classroom teaching.

2 Background

2.1 Mathematics teachers' knowledge

The nature of the knowledge essential for successful mathematics teaching has been widely explored and debated within educational research. Several studies have shown that teachers' professional competence encompasses more than their knowledge of the subject matter. It includes substantive knowledge of the formal content, specific pedagogical approaches relevant to the subject (such as mathematics pedagogy), and general pedagogical strategies applicable to teaching in general (Blömeke et al., 2011; Schmidt et al., 2008). Exploring the interplay between knowledge components in enhancing teaching competencies began as early as the 1980s. One of the pioneering figures in this study area was Shulman (1986), who introduced the concept of PCK as a fundamental requirement for effective teaching. Shulman (1986, 1987) defined PCK as the integration of content and pedagogy, enabling teachers to effectively structure, modify, and deliver content in ways that align with students' varying interests, abilities, and learning needs.

The concept of PCK has been explored across multiple disciplines, including mathematics. For example, Phelps and Schilling (2004) applied the concept in language education to develop survey tools to assess teachers' understanding of the content required to teach early reading. In physics, Halim and Meerah (2002) used PCK to interpret educators' instructional knowledge of specific physics topics, illustrating its broad applicability. Within mathematics education, scholars such as Ball and colleagues (2008) and Park and colleagues (2011; Park & Oliver, 2008) have underscored the critical role of PCK in

effective teaching. However, Ball et al. (2008) cautioned that a generalised view of PCK in mathematics might overlook the depth of subject-specific understanding required. To address this, they proposed distinguishing between mathematical knowledge itself and the knowledge involved in engaging with mathematical practices. They suggested that mathematics teachers need both forms of knowledge to excel in their profession.

Rowland et al. (2005, 2009) identified four distinct knowledge categories essential for teaching mathematics. These categories, collectively known as the Knowledge Quartet (KQ), were derived from in-depth analysis and comparison of 24 videotaped lessons delivered by primary pre-service teachers nearing the completion of their teacher education program. The KQ classifies situations and behaviours in which mathematics-related knowledge surfaces in teaching into four dimensions, as discussed in the following section.

The various knowledge frameworks and distinctions proposed by researchers emphasised the importance of understanding the distinct knowledge required for teaching mathematics at different levels. In light of the implications of these knowledge frameworks, institutions involved in teacher education play a crucial role in preparing prospective mathematics teachers. Understanding PCK has been acknowledged as helpful for mathematics educators in designing professional development programs tailored to teachers' needs (Ball et al., 2008). The knowledge categories associated with teaching can inform the development of support materials, teacher preparation programs, and targeted professional development initiatives (Ball et al., 2008; Grossman et al., 2009). It is important for those involved in teacher education to highlight the link between a teacher's mathematical knowledge and their ability to teach it effectively. The consensus among researchers is that mathematics teachers should be educated about the mathematical content and the pedagogical strategies specific to mathematics education (Krauss et al., 2008; Turnuklu & Yesildere, 2007). This understanding should be integrated into the teaching practices of teacher educators, ensuring that future mathematics teachers can learn from and apply their knowledge effectively in primary and secondary school settings (Loughran & Berry, 2005).

2.2 Knowledge Quartet Framework

Rowland et al. (2005, 2009) designed and presented a framework based on the analysis of videotaped mathematics lessons taught by pre-service primary teachers during their initial training. Rowland et al. (2005) aimed to create a conceptually grounded framework for conducting lesson reviews. The focus of the framework was on exploring the mathematical content within the lessons and examining pre-service teachers' mathematics Subject Matter Knowledge (SMK) and PCK and how this knowledge can be observed to "play out" in practical teaching during school-based placements. The KQ was also intended to support aspects of reflective practice in the classroom (Rowland et al., 2005, 2009). Importantly, it was not designed to assess and grade teaching performance. Rowland et al. (2005) employed a grounded theory methodology to examine the data, beginning with an open coding process that identified 18 distinct ways in which teachers' knowledge is manifested during classroom teaching. An indicative code (such as "choice of examples") was assigned to each of these ways. These 18 codes were then grouped into four overarching categories of the Knowledge Quartet, each category representing a shared characteristic among related codes. This process of "axial coding" eventually led to the identification of the four "dimensions" that they named the Knowledge Quartet. Each is presented in Table 1, followed by a description.

Table 1 The Knowledge Quartet (Rowland et al., 2005, 2009, p. 9)

Knowledge Categories	Descriptive codes
Foundation	Adherence to a textbook, Awareness of purpose, Concentration on procedures, Identifying errors, Overt subject knowledge, Theoretical underpinning
Transformation	Use of terminology, Choice of examples, Choice of representation, Teaching Demonstration
Connection	Anticipation of complexity, Decisions about sequencing, Making connections between procedures, Making connections between concepts, Recognition of conceptual appropriateness
Contingency	Deviation from agenda, Responding to children's ideas, Use of opportunities

Foundational knowledge relates to a teacher's conceptual grasp of mathematics, including their beliefs and understanding, regardless of whether it is directly applied in classroom practice (Rowland & Turner, 2007; Rowland et al., 2005). This dimension includes multiple facets of teacher knowledge (see Table 1) and serves as a critical base for effective mathematics teaching.

The Transformation dimension focuses on teachers' skills in making strategic choices in their teaching by translating foundational knowledge into classroom practice through relevant representations and examples (Rowland & Turner, 2007). This includes selecting examples that illustrate mathematical concepts, choosing representations to enhance students' understanding, and demonstrating mathematical procedures. By employing effective transformation strategies, teachers can facilitate students' conceptual development and problem-solving skills (Rowland & Turner, 2007).

The Connection dimension emphasises the importance of establishing meaningful connections within and across mathematical topics (Rowland et al., 2005, 2009). This dimension involves anticipating the complexity of mathematical concepts and making informed decisions about the sequencing of teaching materials (see Table 1). Turner (2012) explained that the Connection dimension of teacher knowledge centres on the teacher's capacity to integrate mathematical knowledge and procedures that help students make sense of the material presented in the classroom.

The Contingency dimension addresses teachers' flexibility and responsiveness to classroom situations that were anticipated in the planning process (Turner, 2012). It encompasses deviating from planned agendas when necessary, responding to students' ideas and questions, and seizing opportunities for teaching enrichment. Teachers who can respond to contingencies in the classroom can create a dynamic, engaging learning environment that caters to students' needs and interests (Rowland, 2013).

Rowland et al. (2005, 2009) provided a detailed description of each component of KQ and analysed a specific segment from one of the recorded lessons. This examination illustrates how the different dimensions of the framework can be identified and applied within the context of mathematics teaching. The KQ is a valuable tool for observing and understanding the various facets of mathematics teaching, as demonstrated in various classroom contexts (see <http://www.knowledgequartet.org/>). It enables educators and researchers to explore the foundational knowledge, transformative practices, connections between mathematical concepts, and the contingencies involved in teaching mathematics. It is acknowledged, however, that the framework is open to interpretive risks and misappropriation, and the recommendation is to define each of the underlying codes to avoid such risks (Weston et al., 2012). Furthermore, Ruthven (2011) highlights that the grouping of the codes into the

discursive categories of Foundation, Transformation, Connection and Contingency “appears to risk introducing too great an interpretive flexibility ...” (p. 85). For example, when considering a lesson excerpt, examining the teacher’s approach will likely contain connected elements from all four dimensions. The interconnectedness of knowledge categories is emphasised in the teaching scenario presented by Weston et al. (2012). An episode from a Year 3 mathematics lesson in 2002 (Turner & Rowland, 2011) illustrates how responding to students’ ideas during lessons can involve multiple KQ dimensions and how these dimensions can interact. During the lesson, Jason asked students to divide their individual whiteboards into two parts. Most children drew a vertical or horizontal line through the centre, but one student, Elliot, drew a diagonal line. Jason acknowledged Elliot’s creative approach and then asked the class to divide their boards into four sections. While most students added two perpendicular lines, Elliot drew two diagonals instead. Rather than dismissing this unconventional solution, Jason highlighted Elliot’s work and invited the class to consider whether it was correct.

This case demonstrates the Foundation dimension through Jason’s understanding of fractions and his transformation in using whiteboards as a visual tool to make abstract ideas concrete. Connection is evident in the way the activity links to prior learning about equal parts. Most importantly, Contingency emerges when Jason adapts his teaching in response to Elliot’s unexpected approach, choosing to explore its validity with the class. This decision required Jason to draw on both SMK and PCK, which involves determining whether Elliot’s noncongruent parts could be considered equal, deciding what “equal” means for young learners, and framing arguments about area in ways accessible to them. The episode also illustrates how some codes, such as responding to an unexpected idea, can operate across other dimensions, underscoring the interconnected nature of the KQ dimensions (Weston et al., 2012).

3 Method

This study, inspired by Newman and Gough (2019), reports findings from a rigorous systematic review to examine the trend in the use of the KQ in educational research. To ensure transparency and replicability in the data collection, the researchers followed the guidelines outlined in the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA). Subsequently, we followed a step-by-step methodology for the systematic review of articles (Selcuk, 2019).

3.1 Search strategy

This systematic review conducted a thorough, rigorous search for articles using the research terms. To ensure a comprehensive search, we used a combination of the wildcard (*) function and the Boolean operator “AND” to develop the search terms: “knowledge*” AND “quartet*” AND “framework*” in a pilot search. This pilot search allowed us to capture variations of the term “knowledge quartet,” including “framework” and “model,” as some articles used different terminologies to refer to it. However, the results showed that the term consistently used across all relevant articles was “Knowledge Quartet.” Based on this finding, we adjusted the search term to “Knowledge Quartet” AND “Education” for the review. To ensure a sensitive search across databases with heterogenous indexing, the broader term “Education” was used, and non-mathematics records were excluded at

screening to meet the predefined eligibility criteria. We applied these search terms to four databases: ERIC, SCOPUS, Web of Science, and Science Direct, which were selected based on the initial search results and their relevance to the research. We included all pertinent articles that used the KQ framework. For example, studies that mentioned the KQ but did not employ it as a theoretical framework or method (e.g., in data gathering or analysis) were not considered.

3.2 Eligibility criteria

To ensure the inclusion of only pertinent and qualified studies, strict criteria were applied to both selection and exclusion, given that the databases also contained materials unrelated to the field of education. The following carefully selected inclusion criteria were used to determine which articles would be considered for the review. Table 2 shows the inclusion and exclusion criteria for the systematic literature review.

Based on these inclusion criteria, certain types of articles were excluded from the review to maintain scientific originality and avoid potential bias. Specifically, conference proceedings and grey literature, such as reports, theses, and dissertations, were not considered for inclusion.

3.3 Article screening and quality assessment

The retrieved articles were screened for keywords, titles, and abstracts, with duplicates removed (see Fig. 1). The remaining studies were assessed using the established inclusion criteria, resulting in a final selection of 36 peer-reviewed articles that met the eligibility requirements. Full papers were obtained for these selected articles; each article underwent an independent quality assessment conducted by Authors 1 and 2. Then, all the authors compared and finalised the results of independent assessments, focusing on evaluating the methodological rigour, research design, relevance, and overall risk of bias for each article using the standardised Joanna Briggs Institute (JBI) critical appraisal tool. Various sources of bias were considered, including methodological bias (inappropriate methodology), selection bias (unfair representation of the target population), detection or measurement bias (relating to the validity of the research instrument), and performance bias (other factors that could influence the results due to lack of control or blinding). This thorough evaluation was essential to ensure the trustworthiness and

Table 2 Inclusion and exclusion criteria

S/no	Inclusion criteria	Exclusion criteria
1	The article must contain the keywords <i>KQ</i> and <i>Education</i> and must use KQ in a way that is relevant to the research questions	Irrelevant articles not including key concepts or not using KQ in a way that is relevant to the research questions
2	The article must be a scientific paper published in a peer-reviewed journal	Articles not published in a peer-reviewed journal
3	The article must have been published from 1st Jan 2005 to July 2023	Articles published outside the time range of Jan 2005 to July 2023
4	The full text of the article must be available	Unavailable articles
5	The article must be written in English	Articles written in other languages other than English

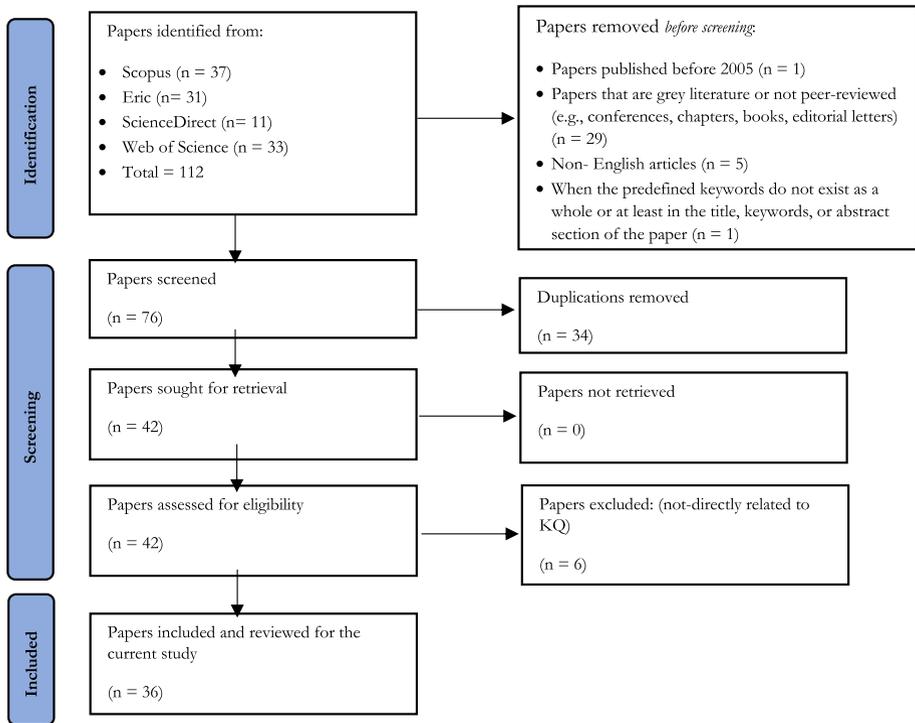


Fig. 1 The study screening process

reliability of the articles selected for the review, rather than outrightly excluding studies. Authors 1 and 2 independently reviewed these selected articles, and information from each article was extracted into an Excel file using a predefined template. The extracted data included details about the study (e.g., authors, publication year), the purpose of the article, research design, key findings, employed methodology, subject areas covered, educational level, study country, implications and recommendations, and suggestions for future studies. This data extraction aimed to facilitate the synthesis and identification of themes, changes, and adaptations in applying the KQ across the studies. Excel was also used to visualise the themes that emerged from using KQ. In accordance with the steps outlined by Thomas and Harden (2008), our approach to thematic analysis encompasses three distinct phases: firstly, the use of KQ in each paper was systematically reviewed and coded; secondly, the formulation of descriptive themes derived from this coding; and thirdly, the creation of common themes that specifically show the application of the KQ within the scope of the studies under review. Figure 1 summarises the search results using a PRISMA flow diagram.

As shown in Fig. 1, the initial search across four databases retrieved 112 relevant articles. During the first screening phase, 36 articles were excluded based on specific criteria, such as a lack of peer review, non-English language, or the absence of key terms defined for the study. For example, our decision not to include them was driven by the need for methodological clarity and reproducibility rather than convenience. Specifically, the ambiguity regarding the peer-reviewed status of conference proceedings, which is

inconsistently reported, impedes transparency. Many proceedings are precursors to journal publications, creating the risk of duplication. Book chapters are often invited and edited rather than peer-reviewed externally, which may lack the methodological detail and rigour required for this type of paper. While we acknowledge the limitations of this decision in the discussion, it enhances internal validity. A future study could include conference proceedings and chapters to assess whether the conclusions align with the present synthesis. The remaining 76 articles were screened, and 34 duplicate articles were removed. All 42 articles that satisfied the inclusion criteria were successfully located and retrieved. During the final screening stage, six studies were excluded based on the previously established exclusion criteria, as illustrated in Fig. 1. Six studies referenced the KQ only tangentially (for background or citation) and did not employ it as an analytic or developmental framework. Because our review included only work that applied the KQ to analyse or improve mathematics teaching, these studies did not meet the inclusion criteria and were excluded. For instance, O'Shea et al. (2018) acknowledged the framework's relevance, particularly in highlighting the roles of both teacher and student in a contingent moment, but did not actually apply it in their analysis. In the end, 36 studies were included in the review and synthesised using a thematic approach, with each article assigned to one primary theme rather than multiple categories.

4 Results

The results of synthesising and analysing the 36 articles included in this review are presented in four clusters: trends in the use of KQ, the educational context of KQ research, the research methods used in KQ-related studies, and how KQ is used in the studies. Additional supplementary material is provided to illustrate these relationships.

4.1 Trends in the use of KQ

Figure 2 presents the distribution of studies included in a systematic literature review focusing on the application of the KQ from January 1, 2005, to July 17, 2023.

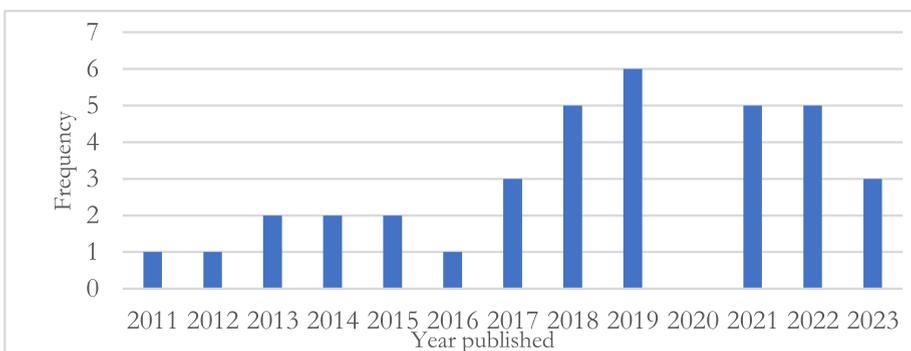


Fig. 2 Published years and frequency

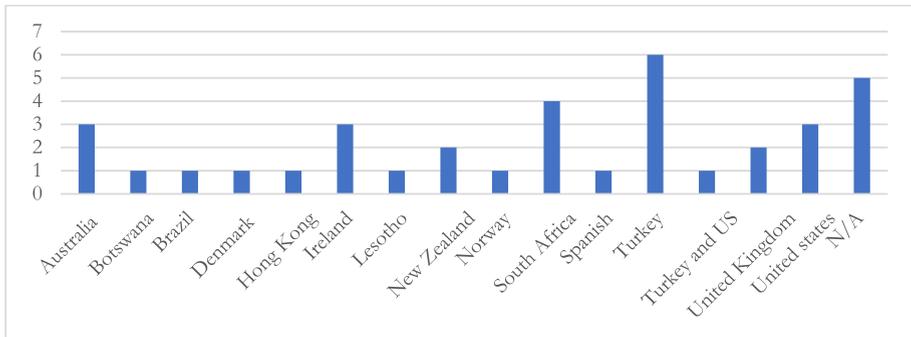


Fig. 3 Frequency of study areas (countries) using the KQ

Research activity using the KQ showed notable fluctuations, peaking between 2017 and 2022. No studies from 2005 to 2010 met the inclusion criteria, possibly indicating limited awareness or adoption of the framework during that period. The increase in publications from 2017 onward indicates growing recognition and application of KQ within mathematics education research. No qualifying studies appeared in 2020, and a slight decline followed in 2021–2023. This trend may be due to: (1) natural variation in publication volume; (2) reduced classroom access during the COVID-19 pandemic, which impacted observational research; and (3) studies could be published while conducting the current study.

Figure 3 illustrates the geographical distribution of KQ-related research, reflecting its expanding global relevance. Despite its UK origins, KQ has been applied internationally. Turkey leads in usage with six studies (e.g., Dogan & Bostan, 2018, 2019; Unver & Guzel, 2019), followed by South Africa with four (e.g., Moodliar & Gopal, 2021). One study drew data from both Turkey and the United States (Dogan & Bostan, 2022). Turkey's strong representation may be linked to an international collaboration involving researchers from Dokuz Eylul University (Weston et al., 2012). Five studies applied the KQ without specifying a country context. For example, Neubrand (2018) employed KQ to develop a conceptual framework that outlines the components of teacher knowledge for mathematics instruction, incorporating professional awareness. Similarly, Hurst (2017) employed the contingency dimension of KQ to propose that powerful teaching involves deliberately provoking contingent events, also without reference to a specific national or educational setting.

4.2 Context of education levels

Since the KQ initially emerged from an attempt to understand how teacher knowledge can be applied in the classroom, this study presents the educational levels for which the KQ was used during the years under consideration. Figure 4 summarises the education levels at which the selected studies for the systematic literature review on the use of the KQ were conducted.

Most studies were conducted in primary school contexts ($n=12$), such as those by Dogan and Bostan (2019), Dennis et al. (2017), and Abdulhamid and Venkat (2018), followed by secondary school settings ($n=10$) (e.g., Lai et al., 2023; Moodliar & Gopal, 2021; Thwaites et al., 2011). Additionally, eight studies focused on higher education,

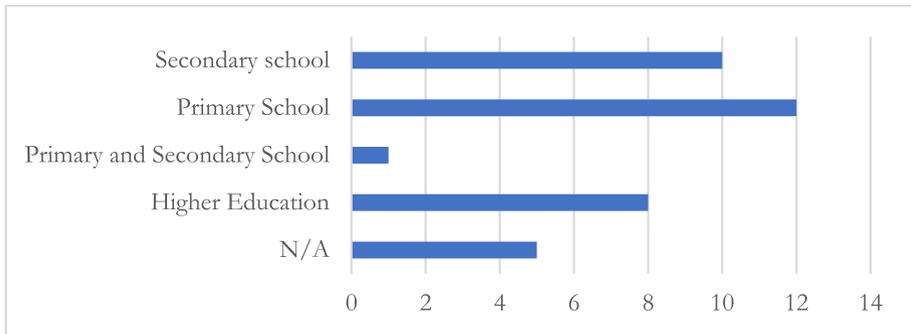


Fig. 4 The frequency of studies focusing on the education levels

involving pre-service teachers working in primary or secondary classrooms (e.g., Lane et al., 2019; Maher et al., 2022; Salani & Jojo, 2023).

In primary school studies, the KQ was applied in various ways, including examining how teachers’ pedagogical efficacy and subject knowledge influence students’ mathematical achievement (Dennis et al., 2017). Dogan et al. (2021) used KQ to explore the link between teachers’ responses to contingent moments and their mathematical knowledge, highlighting KQ’s value in understanding both spontaneous teaching decisions and their pedagogical significance.

At the secondary level, Moodliar and Gopal (2021) combined KQ with Ryle’s (1949) framework to investigate how mathematics teachers responded to unexpected incidents during algebra lessons. In higher education, KQ has been used to demonstrate the interdependence between pre-service teachers’ professional development and classroom practice (e.g., Lai et al., 2023; Livy et al., 2019), suggesting that its application has expanded beyond its original primary school focus to enhance teaching practices across educational levels. Further analysis of the data reveals some notable associations between country, education level, and participant type. For example, studies conducted in Turkey ($n=6$) and South Africa ($n=4$) predominantly focused on secondary school education, whereas studies from Australia ($n=3$) and Ireland ($n=3$) were more evenly distributed across levels, including higher teacher education.

Furthermore, the types of study participants in the reviewed articles provide insight into the use of KQ. Table 3 shows the distribution of participants in KQ research studies.

Pre-service teachers were the most common participants, appearing in 19 studies, followed by in-service teachers, who appeared in 11 studies. Fewer studies involved beginning teachers or higher education instructors. Three conceptual papers did not identify

Table 3 Frequency distribution of participants involved in the studies

Participants	Frequency
Pre-service teachers	19
In-service teacher	11
Not applicable	3
Beginning teachers	2
Higher education instructor	1
Total	36

specific participants (see Table 3), except for Turner (2012), who applied the Knowledge Quartet (KQ) framework as an analytical lens to examine elements of mathematical content knowledge evident in teaching practices, classroom discussions, and written reflections by pre-service teachers. One study (Walsh, 2017) applied the KQ to examine the pedagogical practices of three higher education instructors without a background in mathematics, using video analysis. The predominance of pre-service teacher participants aligns with KQ's original focus on how they apply pedagogical and subject knowledge in classroom settings. In terms of participant type and country, pre-service teachers were the most common focus overall ($n=19$), particularly in studies from Australia, Turkey, and the United States, whereas higher education teachers appeared only in Denmark and Turkey. These patterns suggest that research emphasis varies by geography, with certain researchers prioritising teacher preparation at different stages of professional development.

4.3 Methods used, subject areas, and mathematical concepts in the studies

From its inception, studies involving the KQ have predominantly utilised qualitative research methods through observation of teaching practices in lessons. This study also identified the research methods other researchers have adopted when using the KQ. Table 4 summarises the research methods utilised in the reviewed articles.

Of the 36 reviewed articles, 25 employed qualitative methodologies, most commonly observational techniques similar to those described by Rowland et al. (2005). These studies often combined classroom observations with complementary data collection methods, such as interviews, lesson evaluations, and field notes (e.g., Dogan & Bostan, 2019; Livy et al., 2019). For instance, Dogan and Bostan (2019) examined an in-service teacher's mathematical understanding of measurement by integrating observational data, interviews, and field notes. Their findings illustrated how mathematical knowledge supports the link between lesson planning and instructional practice.

Other qualitative techniques included video/audio recordings (e.g., Getenet & Callingham, 2021), lesson note analysis, and document analysis (e.g., Salani & Jojo, 2023). Surveys and interviews were also frequently used (e.g., Ünver, 2018; Maher et al., 2022). For example, Getenet and Callingham (2021) adopted a mixed-methods approach, using multimedia recordings to explore fraction teaching through the lens of KQ and Behr et al.'s (1983) constructs. In the quantitative component of their study, they conducted frequency counts to quantify occurrences of each KQ dimension, supported by illustrative examples that provided a descriptive overview of teachers' pedagogical content knowledge in practice.

Salani and Jojo (2023) employed document analysis to investigate teachers' perceptions and knowledge, demonstrating how KQ can help navigate the complexities of teaching

Table 4 Methods used in the studies

Research method	Frequency
Conceptual	4
Literature Review	1
Mixed Method research	4
Qualitative research	25
Quantitative research	2
Total	36

algebraic equations. They recommended integrating reflective teaching and collaborative planning to strengthen the KQ framework.

Overall, four studies used mixed methods (e.g., Getenet & Callingham, 2021; Livy et al., 2019), four were conceptual (e.g., Hurst, 2017; Kula Ünver, 2018), and one was a literature review (Pineiro et al., 2021). Two studies employed quantitative methodologies (Dennis et al., 2017; Weston, 2013). For example, Weston (2013) developed a scaled coding protocol based on KQ to assess the quality and frequency of Mathematical Knowledge in Teaching (MKiT). Each aspect was rated across six levels, combining presence and quality, enabling a structured evaluation of teaching practices.

Again, since the KQ was designed to analyse and improve mathematics teaching, we examined the subject focus of the KQ articles to determine if the KQ has been used outside of mathematics. This review indicates that 31 of the articles focused on mathematics content, and five had unspecified focus. Although the mathematics-focused articles covered a range of topics including mathematical content, pedagogy, and instructional strategies, one study extended its scope by integrating mathematics with other disciplines such as science, health and physical education, humanities, languages, and design technology (O’Keeffe & White, 2022). Another study concentrated solely on mathematics and science (Amador et al., 2022). These examples suggest that the KQ framework is beginning to be explored in educational contexts beyond mathematics alone.

In mathematics teaching using KQ, we analysed the concepts and classified them into six areas: number, algebra, measurement, space, statistics and probability. Of the reviewed articles, 18 focused primarily on conceptual, literature-based discussions and general mathematics topics, without specifying a particular strand (e.g., studies that used the KQ to explore how PSTs develop mathematical content knowledge). The remaining 18 articles focused on specific strands: seven on algebra (e.g., Moodliar & Gopal, 2021; Salani & Jojo, 2023), three on number concepts such as fractions (e.g., Getenet & Callingham, 2021; Lane et al., 2019), four on space (e.g., Lai et al., 2023), and four on measurement (e.g., Dogan & Bostan, 2018, 2019; Dogan et al., 2021). No articles were found to focus on probability or statistics. Figure 5 provides a summary of these mathematics strands represented in the reviewed studies.

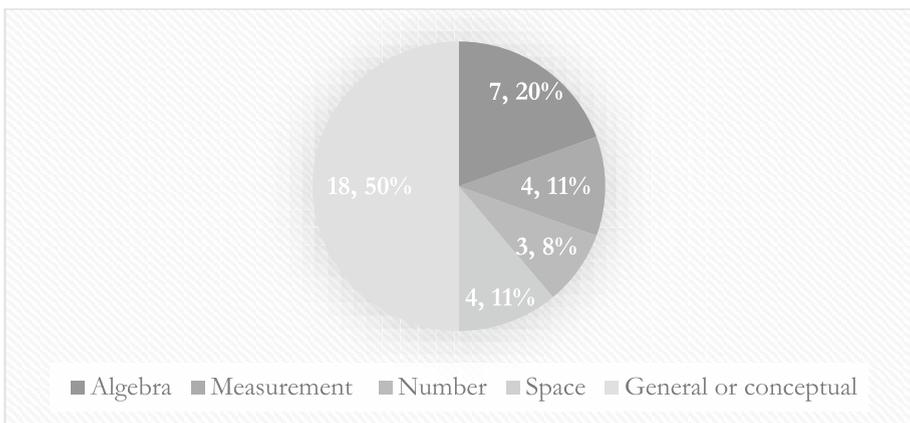


Fig. 5 Mathematics concepts found in KQ articles

4.4 How KQ was used in the studies

The analysis results revealed major themes in the applications of the KQ within mathematics education research. Each theme represents a distinct area of investigation where KQ has been used to gain insights into various aspects of teaching mathematics. Table 5 summarises the five themes that emerged, the number of studies (frequency), descriptions, and examples of relevant studies, providing an overview of the applications of the KQ in mathematics education research.

4.4.1 Direct application for observation and analysis

The KQ was directly applied as a methodological tool to observe and analyse teaching practices in 11 studies. These studies predominantly involved using the KQ to code classroom observations, assess teaching methods, or identify specific pedagogical actions in mathematics teaching. For instance, in Walsh's (2017) study, the KQ was used in conjunction with general pedagogical knowledge to code various teaching dimensions. Similarly, studies such as those by Getenet and Callingham (2021), Dogan and Bostan (2019), and Callingham et al. (2019) relied on the KQ codes to analyse teacher–student interactions and the enactment of mathematical knowledge during lessons. For example, Dogan and Bostan (2019) employed KQ codes to demonstrate that teachers' mathematical knowledge in teaching influenced the implementation of tasks. Lai et al. (2023) examined prospective secondary teachers' use of Foundation and Contingency knowledge when working with

Table 5 Themes that emerged on how KQ is used

Theme	Frequency	Description	Relevant studies
Direct Application for Observation and Analysis	11	Direct use of KQ for observing and analysing teaching practices	Walsh (2017); Dogan & Bostan (2019); Callingham et al. (2019); Mpalami and Moleko (2022); Dahl et al. (2019); Lane et al., (2019); Lai et al. (2023); Ünver (2018); Getenet and Callingham (2021); Weston (2013); Thwaites et al. (2011)
Combination with Other Frameworks or Models	8	Integrating KQ with other frameworks/models for inclusive analysis	Skott (2023); Pineiro et al. (2021); McAuliffe and Lubben (2013); Gumiero and Pazuch (2021); Akar (2016); Dennis et al. (2017); Moodliar and Gopal (2021); Amador et al. (2022)
Development or Modification of Tools/Tests	6	Using KQ to create or adapt tools/tests to assess mathematical teaching	Livy et al. (2019); Dogan and Bostan (2022); Ünver and Guzel (2019); Dogan & Bostan (2018); Maher et al. (2022); Turner (2012)
Conceptual or Theoretical Exploration	6	Exploring the foundational principles, implications, or modifications of KQ for theoretical explorations	Neubrand (2018); Hurst (2017); Rowland et al. (2014); Kula Ünver (2018) Ünver (2018); Salani and Jojo (2023); Abdulhamid and Venkat (2018)
Reflection and Professional Development	5	Utilising KQ to support reflection and guide professional development in teaching practices	Dogan et al. (2021); Liston (2015); Rowland et al. (2015); O'Keeffe and White (2022); Abdulhamid and Venkat (2014)

definitions of transformations and found no statistically significant correlation between the two dimensions. While both types of knowledge were observed in practice, having strong Foundational knowledge did not predict the likelihood of demonstrating Contingency knowledge in the tasks analysed, indicating that these dimensions can operate independently rather than in tandem, which is consistent with the KQ. To illustrate the complexity of classroom knowledge, McAuliffe and Lubben (2013) and Salani and Jojo (2023) provided evidence that teachers have more difficulty connecting concepts and procedures, emphasising reasoning and problem solving (connection knowledge dimension), but are mainly observed to have Foundational knowledge. These are consistent with Rowland et al.'s (2009) suggestion that the KQ was not designed as a hierarchy. The four dimensions — Foundation, Transformation, Connection and Contingency — are intended to represent different but interconnected aspects of a teacher's knowledge as it is deployed in the classroom. Each dimension can manifest independently, and its presence may vary depending on the nature of the lesson, the content, and the classroom context. It is not expected that all four dimensions will be observable in every lesson.

4.4.2 Combination with other frameworks or models

Another theme identified in the review was the use of the KQ alongside other theoretical frameworks to provide complementary perspectives on teaching practices. For example, Skott (2023) combined Patterns of Participation (PoP) with the KQ to evaluate teachers' knowledge during classroom observations, offering a more comprehensive view of teacher learning that reveals complementary insights into both the process and content of learning, as well as the challenges faced by novice teachers. Similarly, the research by Pineiro et al. (2021) integrated the KQ with Mathematical Knowledge for Teaching (MKT) to explore the limitations of knowledge needed for teaching problem solving. Their findings indicated that MKT overlooks both the nature of problem-solving processes and insights from problem-solving research. Similarly, while KQ offers a useful framework, it suffers from overlapping elements. These collaborative approaches underscore the versatility of the KQ when integrated with other conceptual frameworks to elucidate multifaceted aspects of teaching.

4.4.3 Development or modification of tools/tests

A distinctive set of studies focused on leveraging the KQ to develop or modify specific tools or tests tailored to assess mathematical teaching. For instance, Livy et al. (2019) developed an assessment instrument with sections aligned to the KQ dimensions, aimed at measuring pre-service teachers' mathematical understanding. Dogan and Bostan (2022) similarly prepared tests aligned with specific KQ dimensions. These studies demonstrate the KQ's potential to structure and refine assessment tools for evaluating pedagogical effectiveness and mathematical knowledge in education.

4.4.4 Conceptual or theoretical exploration

A series of studies undertook a more conceptual or theoretical exploration of the KQ. These investigations explored the foundational principles of the framework, its implications, or potential modifications to better suit specific research objectives. For instance, Neubrand's (2018) research employed the KQ to develop a conceptual framework that encapsulates the

knowledge teachers require for mathematically conscious teaching. Similarly, Hurst (2017) adapted the contingent code of the KQ to emphasise the role of “powerful teaching” in provoking contingent events. Such studies illuminate the adaptability of the KQ in framing and refining theoretical discussions surrounding mathematics education. Although Rowland et al. (2014) conducted a qualitative study, it is included in this category due to its emphasis on the conceptual interplay between classroom-based research on teaching and learning and the professional growth of teacher educators. The study illustrates how the roles of researcher and teacher educator can complement and reinforce one another.

4.4.5 Reflection and professional development

Lastly, five of the reviewed articles emphasised the importance of the KQ in fostering reflection and professional learning. O’Keeffe and White (2022) employed the KQ as a scaffolding structure to facilitate pre-service teachers’ reflections on their pedagogical evolution and in reflecting on their developmental trajectories. In another study by Dogan et al. (2021), the KQ was helpful in analysing teachers’ responses, especially concerning the contingency dimension. These studies underscore the utility of the KQ not only as an analytical tool but also as a reflective instrument to guide teachers’ professional growth and development.

5 Discussion

The KQ has demonstrated its relevance, expanding its horizons both geographically and thematically. The following discussion explores the various facets of KQ’s growth and application.

5.1 Trends, year level, and participants focus of KQ

The global use of the KQ demonstrates its broad relevance in mathematics education, though its application often requires contextual adaptation. Studies from diverse regions, including Turkey (Dogan & Bostan, 2018, 2019), the United States (Lai et al., 2023), South Africa (McAuliffe & Lubben, 2013), Ireland (Walsh, 2017), and Australia (Getenet & Callingham, 2021), illustrate the framework’s adaptability across educational systems. However, several of these studies explicitly noted the need to elaborate or tailor aspects of the KQ to local contexts. For instance, Dogan and Bostan (2019) applied the KQ to analyse secondary school teaching practices in Turkey, while Getenet and Callingham (2021) focused on school teacher education in Australia. Similarly, McAuliffe and Lubben (2013) highlighted challenges in connecting concepts and procedures within South African classrooms, suggesting that certain dimensions of the KQ required emphasis in specific contexts. These variations indicate that while the KQ offers a robust conceptual structure, its implementation is shaped by local pedagogical priorities and cultural nuances. This evidence points to KQ being a flexible framework that can be adapted to diverse contexts while maintaining its core dimensions.

The KQ was initially designed in the setting of primary school teaching practices (Rowland et al., 2005); however, this systematic literature review shows that it has been adapted and applied across various educational settings—including primary, secondary, and higher education. Although we do not claim that the KQ has directly transformed mathematics teaching practices, its widespread application across diverse studies demonstrates its value

as an analytical tool. Our review shows that researchers have used the KQ extensively, particularly in primary settings, to investigate relationships between teachers' classroom practices, their mathematical content knowledge, and student learning outcomes (e.g., Dogan et al., 2021; Dogan & Bostan, 2019; Dennis et al., 2017; Abdulhamid & Venkat, 2018). At the higher education level, KQ has been used to illustrate the interdependency between professional learning of teacher educators and the practice of teaching and learning. These results suggest that KQ has been successfully extended beyond the primary school educational levels (for which it was originally designed) to improve teaching practices in secondary and higher education.

Furthermore, the varied contexts of the reviewed articles provide insight into the use of KQ, which was initially designed for pre-service teacher education. Similar to the study context of KQ, the current research identified articles situated in contexts involving pre-service teachers ($N=19$) and in-service teachers ($N=11$). Beginning and higher education teachers each account for a smaller number. These results demonstrate that, while the KQ has had an impact on primary education, particularly in the nexus of teachers' pedagogical skills, knowledge, and student achievement in mathematics, its influence is also evident in secondary and higher education. The wide-ranging use of KQ across educational levels underscores its flexibility and relevance.

5.2 Methods and broadening the scope of KQ

The methodological choices across the reviewed studies reflect both continuity and innovation. Consistent with the foundational work of Rowland et al. (2005), qualitative approaches, particularly classroom observations, remain central. However, several studies have extended this tradition by integrating multiple data sources such as interviews, video analysis, and document reviews, enriching the depth and triangulation of evidence (e.g., Maher et al., 2022; Dogan & Bostan, 2019; Salani & Jojo, 2023). Although mixed-method designs were less common, their presence signals the adaptability of the Knowledge Quartet (KQ) to diverse research paradigms. For instance, Getenet and Callingham (2021) demonstrated how KQ codes can be quantified to explore patterns in teaching practice, illustrating methodological flexibility.

Beyond methodological diversity, an important trend is the expansion of KQ beyond its original mathematical context. While mathematics remains the primary domain, covering topics such as algebra, measurement, fractions, and spatial reasoning, recent studies have applied KQ in science, technology education, physical education, and even language teaching (e.g., O'Keeffe & White, 2022; Amador et al., 2022). This cross-disciplinary uptake suggests that the framework's conceptual structure resonates with broader pedagogical concerns, offering a lens for analysing teacher knowledge in varied settings. Notably, no studies addressed probability or statistics, highlighting a gap and an opportunity for future research. Taken together, these patterns underscore the versatility of KQ—not only methodologically but also in its capacity to inform research across subject boundaries.

5.3 Trends of applications of KQ

The results of this study provided valuable insights into the uses and implications of the KQ in mathematics education and beyond. As shown in Table 5, the KQ was applied

across five distinct thematic areas in the reviewed studies, each with its own significance and contributions to the broader discourse.

First, the dominance of studies that applied KQ directly to observe and analyse teaching practices indicates the inherent strength of the framework in codifying classroom dynamics. As highlighted by Walsh's (2017) research, the KQ proves important in discerning nuanced pedagogical actions, emphasising its value in producing in-depth, context-specific analyses of teaching methods. Second, the emergence of this theme using KQ in combination with other frameworks underscores its adaptability. By integrating different frameworks, as evidenced by Skott's (2023) and Pineiro et al.'s (2021) studies, the KQ demonstrates its applicability in offering richer, multidimensional insights into teaching practices. This suggests that while the KQ is used independently in several studies, its compatibility with other frameworks enhances its analytical power. Third, the focus on creating and refining assessment tools inspired by the KQ underscores its importance in shaping evaluation standards in mathematics education. Studies by Livy et al. (2019) and Dogan and Bostan (2022) provide evidence of the KQ's usefulness, indicating its potential to set benchmarks for pedagogical proficiency in mathematics. Fourth, the use of KQ by researchers such as Neubrand (2018) and Hurst (2017) to develop conceptual understanding highlights the KQ's potential not just as an observational tool but also as a useful framework for academic discourse, pushing the boundaries of our understanding of the nuances of effective mathematics teaching. Hurst's (2017) findings showed that a deeper level of teacher knowledge can enhance the ability to anticipate and plan for contingent moments. Ultimately, utilising the KQ as a catalyst for reflection and professional growth highlights its potential in the pedagogical realm. The studies by O'Keeffe and White (2022) and Dogan et al. (2021) demonstrate that the KQ can extend its analytical functions to support educators' professional development. Dogan et al. (2021) demonstrated that applying the KQ as a professional learning framework offers a more holistic understanding of teacher learning, highlighting both the process and content of learning, as well as the challenges faced by novice teachers.

The findings from this research carry significant implications for both mathematics teaching and the preparation of future educators. First, while the KQ is valuable for its multidimensional insights and flexibility in being used alongside other models, revising the framework to better meet the evolving needs of mathematics teaching and teacher education could be beneficial. One suggestion for enhancing the framework is to make it more self-contained by including richer explanations of the four knowledge dimensions and incorporating insights into why teachers select pedagogical strategies. While the 18 descriptive codes provide detailed categorisation of classroom actions, they do not always capture the underlying reasoning behind those choices. This idea is raised by Akar's (2016) study, which found that although algebra and geometry teachers exhibited similar knowledge dimensions and teaching practices, their reasons for choosing these practices differed. Including these reasons could help reduce the risk of misinterpreting the knowledge dimensions, and KQ could be enhanced by incorporating reflective and collaborative practices, as suggested by Salani and Jojo (2023). Second, considering how to link the four knowledge dimensions is important for providing a holistic insight into teaching mathematics. This suggestion is supported by McAuliffe and Lubben's (2013) study, which demonstrated that the Mathematical Knowledge for Teaching (MKfT) framework complements the KQ by offering detailed descriptions of the connections among the knowledge dimensions. For example, from the reviewed studies, it was observed that having Foundational knowledge does not necessarily influence other dimensions of knowledge. Many studies reported that teachers struggle with Connection knowledge (McAuliffe & Lubben, 2013; Salani & Jojo, 2023). This suggests a need for further discussion of the interaction among the four

quadrants to better understand their interrelationships and enhance the framework's applicability in real-world teaching scenarios. Finally, given the significant impact of technology on teaching mathematics, the KQ could be updated to include a knowledge dimension explicitly related to the use of technological tools for teaching mathematics. This inclusion would reflect the modern educational landscape and better prepare teachers to incorporate technology successfully into their teaching practices.

6 Limitations

This study acknowledges certain limitations. Firstly, our review focused solely on peer-reviewed journal articles to ensure quality. Incorporating conference papers, books, book chapters, and reports could have added depth to our findings. Secondly, while our review and search strategies adhered to established scientific methodologies, the choice of keywords, sources, inclusion and exclusion criteria, and database selection was driven by our discretion and may have inherent limitations. Thirdly, this review did not attempt to systematically examine relationships between themes and other variables such as participant type, education level, or methodological approaches, as doing so would have significantly expanded the scope and complexity of the paper beyond its primary aim. Future research could explore these connections in greater depth. Nonetheless, these limitations also provide avenues for future research, offering a foundation for expanding and exploring new directions on the topic or similar frameworks. While KQ remains relevant to mathematics teaching, future studies may seek to assess teachers' conceptions of the dimensions, examine the consistency of teachers' knowledge and practices, and examine how these influence students' proficiency.

7 Conclusion

The use of KQ in various contexts (such as grade levels, countries, and methodologies) as an instrument in mathematics education research and classroom practices underscores its adaptability, significance, and broad applicability in shaping mathematics teaching practices. Originating in the context of primary mathematics education in the UK, the framework has broadened its scope, finding relevance across diverse geographic regions and thematic areas. The results of this study have evidenced KQ's impact, not only in primary education settings but also in its expansion to higher educational echelons. The methodology employed in KQ-oriented research, primarily qualitative, has evolved to integrate innovative methods, demonstrating the flexibility of the KQ in accommodating diverse research approaches. Furthermore, its application beyond mathematics into disciplines such as science, physical education, and languages highlights its flexibility and potential for broader application across disciplines. Thematically, KQ's applications span from direct observation of teaching practices to integration with other frameworks for a more comprehensive description of classroom practices, to benchmarking for pedagogical proficiency, to providing a foundation for theoretical discourse, and to serving as a catalyst for teachers' and educators' professional development. This vast spectrum of applications underscores KQ's flexible nature, establishing it as an important framework in both pedagogical research and practice. Its adaptability, use, methodological flexibility, and diverse applications make it

an important framework for shaping and enhancing mathematics and other non-mathematics teaching practices across various contexts.

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