A Synthesis of Linguistic and Semiotic Structures that Create Problems for Adult-CALD Bilingual Learners when Decoding and Learning M2 (Mathematics in English as an Additional Language)

A thesis submitted for Doctor of Education

2015

Faculty of Business, Education, Laws, and Arts University of Southern Queensland

By Anthony Rigoni

Bachelor of Economics, Graduate Diploma in Education, Diploma of European Languages, Masters of Applied Linguistics

Abstract

This study employs linguistics and semiotic science, and interpretive and synthetic logics to synthesise the *problem of teaching and researching adult-CALD bilingual learners when decoding and learning M2 (Mathematics in English as an Additional Language)*. The findings identify the philosophical and structural conventions, and bilingual decoding and learning behaviours that emerge and create problems when decoding and learning M2 in a cross-cultural, linguistic, and semiotic context. The thesis advocates (1) structural and language-based teaching strategies and (2) poststructuralists' interpretive and synthetic logics to address the complex language-learning problem.

Key words

Linguistics, semiotics, codes, culture, language, education, bilingual, interpretive, synthetic, mathematics, structural and language-based teaching, poststructuralism

Certification of Dissertation

The work contained in this thesis has not been submitted previously for the requirement of an award or publication. To the best of my knowledge all references have been duly cited. Section 4.2.3 defines the logics employed to reference authorship in the synthesis.

Candidate: Anthony Rigoni Date: 25/11/2015

Principle Supervisor: A/Professor Dr. S. O'Neill Date: 21/12/2015

Associate Supervisor: Dr. L. Abawi Date: 27/12/2015

Acknowledgement

I acknowledge the support of the Faculty of Business, Education, Law, and Arts at University Southern Queensland in allowing this study. I also acknowledge the contribution of Associate Professor Shirley O'Neill, Dr. Lindy Abawi, and Dr Mark Dawson for their academic and professional guidance. I thank QSR International Pty. Ltd., for the free use and support of NVivo 10 Data Management Software in the data synthesis. Their product was greatly appreciated and highly recommended. I thank my participants for their collaboration, and my sister Maria Brancalion for her advice and introduction to the study of Linguistics. To my mother and father, wife Sharon, and daughters Annelisa and Leeanna thankyou for your time and patience.

Table of Contents

List of tables

List of figures

List of acronyms

ABS (Australian Bureau of Statistics)

AIR (The American Institute for Research)

ALLS (Adult Literacy and Life Skills Survey)

AMEP (Adult Migrant Education Program)

BL (Bilingual)

CALD (Culturally and Linguistically Diverse)

C, L, and S (Cultural, Linguistic, and Semiotic)

COAG (Council of Australian Governments)

EAL (English as an Additional Second Language)

L1 (First Home Language)

L2 (English as an Additional Second Language)

LLNP (Language Literacy and Numeracy Program)

ML (Mathematical Language)

M1 (Mathematics in a First Home Language)

M2 (Mathematics in English as an Additional Second Language)

PIAAC (Programme for the International Assessment of Adult Competencies)

SEE (Skills for Education and Employment)

SFL (Systemic Functional Linguistics)

VET (Vocational, Education, and Training)

Chapter 1 Research Problem and Conceptual Framework

> To study adequately any breakdown in communications we must first understand the nature and structure of the particular mode of communication that has ceased to function.
>
> (Jakobson & Halle, *Fundamentals of language,* 1956, p. 55)

1.1 Background

Australian national survey data identified illiteracy and innumeracy are linked to unemployment and poverty, and CALD (Culturally and Linguistically Diverse) people over-represent illiterate and innumerate people (ABS, 2006a; ABS, 2006b; ABS, 2013; NCVER, 2006; Shomos & Forbes, 2014). Little is known in Australia or overseas about *the problem of teaching and researching adult-CALD learners M2* (Mathematics in English as an Additional Language) as an educational construct and more research is needed (AIR, 2006; COAG, 2008; NCVER, 2006). The following study conceptualised the problem from the ground up by interpreting the L and S (Linguistic and Semiotic) structures, and adult-CALD BL (Bilingual) decoding and learning behaviours that created problems when learning M2. The research findings articulate structural and language-based teaching strategies to address the problem. The case-based interpretive research procedures are advocated for other areas of education that also have complex language-learning problems to resolve.

There are significant numbers of CALD learners affected by the research problem. The 2011 ABS Population Census Data (ABS, 2011), for example, identified 28% of Australians come from a CALD background, 18% speak a language other than English at home, and 26% (523,000) of these revealed they do not speak English well or at all. The 2006 ALLS (Adult Literacy and Life Skills Survey) (ABS, 2006a) and 2011-12 PIAAC (Programme for the International Assessment of Adult Competencies) (ABS, 2013) identified that CALD minorities perform well below average in national literacy and numeracy levels. It has been estimated that which 47% of Australians already have literacy and 54% numeracy skills that are not good enough to meet the requirements of everyday living and work (ABS, 2006a; ABS, 2013).

Adult-CALD learners also accounted for approximately 70% of the 80,000 learners enrolled in 2009 in foundational literacy and numeracy programs in Australia such as AMEP (Adult Migrant Education Program) and SEE (Skills for Education and Employment) (NCVER, 2011). These programs focus on building employability skills and educational opportunities for low literacy and numeracy learners (Miralles, 2004; NCVER, 2011; Stapar, 2014). Foundational literacy and numeracy programs also help build self-esteem and social cohesion in minority groups (Balatti, Black, & Falk, 2006; Papen, 2005). The AMEP and SEE programs cost approximately $250 million a year to deliver between 2009-2011 (NCVER, 2011).

The transition of adult-CALD learners into work and further education is, however, problematic (Black, 2002; Griffin, 2014; Miralles, 2004; Stapar, 2014). The problem emerges, in part, because of poor learner experiences when learning mathematics (AIR, 2006; COAG, 2008). CALD children also display similar problems when they take up higher studies (COAG, 2008; Rothman & McMillan, 2003).

1.2 Focus

The research problem was conceptualised as a sign and language-learning problem. Signs defined the symbolic objects in M2 that generated meanings and problems for adult-CALD learners when decoded in a cross-C (Cultural), L, and S context. Peirce (1931, p. 139) asserted mathematical signs articulated three "manifestations" or levels of meanings that need to be decoded: the symbol; the first order meanings; and the cognition produced in the mind. For example, the symbol '7' signifies the quantity 'seven' and 'good luck' in certain social contexts. Problems emerge because, as Schleppegrell (2007) stated, "it is not enough to be able to work with the language alone; mathematics draws on multiple semiotic meaning-creating systems to construct knowledge: symbols, oral language, written language, and visual representations such as graphs and diagrams" (p. 142). The problem is exacerbated when signs are decoded and learnt differently in a cross-C, L, and S context (Arzarello, Robutti, & Bazzini, 2005; Whorf, 1956).

The research problem was, however, inadequately addressed as a semiotic problem in the extant literature and research. Firstly, the study of M2 as a semiotic construct is highly theoretical, and more applied research is needed to interpret learner behaviours

in this field of study (Ernest, 2006; O'Halloran, 2005; 2010; Presmeg, 2006; Trinick, Meaney, & Fairhall, 2014). Secondly, there was a distinct lack of both theory and applied research available to specifically interpret the research problem as an adult-CALD learner problem. The study found that both CALD and English monolingual learners share problems in this area, however, adult-CALD learners are affected by cross-C, L, and S differences in M1 and M2 (Barton, 2009; Whorf, 1956).

Two pedagogic and one methodological research questions interpreted the research problem from a cross-C, L, and S perspective. The first two questions interpreted the research problem as a semiotic and educational problem, and the third as a methodological concern. The review of literature and research has found existing knowledge was inadequate in both areas of the research. A deeper synthesis was needed to interpret and articulate solutions. The term BL (Bilingual) emerged in this context and more precisely define adult-CALD learner decoding and learning behaviours in M2. The respective Research Questions being asked in this thesis are as follows:

1. *What are the linguistic and semiotic structures in mathematical language that create problems for adult-CALD bilingual learners when learning mathematics in English as an additional language?*
2. *How can knowledge of adult-CALD bilingual learner-related problems in linguistic and semiotic structures in mathematics be used to enhance mathematics education in Australian classrooms?*
3. *How can case-based interpretive research methodology deepen our understanding of adult-CALD bilingual learners who are engaging in linguistic and semiotic structures in mathematics as an additional language?*

1.3 Conceptual framework and methodological overview

The review of existing literature and research identified that the elements and constructs of the research problem were, in the main, inadequately defined and therefore could not generate a holistic answer and solution to the problem. The study, as a result, interpreted the nominal clauses (Halliday, 2006) {mathematics, language, adult-CALD bilingual learners, culture, Australian, and education} in the research problem and questions as the elements needing to be explored in greater depth.

Constructs identified, as MacInnis (2011, p. 137) stated, the factors that helped "conceptualise" and/or "measure" the causes, effects, and relationships in the research problem. The approach generated a more precise interpretation of the elements and constructs of the research problem.

The elements were, however, inconsistently and imprecisely defined in national research data such as the 2006 ALLS (ABS, 2006a) and PIAAC (ABS, 2013), and could not generate a sophisticated interpretation and resolution to the problem. The 2006 ALLS (ABS, 2006a) survey, for example, broadly defined its CALD participants as speakers who acquired English as an additional language. It did not distinguish, for example, between the cause and effects coordinate and compound BL experiences (Hamers & Blanc, 2000) had on assessing numerical competencies in an additional language context. Coordinate BL experiences identified M1 and M2 as shared meanings and processes, and compound experiences identified M2 was learnt as a new L and S experience. The ALLS (ABS, 2006a) survey was problematic to interpret in this context because, firstly, it tested CALD learners in English as an additional language and, secondly, it did not distinguish the effect BL experiences in M1 had on assessing competencies in M2 (Solano-Flores, Barnett-Clarke, & Kachchaf, 2013; Solano-Flores, 2014).

The elements {mathematics, language, adult-CALD bilingual learners, culture, Australian, and education} also helped search, synthesise, and classify existing knowledge more precisely in the literature review. The resulting typology (Onwuegbuzie, Leech, & Collins, 2012) articulated into the battery of open-ended questions (Appendix D) that helped collect and interpret data. The questions were coded a posteriori (Kant, 1788) in this context to interpret the unknown relationships and factors that affected cross-C, L, and S behaviours in the study (van de Vijver & Lueng, 2011) in M2.

Case-based interpretive synthetic poststructuralist epistemologies were employed to collect and interpret data in the battery of open-ended questions. Interpretive logics were applied to "watching", "listening", "asking", "recording", and "examining" the intricate and unknown relationships (Schwandt, 1994, p. 119) that emerged in the cross-C, L, and S study. A poststructuralist approach made it possible to explore

meanings and relationships in greater depth that lay outside the assumptions that were made in existing research (Kant, 1788; Guba & Lincoln, 1994).

Synthetic logics interpreted and articulated data into holistic solutions rather than individual analytic propositions (Gharajedaghi & Ackoff, 1985; Kant, 1788; Patton, 1990). A synthetic judgement questioned subjects and phenomena from outside the boundaries and parameters of what was known. For example, the synthetic proposition "all bodies are extended" is interpreted open-endedly by factors, for example, the space that exists outside the human body (Cleve, 2003, p. 230). Alternatively, the analytical proposition "all bodies are heavy" is interpreted by what is known about what is stated about the subject: the human body (Cleve, 2003, p. 230). A poststructuralist perspective identified that not all propositions were, necessarily, known or understood beforehand in a cross-C, L, and S study (Schutz, 1932; van de Vijver & Lueng, 2011) and, therefore, the propositions and questions were synthetic in nature.

Synthetic logics also generated a more comprehensive review of existing literature and research. As Onwuegbuzie, Leech, and Collins (2012) stated:

> By comprehensive, we mean that from the literature review, researchers obtain a complete picture of what has been conducted before, the inferences that have emerged, the inter-relationships of these inferences, the validity of these inferences, the theoretical and practical implications stemming from these inferences, and the important gaps in the literature, as well as positions them to select the most appropriate methodologies for their studies by allowing them to identify the strengths and weaknesses of approaches used in previous studies. (p. 4)

A case-study approach helped interpret the variances and relationships that were empirically underrepresented and quantitatively difficult to reconcile in the study (Gerring, 2007; Stiles, 2009; Yin, 1994). The technique identified what was both "common" and "distinctive" (Stake, 1994, p. 238) between the CALD BL participants when decoding and learning M2.

Linguistic and semiotic science and language interpreted and articulated the research problem into a sign and language-learning problem (Bloomfield, 1939; de Saussure, 1910; Halliday, 2006; Peirce, 1931; Presmeg, 2014). This lent to more precisely interpreting the L and S characteristics of the M2 sign system, and adult-CALD BL decoding and learning behaviours in it.

Thematic coding techniques, respectively, synthesised and articulated data into research findings (Attride-Stirling, 2001; Braun & Clarke, 2006; Saldana, 2009). Processes were also coded to address the criticisms that are often made about interpretive and qualitative-based education research (Cohen, 2007; Onwuegbuzie & Daniel, 2003). Coding enhanced the quality, validity, and reliability of the findings that emerged.

Interpretive, synthetic, and thematic procedures were, however, challenging to undertake in a cross-C, L, and S study. Firstly, there were no ideal models available in existing literature that suited the specificities of the study. As a result, the research design and coding techniques were conceptualised, in the main, from the ground up. Secondly, the study's exploratory nature meant the open-ended data collection questions needed to be precisely coordinated to produce meaningful observations. As a result, the theoretical and technical specificities of the research design were vigilantly reviewed throughout the study to produce meaningful findings. The procedures generated a more precise understanding and resolution of the research problem. The following case-based interpretive synthetic data collection procedures were sequentially developed for the synthesis:

1. Procedures for interpreting the elements and constructs of the research problem.
2. Procedures for synthesising and articulating existing literature and research into a battery of opened-ended questions to help collect and interpret data.

3. Procedures for synthesising data from multiple contexts and theoretical perspectives. The study collected data holistically through three different stages:
    a. Stage 1 conducted a semiotic synthesis on the 15 mathematical tasks that were employed to interpret participant in-tasks decoding and learning behaviours in Stage 3.
    b. Stage 2 conducted open-ended background interviews on participant BL experiences that affected decoding and learning M2.
    c. Stage 3 conducted participant in-tasks observations and interviews.
4. Procedures for recording and transposing interviews conducted in English as an additional language into text.
5. Procedures for synthesising and articulating data into coded thematic propositions. The data were coded thematically into a multidimensional Data Matrix (Appendix F) to synthesise and articulate solutions.

The findings and thesis advocate (1) structural and language-based teaching strategies and (2) poststructuralists' interpretive and synthetic logics to address the complex language-learning problem.

1.4 Parameters

The study was conducted over 4 years (3200 hours). Participant interviews were undertaken at a migrant language-learning centre in a TAFE college located in regional Australia. The city's population was approximately 60,000, and 25% the inhabitants came from CALD backgrounds (ABS, 2006b). There were over 200 students enrolled at the centre at the time of the interviews. Five participants were recruited because of their availability and ability to participate in the in-depth one-on-one interviews without BL support.

Participant interviews were originally envisaged in in-group situations such as classrooms, workplaces, and focus groups, however there were distinct disadvantages in initiating the exploratory study from this perspective. Anecdotally, not all ethnic, cultural, and gender groups communicate openly about their personal experiences and behaviours in in-group situations. The one-on-one interviews provided a space, time, and place for individual participants to speak freely, without the influence and intimidation of other participants. As a result, clinical conditions were generated to

interview CALD participants in a cross-C, L, and S context. It is envisaged, however, applying what has been learnt from the one-on-one interviews into actual classroom teaching episodes in the future, as part of ongoing post-doctoral research.

The respondents' capacities to participate in both the background and in-tasks interviews for extended periods of time were affected, firstly, by their competencies in English and, secondly, their availability due to factors such as family, work, and settlement commitments. The interviews were, therefore, consecutively conducted a week apart over a 2-3 hour session each, and concluded within 12 weeks. In total the interviews generated 146,000 words for the transcripts for this study.

Participants were selected through criteria to help manage the logistical and ethical issues (Levinson, 2004; Robinson-Pant, 2005; van de Vijver & Lueng, 2011) that challenged conducting the cross-C, L, and S study. Protecting the rights of the CALD participants was a foremost consideration, and Kant maxims (Kant, 1788) were employed to reconcile the ethical dilemmas that emerged. As a result, the location, language, gender, and ethnicity of the participants are not revealed, even if it would have benefited the outcomes of the study (Howe & Moses, 1999; Kant, 1788). Third party BL support was also not employed to interview participants. The criteria and procedures employed to interview participants were:

1. The participants were known and approached directly by the author to maintain their anonymity.
2. A mix of present and past students was recruited to make it difficult to identify participation.
3. Participants were not informed of each other's identity.
4. Participants needed to have completed intermediate-level English classes to participate in the study without third party involvement. Maintaining anonymity, confidentiality, and freedom to speak openly meant one-on-one interviews were conducted and BL support was not employed.
5. Competency in mathematics, however, was not a criterion for selection. The design of the mathematical tasks for Stage 3 also meant pre-testing was not needed.

Different mathematical tasks were employed to collect and interpret data (Solano-Flores, 2014). The objective of the tasks, however, was not to quantify competencies, but rather synthesise in qualitative terms the L and S structures, and BL behaviours and experiences that affected decoding and learning M2. The tasks represented the type of M2, for example, graphs, charts, symbols, and algebra, found in VET and middle year high school text. The text was employed to elicit behaviour and compare competencies in M2. Four types of knowledge were generated for the study:

1. L and S structures that defined M2 as a sign system.
2. Competencies needed to decode and learn M2.
3. Problems emerging when M2 was decoded and learnt in a cross-C, L, and S context.
4. Strategies for teaching adult-CALD BL learners M2.

Case-based interpretive synthetic methodology, and L and S science and language interpreted a range of factors that affected the research problem. An alternative post-positivist analytical approach was not used, because it would have conceptualised a narrow set of theoretical propositions (Guba & Lincoln, 1994; Creswell, 2003; Gharajedaghi & Ackoff, 1985; Patton, 1990). An interpretive synthetic approach generated a more holistic interpretation and set of findings (Kant, 1788; Gharajedaghi & Ackoff, 1985; Patton, 1990).

1.5 Chapter summaries

The chapters are sequentially arranged to interpret the research problem through synthesis. The research methodology and data collection instruments were conceptualised after a preliminary review of the research problem, and existing literature and research. Procedures were then employed to synthesise and articulate existing literature and research into the battery of open-ended that helped collect and inform the data collection. In turn the data were evaluated, synthesised, and coded into: firstly, findings; and secondly, implications, recommendations, and conclusions to address the research problem and questions. A diagram is included in the beginning of each chapter (Figures 2.1, 3.1, 4.1, 5.1, and 6.1) to identify the chapter's methodological position and role in the synthesis.

Chapter 1, *Research Problem and Conceptual Framework,* defines the background and rationale for studying the research problem. It concludes with a summary of the findings, implications, and recommendations that emerged.

Chapter 2, *Research Methodology and Preliminary Review of Existing Literature and Research,* recounts the background, methodology, and focus of the study. A preliminary review of existing literature and research is conducted to identify the limitations of existing research, and the vectors that affected the study's focus, research questions, and methodology. Interpretive synthetic methodology emerged to generate a more "genuine" (Schutz, 1932, p. 36), "complete" (Onwuegbuzie, Leech, & Collins, 2012, p. 14), and "holistic" (Patton, 1990, p. 79) interpretation of the research problem.

Chapter 3, *Data Collection Description, Instruments and Techniques*, defines the data collection rationale and procedures employed. The distinct lack of existing research in the area meant the instruments and techniques were conceptualised and developed, in the main, from the ground up. They are defined in sufficient detail for redeployment. A critique of the research design is conducted to identify areas for ongoing improvement in future research.

Chapter 4, *Synthesis of Existing Literature and Research*, recounts the synthesis and articulation of existing literature and research into the battery of open-ended questions that helped collect and interpret data. The chapter identifies the application of MacInnes's (2011, p. 137) Framework for Conceptual Thinking in Marketing in interpreting and classifying this knowledge. The framework, respectively, identified:

1. The constructs being addressed by existing literature and research.
2. The research divisions and disciplines that affected the findings.
3. The research domain that governed the science of the study.
4. What constituted science.
5. The procedures that were followed or recommended for conducting research.
6. The theoretical relationships that were claimed.

The resulting taxonomy was coded and articulated a posteriori (Kant, 1788; Onwuegbuzie, Leech, & Collins, 2012) into the battery of open-ended questions which informed the data collection. The framework also identifies the areas in which

the findings of the proposed study contributed its own knowledge to the existing research domains.

Chapter 5, *Data Evaluation, Synthesis, and Findings*, recounts the examination of the findings that emerged from the synthesis of the three stages of data collection. Data patterns were thematically coded (Attride-Stirling, 2001; Braun & Clarke, 2006; Saldana, 2009) and summarised into a Data (Appendix F) and Summary (Appendix G) Matrix to articulate answers to the research problem and questions.

Chapter 6, *Implications, Recommendations, and Conclusions*, defines the theoretical, policy, and practical implications that emerged. The thesis advocates structural and language-based teaching strategies, and interpretive synthetic research techniques to address the research problem. The findings also advance knowledge for other areas of education that have complex language-learning problems to resolve.

1.6 Summary of findings and recommendations

The following summary identifies the findings that emerged with reference to the research problem and questions.

Research Question 1:

> *What are the linguistic and semiotic structures in mathematical language that create problems for adult-CALD bilingual learners when learning mathematics in English as an additional language?*

The following conventions emerge and encode L and S structures in M2:

1. Philosophical conventions that encode ideological values and beliefs.
2. Structural conventions that encode meanings through the form and function of the M2 sign system. The structural codes identified are:
    i. Paradigmatic and syntagmatic codes
    ii. Iconic, symbolic, and indexical codes
    iii. Opposition, marking, and contrasting codes
    iv. Physical codes (shape, position, energy, motion, force, synergy, timing, and aesthetic relationships)
    v. Agentic codes

vi. Metaphoric codes
vii. Axiomatic and logic codes
viii. Discursive codes
ix. Mediums

The following BL behaviours and experiences emerge and create problems when decoding and learning M2 in a cross-C, L, and S context:

1. BL decoding behaviours
2. BL humanistic behaviours
3. BL cultural behaviours
4. BL educational experiences

Research Question 2:

> *How can knowledge of adult-CALD bilingual learner-related problems in linguistic and semiotic structures in mathematics be used to enhance mathematics education in Australian classrooms?*

Knowledge from Research Question 1 identified the L and S competencies and strategies needed to teach and learn M2 in a cross-C, L, and S context. The thesis advocates structural and language-based teaching strategies to address *the problem of teaching adult-CALD learners who are learning M2*.

Research Question 3:

> *How can case-based interpretive research methodology deepen our understanding of adult-CALD bilingual learners who are engaging in linguistic and semiotic structures in mathematics in an additional language?*

Knowledge from conducting case-based interpretive research in Questions 1 and 2 enhances the capacities of teachers and researchers to:

1. Interpret intricate L and S structures, and complex BL decoding and learning behaviours in M2 that were, otherwise, difficult to interpret via non-interpretive research techniques.

2. Articulate new knowledge, theory, and procedures for teaching M2 as a structural and language-based problem and solution.
3. Develop teaching and research strategies that are transferable to other areas of education that have complex language-learning problems to resolve.

The thesis advocates case-based poststructuralist interpretive and synthetic research procedures to study complex language-learning problems such as M2.

Chapter 2: Research Methodology and Preliminary Review of Existing Literature and Research

> It is a capital mistake to theorise before one has data. Insensibly one begins to twist facts to suit theories, instead of theories to suit facts.
>
> (Doyle, *The adventures of Sherlock Holmes: A scandal in Bohemia*, 1928, p. 7)

2.1 Chapter introduction

Interpretive and synthetic logics synthesised data from three stages of observations into research findings. Data were conceptualised as signs, and synthetic logics interpreted their meanings as part of a whole sign system (Schutz, 1932). The logics generated a more comprehensive interpretation of the research problem (Gharajedaghi & Ackoff, 1985; Onwuegbuzie, Leech, & Collins, 2012; Patton, 1990; Schutz, 1932).

Interpretive and synthetic logics also guided the literature and research review (Onwuegbuzie, Leech, & Collins, 2012). The findings were articulated into a series of open-ended questions that helped collect and interpret data. Open-ended questioning techniques were also applied to elicit qualitative data inductively from the observations that were made. Existing theory was coded a posteriori to suit the data, and not data to suit the theory (Kant, 1788).

The data collection instruments and techniques were developed from the ground up, because little knowledge existed on how to best study the research problem. The research design was also mindful of the criticisms leveraged against interpretive and qualitative-based research methodologies (Cohen, 2007; Onwuegbuzie & Daniel, 2003). Thematic coding techniques were, therefore, employed to maintain links between the data observations, procedures, and the findings that emerged (Attride-Stirling, 2001; Braun & Clarke, 2006; Saldana, 2009). The coding procedures enhanced the study's quality, validity, and reliability as science.

The chapter sections, sequentially, recount:

1. The meaning and significance of the elements and constructs that defined the research problem.
2. The preliminary review, limitations, and implications of existing literature and research.
3. Interpretive and synthetic ontologies and epistemologies that emerged.
4. The research questions.
5. L and S science and language employed.
6. Research panel and supervisor input.
7. Overview of data collection stages.
8. Participants and site selection.
9. Ethics.

The position of the study's methodological development is depicted in Figure 2.1.

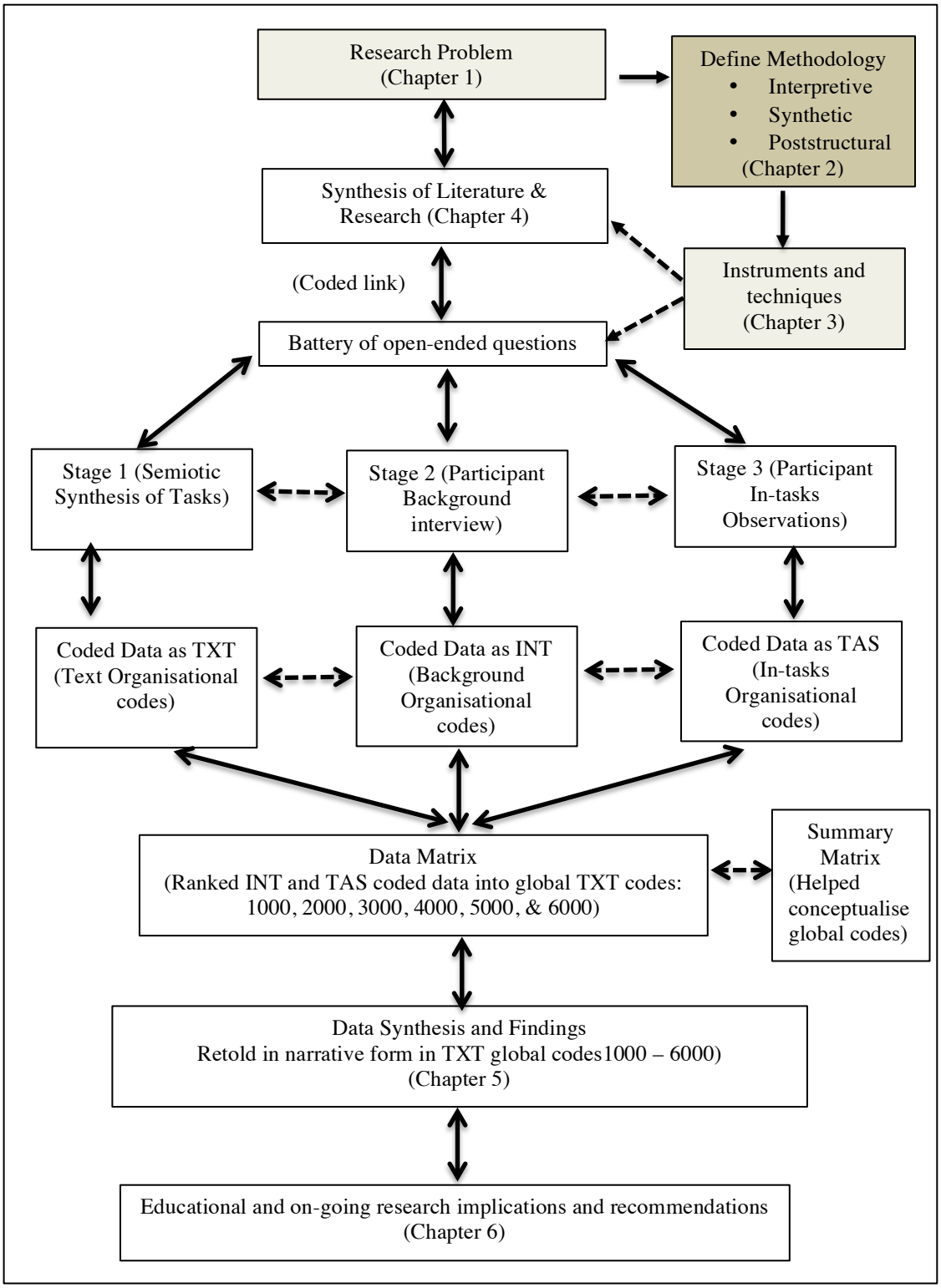


Figure 2.1: The position of the research methodology in the study.

2.2 Research problem constructs

The terms mathematics, language, adult-CALD bilingual learner, culture, Australian, and education were interpreted as the elements constituting the research problem and questions, and their relationship helped conceptualised the constructs of the research problem. Conceptualising was defined by MacInnes (2011) as, "the process of understanding a situation or problem abstractly by identifying patterns or connections and key underlying properties" (p. 140). Section 3.2 defines in detail the logics and techniques employed to interpret and articulate the elements and constructs of the research problem.

Constructs reflected what researchers and stakeholders believed were important for interpreting and resolving the problem. COAG (2008), for example, defined numeracy as "mathematical knowledge applied in real life contexts" (p. xi). This approach signalled *mathematics* should be taught through everyday humanistic experiences such as work, rather than as an abstracted idea in the classroom. Alternatively, Devlin (2000) defined mathematics as "the science of patterns" (p. 1). This approach signalled mathematics should be taught as a cognitive construct that enhances the mental capacity of people to recognise patterns in phenomena. Both COAG (2008) and Devlin (2000) identified legitimate, but divergent ways of studying and conceptualising mathematics and education as constructs.

A preliminary review of existing literature and research found the elements were, in the main, too inconsistently and imprecisely defined to generate a holistic interpretation of the research problem. COAG (2008), for example, provided an in-depth analysis of mainstream English adult learners in M2, yet very little of the analysis accounted for adult-CALD learners' behaviours and experiences. The 2006 ALLS (ABS, 2006a), for example, identified its adult-CALD participants as people who acquired English as an additional language. The survey did not reconcile the effect of testing in English as an additional language or what impact coordinate and compound bilingual experiences (Hamers & Blanc, 2000) in M1 and M2 had on the results (Solano-Flores, Barnett-Clarke, & Kachchaf, 2013; Solano-Flores, 2014). Research such as the 2006 ALLS (ABS, 2006a) and COAG (2008) generated an incomplete and problematic interpretation of the research problem. A more precise synthesis was required to interpret the existing relationships and solutions.

2.3 Preliminary review of existing literature and research

A preliminary review of literature and research identified the limitations and potential vectors for interpreting the constructs that made up the research problem. The findings articulated the parameters and methodologies that were later employed to conduct a more strategic synthesis of existing literature and research (Chapter 4). The typology then generated the series of open-ended questions that helped collect and interpret data (Chapter 5 & Appendix D).

A small number of researchers identified that an inverse relationship existed between CALD children and their success in learning M2 at school (Trinick, Meaney, & Fairhall, 2014). Rothman and McMillan (2003) conducted a statistical analysis on the Longitudinal Surveys of Australian Youth between 1995 and 1998 and found the social, economic, linguistic, and cultural background of school children significantly affected academic achievement. Specifically, one-sixth of variations between literacy and numeracy emerged due to differences between schools. More than half of this variance emerged as a result of the socio-economic status, proportion of ESL students, and overall school climate. At a student level, girls scored higher in literacy, boys higher in numeracy, and indigenous and ESL students lower in literacy and numeracy. Rothman and McMillan's (2003) findings were difficult to apply to adult-CALD learners because, anecdotally, adults were affected in different ways by age, maturity, settlement, and cultural experiences.

The review of existing literature and research did not identify longitudinal studies such as Rothman and McMillan (2003), on the subject of adult-CALD learners in M2. Condelli, Wrigley, and Yoon (2003) studied 495 foundational adult-ESL learners in the USA between 1999 and 2001, and found classrooms that were well organised, had learning materials that connected students to the outside world, and included bilingual support contributed more to learning L2. Condelli, Wrigley, and Yoon (2003), however, did not specifically identify how these factors might relate to teaching and learning M2 as a construct.

Prins and Ulijin (1998) investigated the effect the C and L backgrounds of indigenous South African high school students had on learning M2. Three C and L groups were identified in the study: E1 (Western European and English-only speakers); E2

(Western European and Afrikaans); and, E3 (Indigenous Africans). Mathematical tasks were modified in three ways to interpret CALD behaviours and competencies in this context: highly verbal English tasks; modified with less verbal content; and, non-verbal and highly precise mathematically language. Prins and Ulijin (1998) found through a mix of interviews and assessments that the highly verbal and poor readability tasks detracted from all three students groups answering questions correctly. However, the indigenous group displayed the greatest level of anxiety and lowest overall success in completing the tasks in L2/M2.

Prins and Ulijin (1998) believed the text readability was not the only factor that detracted the E3 group from answering tasks. Better results for the E1 and E2 groups in the second and third type of assessments identified a form of linguistic determinism as identified in the Sapir-Whorf hypothesis (1949, 1956) existing in M2. The Sapir-Whorf hypothesis (Whorf, 1956) proposes that a person's linguistic and cultural experience bounds their perception and interpretation of the world. According to Prins and Ulijin (1998) their indigenous students found Westernised mathematics difficult to read and learn in this context. The authors concluded M2 text and assessments should be modified to accommodate for cross-C and L differences in learners.

The strength and weakness of the Sapir-Whorf hypothesis (1949, 1956) is, however, extensively debated in literature and research (Fishman, 1980). The proposition that language and culture generated different ways of thinking in mathematics (Barton, 2009; Krummheuer, Leuzinger-Bohleber, Muller-Kirchof, Munz, & Vogel, 2013; Whorf, 1956) is complex to interpret, and it is not sure if or to what extent a person's C and L background specifically affects learning M2. Firstly, while M1 and M2 may employ different symbols, they may still share the same meanings and processes (de Saussure, 1910; Durst-Andersen, 2011). In this context, as Fromkin et al (1999) stated, "if it can be expressed in one language or one dialect, it can be expressed in another language or dialect. It might involve different means and different words, but it can be expressed" (p. 11). Secondly, a more pragmatic view believed ML is always open to cross-C, L, and S differences, because its meanings and processes are not universally shared across languages and cultures (Barton, 2009; Whorf, 1956).

Parvanehnezhad and Clarkson (2008) studied the BL behaviours and competencies of a group of 4-5 year-old Iranian students in M2. The study found students having lower competencies in L1 (Persian) and L2 (English) were also less competent in learning M2. Parvanehnezhad and Clarkson (2008) identified with Cummins's (2000, 2003) Interdependency and Threshold Hypothesis to explain their findings. Cummins (2003) distinguished BICS (Basic Personal Communicative Skills) and CALP (Cognitive Academic Language Proficiency) in this context to propose BL learners need to be academically proficient in at least one language first, before becoming proficient in an additional language. The review of literature and research did not find Cummins' (2003) hypothesis applied in the context of adult-CALD learners in M2.

Researchers also have alternative and conflicting views about the cause and effect of the research problem. Rowland (1999) believed ML was vague, in part, because it reflects the way children learnt M2 in the classrooms. Pietarinen (2006) believed this type of discourse is strategic in nature, as meanings are acquired and exchanged through strategic types of linguistic behaviour (Section 4.4.3). Rowland (1999) and Pietarinen (2006) signalled the research problem is not, as suggested by Prins and Ulijin (1998), necessarilly a text readability problem, but rather CALD learners lack appropriate discourse strategies in learning M2 (Section 5.3.8).

Reserachers such as Gillard, Van Dooren, Schaeken, and Verschaffel (2009), and Leron and Hazzan (2006), identified learners needed to employ a combination of heuristic and analytic thinking behaviours to solve problems mathematically. Heuristics describes thinking that is quick and responsive to solve problems in a limited time frame (Gigerenzer & Goldstein, 1999). The preliminary review of existing literature found very little research that studied the heuristic and analytical thinking behaviours of adult-CALD BL learners in M2.

Feist and Gentner (2007) and O'Neill (2009) studied the effect symbolic logics had on thinking and learning English as an additional language construct. The prepositions 'in' and 'into', for example, generate distinctive meanings and relationships in L2 such as 'the bird is in the sky' and '3 goes into 6 twice'. These meanings are not universally shared across languages. Not all cultures and languages see the sky or numbers as containers in which objects can be symbolically placed (O'Neill, 2009;

Whorf, 1956). There was very little research that specifically interpreted the symbolic interactions of adult-CALD BL learners in M2.

Ernest (2006), Schleppegrell (2007), and Presmeg (2006; 2014) studied M2 as a semiotic learning experience. Crystal (1992, p. 339) defined Semiotics as the study of signs in the production of "human creativity", "social rules", and "behaviours". The term semiosis identifies the "theory and analysis" employed to interpret "signs and signifying practices" (Crystal, 1992, p. 561). In this context, Vile (2009) stated that the study of semiotics in M2 asked:

> What does that sign mean to that student in that setting? Questioning, observation and experience may give rise to answers which would lead to solutions, to ways of helping a student redefine for themselves the nature of the signs that they are using in a way that will help them cope with signs of higher semiotic demand. (p. 102)

Semiotics identifies the distinctive signs that emerge and create problems when learning M2 (Schleppegrell, 2007). Duval (2006) identified, for example, that unlike the applied sciences (Astronomy, Physics, and Chemistry) that employ instruments such as microscopes and telescopes, mathematics is only accessible through the perceptions generated by the sign system. In this context, Duval (2006) stated:

> The crucial problem of mathematics comprehension for learners, at each stage of the curriculum, arises from the cognitive conflict between these two opposite requirements: how can they distinguish the represented object from the semiotic representation used if they cannot get access to the mathematical object apart from the semiotic representations. (p. 107)

The interpretation of M2 as a semiotic construct was, however, highly theoretical, and more applied research is needed to interpret its implications for teaching M2 (Ernest, 2006; O'Halloran, 2005; O'Halloran, 2010; Presmeg, 2006). The study of adult-CALD BL learning behaviours in this area was negligible.

Farrugia (2003) studied the discourse strategies of a group of BL 7-8 year old students in Malta when learning M2 as a semiotic construct. The study found that

both the teacher and students employed distinctive code-switching behaviours. M2 concepts were introduced in L1/M1 first, and once the concepts were familiar to the learner then M2 symbols were introduced to interpret more complex problems in M2. For example, multiplication was taught as a construct in L1/M1 first then replaced with M2 symbols and discourse.

Research such as Farrugia (2003) and Parvanehnezhad and Clarkson (2008) identified an Applied Linguistics approach was employed to study M2 as a sign and language-learning problem. Applied Linguistics as a research domain has a theory and research base dedicated to interpreting and resolving complex language-learning problems through educational practices (Crystal, 1992; Ivanic & Tseng, 2005; O'Neill, 2009). The implication of Applied Linguistics in shaping the methodology of the proposed study is defined in greater detail in Section 2.6.

In summary, the preliminary review of existing literature and research identified:

1. The elements and constructs of the research problem were too imprecisely and inconsistently defined to articulate global solutions. For example, learner differences such as age, language, culture, gender, economic background, and cognitive behaviours were rarely interpreted as interrelated factors when teaching and learning M2 as an adult-CALD BL construct.
2. Research was conducted mostly on children and adult-monolingual English speakers. However, adult-CALD BL learners have their own specificities that need to be studied.
3. Alternative and conflicting theories emerge to interpret the research problem. For example, there was no certainty if adult-CALD learner problems in M2 emerge because of poor text readability or inappropriate discourse strategies. This made it difficult to interpret the problem without conducting a deeper synthesis of the factors (Section 2.4).
4. Semiotics and Applied Linguistics interpreted the research problem more deeply as a sign and language-related learning problem. Semiotics is dedicated to studying the form and function of the M2 sign system (Ernest, 2006), and Linguistics the role and function of language in articulating those meanings (Bloomfield, 1939; Durst-Andersen, 2011). Applied Linguistics

merged semiotic and linguistic observations into language-based educational solutions.

5. Cross-C, L, and S differences also challenged the validity and ethics involved in conducting a study in English as an additional language. The study could, for example, misinterpret the meanings that emerge from interviews conducted in a second language (Levinson, 2004; Robinson-Pant, 2005; van de Vijver & Lueng, 2011). Applied Linguistics research offered a research base also dedicated to interpreting and resolving these types of problems (Section 2.10).

2.4 Epistemologies and ontologies

Interpretive and synthetic epistemologies and ontologies were employed to interpret the research problem. Epistemology defined how and what was learnt, and ontology what was real and truthful as a result of the inquiry (Creswell, 2003; Guba & Lincoln, 1994). The study's interpretive synthetic approach emerged through contrasting alternative research paradigms. Paradigms identified the metaphysical, ultimate, and first principle questions that emerged (Ernest, 2012; Guba & Lincoln, 1994), for example: the nature of the problem and the researcher's role in studying it. Metaphysics identified the abstracted concepts that needed to be conceptualised, for example: the role of being, knowing, identity, substance, time, and space (The Shorter Oxford English Dictionary, 1990). Epistemological and ontological choices were, however, human constructs and therefore subject to interpretation, error, and debate (Guba & Lincoln, 1994). The following section defines the rationale for the study's case-based interpretive synthetic vector.

Post-positivism, Socially Construed, Pragmatism, and Advocacy identified the dominant paradigms available to interpret the research problem (Creswell, 2003, p. 6; Ernest, 2012). The post-positivist approach reflected a more traditional scientific choice (Creswell, 2003), because it valued empirical and quantitative data, theory verification, and closed questioning techniques (Ernest, 2012; Heywood & Stronarch, 2005). The approach, however, also proposed the study to work within existing assumptions to interpret the sign system and human behaviour in it (Heywood & Stronarch, 2005; Schutz, 1932). This created an epistemological dilemma, because

human behaviours are difficult to predict in a cross-C, L, and S context (Blumer, 1969; Geertz, 1973; Schutz, 1932). Guba and Lincoln (1994, p. 106) identified post-positivist research has having internal "intraparadigm" problems, because it predetermines the propositions, contexts, and contents that are employed to interpret human behaviour. For example, applying knowledge based on monolingual adults and children to define adult-CALD BL learner behaviours in M2. Guba and Lincoln (1994, p. 106) identified that post-positivist research also encounters external "extraparadigm" problems. This is because it values observations through quantity and theory verification, rather than inductively through qualitative observations that are also important. It was difficult to predispose the proposed study to verify theory quantitatively, when not much was known about adult-CALD BL learning behaviours in M2.

Qualitative research lent to interpreting the meanings and relationships that emerge from outside the assumptions that were made in existing theory (Guba & Lincoln, 1994). Creswell (2003, p. 6) contended that an interpretive qualitative approach also reflects a "Socially Construed Paradigm", because it observes meanings and relationships through social interactions and discourses. The approach enabled subjective meanings that were difficult to predict in a cross-C, L, and S study, to be interpreted, for example: the role and function of the teacher becoming a researcher, and the meaning and significance of BL experiences in shaping learning behaviours in M2.

Constructivist and interpretive methodologies are employed as alternative approaches in a Socially Construed Paradigm (Creswell, 2003). Schwandt (1994) stated, however, a constructivist inquiry believes "knowledge and truth are created, not discovered by mind" (p. 124). According to Piaget (1972) cognitive theories reflected a constructivist approach, because it believed knowledge and learning are scaffolded in the mind through experiences. Alternatively, an interpretive approach believes knowledge is acquired, for example, by "watching", "listening", "asking", "recording", and "examining" new experiences (Schwandt, 1994, p. 119) that are, in part, extricated from old experiences. An interpretive approach lent to identifying the symbolic interactions and behaviours (Blumer, 1969) that were difficult to predict in a cross-C, L, and S context (Schutz, 1932).

A poststructuralist approach was employed to question the hidden and unknown relationships that existed in the research problem. Alternatively, a structuralist approach would have interpreted semiotic behaviours through the form and function of the sign system only (Chandler, 1994). Halliday's (2006) SFL Model, for example, interpreted the M2 sign system through its forms and functions, without reconciling the broader social contexts that shape the human behaviours within M2 (Section 4.4.2). A poststructuralist perspective identified the M2 sign was more difficult to predict in a cross-C, L, and S context than Halliday's (2006) model suggested (Section 5.2). For example, the M2 sign system might advocate the most efficient way to navigate between two points was a straight line, however, some non-western cultures employ more dynamic and non-linear ways for interpreting the same signs such as navigating by tides, currents, sense, and feelings (Barton, 2009; Whorf, 1956). This made it difficult to define human behaviours in M2 from a purely structuralist and functionalist perspective.

A poststructuralist approach helped interpret the unpredictable, expressive, and non-functional relationships that emerged form studying learning behaviours that emerged in M2. For example, M2 meanings are affected, firstly, by a "significative function", where they emerge from what the learner already knows about the sign system and, secondly, a "subjective and expressive function", where meanings emerge as a new and occasional discourse (Schutz, 1932, p. 38). The expressive function identifies there was always a chance the learner might miscommunicate meanings in M2, as they were not universally shared (Whorf, 1956).

Antecedent propositions were not employed to interpret and verify learner behaviours, but instead they were observed and interpreted holistically within the context they were communicated (Denzin & Lincoln, 1994; Kant, 1788; Schutz, 1932). As Denzin and Lincoln (1994) stated, "there are no objective observations, only observations socially situated in the world of the observer and the observed" (p. 12).

An advocacy paradigm was explored but not chosen, because it also presupposed propositions and conditions that were not, necessarily, truthful. For example, an advocacy paradigm would have proposed that the M2 sign system and existing

educational practicediscriminatory, and that a participatory education would help resolve CALD learners' problems – when neither proposition was, necessarily, true. The study envisaged adult-CALD BL learners were vulnerable learners and educational reforms might resolve the problem, however, the propositions did not define the processes that were employed to interpret the problem. Nonetheless, the study reconciled, as Howe and Moses (1999) stated, "education is always advocacy research in-as-much as it unavoidably advances some moral-perspective" and "deals with vulnerable student populations" (p. 56).

Case-based methodology was chosen to collect data in the cross-C, L, and S study. Case-based methods lent, firstly, to reconciling and managing the distinct lack of research in the area and, secondly, the intricate relationships and variances that emerged between cases (Gerring, 2007; Stiles, 2009; Yin, 1994). The method interpreted what was "common" and "distinctive" (Stake, 1994, p. 238) in the CALD participants. However, case study was more about selecting the subjects and objects that were studied than an ontological and epistemological choice (Yin, 1994). Case study methods are, for example, employed in both positivist and qualitative research. A qualitative vector was chosen, because it was difficult to be statistical about the large quantity of non-discrete qualitative data that was envisaged from conducting a synthesis.

Synthetic logic interpreted datum as part of the whole system, and not as individual analytic propositions. As Cleve (2003) stated, "in an analytic judgement, the predicate belongs to the concept or subject being studied (as in 'all bodies are extended[1]'), and in a synthetic judgement "the predicate lies outside the subject concept (as in 'all bodies are heavy')" (p. 230). Not all propositions were, necessarily, known before or after this study, and therefore the problem was synthetic in nature (Kant, 1788).

The Oxford Shorter Dictionary (1990) defined, the term analysis in this context as "the resolution of anything complex into simple elements" (p. 69), and 'synthesis' the "action of preceding in thought from causes to effects or from laws or principles to consequences" (p. 2225). The synthetic approach identified the common truths that defined decoding and learning behaviours in M2, and their synthesis gathered as much data as possible to articulate a holistic resolution to the research problem

[1] Wording cited in original text.

(Gharajedaghi & Ackoff, 1985; Patton, 1990). A constructivist approach also appeared analytic in this context, because it would have interpreted new knowledge through existing propositions. An interpretive approach was synthetic in nature, because it accommodates knowledge through new and open-ended propositions and experiences.

In summary, a case-based interpretive synthetic approach emerged to address the ontological and epistemological challenges faced in conducting a cross-C, L, and S study. Interpretive logics lent to exploring and observing unknown behaviours in the symbolic interactions of the participants (Blumer, 1969; Schutz, 1932) in the M2 sign system. Synthetic logics lent to interpreting and articulating data into holistic solutions for research problem (Gharajedaghi & Ackoff, 1985; Kant, 1788; Patton, 1990). Case-study methodologies helped reconcile the cross-C, L, and S variances that emerged between participants as cases.

2.5 Research questions

The research problem was interpreted from the ground up by, respectively, questioning the L and S structures and adult-CALD BL learner behaviours that emerge and which create problems when decoding and learning M2. The findings articulate the competencies and strategies needed to teach and learn M2 in a cross-C, L, and S context. The implications of the findings also enhance the capacity of teachings institutions to plan and deliver M2. Mathematics education in VET is, for example, merged into mainstream courses and, as a result, rarely studied and taught as a standalone specialist subject (Fitzsimons, 2002). This creates problems, because VET institutions and teachers are ill prepared to teach M2 as a construct (COAG, 2008; Fitzsimons, 2002). The proposed research questions focused on enhancing teacher, institutional, and curricular capacities. The knowledge is also envisaged to help other areas of education that have teaching problems in M2 to resolve.

Two pedagogic and one methodological question emerged to interpret the research problem. The two pedagogic questions examined: firstly, why M2 was problematic to decode and learn in a cross-C, L, and S context; and secondly, how would this knowledge enhance mathematics education. The methodological question examined the meaning and significance of a case-based interpretive research design in studying a complex language-learning problem such as M2. The term bilingual emerged in the

synthesis to more precisely define adult-CALD decoding and learning behaviours in M2.

The respective research questions addressed:

1. *What are the linguistic and semiotic structures in mathematical language that create problems for adult-CALD bilingual learners when learning mathematics in English as an additional language?*

2. *How can knowledge of adult-CALD bilingual learner-related problems in linguistic and semiotic structures in mathematics be used to enhance mathematics education in Australian classrooms?*

3. *How can case-based interpretive research methodology deepen our understanding of adult-CALD bilingual learners who are engaging in linguistic and semiotic structures in mathematics as an additional language?*

2.6 Science and language

Semiotic and linguistic science and language interpreted and articulated solutions for the research problem. The study defined the term science as the procedures that acquire knowledge through studying a particular problem (Reber, 1984). Semiotic science studies the meaning and significance of the sign system in human activity (Crystal, 1992; Ernest, 2006), and Linguistics the sign system through language and discourse behaviour. Psycholinguistics, Structural Linguistics, and Sociolinguistics are specialist areas of Linguistics that interpret human behaviour through language (Crystal, 1992). Applied Linguistics articulates the broad application of this knowledge into language-based teaching solutions (Crystal, 1992).

Applied Linguistics can employ quantitative, qualitative, and mixed research methods to study language-learning problems. Qualitative approaches are, however, criticised in the Education domain, because their findings are difficult to prove and often provide little useful information outside the context of the study (Cohen, 2007; Onwuegbuzie & Daniel, 2003). Criticisms levelled, specifically, against interpretive research approaches included, for example: lacking criteria; making unsubstantiated

claims; and, resorting to nihilism by exploring facts to an infinite degree without actually resolving problems (Schwandt, 1994).

Nonetheless, these criticisms also emerge from a quantitative perspective, because they believe "social observations" should be treated more like the "physical scientists treats physical phenomena" (Johnson & Onwuegbuzie, 2004, p. 14). Criticisms are also leveraged against quantitative methods, because they often employ inappropriate empirical and statistical techniques to make claims (Cohen, 2007; Onwuegbuzie & Daniel, 2003). The study shows mindfulness of the criticisms that were leveraged against both qualitative and quantitative research within its own research design. The following subsection defines the science and language employed to reconcile these problem.

2.6.1 Science

The data collection procedures were coded to enhance the validity, reliability, and generalisability of the findings that emerged (Braun & Clarke, 2006; Saldana, 2009). Validity defined the capacity of the study to answer the research questions; reliability the truth that was attached to the findings; and, generalisability the ability to depict what was happening outside the context of the study (Somekh & Lewin, 2005, p. 345). The science employed to encode and articulate data observations into findings is summarised below. Chapter 3 defines in greater detail the data collection instruments and techniques that were employed.

1. Research Problem:

1. A preliminary review and critique of existing literature and research was undertaken to identify its limitations, benefits, and potential foci for conducting the study (Chapter 1 & Section 2.3).
2. The elements of the research problem were synthesised and articulated into parameters, vectors, and research questions to interpret their relationships and constructs (Sections 2.2, 2.5, & 3.2).
3. The study defined its scientific and linguistic procedures for interpreting the problem (Section 2.6).

4. Existing knowledge was synthesised, classified, coded, and linked in taxonomic form to interpret the research problem. Coding linked the knowledge that emerged from studying the problem to existing research domains (Section 3.2).

2. Data collection instruments and techniques:

5. A synthesis of existing literature and research (Section 3.2 & Chapter 4) codified the series of open-ended questions that helped collect and interpret data (Appendix F). A Poststructuralist epistemology meant existing theory was coded a posteriori without being bound to verifying its validity (Section 2.4). New knowledge and theory emerged inductively from the open-ended questioning techniques and data observations that emerged.
6. Data were collected holistically through three different stages of collection. This generated a more complete interpretation of the research problem (Sections 2.8 & 3.4-5).
7. Data were synthesised and coded semiotically into a three-tier observation, interpretation, and implication structure to interpret the research problem (Section 3.2).
8. Organisational and sub-organisational patterns were thematically coded in the data to interpret relationships (Section 3.2) (Attride-Stirling, 2001; Braun & Clarke, 2006; Guest, MacQueen, & Namey, 2012).
9. Coded themes were transposed onto a Data (Appendix F) and Summary Matrix (Appendix G) to help conceptualise global themes, answers, and solutions to the research problem and questions. The matrices are attached as appendices for scrutiny and/or further interpretation.
10. The Data Matrix can be ranked and pivoted from different theoretical perspectives to interpret the research problem (Sections 3.2 & 6.4). The science, as a result, offers more than "one lens on truth and reality" (Sommerville, 2006, p. 86). In this context, the research problem was interpreted and coded, foremost, as a sign and language-learning problem (Eco, 1981).

3. Verification:

11. The study coded its narrative of the research problem (Section 3.2 & Chapter 5). Coding maintained an important link between the recount of existing knowledge, data observations, procedures, findings, and thesis that emerged (Section 6.2).
12. The data collection instruments and techniques were critiqued to interpret their capacity and alignment in answering the research problem and questions. This helped identify areas for ongoing improvement in future research (Section 3.9).

4. Findings:

13. Answers to Research Questions 1 and 2 contributed knowledge at a conceptual level (MacInnes, 2011) by proposing new ways to teach mathematics.
14. Answers to Research Question 3 contributed knowledge at a procedural level (MacInnes, 2011) by enhancing the capacity of case-based interpretive research to represent legitimate research (Onwuegbuzie & Johnson, 2006) and science. As Onwuegbuzie and Johnson (2006) stated, "the problem of legitimation refers to the difficulty in obtaining findings and/or making inferences that are credible, trustworthy, dependable, transferable, and/or confirmable" (p. 52).
15. The pedagogical propositions that emerged can be verified, rejected, and/or enhanced through ongoing research. The propositions are, however, qualitative in nature, because cross-C, L, and S relationships were statistically difficult to interpret in this context. Qualitative procedures, for example, enhanced the capacity of the study to identify the aberrant decoding and learning behaviours (Jakobson & Halle, 1956; Durst-Andersen, 2011) that were statistically difficult to quantify in a cross-C, L, and S study (Sections 5.4-5.5).

2.6.2 Language

Semiotic and linguistic language interpreted and articulated findings (Bloomfield, 1939) in the study. However, as Halliday (2006, p. 153) identified, scientific language

created problems for readers, because it removes verbs at clause level to explain meanings. This made it difficult to unpack and interpret the message (Halliday, 2006). For example, the phrase 'the aeroplane departs early in the morning for Sydney' is sequentially reworded into scientific language by removing the adverbial clauses to 'early flight for Sydney' and early Sydney flight'. This created a message that was metaphorically disconnected from its original form and sense for the reader (Halliday, 2006, p. 87). As Halliday (2006) stated:

> The verbal group signals that the process takes place; or, substantively, sets up the logical relationships of one process to another, either externally (a causes x), or internally (b proves y[2]). Concepts are organized into taxonomies, and constructs of concepts (processes) are packaged into information and distributed by backgrounding and foregrounding; and since the grammar does this by normalising, the experiential content goes into nominal groups. (p. 153)

The study, however, coded its metaphoric message and articulation processes so that its meanings and message can be unpacked by the reader. As Chandler (1994) stated:

> A message is articulated if it can be broken down into elements which are themselves significant. All semiotic elements must be significant. Thus the lorry on the traffic sign (Figure 2.2 below) can be broken down into wheels, chassis, cabin, etc., but the presence of these elements does not modify the sign. (p. 395)

Figure 2.2: Coding L and S elements that made up a traffic-warning sign
- Adopted from Chandler (1993, p. 396).

[2] Wording cited in original text

The message in Figure 2.2 was encoded by the elements and context that make up the traffic warning sign. However, the message can be interpreted from alternative and aberrant perspectives unless the context and procedures are explicitly coded for the reader (Jakobson & Halle, 1956). The aforementioned sign can be interpreted through different contexts such as 'trucks as targets' and/or 'a shooting solution'. Coding helped define the respective message that emerged from reading the sign from different perspectives.

Peirce's (1931) Triadic model of the sign system was employed to encode the interpretation and articulation of datum. Respectively, the terms *observation* defined the physical representation of datum; *interpretations* the first sense and meanings generated; and *implications* the message that was encoded in the mind of the interpreter. Example one: 'smoke' as the physical representation; 'fire' the first order of interpretation; and, 'danger' the message and implication when it is interpreted in context of a hot and windy day. Example two: the symbol '7' as datum; the 'quantity seven' the first order of interpretation; and, 'good luck' in the context of gambling. Sections 3.2-3.6 define in greater detail the procedures employed to encode and articulate data holistically as a sign system.

The terms *observations, interpretations,* and *implications* were reworded in the synthesis to more precisely interpret data in the context of the research problem (Section 3.2-3.6 & Chapter 5). Respectively, the term *data observation* was reworded to *meaning and significances* to articulate datum as problem statements; *interpretations* to *incongruences* to define the nature of the problem; and, *implications* to *educational implications* to reconcile the solutions emerging in context of the research problem and questions. *Educational implication*s were also reworded (Sections 3.2-3.7) with if and then propositions to generate more precise interpretations. The following statement emerged from the data observations, for example, to help conceptualise and define, in part, the educational implications of learner discourse problems in M2 (Figure 3.18, Section 3.6). The wording has not been edited and reflects the in vivo processes that were employed to articulate observations into teaching implications.

> *L and S structures in M2 ~~can be~~ are problematic to read if BL readers do not interact with questions and discussions, even if they* (learners[3]) *have a sophisticated understanding of Mathematical ideas. An (educational) appropriate space is needed for M2 readers* (BL learners) *to ask questions and have* (structured) *discussions when decoding M2 - at an unsophisticated level.*

Chapter 3 defines in detail the data collection instruments and techniques employed to interpret, code, and articulate data into research findings. Sample texts also identify how the text was employed to interpret and articulate the findings into propositions for teaching M2. For example, Task 10 (Appendix E) interpreted decoding and learning behaviours using mathematical formulas. However, the formula in Task 10 was, inadvertently, written incorrectly. The error generated two additional observations for the synthesis. Firstly, it identified how the participants and author negotiated formulaic errors in M2. Secondly, it helped interpret the meaning and significance of the text in stimulating these responses. As Eco (1981) stated:

> A text can be used as criminal or psychoanalytical evidence, as hallucinatory device, or as a stimulus for free association. But all of this has nothing to do with the interpretation of text qua text. Now, this does not mean that a text is a crystal-clear structure interpretable in a single way; on the contrary, a text is a lazy machine which forces its possible readers to do part of its textual work, but the modalities of the interpretive operations – albeit multiple, and possibly infinite – are by no means indefinite and must be recognized as imposed by the semiotic strategies displayed by the text. (p. 36)

The participants' decoding and learning behaviours were interpreted and coded from different linguistic and research domain perspectives to generate a holistic interpretation of the research problem. For example, questions that emerged from the Behavioural and Cognitive research domain (Arzarello, Robutti, & Bazzini, 2005; Dockens, 2008; Gillard, Van Dooren, Schaeken, & Verschaffel, 2009; Shreyar, Zolkower, & Perez, 2010) were coded to interpret factors, such as: sense and perception, learning memory, cognition, and neuropsychological behaviours in M2

[3] Comments in brackets added here to clarify meanings.

text. Cultural and Sociology research domains helped code and interpret cross-C, L, and S behaviours in M2 text (Geertz, 1973; Holmes, 2001; Krummheuer, Leuzinger-Bohleber, Muller-Kirchof, Munz, & Vogel, 2013; Schutz, 1932; Trinick, Meaney, & Fairhall, 2014). Merging the two respective sciences and questions generated a more holistic interpretation of BL decoding and learning behaviours in M2.

2.7 Research panel and supervisory input

The Supervising Panel and Academic Supervisors played an important role in helping to conceptualise and refine the research design by, respectively, questioning:

1. The mixed structuralist-poststructuralist paradigm that was originally proposed for the study.
2. The methodologies envisaged for assessing mathematical competency in a cross-C, L, and S context.
3. The rationale for conducting one-on-one interviews instead of group or classroom observations.
4. The importance of conducting future follow-up classroom observations.
5. The significance of a methodological-related research question to help guide the findings of the study.

A mixed structuralist-poststructuralist methodology was originally proposed to interpret the structural and pragmatic relationships that might emerge in M2. However, the panel's challenge of a mixed proposition, and a subsequent review, identified that the two paradigms were, in fact, conceptually incongruent, because they would have interpreted the data from different methodological perspectives (Miller, Whalley, & Stronach, 2005). A structuralist approach would have extricated the role and effect, for example, of the teacher becoming a researcher in the study and the meanings that emerged. As Chandler (1994) stated:

> The primary concern of the Structuralists is with systems or structures rather than with referential meaning or the specificities of usage (Langue and Parole). Structuralists regard each language as a relational system or structure and give priority to the determining power of the language system (a principle shared by poststructuralists). (p. 570)

A poststructuralist approach, however, interpreted the non-structural contexts and relationships that generated meanings in the sign system. For example, this includes the effect that teacher motivations had on conducting the study, and the participant and researcher relationships that emerged. These meanings and behaviours were difficult to interpret through the form and function of the sign system alone. As Chandler (1994) stated:

> Some poststructuralist semioticians are social semioticians who are concerned with signifying practices in specific social contexts. Such semioticians have extended Saussure's emphasis on meaning as relational to include not only relationships within a self-contained linguistic system, but also the interpretive importance of such broader contexts of language use. (p. 538)

A poststructuralist social semiotician perspective was employed in this study to interpret and define social, ideological, and cognitive behaviours in the M2 sign system (Ernest, 2012; de Freitas & Zolkover, 2011).

Accounting for competency in mathematics when, for example, there are many native English speakers who also struggle with learning mathematics (ABS, 2006a; COAG, 2008) was conceptualised as follows. English (native) monolingual and CALD BL learners may share problems decoding and learning M2. However, the latter faced different types of problems when decoding and learning M2 in a cross-C, L, and S context (Solano-Flores, 2014). For example, an unsophisticated discourse in M2 might reflect innumerate behaviour in a learner, however, this did not mean BL learners have an unsophisticated understanding of mathematical concepts in M1 or M2 (Solano-Flores, Barnett-Clarke, & Kachchaf, 2013).

BL learners face different kinds of cross-C, L, and S ambiguities and problems decoding and learning M2. The aim of the proposed tasks was, therefore, not to test competencies in M2 in quantitative terms, but instead to stimulate discourse, explore, and interpret BL behaviours and problems from a qualitative perspective to differentiated competencies.

The nature of the proposed one-on-one interview techniques instead of a classroom focused learning environment emerged as follows. There were merits in studying group behaviour, however, there were also distinct disadvantages and limitations in

initiating a cross-C, L, and S study from a group perspective. Anecdotally not all gender, cultural, and ethnic groups participated freely and equally in a classroom situation. An initial clinical one-on-one approach was adopted as a response to provide a space, time, and place for the CALD BL participants to contribute data freely, without the stifling influences of other participants. The inclusion of applied classroom teaching episodes in the future, however, forms, as stated by the research panel, "an excellent post-doctoral study that builds upon the evidence base uncovered in this project".

The study adopted a third methodological orientated research question to explore the role and significance of a case study and interpretive inquiry in studying a complex language-learning problem such as M2. This question enhanced the study's capacity to generate methodologically driven findings and recommendations for ongoing research implications (Section 6.8).

2.8 Overview of data collection stages

Data were collected through three stages of observations. Stage 1 interpreted the L and S structures in the M2 sign system that encoded meanings. The knowledge was then employed to observe and interpret the participant in-tasks decoding and learning behaviours in Stage 3 that created problems. Stage 2 collected background data that affected the participants' decoding and learning experiences in M2. Data from the three stages were synthesised and coded individually as sets and then globally to interpret the research problem. The three stages of data collection were also conceptualised before the recruitment of participants, reconciliation of ethical issues, and the development of data collection instruments and techniques. The logics that underpinned to the data collection stages emerged as follows.

Stage 1 was conceptualised from reading literature and research that reflected, as Eco (1981) stated, "a theory of communication is dialectically linked to a theory of signification, and a theory of signification should be first of all a theory of signs" (p. 37). Stage 1 interpreted:

1. The meaning and significance of L and S structures in the M2 sign system.

2. Points and junctures where BL learners might have problems decoding and learning M2 in a cross-C, L, and S context.

In this context Bloomfield (1939) stated, "for the most parts, our statements of meanings are makeshift. Even if this were not the case, linguistics would still study forms first and then look into their meanings (p. 55)".

A battery of open-ended questions (Section 3.2 & Appendix D) emerged from the synthesis of existing literature and research (Chapter 3) to help question and interpret L and S structures, and BL decoding and learning behaviours in the M2 that created problems. The respective battery questions, therefore, formed the important sub and incidental questions and discussions that sequentially helped synthesise answers to the research problem and questions. For example, from reading:

1. Jakobson and Halle (1956) the questions and answers emerged to *if everyday language generates meanings through binary opposition (short/tall, large/small) and markedness (happy/un-happy), then does ML function (for example, as in the sign '-' in '$3^{2\prime}$ and '$3^{-2\prime}$) the same way? If so, then does this create problems for BL learners?*
2. Eco (1981) the questions and answers emerged to *what reading (decoding and thinking) behaviour does the sign system expect from its/the reader*?

Stage 2 was conceptualised from reading literature and research that reflected, as Bortolussi and Dixon (2003) stated, a "narrative discourse seems to be intrinsic to our ability to use language to explain and interpret the world around us" (p. 1). For example, from reading:

1. Presmeg (2006) the questions and answers emerged to *how do background behaviours and experiences, for example, prior knowledge, culture, and immersion, affect* (the participant) *reading (decoding and learning) M2*?
2. Barton (2009) the questions and answers emerged to *if ML is heterogeneous, as it consists of many types and threads, then how does the participant define/interpret what is mathematics*?

Stage 2 conducted one-on-one interviews to articulate a narrative of BL experiences that affected decoding and learning M2. The participant recounts were synthesised and coded along with Stage 1 and 3 to articulate a holistic recount of the research problem.

Stage 3 observed and interpreted the individual participants' in-tasks decoding and learning behaviours. The terms reading and decoding emerged simultaneously in this context, however, decoding more precisely defined the L and S processes (Jakobson & Halle, 1956) employed to interpret M2 as a sign system. The term BL learner emerged to more precisely define, as Hamers and Blanc (2000) stated, "individuals who have access to two or more linguistic codes" (p. 368). For example, from reading:

1. Schleppegrell (2007), O'Halloran (2005), and Halliday (2006) the questions and answers emerged to *does the sign system signify systemic structures for expressing meanings at a metafunctional level, or is meaning making bounded by the sign system?* A participant directed question asked, *looking at the different things (objects) you (participant) see in the tasks - can you tell me why they are included and why they are placed there and/or not somewhere else on the page?*
2. Leslie (1993), Tylen (2007), and Ernest (2006) the questions and answers emerged to the construct of agency in the M2 sign system, whereby, as Tylen (2007) stated, "the very act of signification must be physical" (p.91). Respective, participant directed questions discussed, for example, *do you (create a) picture or image in your head to help you do these tasks? Can you describe what you see (in your mind) as you look at these symbols and figures?*

Data from Stages 1, 2, and 3 also identified the competencies needed to decode and learn M2 in a cross-C, L, and S context. Competency identified two types of cross-C, L, and S learning behaviours and cognitions in M2. Firstly, competent behaviour identified the participants who decoded and answered the tasks independently from prior C, L, and S experiences. Secondly, competency identified the type and accuracy of C, L, and S response displayed by the participants. Some participants gave correct

answers, however, were linguistically and semiotically inaccurate, because they wrote, for example, responses outside the spaces and cues provided by the text (Section 5.3.9). These behaviours identified the type of cross-C, L, and S support that was needed to decode and learn the tasks as text.

The data collection stages were complementary and holistic. Respectively, Stage 1 identified that the L and S structures envisaged from a theoretical perspective that were problematic to decode and learn in M2, Stage 2 identified the BL experiences that affected decoding and learning, and Stage 3 the cross-C, L, and S behaviours and problems that emerged. The three stages helped verify ambiguous observations. For example, Participant 1 stated in Stage 2, "*they are not good at mathematics*". However, the response was ambiguous given Participant 1 in Stage 2 also said they spent "*many years learning M1*", and in Stage 3 answered tasks independently and accurately. Synthesising the three observations identified, what Participant 1 meant to say was, they *didn't like doing mathematics at school in M1* and this affected, in part, how they approached learning M2 in a second language context. As Yin (1994) stated, conducting a case study was rather like "the detective carries out this work in (a) constructing an adequate explanation for each case, and (b) knowing the acceptable levels of modification in the original explanation as new cases are encountered" (p. 63). The study synthesised data from three sets of observations to articulate a more complete interpretation of the M2 sign system and BL decoding and learning behaviours within it.

2.9 Site and participants

Five participants from a migrant language-learning centre at a regional TAFE college in Australia were recruited for the study. There were over 200 students enrolled at the centre at the time of the study, and their C, L, and S backgrounds varied significantly. The broader student population included learners from the Middle East (Turkey, Iran, Iraq), Subcontinent (India, Sri Lanka), Central and North Africa (Congo, Burundi, Sudan), Asia (Afghanistan, China, Thailand, South Korea, Philippines), and Europe (Italy, Albania). It was difficult, however, to classify the participants in this context into discrete groups for the purpose of the study. Even if the participants came from the same country, spoke the same language, and had similar educational experiences,

there were, for example, ethnic and gender differences affecting their behaviours and experiences in M2. The participants were, as a result, interpreted individually on a case-by-case basis rather than as a group. Two participants came from similar C and L backgrounds. One participant had been in Australia for approximately 10 years, whilst the others had been in Australia for less than 2.

Seven participants were originally recruited, however, two withdrew because of unforeseen family and settlement commitments. This signalled the one-on-one interviews had to be expedited quickly to suit the participants' availability and to capture the data that was available. Stage 2 and 3 interviews were conducted in sequence a week apart over a 2-3 hour session each. The interviews were finalised within twelve weeks and subsequently generated 146,000 words of typed transcripts. This was sufficient for the purpose of the study.

The participants were selected through specific criteria for the study. The small number of participants meant they were approached individually to maintain their anonymity and confidentiality. The author knew the participants beforehand, because they were either current or past students at the college. The mix made it more difficult to identify individual participation. Participants were not informed of each other's identity.

Maintaining anonymity and confidentiality also meant BL-interpreters were not used, however, the participants needed to participate independently in the interviews and tasks. The participants had completed Certificate III in Written and Spoken English: NAT10364 (AMES, 2013). The certificate identified the participants had an ISLPR of 2 in the four macro-skills (speaking, listening, reading, and writing) and were functional in English. Competency in mathematics, however, was not a criterion for selection, and the design of the proposed tasks meant pre-testing was not required.

Maintaining the participants' anonymity and confidentiality meant that personal information such as gender, socio-economic status, age, and ethnicity, was not disclosed in the compiled transcripts. The study also avoided asking the participants personal questions about their ethnicity and personal experiences, such as refugee experiences, unless it was volunteered. These insights were synthesised and coded through less intrusive lines of questioning and observations. For example, the

participants in Stage 2 were asked *what is mathematics and what type of mathematics did they learn at school?* The response identified, firstly, the level and type of M1 and M2 education the participants had received and, secondly, with additional who, what, how, where, and why questions, the background to these experiences. Personal information that identified, for example, political and refugee experiences were recorded in separate memos as contexts rather than individual experiences. This was sufficient to identify the broader respective contexts influencing the research problem.

The participants were recruited because of their BL characteristics, availability, and interest in the study. They were briefed about its aims, their role, and the intent to maintain anonymity and confidentiality. All participants stated they were happy to participate in the study.

2.10 Ethics

The study interpreted its ethics in four ways:

1. Values identified what the participants prized as human beings (Preston, 1996, p. 17), for example: privacy, freedom of speech, and respect as an adult learner.
2. Empathy because, as Sommerville (2006) stated, it identified "the ability to vicariously experience another's feelings" and "elicit compassion" (p. 223). For example, interpreting the participant's feelings in learning M2 as a new C, L, and S experience rather than as a school subject.
3. Morality identified the values required to make empathetic judgments in a cross-C, L, and S study. For example, understanding the effect that the teacher becoming a researcher had on the study and the participants.
4. Virtues identified the practices, competencies, and dispositions a 'moral' researcher needed to conduct an ethical study (Loue, 2002; Preston, 1996).

Stage 1 did not encounter any specific ethical dilemmas, as it involved laboratory-type work that extricated the study and researcher from human contact. However, Stages 2 and 3 were more complex to negotiate, because they involved dealing with human subjects in a complex language-learning situation. Learning a new language

such as M2 potentially threatened the participants' C, L, and S beliefs, identity, and psychological behaviours (Hamers & Blanc, 2000; Tabouret-Keller, 2000).

Kant maxims (Kant, 1788) helped guide the ethical choices by identifying that the individual participant's interests and respect were more important than that of the group outcome (Howe & Moses, 1999; Kant, 1788). An alternative Utilitarian approach would have waived individual rights and respect, and exposed, for example, the participants' individual humanitarian experiences for the purpose of the study (Howe & Moses, 1999).

The interviews were digitally recorded, however, the recordings were conducted in private and personal information was deleted from the coded text. The participants were also given numbers 1-5 and a generic 'they' pronoun to protect their identity.

Kant maxims (Kant, 1788) also helped interpret the role of the teacher becoming a researcher in the study. The participants might have, for example, inappropriately interpreted the researcher as an authority in M2 and, respectively, positioned them into a subservient rather than a participatory role. The participants were informed throughout the study that the researcher was also a participant in the study, because he had as much to learn about learning M2 in a cross-C, L, and S context as they did. Respectively, a Professor Henry Higgins's (Shaw, 1912) authoritarian approach in studying linguistic behaviour was avoided. As Kant (1788) stated:

> In short: science (critically sought and methodically introduced[4]) is the narrow gate which leads to teachable wisdom, if with this we mean not merely what we are supposed to do, but rather what is supposed to serve teachers as indicators for the good and discernible cleaving of the way to wisdom which everyone is supposed to take, and also for securing others from false ways. This is a science whose keeper must always remain philosophy, and with whose subtle investigations the public has no part to play, though indeed with the teachings which first after such a treatment can be quite clearly evident to it. (p. 199)

[4] Comment cited in original text.

Three official instruments were employed to help protect the rights of the participants and researcher in the study:

1. The Supervising University Ethical Clearance Form: H12REA126 (Appendix A:).
2. Site location clearance (not included).
3. Blank signed Participant Consent Form (Appendix B).

The participants were informed the TAFE college were the interviews were conducted also had protocols that protected their rights and avenues of appeal. This avenue identified, for example, the location provided:

1. An open-door policy, in which the participants could speak to the Migrant Education Centre Manager about issues. However, the manager was not informed of the names and details of the participants.
2. The participants could take up concerns with the college student welfare counsellor if needed. This service was free and confidential.
3. The college had its own legislated code of human ethics that had to be respected and followed.

Nonetheless, signing consent forms and being informed of avenues of appeal did not mean the CALD participants, necessarily, fully understood their rights, obligations, and risks involved. These ethical issues were also reconciled and managed during the study. For example, the participants were reminded during and after the interviews they had the right to add and change their responses, and/or withdraw from the interviews if they wanted. However, once the interviews were finalised, transcripts synthesised and coded, and personal information, for example, name, age, language, gender, culture, and ethnicity, removed, then their information for practical reasons could not be withdrawn.

The study was cognisant of the ethical issues that emerged and challenged a cross-C, L, and S study. Personal information such as gender, ethnicity, and family relationships, were reconciled and managed during the study. As Piper and Simons (2005) stated, “the uniqueness and complexity of each situation and any ethical decision needs to take cognisance of the precise way many of the factors are played

out in the specific socio-political context" (p. 58). Personal information was deleted from transcripts, and broader refugee and settlement issues, for example, were interpreted globally in the context of their effects on decoding and learning M2 in a cross-C, L, and S context. The participants were informed the intent of the tasks was not only about measuring their intelligence, but instead generate insights into the BL behaviours and experiences that created problems when decoding and learning M2. This intent became clearer during the in-tasks activities.

ML was also given a participatory "voice" in this context (Holmes, 2001, p. 56). The outcome identified the "agentic" characteristics of the M2 sign system as a free agent in shaping the participants' decoding and learning behaviours (Ernest, 2010, p. 69). The meaning and significance of agency emerged from reading Leslie (1993) and Tylen (2007) who asserted the sign system has its own goals, reactions, and cognitive properties that needed to be respected. These finding are synthesised in greater detail in Sections 4.4.2 (Synthesis of Existing Literature and Research: mathematical language) and 5.3.5 (Data Synthesis: structural codes).

Kantian maxims (1788) guided the ethical choices made in the cross-C, L, and S study. The participants were informed of their obligations, anonymity, and confidentiality: however, cooperation, openness, and empathy helped build a more trustworthy relationship between the researcher and CALD participants. M2 was also given a participatory "voice" in the study (Howe & Moses, 1999, p. 56) to articulate its agentic characteristics and effects on participant behaviours (Leslie, 1993; Tylen, 2007; Ernest, 2010).

2.11 Chapter summary

A preliminary review of existing literature and research determined the constructs of the research problems were imprecisely and inconsistently defined to generate a holistic interpretation of the research problem. A deeper and more precise synthesis was, therefore, required to interpret and articulate solutions. Alternative epistemological and ontological paradigms were examined to interpret the methodological choices that were available to guide the study. Positivist methodologies were inappropriate for interpreting the complex and unpredictable relationships that were envisaged in the cross-C, L, and S study. Case-based

interpretive synthetic methodologies lent to more precisely interpreting and defining the M2 sign system and human behaviours in it.

The three research questions emerged from the preliminary review of literature and research to define the focus for the study. The research problem was interpreted from:

1. A semiotic and linguistic perspective, by questioning the L and S structures in M2 that created problems when decoded and learnt in a cross-C, L and S context.
2. A pedagogic perspective, by questioning how knowledge from Question 1 enhanced mathematics education.
3. A methodological perspective, by questioning how knowledge from an interpretive case-based research design deepened our understanding of a complex language-learning problem, such as, M2.

The study employed the following logics, science, language, location, participants, and ethics to interpret and articulate answers to the research problem and questions:

1. Linguistic and semiotic science and language defined, coded, and articulated the research problem as a sign and language-learning problem.
2. Applied Linguistics articulated the findings into structural and language-based teaching solutions.
3. A synthetic rather than an analytic approach interpreted the data holistically (Gharajedaghi & Ackoff, 1985; Patton, 1990). An alternative analytic approach was deemed incongruent for the study's poststructuralist's belief in which knowledge emerged inductively through observation and open-ended questioning. Analytic logics predisposed the study to a priori coding and closed questioning techniques that were considered to be inappropriate for interpreting the sign system and cross-C, L, and S behaviour in it.
4. Kant maxims (1788) helped reconcile the ethical threats that challenged a cross-C, L, and S study. This meant paying attention to a participant's unique situation (Piper & Simons, 2005). The approach built a more trustworthy and productive outcome for the study.

Chapter 3: Data Collection Description, Instruments, and Techniques

> Having established that all genuine understanding of the other person must start out from Acts of explication performed by the observer on his own lived experience, we must now proceed to a precise analysis of this genuine understanding itself. From the examples we have already given, it is clear that our inquiry must take two different directions. First we must study the genuine understanding of actions which are performed without any communicative intent. Second we would examine cases where such communicative intent was present. The latter type of action involves a whole new dimension, the using and interpreting of signs.
>
> (Schutz, *The phenomenology of the social world*, 1932, p. 36)

3.1 Introduction

This chapter defines the technical aspects of the data collection instruments and techniques that were employed to synthesise the research problem. The distinct lack of established research in the area meant, however, the instruments and techniques were conceptualised and developed, in the main, from the ground up. Their rationale, procedures, and implications are recounted in detail. Samples of transcripts and associated texts are included in the chapter sections and appendices as supporting resources. The transcripts and texts are unedited and replicated in italics, however, additional comments are added in brackets and regular font in this and subsequent chapters to clarify the more ambiguous meanings that were interpreted. The chapter concludes with a critique and recommendations for ongoing improvements in future research. It is envisaged the instruments and techniques developed in this study will help researchers in other areas of education where complex language-learning problems need to be solved.

The position and development of the data collection instruments and techniques in the study is depicted in Figure 3.1.

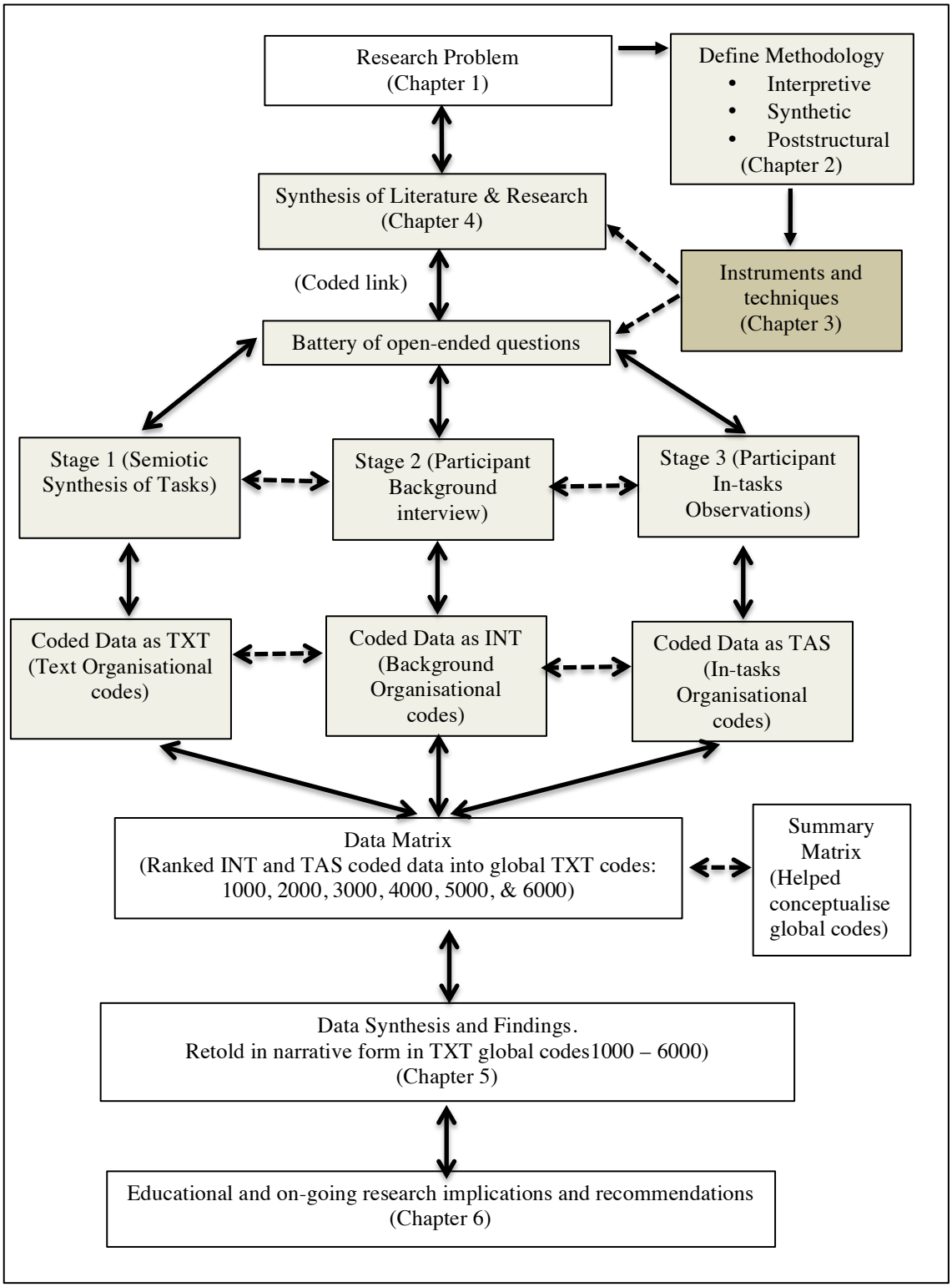


Figure 3.1: Position of the data collection instruments and technique employed in the study.

3.2 Battery of open-ended and micro questions

A battery of open-ended questions was synthesised from the review of existing literature and research (Chapter 4) to generate a more precise and holistic interpretation of the research problem.

Rationale

The rationale for the battery of open-ended questions emerged in three steps:

1. A preliminary review of existing literature and research (Section 2.3) identified there were no ideal models of analysis or series of questions published that suited the specificities of study. For example, Halliday's (2006) model of SFL helped interpret and define the functional and structural relationships that created problems in M2. However, it was inadequate for interpreting the cross-C, L, and S variances and behaviours that also affected decoding and learning M2 (Section 4.4.2). The battery of open-ended questions questioned human behaviour in greater depth.
2. The study's poststructuralist approach did not predispose the battery questions a priori to verifying existing theory and knowledge. Rather, as Walsham (1995) stated, existing knowledge was applied "without being trapped in the view that it represents final truth in that area" (p. 77). The study interpreted the research problem holistically from alternative theoretical and domain perspectives. Micro questions such as who, what, how, when, where, and why were added to the model to generate more precise observations.
3. The battery of open-ended questions reflected a broad range of knowledge in the area, however, there were methodological challenges that emerged in its design. For example, there might be relevant theory and knowledge that were inadvertently overlooked in the synthesis and articulation of existing literature and research into respective battery questions (Chapter 4). In this context, existing knowledge was coded thematically into themes and research domains to identify: firstly, the broad contexts and reasons for asking the data collection question; and secondly, identify potential the questions that might have been overlooked in the synthesis of existing literature and research. The

procedure generated a more comprehensive line of questioning and linking of existing knowledge to the findings that emerged.

Methods

Figure 3.2 summarises the five steps employed to conceptualise and develop the battery of open-ended questions.

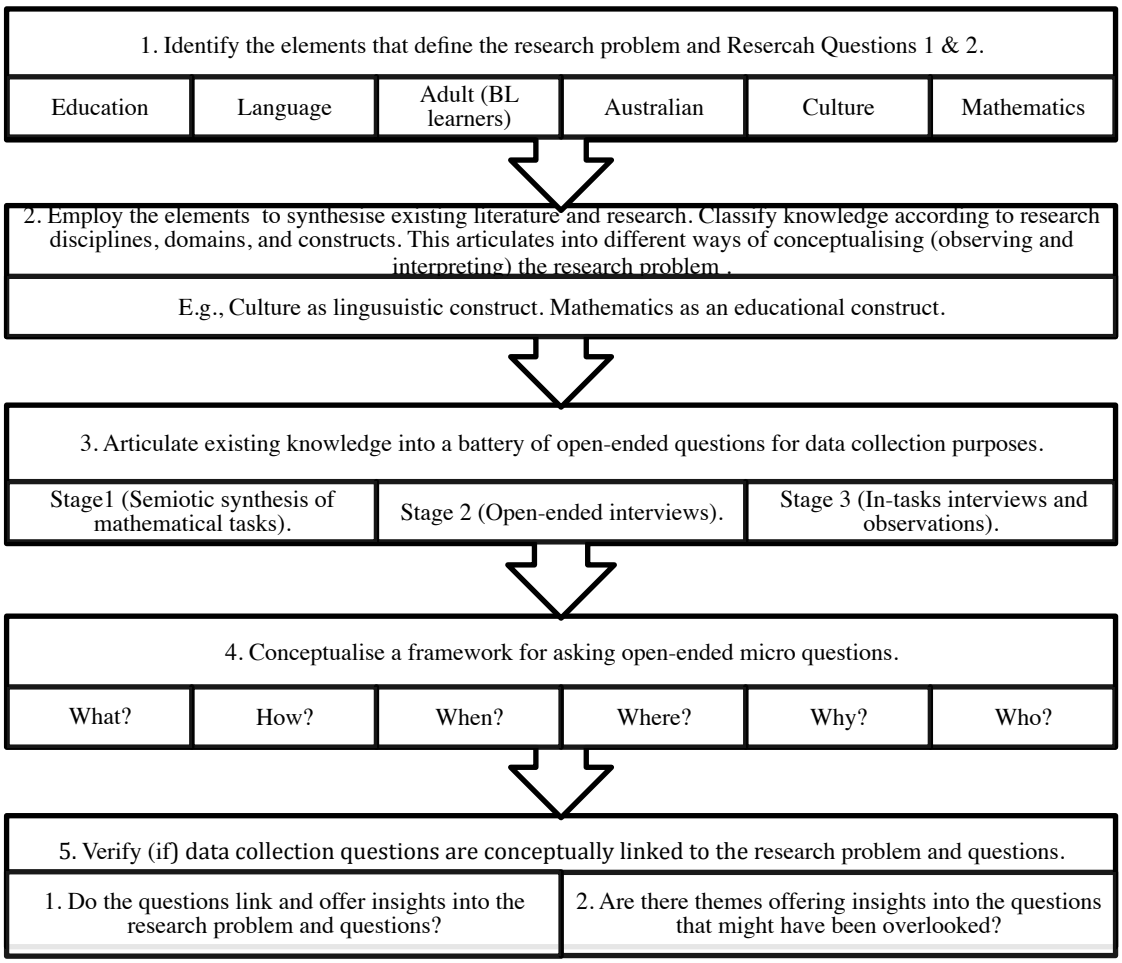


Figure 3.2: Five-step model for building a battery of open-ended questions.

Step 1 interpreted the elements of the research problem and Research Questions 1 and 2 through their nominal clause structures. Nominal clauses identified the subjects and entities (Halliday, 2006) helping to define the research problem and its respective constructs. Constructs identified relationships, as MacInnes (2011) stated, "in a sufficiently precise manner to be operationalized or measured" (p. 141). For example, this can refer to conceptualising adult-CALD learners differently from monolingual English learners. Conceptualising generated a different way of thinking about

relationships, entities, and potential solutions (MacInnes, 2011; The Shorter Oxford English Dictionary, 1990), for example: comparing competencies and strategies needed to teach and learn M2 as a sign and language-based learning problem.

The nominal clauses (elements) as an aide helped define the parameters of the study, otherwise the study was open to an infinite and potentially inconclusive line of questioning (Section 4.2). Figure 3.3 depicts the techniques employed to interpret and articulate the wording of the research problem and Research Questions 1 and 2 into the elements that were further questioned and synthesised in the study: *Education, Language (English, mathematical, additional/second), BL Adult learners, Australian, Culture,* and *Mathematics*. Research Question 3 was not interpreted in the same way, because it was envisaged as generating a different focus for the study. The distinction was, however, reviewed and recommendations are made to integrate and treat methodological questions in the same context in future research (Section 3.9).

Part 1. Identify the key concepts by underlining nominal clauses and then articulating them as elements. (Adverbial phrases identified propositional relationships).

$\Longrightarrow$ *Research Problem*

The problem of teaching mathematics to adult-CALD BL learners in English as an additional language

$\Longrightarrow$ *Research questions*

1. *What are the linguistic and semiotic structures in mathematical language that create problems for adult-CALD BL learners when learning mathematics in English as an additional(second) language?*

2. *How can knowledge of adult-CALD BL learner related problems in linguistic and semiotic structures in mathematics be used to enhance mathematics education in Australian classrooms*?

$\Longrightarrow$ *Nominal clauses,elements,and constructs that emerge*

1. *Linguistic structures - semiotic structures – mathematics -adult – CALD – learners – learning - mathematical language – English – additional (second) language – knowledge -learner problems - educational outcomes –Australian – classroom*

Part 2. Summarise and articulate nominal clauses as elements for questioning and exploring the constructs of the research problem.

$\Longrightarrow$ *Focus for questioning and synthesis of literature and open−ended question battery*

1. *Education.*
2. *Language (English, mathematical, additional/second).*
3. *Adult learners (BL).*
4. *Australian.*
5. *Culture.*
6. *Mathematics.*

Figure 3.3: Methodology employed for interpreting and defining elements and constructs in the research problem.

Step 2 articulated the elements into the parameters that also helped search and synthesise existing literature and research. The elements identified, for example, key words that, respectively, guided the electronic search for literature and research, and synthesised their findings into taxonomic form. The approach generated a more comprehensive picture of the existing research, such as: its strengths, weaknesses, and implications for conceptualising the research problem, design, and focus (Onwuegbuzie, Leech, & Collins, 2012, p. 4).

The study adopted MacInnes's (2011, p. 137) Framework for Conceptual Contributions to Marketing to help interpret and classify existing literature and research in this context (Chapter 4). The framework questioned:

1. What were the constructs identified?
2. What were the divisions and disciplines that affected the findings?
3. What research domain governed the study?
4. What constituted science?
5. What procedures were followed or recommended for conducting research?
6. What theoretical relationships were claimed from the study?

Step 3 articulated existing knowledge into the battery of open-ended questions in two sub-steps. Sub-step 1 generated an Annotated Summary (Appendix C) of 104 theoretical ideas and associated rough drafted questions that were later refined for the battery of open-ended questions. The questions were reworded by the researcher into different lines of questioning for the three stages of data collection. The questions were also numbered to code a link between their theoretical underpinning and the data observations and findings that emerged. Figure 3.4 depicts an example of the questions that emerged for the term *culture* for the three stages of data collection. The wordings of the questions were refined during the study to more precisely interpret the research problem.

Culture

Theoretical Idea 12: Some languages and cultures have different ways of conceptualising mathematics. For example, spatial and temporal reasoning varies between languages (Whorf, 1956; Barton, 2009).

- Stage 1 (Semiotic Synthesis of Mathematical Tasks) > *Does the sign system signify/make cultural assumptions?*
- Stage 2 (Open-ended Interviews) > *Do you (*participant*) think the way we use numbers and mathematics in Australia is different to the way you use them in your home country?*
- Stage 3 (In-task Interviews and Observations) > *Does the task look different to the way you (*participant*) may have seen it written in your first language.*

Theoretical Idea 13. Culture is the 'know-how' needed for daily living (Wardaugh, 2002), and Dokery (2009) explains culture is the set of beliefs and values that are transmitted through generations to identify groups of people.

- Stage 1: > *What culture underpins the sign?*
- Stage 2: > *Do you think it is important to remember your language and culture?*
- Stage 2: > *Do you think it is important to teach (your-CALD) children how to live and solve everyday problems the way your parents taught you?*

Figure 3.4: Sample of cultural-related data collection questions generated from the synthesis of existing literature and research.

Sub-step 3 refined the rough draft of questions (Appendix C) into a battery of open-ended questions (Appendix D). The questions were respectively documented onto Excel spreadsheets, reworded, sorted, and coded into themes for the three stages of data collection. The theme in this context helped, and as Braun and Clarke (2006) stated, to "capture something important about the data in relation to the research question" (p. 82).

The questions were grouped and coded into themes to identify, firstly, patterns in the lines of questioning and, secondly, did their findings contribute to existing research domains. The themes emerging in the battery identified:

1. Stage 1 (Semiotic Synthesis of Tasks) the elements of the research problem and questions that affected decoding and learning M2: Education, Language, Adult (BL), Australian, Culture, and Mathematics.
2. Stage 2 (Background Interviews) the participant perceptions and experiences that affected decoding and learning M2: experiences in education, language,

and mathematics; perceptions of language, education, and culture in Australia; perceptions of mathematics; experiences in learning mathematics; structural perceptions of mathematics; cognitive related habits and perceptions in decoding mathematics; and, behavioural perceptions of age, thinking, and adulthood.

3. Stage 3 (In-tasks Observations and Interviews) themes emerged in two ways. Firstly, they questioned the participants directly, when possible, about the factors that affected their in-tasks decoding and learning behaviours: thinking with signs; interpreting sign meanings, flows, and, origins. Secondly, the study was mindful of time and operational constraints limiting the number of (59) questions that could be asked during Stage 3 in-tasks interviews. Asking all the questions would have detracted the participants from focusing on and completing the tasks. Questions were addressed as needed in the context of the tasks and participant behaviours in it. As a result, Stage 3 added 13 observational questions to interpret and summarise the participants' decoding and learning behaviours in the tasks. The summaries identified: participant perceptions of ML; competencies; support; decoding; questioning and discussion processes; modalities; physical interaction; reader characteristics; ability to read semiotic cues; decoding reaction; discomfort; and anxieties in M2. Merging the two sets of observations and questions for Stage 3 generated a more practical and complete interpretation of BL decoding and learning behaviours, and problems in the tasks.

Step 4 tagged micro who, what, how, when, where, and why questions to the battery to enrich the type and depth of data collected. Figure 3.5 depicts the micro-type questions generated for Stage 1 Question 69.

Stage 1: Semiotic Synthesis of Mathematical Tasks

Primary Question 69: Does the sign system signify assume social and cultural conventions as codes (Derrida, 1967)?

Micro (follow-up) questions:

- ***What** are the codes that are assumed as conventions within the sign system?*
- ***How** are (coding) conventions signified?*
- ***When** does the sign system signify patterns in which more or less meanings are culturally dependent?*
- ***Where** within the sign system are social and cultural conventions assumed as codes?*
- ***Why** are there social and cultural conventions in this task - what role do they perform - are they important?*
- ***Who** would think social and cultural conventions are or are not important?*

Figure 3.5: Sample micro questions employed in Stage 1 Question 69.

Step 5 questioned and refined the battery's potential to interpret the research problem. The questions were, in the main, relevant and only minor rewording changes were made to enhance their overall precision. Step 5 also questioned the battery's ability to interpret the research problem holistically. The review identified the M2 sign system and participants were interpreted from a diverse range of theoretical and domain perspectives. Theming questions also saved time in conducting interviews by, for example, not asking questions that were thematically addressed through other lines of open-ended questioning.

Implications

The battery of open-ended questions helped interrogate and interpret the research problem from different theoretical perspectives (Section 4.2). The questions were themed and coded a posteriori to build on existing knowledge frameworks, without being committed to proving or disproving existing theory.

3.3 Mathematical tasks

15 mathematical tasks (Appendix E) were developed to stimulate data collection in Stage 1 (Semiotic Synthesis of Mathematical Tasks) and Stage 3 (In-task Interviews and Observations).

Rationale

The tasks represented the type of M2, for example, graphs, table, charts, symbols, and everyday language, found in VET and Middle-year Secondary School mathematics texts. Anecdotally, the tasks were envisaged as a benchmark for reading mathematics in VET. Figure 3.6 summarises the logics that underpinned the design and modelling of the individual tasks. The observations emerged from the responses generated in Stage 1 (Semiotic Synthesis) Question 1: *Why choose the text (Chandler, 1993)*?

Stage 1 Question 1: Why choose the text (Chandler, 1993)?

- *Tasks 1 and 3 observed decoding symbolic and abstracted language in mathematics.*
- *Task 2 observed decoding symbolic logics in ML.*
- *Tasks 4 and 5 observed decoding graphical representations and chance/probability concepts.*
- *Task 6 observed decoding irrational concepts across different tables and resources to generate meanings.*
- *Task 7 observed decoding ML with higher lexical content, and real life meanings. Some Tasks were designed to help identify decoding everyday experiences, and others were more abstracted in nature.*
- *Task 8 observed decoding ML as an oral and textual experience. It also generated, for example, inductive, deductive, heuristic, and abductive (Peirce, 1931) decoding behaviours.*
- *Task 9 observed decoding ML as distinct (algebraic) sign systems. The signs were clustered and decoded across several figures and resources.*
- *Task 10 observed decoding formulas in L and S structures.*
- *Task 11, 12, 13, and 14 observed decoding intricate and densely packaged ML (Halliday, 2006) that were, in part, articulated out of real life concepts. The formulas were representations of the ML found in middle year school* (Years 9-10)*, and the multiple-choices constructs signified decoding ML as a heuristic experience. The answers looked similar, and it is of interest to observe how and why some BL readers choose certain answers.*
- *Task 15 is observed in the context of potential real life problems that involve a combination of symbolic, iconic, and indexical signs, such as, calculus and graphic representation. The multiple focuses observed ML generating similar meanings with different types of semiotic resource (O'Halloran, 2005, 2010).*

Figure 3.6: Why choose the tasks as text?

The rationale for the modelling emerged as follows:

1. There was no ideal model of tasks published that suited the specificities of the study. The model was therefore conceptualised in the main from the ground up.
2. The heterogeneous nature of ML (Barton, 2009; Devlin, 2000) meant the tasks could vary in their form and function. As a result, the tasks were, in part, modelled to help interpret the research problem and the questions. The objective of the tasks was not to quantify competency in this context, but instead interpret its qualitative meaning and significance in the research problem.
3. The tasks needed to maintain a theoretical link between the text, the observations that emerged in Stage 1 (Semiotic Synthesis), and their applied contexts in Stage 3 (In-Tasks Interviews and Observations). Respectively, Stage 1 identified potential points and junctures envisaged as being problematic to decode and learn in M2, and Stage 3 interpreted the points and junctures through the participants' actual decoding and learning behaviours.

Method

The tasks were organised in order of L and S complexity to help observe and interpret participant L and S competencies and decoding thresholds. Participants 1 and 2, for example, understood the mathematical concepts identified in Tasks 14 and 15, however, they needed support to interpret the respective tasks as a discourse in English. Alternatively, Participants 3, 4, and 5 found the two tasks challenging to interpret, because they introduced new concepts and discourses they were unfamiliar with.

Decoding and learning behaviours were interpreted through the participants' physical reactions and responses in the text, for example: questioning, writing, and engaging in paralinguistic activities. Physical responses also identified the contexts, contents, and processes in M2 that created problems. Figure 3.7 depicts an example in which Participant 3 physically responded to Task 3 by ticking and stating: "the word apple" was "not as a number", however adding 's' as in apples made it sound more "like a number". Ticking in this context identified the text and how Participant 3 interpreted English plurals as mathematical signs.

Task 3. Tick ✔ which symbol is not a number.

1	*六*	*Apples*	*A*	*π*	$22/7$	*200,000*	*Apple*	*+*	*Five*
		✔					**X**		

Figure 3.7: Example of Task 3 stimulating physical BL decoding and thinking behaviours in M2 text.

Physical evidence helped interpret the cognitive behaviours that were difficult to explain in M2. Cognitive theory was problematic to articulate with out physical evidence, because it involved interpreting abstract ideas and behaviours that occurred in someone else's mind (Ramachandran, 2011). Section 5.5 details in greater depth the cognitive behaviours that emerged through these types of observations and have affected the research problem.

Implications

The tasks fulfilled two functions. Firstly, they "act" (Schutz, 1932, p. 36) as the semiotic artefacts that defined M2 as a sign system. Secondly, the tasks functioned as a semiotic stimulus (Eco, 1981) to interpret BL decoding and learning behaviours in it. The tasks varied in their form, function, and structure to stimulate different types of responses and behaviours. Task 6, for example, helped interpret decoding and learning behaviours in highly symbolic language, and Task 7 in problematic everyday language. The tasks enabled the synthesis to, respectively, merge, contrast, and compare the different types of L and S structures and BL decoding and learning behaviours that created problems.

3.4 Stage 1 data collection

Stage 1 (Semiotic Synthesis of Mathematical Tasks) synthesised the meaning and significance of L and S structure in M2, and their problematic nature when envisaged in a cross-C, L, and S context.

Rationale

Stage 1 defined the form and function of the M2 sign system. Their findings identified points and junctures that were envisaged difficult to decode and learn in Stage 3 (In-task Interviews).

Method

Stage 1 synthesised data observations in five steps. Step 1 interpreted the meanings and significance of L and S structure in M2 through the 92 questions generated in the battery of open-ended questions for Stage 1 (Appendix D). The responses generated 40,000 words of typed observations in which the researcher systematically answered each question. Figure 3.8 depicts an example of the question and response generated for Stage 1 Question 17. The non-italic wording in brackets is added to clarify the more ambiguous observations.

Stage 1 Question 17: (44) Do the signs signify non-abstract meanings – see indexical signs (Peirce, 1931)?

(Interpretation and responses)

None of the signs are real in the sense they are physical things you can touch. Some signs are indexical and, for example, figuratively resemble some object. Task 14 represents this type of figure - even if there is no house, trees, or supermarket drawn. Nonetheless, while mathematics is often conceptualised as the symbols that represent abstract concepts, all the signs in the tasks have their own identity in which meaning occurs (emerge). *This does not require a man holding a stick is needed to signify the height of a triangle (Devlin, 2000)* (Refers to the removal of humanistic objects such as eyes to interpret and calculate perspectives in ML – See Devlin 2000).

How. By creating their own identity the signs within these tasks work within themselves (independently) *to generate meanings. Other resources, measuring sticks, scales, and cars are not needed to signify meanings. Analogically, film and radio can generate meanings within themselves* (as a medium) *without being real. This creates the notions of mathematics being a highly 'objectified' (Sfard, 2005) educational process. Somehow, the* (learners) *mind allows this to be achieved with symbolic and iconic signs that have replaced indexical ones in these tasks: Task 14 is in a state of transition, where a figure is partially removed from the index.*

Where. This occurs as an evolutionary (by articulation) *part of the sign system within the tasks: some are less indexical than other. Task 10 does not require a picture of a car, but the formulas describe its* (car) *function relative to speed. Task 1, depicts how number symbols can also have indexical meanings.*

When. Indexical signs are used in these tasks to help code processes. They are in fact arbitrary because the tasks undertake this without (having explicit) *codes guide the processes for the*

reader. Signs can also be iconic in this case in signifying meanings. The diagram of share prices going up and then down is iconic representations of the current stock market. The rate of change that plunges from positive feelings into negativity may also be iconic.

Who. The narrator essentially works within existing genres and registers in using indexical signs. However, historically speaking these resources have diminished and mathematics does require the reader within these tasks to read signs that are less so. Some readers, who are unable to read the signs within these codes, may see them as abstract and therefore cognitively challenging readers who have acquired the codes. The product may be taught thru informal learning and experiences in decoding signs that are have their own identity (learnt through repetition and everyday experiences rather than a formal classroom).

Why. The tasks identify the signs generate their own identity, allowing them to function into creating and expressing human abstracted experiences. This signals a degree of cognitive ability within the signs themselves – analogically speaking, programing computers to manipulate and make decisions through signs and logic also occurs in spaces that are not human. How people with CALD related backgrounds interact within the sign system is of interest because they may show and relate to the signs and systems in a different way – (having) *its own way of thinking- and this may be different to the way they* (CALD BL learner) *think – along the lines of what Whorf (1956) hypothesised. Overall, all signs are given* (have) *reality within the systems in these tasks. They do not require indexical signs to do this, and as such are achieved and coded within the logics of decoding the task. It is not clear from an instructional perspective if they* (logics and codes) *help or hinder the process when CALD factors are included in decoding and learning.*

Figure 3.8: Sample of observations generated for Stage 1 Question 17.

Step 2 uploaded the typed observations made in step 1 onto NVivo 10 Qualitative Data Management Software for synthesis and coding. Other data management software programs were explored, but NVivo 10's ability to synthesise and code the large quantities of non-discrete qualitative data emerging from the three stages of collection best suited the study. The program managed an operational file of 5.1MB (870K words) that would have been difficult to reconcile manually. Data codes and themes were, however, conceptualised and articulated mentally through if and then propositions by the author.

Step 3 coded the data observations as follows. Prefix *TXT* identified the observation emerged as a *text-based* observation in Stage 1. The first two digits, for example, '*20*' in *TXT2017*, identified the thematic code that was identified. The last two digits identified the question that generated the observation, for example: Question 17 asked *do the signs signify non-abstract meanings*? The last 2 digits also linked the question to the theory and research domain that generated the observation in the battery of

open-ended questions. The themes were reworded during the synthesis to more precisely reflect the data theme. For example, *TXT2000* was reworded:

From: *Linguistic structures that are problematic to decode in M2*

To: S*tructural codes that are problematic to decode in M2*.

Step 4 synthesised and articulated the coded observations recorded in step 3 into summary *observations, interpretations,* and *implications*. Figure 3.9 depicts the data *observations, interpretations*, and *implications* that emerged for *TXT2017* in Figure 3.8 above. The implications were also reworded to more precisely reflect the research problem and questions. For example, the implication of *TXT2017* was reworded:

To: *L and S structures are problematic to decode in M2 if BL are unable to reconcile M2 as an objectified language*.

Coding Summary By Source

Case study of Semiotic and Linguistic Factors that create problems for adult-CALD Readers.

13/04/2013 1:06 PM

Classification. Aggregate. Coverage. Number of coding. Reference No. Coded By. Modified On

Document
Internals\\Background \\10. Text observations and interpretations

Node

Nodes\\Semiotic summaries\\TAS Tasks Observation Summaries

TXT2017 *Does the sign system signal non-abstracted meanings (e.g., indexical signs such as maps) that affect how ML is read, and how does this factor create problems for BL readers when decoding M2?*

(Observation) *While mathematics is often construed as a highly symbolic and abstracted language, the sign system also importantly employs iconic (e.g., in Task 15 the diagram of share prices going up and then down is an iconic representations of the stock market) and indexical signs (e.g., Task 14 uses a figure for a house although there is no house). Nonetheless, indexical signs are not randomly placed as they are often written within genres and registers. Indexical signs such as eyes and human figures are less likely to be found in contemporary mathematical texts (Devlin, 2000), however, their function are clear* (identifiable).

(Interpretation) *ML appears 'objectifying' (Sfard, 2005) because resources such as measuring sticks, scales, and instruments are not needed to reconcile experiences through ML: e.g., Task 10 does not require a picture or an actual car because the formula symbolically replaces the car and speed as objects. Both monolingual and BL may have problems reconciling ML as objectifying language: however, BL readers need to reconcile the problem across two L and S systems.*

(Implication) *L and S structures are problematic to read in M2 if BL are unable to reconcile M2 as an objectified language. Reconciling objects mentally without instruments from BL perspectives may be complex because it requires readers to symbolically reconcile mathematical meanings in (strangely) abstracted ways.*

Figure 3.9: Sample TXT2017 data observation synthesised and coded for Stage 1.

Step 4 summarised the responses generated by the 92 battery questions into a single text of 16,000 words of summarised coded observations, interpretations, and implications for Stage 1 data collection.

Step 5 coded the thematic patterns found in the summary observations, interpretations, and implications. Themes were coded, firstly, within the three different stages of data collection and, then, together through global themes (Attride-Stirling, 2001, p. 389). Applied Thematic Procedures interpreted and coded the data observations in this context (Braun & Clarke, 2006; Guest, MacQueen, & Namey, 2012). The procedures, as Guest, MacQueen, and Namey (2012) stated:

> Move beyond counting explicit word or phrases and focus on identifying and describing both implicit and explicit ideas within the data, that is, themes. Codes are then typically developed to represent the identified themes and applied or linked to the raw data as summary markers for later analysis. (p. 10)

Thematic codes emerged inductively, because it was difficult to predict relationships and behaviours in this research problem. The approach, as a result, generated a broad range of qualitative data sets that were difficult to interpret through a priori coding techniques (Braun & Clarke, 2006). An a posteriori approach, as Clarke and Braun (2006) stated, emerged:

> Without trying to fit it into a pre-existing coding frame, or the researcher's analytic preconceptions. In this sense, this form of thematic analysis is data-driven. However, it is important to note, as we discussed earlier, that researchers cannot free themselves of their theoretical and epistemological commitments, and data are not coded in an epistemological vacuum. (p. 84)

Data were also interpreted in this context as signs within a broader sign system (Schutz, 1932), and coding defined their semiotic relationships in that system (Section 2.6.2). The coding techniques, however, maintained links between the theory, science, and language employed to generate the data observations and findings that emerged. This helped define the logics that underpinned the data collection, and where the findings would contribute knowledge to the existing research domains.

Figure 3.10 depicts the six data themes that emerged for Stage 1. Stage 1 codes, in the main, emerged from the domain themes identified in the battery of open-ended questions. For example, *TXT1000* was reworded:

> **From:** *The philosophical factors that affect how L and S structures are coded in ML and are problematic to decoding in M2.*
>
> **To:** *Philosophical codes that are problematic to decode in M2.*

The latter form more precisely reflected the effect philosophical conventions had on decoding and learning M2 in a cross-C, L, and S context. Stage 2 and 3 data themes, however, emerged more pragmatically since the participants were different.

1. *(TXT1000) Philosophical codes that are problematic to decode in M2.*
2. *(TXT2000) Structural codes that are problematic to decode in M2.*
3. *(TXT3000) BL decoding behaviours (e.g., thinking, decoding) that affect how M2 is decoded and learnt.*
4. *(TXT4000) BL humanistic behaviours (e.g., age, gender, physical) that affect how M2 is decoded and learnt.*
5. *(TXT5000) BL cultural behaviours that affect how M2 is decoded and learnt.*
6. *(TXT6000) BL educational experiences that affect how M2 is decoded and learnt.*

Figure 3.10: Stage 1 data themes.

The findings from Stage 1 articulated the meaning and significance of L and S structures in M2, and their problematic nature when conceptualised in a cross-C, L, and S context. Coding tracked the logics and processes employed to interpret meanings. This made it easier to identify the source and reasons for the findings that emerged. The data collection instruments and techniques have the capacity to include texts, participants, and collaborations from ongoing research.

3.5 Stage 2 data collection

Stage 2 (Open-ended Background Interviews) collected background data that affected the participants' decoding and learning M2.

Rationale

Stage 2 complemented Stage 1 data by identifying the BL experiences that emerged and created problems.

Method

Stage 2 synthesised participant backgrounds in 4 steps. Step 1 employed the 78 questions generated in the battery of open-ended questions (Appendix D) to stimulate background interviews. Two copies of the interview questions were produced for the interview. One copy was given to the participant to read, and a second was read aloud and rephrased by the interviewer to suit the participant. Anecdotally, a combination of the interviewer reading the questions aloud and the participant reading them quietly at the same time enhanced the participant's capacity to respond to questions. Contextualising and citing examples helped the participants interpret questions. Not all the questions were, however, asked. Questions that appeared ethically intrusive for particular participants, or were answered through other lines of questioning, were not asked. 50 questions on average were addressed and these were sufficient enough to drive the 2-3 hour interview.

The interviews were digitally recorded and written responses were not required. The digital recordings enabled the interviewer and participants to think aloud and

contribute to free-flowing discussions without having to write down notes and responses. Participants were encouraged to contribute, confirm, and amend their responses during and after the interviews. The participants all stated they were happy to participate in this type of interview, and continue to Stage 3 (In-tasks Interviews and Observations) a week later. Reflective notes were recorded after the interviews by the interviewer.

The first participant contributed significantly to refining the interview techniques that were developed. Respectively, timing, questions, and procedures were discussed and refined at the initial interview. The term participant also replaced a case number at this stage to better describe the participatory and cooperative data that emerged.

Step 2 transcribed the digital recordings into text. Oral to text dictation software was trialled, but it was found inappropriate for transcribing long dialogues spoken in English as an additional language. Pronunciation problems, for example, were often transcribed incorrectly into words through this technology. As a result, recordings were transcribed manually, however, it meant it took more time to interpret the interviews more thoroughly. The transcripts also included paralinguistic details, such as timing, laughter, and sarcasms.

The transcripts enabled the interviewer to also reconcile inconsistent and ambiguous responses. These responses were collated with other statements and observations made in the interviews. Anomalies most often emerged because of problems associated with communicating in English as an additional language. For example, Participant 1 stated, they "*were not good at mathematics*". However, when reconciled with their in-tasks responses and backgrounds it was interpreted, as *they didn't like learning mathematics at school*. All background and in-tasks interviews were concluded, transcribed, and checked for accuracy before uploading on the data management system for synthesis.

Figure 3.11 depicts a sample of a transcript and open-ended questioning technique that emerged from interviewing Participant 3 in Stage 2. The response identified, in part, how Participant 3's background experiences and perceptions affected them decoding and learning M1 and M2.

Case/participant 3

19. Did/do you feel happy with the amount of mathematics you learnt in English in these classes?

Time: 43:22.7

(Interviewer) *So, what were you maths teachers like overseas - were they specialist maths teachers?* (An incidental question that flowed on from Question 19)

> (Participant) *Yeah every subject had specialist teacher* (Secondary School).
How did you find them?
>They were organised from the office. For maths they would find special teacher. Every subject separate teacher for each subject.
What about your maths teacher in high school were they good or bad – (can you tell me about) *your experiences in maths classes?*
>Actually, it wasn't good for me - a bad experience for me. The teachers you are not allowed to talk. If you asked can you show me again or can you show me this way they would kick you out of the class.
So, it was pretty tough?
>Yeah, very tough. For example, if the teacher did a mistake and you solved or found the right way they would kick out of the class.
Not like my class (both laugh and smile)? (Participant was enrolled in the Interviewers ESL class for one semester).
>Yeah. Yeah, they don't (pause)
Work like that?
> They would abuse xxxx (comment deleted). *You had to listen and not talk. For example, many years ago when I was in class in 2009* (overseas) *the teacher was teaching mathematics and the student was a very high level in maths, but he saw a mistake and told the teacher 'excuse me but you went the wrong way and this way is better and easier to learn for the class', ' ah', he said, 'you are teaching me. Come and stand here and teach us and I am going to sit in your place'. And, he* (the student said) *said, 'why'. After class he* (teacher) *called the principal and he said, 'this boy is arguing with me as I am teaching the rest of the class'. This is why they kicked the guy out from that class.*
So, you would never say anything in class?
>Even if you say something wrong they would xxxxxx (comment deleted).
Tough?
>Yeah, very tough but not here.

Figure 3.11: Sample Stage 2 transcript Question 42 Participant 5.

Step 3 uploaded the transcripts onto the qualitative data management software for synthesis and coding. The themes identified in the battery of open-ended questions for Stage 2 also functioned as organisational themes to, respectively, help conceptualise, group, and link the data observations from different research domain perspectives. The organisational themes were then interpreted and coded into sub-organisational data themes. Sub-organisational themes identified patterns and relationships in participants' BL experiences that affected organisational themes. The process helped synthesise the large quantity of collected qualitative data.

Figure 3.12 depicts the coded organisational and sub-organisational themes that emerged in Stage 2. The prefix '*INT*' identified the observations that emerged from Stage 2 interviews. The first number in the 4-digit code, for example, *1-7000,* linked the questions and responses to the organisational themes generated by the battery of open-ended questions. The second and third digits identified the sub-organisational themes emerging through synthesising the transcripts. The last two digits, for example, *INT13001-2*, identified the participant and number of contributions they made to the data observation.

Stage 2 Data Themes

INT1000 Participant experiences in education, language, and mathematics that affect how M2 is (problematic to) decoded.

- *INT1100 Participant experiences in decoding L1 that affect how M2 is decoded.*
- *INT1200 Participant experiences in decoding in M1 in a L1 context that affect how M2 is decoded.*
- *INT1300 Participant experiences in decoding L2 in a L1 context that affect how M2 is decoded.*
- *INT1400 Participant experiences in decoding ML in a L2 context that affect how M2 is decoded.*
- *INT1500 Participant experiences in decoding ML across L1 and L2 that affect how M2 is decoded.*
- *INT1600 Participant experiences in a L2 classroom that affect how M2 is decoded.*

INT2000 Participant perceptions on language, education, and Australian culture that affect how M2 is decoded.

- *INT2100 Participant perceptions on language, education, & Australian culture reconciled from a cultural perspective that affect how M2 is decoded.*
- *INT2200 Participant perceptions on language, education, & Australian culture reconciled from an economic perspective that affect how M2 is decoded.*
- *INT2300 Participant perceptions on language, education, & Australian culture reconciled from a personal and behavioural perspective that affect how M2 is decoded*

INT3000 Participant perceptions of mathematics that affect how M2 is decoded.

- *INT3100 Context related factors and beliefs that affect how BL readers read M2.*

INT4000 Participant experiences in learning to read M2 that affect how M2 is decoded.

- *INT4100 Participant M1 learning experiences that affect decoding M2.*
- *INT4200 Participant M2 decoding experiences that affect decoding M2.*

INT5000 Participant perceptions of ML structures that affect how M2 is decoded.

- *INT5100 Participant perceptions on truth and modality in ML that affects how M2 is decoded.*

- *INT5200 Participant perceptions on learning to read M1 compared to L2 that affect how M2 is decoded.*

INT6000 Participant behaviours that affect how M2 is decoded.

- *INT6100 Participant physical type experiences in learning to read M1 that affect how M2 is decoded.*
- *INT6200 Participant physical experiences in learning to read M2 compared to M1 that affect how M2 is decoded.*

INT7000 Participant perceptions on age, thinking, and adulthood that affect how M2 is decoded.

- *INT7100 Participant perception on playing games.*
- *INT7300 Participant perceptions on changing while living in Australia that affect how M2 is decoded.*
- *INT7400 Participant perceptions on changing with age that affect how M2 is decoded.*
- *INT7500 Participant perceptions on creativity that affect how M2 is decoded.*

Figure 3.12: Stage 2 coded organisational and sub-organisational themes.

Figure 3.13 depicts a section of Participant 1's interview that generated *INT13001-2: Participant experiences in decoding L2 in a L1 context that affects how M2 is decoded*. The observations identified, in part, how Participant 1's L1 experiences and behaviours affected decoding and learning M2. The synthesis interpreted data from multiple domain perspectives to code themes in this context.

13/04/2013 1:06

Case study of Semiotic and Linguistic Factors that create problems for adult-CALD

Readers.

Coding summary by source\ INT1300participant experiences in decoding L2 in a L1 context that affect how M2 is decoded

Nodes\\Case 1\\Case 1 interview\\Case 1 interview contexts in which semiotic & linguistic structures in ML create problems for adult-Cald learners when learning M2 in L2\1000 Learner experiences in education, language, and mathematics that affect how M2 is decoded\INT1300participant experiences in decoding L2 in a L1 context that affect how M2 is decoded

Case Interview No 1 .1135 33

4 A 14/12/2012 1:36PM

(Interviewer) *How old were you when you started learning English?*
(Participant) > *Thirteen.*
Okay you started in high school?
>In middle school in XXXX.

26 A 14/12/2102 2:50PM

Did you study mathematics at school?
Yes, since in basic (primary school).
Did you study it in high school?
> Yes, it is kind of compulsory.
What year did you stop doing mathematics (at school)*?*
> No, I didn't finish until I graduated from high school.
Is this compulsory?
> Yeah, until the end of high school.

27 A 14/12/2012 2:57 PM

Did you like doing mathematics at school?
(Dwells and moves her head side ways and indicates no).
Good question, why?

3 A 14/12/2013 5:43 PM
> In XXXX in university when I studied I always-read kind of YYYYY magazines because they show a lot of trends.

30 A 14/12/2013
What type of mathematics did you learn in your English classes?
So what type of mathematics did you learn in your English classes? Did you do any numbers, and I don't mean calculus or anything like that?
> Last time you showed some web sites (had participant only once or twice in my class) *and it showed how to speak with numbers.*
And that was me?
> Yes, it was you.
So, I must have been the only one to introduce numbers (mathematics) in language.
> (Gives a strong laugh). Yeah, usually teachers didn't teach us how to use numbers. But, I think level I or II (ESL course)*. I did some. Yeah, very basic.*

Figure 3.13: Sample synthesis and articulation Participant 1 transcript into INT13001-2.

Step 4 summarised the sub-organisational themes into *observations, interpretations,* and *implications*. Figure 3.14 depicts the summary recorded for Figure 3.13 above: *INT13001-2 Participant experiences in decoding L2 in a L1 context that affect how M2 is decoded* in. Step 4 generated 14,000 words of summarised coded data observations, interpretations, and implications for Stage 2.

Coding Summary By Source

Case study of Semiotic and Linguistic Factors that create problems for adult-CALD Readers.

13/04/2013 1:06 PM

Classification. Aggregate. Coverage. Number of coding. Reference No. Coded By. Modified On

Internals\\Background\\7. Open-ended interview summaries
Node
Nodes\\Semiotic summaries\\INT Open-ended interview summaries

INT13001-2 participant experiences in decoding L2 in a L1 context that affect how M2 is decoded:
(Observation) participant 1 acquired L2 and M2 for short periods of time in two contexts: high school in L1, and now in an adult ESL-migrant class in L2 for less than 2 years

(Interpretation) M2 can be acquired in different contexts over different periods of time. Participant 1 experienced decoding L2 and M2 in a L1 context for a short period of time in secondary school. BL experiences are also affected by contents, experiences, and the amount of time readers are exposed to a language. Factors associated with language immersion are critical in understanding how well L2 and M2 are acquired as joint construct.

(Implication) L and S structures are problematic to read in M2 if BL readers are unable to immerse themselves within an appropriate space when acquiring M2.

Figure 3.14: Sample coding and synthesis INT13001-2.

Implications

Stage 2 observed and interpreted the participants' BL experiences that affected decoding and learning M2. The interviews were recorded, transcribed, and uploaded onto data management software for synthesis and thematic coding. The instruments and techniques have the capacity to include observations and participants from ongoing research. The techniques are envisaged for areas of education research where complex C, L, and S experiences have to be interpreted.

3.6 Stage 3 data collection

Stage 3 collected data from in-tasks observations.

Rationale

Stage 3 complemented Stages 1 and 2 data observations by observing the participants' in-tasks decoding and learning behaviours that created problems in M2.

Method

Stage 3 collected and synthesised data in 4 steps. Step 1 employed the 59 questions generated in the battery of open-ended questions to observe and question participants during their tasks. The 13 observational questions were written up after the in-tasks interviews were completed. The in-tasks interviews were conducted one-on-one and digitally recorded.

The participants were encouraged to read and answer the tasks one at a time, before discussion and support was given for individual tasks. This enabled the interviewer to observe the participants' competencies, physical responses, and discourse behaviours within the different L and S structures. The discourse also identified background information such as differences between M1 and M2. The 2-3 hour in-tasks interviews were conducted a week after Stage 2 background interviews. This enabled follow-up observations, concerns, and discussions to be addressed from the previous week.

Stage 3 employed the battery of open-ended interview questions differently than Stages 1 and 2. Firstly, the participants' capacities varied and, therefore, attracted different lines and levels of questioning. For example, the multilingual Participants 3, 4, and 5 often questioned the tasks from multiple linguistic perspectives by comparing M2 across more than two languages. Secondly, addressing all the battery questions would have distracted the participants from focusing on completing the tasks. As a result, the interview questions were kept nearby to prompt questions when needed, but did not drive the interviews. The 13 observational questions generated for Stage 3 played a more strategic role later on in interpreting behaviours after all the interviews were finalised.

Step 2 transcribed the oral recording made in-tasks interviews onto the participants' tasks response sheets. This generated 15 - 20,000 words of typed transcripts for each participant. The transcripts also recorded response times and paralinguistic details. Figure 3.15 depicts an example of a transcript generated by Participant 2 for part of Task 4. The in-tasks interviews were finalised and transcribed before uploading onto the data management software for synthesis and coding.

***Participant* 2 Task 4**

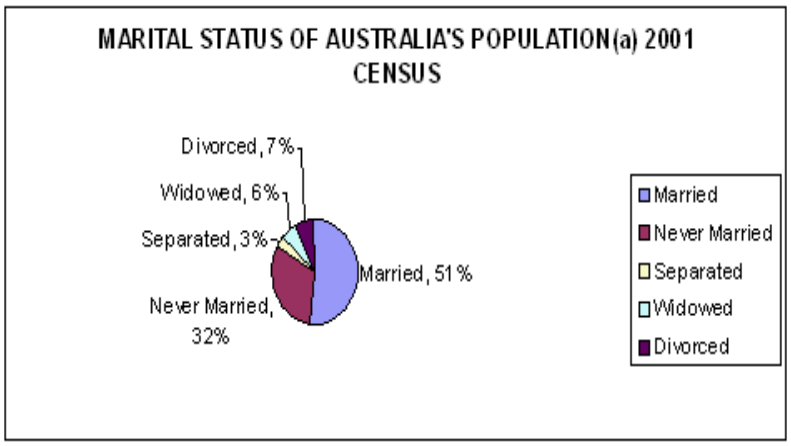
4. Below is a chart showing information about Australians. Answer the following questions:

a. What is the information about?

b. How is the information presented?

c. What is the significance of the information?

e. Are there any trends in the data?



(18:49)

(Participant) > *I can't remember what some of the words.*
(Interviewer) *You can ask me, and maybe I might be able to help you. So, what are you looking at?*
>Percentage of living or lifestyle.
Okay, that is 'marital' we are looking at, and 'status' means how people are. And, is sometimes represented by figures. (Time delay) Marital is how we see people married or not married (Prompt). I did notice you looked at the questions, underlined the how, and what type things, including (the word) *trends. You put a tick next to the symbol* (%) *- how come you put a tick there next to the 32? Do you understand what that symbol is – percentage????*
>Yeah.

So you put a tick, and looked at the grid (legend).
>The first time I match each bit of information.
In the box?
>Yeah, then the pie thing, and then I saw some numbers in which one is the biggest and the lowest one.
Okay, so you went through to the biggest numbers and sorted them out by looking at them (first).
Are you familiar with pie charts (nods)? Have you seen these types of symbols and figures before?
>I think so.
Yeah, I can imagine. So, you didn't have any problems understanding the way the information is presented?
>No (noted at interview moved and nod head instead of speaking).
Lets have a look at some of the things that are actually in the chart. (Delay) Anything surprises you in there, like 51% are married, 32 never married, 3% are separated?
>hmm hm.
6% widowed, and 7% divorced, is there anything surprising about those figures?
>Never married?

(Noted: Participant, generally, had no problems decoding and completing task, just terminology.

Figure 3.15: Sample Participant 2 Stage 3 in-tasks transcript Tasks 4.

Step 3 uploaded the transcripts onto the data management software. Its synthesis and coding, however, emerged more pragmatically than Stages 1 and 2. The participants' decoding and learning behaviours varied in and across the tasks and, therefore, data fitted less precisely into the organisational themes identified in the battery of open-ended questions. For example, not all participants responded to or alike to Question 22: *structural meanings - can you see patterns in the way the symbols are used in this task?*

The 13 post-interview observational questions generated by the battery of open-ended questions emerged and played a more strategic role in coding data into the organisational themes for Stage 3. The questions were originally envisaged to prompt interviewer memos and post-task observations. However, their wording and structure also functioned as organisational codes for interpreting task responses and associated discussions. The 13th organisational theme emerged during the synthesis to identify an additional thematic line of questioning. The observational questions were, respectively, reworded to more precisely reflect the data observation. For example, Stage 3 observation Question 1 was reworded:

> **From:** *In task observation - how, when, and what type of participant questions occurred.*

To: *TAS105A2* the *participant questioning and discussion processes that affected how M2 is decoded.*

Figure 3.16 identifies the 13 organisational codes that emerged as data themes in Stage 3. Stage 3 codes were defined as follows. *TAS* identified the data as in-tasks observations. The first three-digit defined the organisational theme. '*A*' identified the data was interpreted as observational question. The last digit, for example, '*2*' in '*TAS105A2*', identified Participant 2 as the contributor.

Stage 3 codes

1. *TAS101A Participant perceptions of ML that affect how (L and S structures) M2 is decoded (and becomes problematic).*
2. *TAS102A Participant competency factors that affect how M2 is decoded.*
3. *TAS103A Participant support factors that affect how M2 is decoded.*
4. *TAS104A Participant BL decoding processes that affect how M2 is decoded.*
5. *TAS105A Participant questioning and discussion processes that affected how M2 is decoded.*
6. *TAS106A Modalities associated processes that affect how participants read M2.*
7. *TAS107A Participant physical interaction and processes that affect how M2 is decoded.*
8. *TAS108A Participant reader characteristics that affect how M2 is decoded.*
9. *TAS109A The participant's ability to read semiotic cues that affect how M2 is decoded.*
10. *TAS110A Participant decoding reactions that affect how L and S resources are read in M2.*
11. *TAS111A Participant displays of discomfort that affect how M2 is decoded.*
12. *TAS112A Participant displays of specific anxieties that affect how M2 is decoded.*
13. *TAS113A Participant BL code switching processes that affect how M2 is decoded.*

Figure 3.16: Stage 3 TAS coded data themes.

Step 3 transposed (cut and pasted) sections of the transcripts into their respective observations under TAS organisational themes. The organisational themes were reworded to more precisely reflect the emerging data. The process enabled the participants' decoding and learning behaviours to also be compared across tasks and with other participants. Figure 3.17 depicts the summary generated for Participant 2 TAS105A2 Tasks 3, 7, and 8.

Nodes\\participant\Case 2 tasks observations\ processing observation\ tasks observation interpretation

TAS105A2 questioning and discussion related processes that affect how M2 is decoded.

Task 3
~~Case 2~~ Participant 2 asked questions about a significant number of word meanings. Case 2 questions appeared simplistic compared to the M knowledge they had (displayed). They were able to explain themself, nevertheless.

Task 7
~~Case 2~~ Participant 2 did not ask questions and generally responded with 'yeah/no' type answers.

Task 8
~~Case 2~~ Participant 2 did not ask many questions and generally responded with 'yeah/no' type answers. Nonetheless, ~~Case 2~~ Participant 2 was able conceptualised the task at a sophisticated level. Their discourse, however, was relatively unsophisticated.

Figure 3.17: Sample Stage 3 data observations merged for Participant 2 code TAS105A2 Tasks 3, 7, & 8.

Step 4 articulated the coded in-tasks observations into *summary observations*, *interpretations*, and *implications*. Figure 3.18 depicts the summary generated for Participant 2 *TAS105A2: Questioning and discussion related processes that affect how M2 is decoded*. Stage 3 generated a total of 11,000 words of coded summarised observations, interpretations, and implications for this stage of data collection.

Coding Summary By Source

Case study of Semiotic and Linguistic Factors that create problems for adult-CALD Readers.

Number Of Coding References Reference Number Coded By Initials Modified On

13/04/2013 1:06 PM

TAS105A2 questioning and discussion related processes that affect how M2 is decoded.

Data observations:

Participant ~~case~~ 2 did not ask many questions other than for word meanings, and generally responded with 'yeah/no' type answers. The interviewer also appeared to do a fair amount of talking within tasks. Nonetheless, Participant ~~case~~ 2 was able conceptualised the tasks at a sophisticated level while their discourse was relatively unsophisticated.

Interpretation:

It is difficult to interpret BL questioning and discussion related decoding processes in M2 because (for a variety of reasons, e.g., linguistic, competency, experience, and culture) BL readers most often don't say much. Further, the lack of interactive discourse is not a good

indicator of how accurately BLs read M2. However, the problem is articulated when a reader doesn't have a sophisticated level of M, and is unable to interact appropriately to acquire it in M2. Case 2 discourses (in L2 and M2) were relatively unsophisticated compared to how they read M1 and M2. Nonetheless, they were able to read M2 accurately despite the short period of time (2 years) they have lived in Australia.

Implications:
L and s structures in M2 can be problematic to read if BL readers do not interact with questions and discussions, even if they have a sophisticated understanding of M. An (educational) appropriate space is needed for M2 readers to learn how to ask questions and have discussions when decoding M2 (even) at an unsophisticated level.

Figure 3.18: Sample coding summary Participant 2 Code TAS105A2 Stage 3.

Implications

In summary, Stage 1 defined the L and S structures in M2 that were conceptualised as being problematic in decoding and learning M2. Stage 2 defined the BL decoding and learning experiences that emerged and created problems. Stage 3 complemented Stages 1 and 2 observations by interpreting actual BL decoding and learning behaviours. The terms BL and decoding more precisely defined adult-CALD reading behaviours in M2.

The subsequent sections identify the instruments and techniques employed to synthesise and articulate the coded data themes from Stages 1, 2, and 3 into a single data set and research findings.

3.7 Data matrix

A Data Matrix (Appendix F) merged data from the three stages of coded observations into a single data set. The instrument enabled a deeper interpretation of the global themes that emerged and defined the research problem.

Rationale

The Data Matrix provided a coded link between the domains generating the questions, the observations that emerged from the three stages of data collection, and the findings that, respectively, answered the research problem and questions.

Method

The Data Matrix merged organisational and sub-organisational coded summaries into global themes in two steps. Step 1 posted the three stages of data observations, interpretations, and implications in respective order onto a single Excel spreadsheet for synthesis and global coding. The columns of the matrix, respectively, identified the coded sub-organisational which were organised into themes, observation, interpretation, and educational implication.

Step 2 interpreted global themes by ranking and pivoting the rows of coded matrix from different theoretical perspectives. Global themes defined the "macro arguments", "positions", and "assertions" that emerged to interpret and define the research problem and questions (Attride-Stirling, 2001, p. 389). Global themes were first interpreted through word maps and the frequency count tools provided by Data Management Software, however, these observations were fragmented and inconclusive for the large qualitative data sets collected. Ranking, alternatively, repositioned and pivoted the three sets of organisational themes {*TXT1000-6000, INT1000-7000, TAS101A-113*} into different sub-ordinate and super-ordinate positions and relationships in the Data Matrix. For example, complementary observations observed in *TXT* and *INT* were transposed into subordinate data observations under *TAS* (In-task Observations) to interpret the research problem. In this context *TXT* and *INT* decoding and learning experiences helped interpret the participants' behaviours *TAS* in the tasks.

Continuous reinterpretation of the Matrix identified the six *TXT* organisational themes from Stage 1 (Semiotic Synthesis of Mathematical Tasks) as global data themes to collate the data sets into interpreting and answering the research problem and questions. The final proposition articulated the research problem and questions, foremost, as a TXT-based learning problem. Respectively, Research Question 1 asked as a TXT question w*hat are the linguistic and semiotic structures in mathematical language that create problems for adult-CALD bilingual learners when learning mathematics in English as an additional language;* and Research Question 2 addressed *how can knowledge of adult-CALD bilingual learner related problems in linguistic and semiotic structures in mathematics be used to enhance mathematics*

education in Australian classrooms? The choice, however, was theoretically driven to articulate the research problem and questions as a TXT problem and potential solution (Section 3.9). For example, *INT2002-1* (*Participant 2 observation 1 experiences in decoding in M1 in a L1 context that affect how M2 is decoded)* and *TAS106A2* (*Modalities associated processes that affect how participant 2's read M2)* were ranked and interpreted as subordinate relationships in *TXT2000* (*Structural codes that are problematic to decode in M2). TXT2000* identified the global theme that emerged and subsumed data observations from *INT12002-1* and *TAS106A2*.

Figure 3.19 depicts the 6 global themes that emerged and interpreted data as a TXT relationship. Their relationship and implications are defined in greater detail in the Data Synthesis (Chapter 5).

1. ***(TXT1000) Philosophical codes that are problematic to decode in M2.***
2. ***(TXT2000) Structural codes that are problematic to decode in M2.***
3. ***(TXT3000) BL decoding behaviours that affect how M2 is decoded and learnt.***
4. ***(TXT4000) BL humanistic behaviours that affect how M2 is decoded and learnt.***
5. ***(TXT5000) BL cultural behaviours that affect how M2 is decoded and learnt.***
6. ***(TXT6000) BL educational experiences and affect how ML is coded and become problematic to read in M2.***

Figure 3.19: Global data themes.

Implication

The Data Matrix was ranked and pivoted from alternative theoretical perspectives to interpret and articulate global themes and relationships in data. The final relationship, however, reflected the focus of the research questions as a TXT problem and solution. It is envisaged the research questions can also be reworded from a TAS and INT perspective to articulate different global themes and solutions. As Holland (1990) stated, "for the reader-response critic, as for the modern physicist probing the atom, the answer you get depends on the question you ask" (p. 56). The study reconciled its approach by coding its questions and procedures, and linking them to the findings as a TXT problem and solution.

3.8 Narrative

The study coded its narrative to recount the findings and conclusions in Chapters 5 and 6. The procedure enables the reader to interpret and unpack the meanings and propositions that emerged (Section 2.6.2).

Rationale

A narrative, Crystal (1993) stated, most often retells a story in the following way:

> The setting has three components: the characters, a location, and a time. The theme consists of an event and a goal. The plot consists of various episodes, each with its own goal and outcome. Using the distinctions of this kind, simple stories are analysed into these components, to see whether the same kinds of structures can be found in each. Certain similarities do quickly emerge; but when complex narratives are studied it proves difficult to devise more detailed categories that are capable of generalization, and analysis becomes increasingly arbitrary. (p. 119)

The study coded its complex story about learning M2 in a cross-C, L, and S context to enable the reader to interpret the events.

Method

The narrative recounted in Chapter 5 emerged in 2 steps. Step 1 summarised the 40,000-word Data Matrix (Appendix F) into a 3600-word Summary Matrix (Appendix G) of coded observations, interpretations, and implications. This helped interpret the overarching setting, theme, plot, and resolutions (Crystal, 1992, p. 119) that emerged and defined the research problem. The Data Matrix (Appendix F) identified the details of the subplots, contents, circumstances, and process in the story.

The Data Matrix headings *observations*, *interpretations*, and *implications* were reworded to *meaning and significance*, *incongruence*, and *implications for teaching* to more precisely reflect the research problem and questions. For example, *TXT1000 meaning and significance* identified the ideological values and beliefs that encoded meanings and created problems in M2. *TXT1000 incongruences* identified the

characteristics of the problem, such as: the ideological meanings that differentiated the M2 sign system from M1 and created problems for BL learners. *TXT1000 implications for teaching* identified foci for teaching the philosophical conventions that encoded meanings. In this context *TXT1000* addressed Research Question 1 by identifying the philosophical codes *that create problems when decoding and learning L and S structures in M2. Implications for teaching* addressed Research Question 2 by identifying the philosophical codes that needed to be learnt when *teaching M2 in Australian classrooms*.

Step 2 recounted the narrative in Chapters 5 and 6 through the coded global, organisational, and sub-organisational themes that emerged in the Summary and Data Matrices. This enables the reader to, respectively, interpret and scrutinise the meanings that were articulated. As Schutz (1932) stated:

> Now since the words chosen by the speaker may or may not express his meaning, the listener can always doubt whether he is understanding the speaker adequately. The project of the speaker is always a matter of imaginative reconstruction for his interpreter and so is attended by a certain vagueness and uncertainty. (p. 39)

Coding, however, enables the reader to reconcile the inter-subjective meanings that were articulated in the qualitative narrative.

Implications

Global, organisational, and sub-organisational data themes helped recount the setting, theme, plot, and resolutions emerging in the research problem. It is envisaged the procedures help other areas of education also having complex stories to recount.

3.9 Data collection critique

Two checklists were developed to critique and make recommendations for enhancing ongoing research. The implications also helped answer Research Question 3: *how can case-based interpretive research methodology deepen our understanding of adult-CALD bilingual learners who are engaging in linguistic and semiotic structures in mathematics as an additional language?*

Checklist 1 Table 3.1was adapted from Braun and Clarke's (2006, p. 96) 15-point Checklist of Criteria for Good Thematic Analysis to critique and offer advice on thematic coding. Braun and Clarke's (2006, p. 96) checklist was, however, modified in two ways to suit the specificities of this study. Firstly, the term synthesis replaced analysis to reflect the study's synthetic methodology. Secondly, a summary column was added to critique and make recommendations for enhancing future research.

Table 3-1: *15-point checklist for conducting a good thematic.*

Process/activity	**No.**	**Criteria**	**Summary**
Transcription	1	The data have been transcribed to an appropriate level of detail, and the transcripts have been checked against the tapes for 'accuracy'.	✔ Yes: Transposed per verbatim (in English) in a cross-C, L, & S study.
Coding	2	Each data item has been given equal attention in the coding process.	✔ Yes: **But** thematic coding e.g., global and organisational themes ranked data from a theoretical perspectives rather than their ordinal significance in causing the problem.
	3	Themes have not been generated from a few vivid examples (an anecdotal approach), but instead the coding process has been thorough, inclusive and comprehensive.	✔ Yes: Codes emerged equally and synthetically from multiple stages and perspectives. E.g., ML reflected agency and a participatory voice.
	4	All relevant extracts for all/each theme have been collated.	✔ Yes: **But** a synthesis meant extracts merged together rather than analysed through discrete elements.
	5	Themes have been checked against each other and back to the original data set.	✔ Yes: Coded, linked and read backwards. **But** propositions could be more vigorously checked in the future via logic software.
	6	Themes are internally coherent, consistent, and distinctive.	✔ Yes: **But** language articulates meanings that may sometimes be lost.

Synthesis (analysis)	7	Data have been synthesised -interpreted, made sense of - rather than just paraphrased or described.	✔ Yes: Synthesis generated a holistic interpretation of data.
	8	Synthesis and data match each other - the extracts illustrate the synthetic claims.	✔ Yes: Data Matrix and Summary maintained links.
	9	Synthesis tells a convincing and well-organized story about the data and topic.	✔ Yes: **But** meanings always have a subjective element (Schutz, 1945), however coding the narrative helped.
	10	A good balance between synthetic narrative and illustrative extracts is provided.	✔ Yes: research findings are coded and examples cited.
Overall	11	Enough time has been allocated to complete all phases of the synthesis adequately, without rushing a phase or giving it a once-over-lightly.	✔ Yes: Through a combination of manual and data software.
Written report	12	The assumptions about, and specific approach to, thematic synthesis (analysis) are clearly explicated.	✔ Yes: **But** a synthesis approach works differently with the assumptions that are made.
	13	There is a good fit between what you claim you do, and what you show you have done – i.e., described method and reported synthesis are consistent.	✔ Yes: methodology cited and made explicit.
	14	The language and concepts used in the report are consistent with the epistemological position of the analysis.	✔ Yes: Language vigorously and consistently defined.
	15	The researcher is positioned as active in the research process; themes do not just 'emerge'.	✔ Yes: Declared, **but** not always made explicit.

-Adapted from Braun and Clarke's (2006, p. 96) 15-point Checklist of Criteria for Good Thematic Analysis

A review of Criterion 2 identified thematic coding transposed qualitative data into ordinal and causal relationships that were pseudo-quantitative in nature. However, coding and ranking qualitative data was a theoretical and not a quantitative choice. For example, the study interpreted the research problem as a TXT problem, and INT and TAS observations were coded as subordinate ordinal themes in TXT. This did not mean INT and TAS observations were empirically less important or problematic than TXT. Studying the effect interpretative and thematic coding techniques has on

transposing qualitative data into ordinal and pseudo-quantitative relationships forms part of ongoing research.

Criteria 4 and 12 identified that merging data through synthesis was intricate and complex to undertake. An analytical approach might have, for example, been quicker and less complex, as it interprets data from discrete and predetermined theoretical propositions (Gharajedaghi & Ackoff, 1985). However, an analytical approach would also emerge at an epistemological cost, because it pays less attention to merging, contrasting, and comparing data (Gharajedaghi & Ackoff, 1985). A synthetic approach, as a result, generated 158 interrelated teaching implications in the Data Matrix. The implications were synthesised as individual lines of questioning and reasoning with if and then logic statements, and then globally across the different implications that affected teaching M2. For example, the meaning and implication of the English language as a medium for teaching and learning M2 was synthesised, in part, as follows:

1. Stage 1 battery Question 25 (*Do the signs adopt opposition and markedness to generate meanings, and how does this factor create problems for BL readers when decoding M2?*) articulated horizontally into the teaching implication *TXT2025* (*L and S structures are problematic to read in M2* ***if*** *BL readers are unable to reconcile how the English medium generates meanings through opposition and markedness).*
2. The implication of *TXT 2025* was contrasted and merged vertically with similar observations in the data matrix, for example: *TXT2029 (L and S structures are problematic to read in M2 if BL readers are unable to reconcile the sign system most often can be unpacked and read backwards, however, the context and medium is problematic because they elicit irreversible meanings).*
3. While there was a subtle difference between the implications generated by *TXT2025* and *TXT2029*, both identified the problematic nature of English as a medium for teaching M2 in a cross-C, L, and S context.
4. The meaning and significance of the medium in *TXT2025* and *TXT2029* was then interpreted in the context of the other L and S conventions that encoded M2. Their synthesis articulated into the global implications that affected teaching and learning (Chapter 5).

Criterion 5 identified that the propositions were synthesised mentally with if and then statements, however, digital software would have enhanced the speed and accuracy of this part of the synthesis. Suitable software was not found for this area of the study, and its development forms part of enhancing future research.

Criteria 6, 9, and 15 identified the L and S science and language chosen to interpret data was significant in shaping the meanings that emerged. The form and function of L and S as a science and language, however, needs to be further explored to interpret their full implications for conducting case-based interpretive research. This investigation serves to substantially improve ongoing research.

Checklist 2 Table 3.2 was developed to conceptualise and evaluate the alignment of the data collection stages, instruments, and techniques that were employed. Two critical observations and recommendations emerged from this checklist to enhance ongoing research.

Research Question 3 was interpreted through the findings of Research Questions 1 and 2, however, it could also have been more effectively interpreted and questioned through the battery of open-ended questions generated to collect and interpret data. For example, *TXT2025 (Do the signs adopt opposition and markedness to generate meanings, and how does this factor create problems for BL readers when decoding M2?*) could have been articulated into a methodological-related battery question: *Does the M2 medium affect the interpretation and case-based research methodology when studying the problem of adult-CALD bilingual learners who are engaging in linguistic and semiotic structures in mathematics as an additional language?* The question would have more precisely interpreted and defined the role of the medium in answering Research Question 3.

Table 3-2: *Data critique and alignment table.*

Data activity **Research questions**	**1. Synthesis of existing literature and research. (200 Hrs.)**	**2. Compile battery of opened-ended questions. (120 Hrs.)**	**3. Stage 1 Semiotic synthesis of 15 M2 Tasks. (120 Hrs.)**	**4. Stage 2 open-ended interviews 5 participants. (40 Hrs.)**	**5. Stage 3 In tasks observations 5 participants. (40 Hrs.)**	**6. Transpose 10 oral recordings into transcripts. (80 Hrs.)**	**6. Synthesise data with management software. (200 Hrs.)**	**7. Compile Data and Summary Matrix. (200 Hrs.)**	**8. Synthesise the narrative of research findings. (200 Hrs.)**
1. What are the linguistic and semiotic structures in mathematical language that create problems for adult-CALD?	✔ **(Yes)** Identified, classified, and characterised potential ways of interpreting data.	✔ **Yes)** Articulated existing knowledge into data collection instruments.	✔ Defined M2 contents.	✔ Identified backgrounds experiences as BL contexts.	✔ Identified BL behaviours, and competency related factors as processes.	✔ Generated a space to conceptualise and interpret data more deeply.	✔ Enhanced the interpretation of complex and non-discrete data.	✔ Enhanced the synthesis and articulation of data into research answers.	✔ Articulated data observations and synthesis into narrative form.
2. *How* can knowledge of adult-CALD *bilingual* learner related problems in linguistic and semiotic structures in mathematics be used to enhance mathematics education in Australian classrooms?	✔ Identified, classified, and characterised existing knowledge.	✔ Articulated existing knowledge into data collection instruments.	**(Partially answered)** Identified potential points and junctures were problems arise.	✔ Direct relevance. Identified background related factors.	✔ Direct relevance. Identified process, competency and performance related factors.	✔ Enhanced the interpretation of cross-C, L, and S data observations.	✔ Enhanced the interpretation of quantity and quality of data.	✔ Enhanced the synthesis and articulation of data into research answers.	✔ Articulated observations and synthesis into narrative form
3. How can case-based interpretive research methodology deepen our understanding of adult-CALD bilingual learners who are engaging in linguistic and semiotic structures in mathematics as an additional language?	✔ Identified, classified, and characterised existing research approaches.	**X** Could have addressed Question 3 more directly through the battery of open-ended questions**!**	**(Summative)** Contributed to study's overall interpretation of the research question.	**(Summative)** Contributed to study's overall interpretation of the research question.	**(Summative)** Contributed to study's overall interpretation of the research question.	✔ Enhanced the interpretation of cross-C, L, and S based observations.	✔ Enhanced the interpretation of cross-C, L, and S based observations.	✔ Enhanced the synthesis and articulation of cross-C, L, and S based data.	✔ Contributed to interpreting the role and meaning of the narrative.

Table 3.2 also summarises the time spent on conducting the study. The data collection and synthesis stages accounted for 40% of the total 3200 hours spent on the project. The balance involved undertaking associated tasks such as writing a research proposal, editing chapters, and exploring and developing the data collection instruments and techniques that were employed. The lack of established research in the area meant, however, a significant and unforseen amount of time (350-400 hours) was spent on developing the data collection instruments and techniques. Ongoing research will appropriately factor the time required for refining the data collection instruments that were developed in this study.

In summary, the two checklists identified appropriate interpretive and thematic research procedures were employed in the study. The potential areas identified for ongoing improvement are synthesised in greater depth in Chapter 6 Section 6.8 Implications for Future Research. The areas include:

1. Defining and exploring the meaning and significance of ranking thematically coded qualitative data into pseudo-quantitative observations.
2. Developing logic-based software to help synthesise and articulate qualitative data into theoretical propositions.
3. Studying in greater depth the role and function of L and S science and language in the interpretation and articulation of qualitative data.
4. Aligning methodologically related research questions more precisely to the research problem. Battery questions could also be generated to interpret methodologically driven problems and questions.
5. Refining the time and resources required to develop interpretive and synthetic research instruments and techniques.

3.10 Chapter summary

The battery of open-ended questions served to collect and interpret data through multiple stages and theoretical perspectives to understand the research problem holistically. Stage 1 conducted a semiotic synthesis on the mathematical tasks employed to observe and interpret participant in-tasks decoding and learning behaviours in Stage 3. Stage 2 interpreted the participants' background BL experiences that affected decoding and learning M2.

Five adult-CALD BL participants were recruited from a regional TAFE college in Australia for Stages 2 and 3 of the study. The one-on-one interviews generated a more precise interpretation of the BL behaviours and experiences in the research problem.

Thematic coding techniques synthesised and articulated the data into global observations, interpretations, and implications. A thematic approach, as Braun and Clarke (2012) stated, "summarizes key features of a large body of data" and offered "a thick description of the data set" (p. 97). The Data Matrix interpreted the coded data from different theoretical perspectives to articulate its narrative of research findings (Chapters 5 & 6).

Two observational checklists confirmed that appropriate interpretive and thematic coding procedures were followed. Their critique identified areas for ongoing improvement in case-based interpretive synthetic research. It is envisaged the case-based interpretive procedures and techniques developed in this study also help other areas of education that have complex language-learning problems to resolve.

Chapter 4: Synthesis and Articulations of Existing Literature and Research into the Battery of Open-ended Data Collection Questions

> We are thus introduced to a new principle of relativity, which holds that all observers are not led by the same physical evidence to the same picture of the universe, unless their linguistic backgrounds are similar, or can in some way be calibrated.
>
> (Whorf, *Language, thought, and reality: selected writings of Benjamin Lee Whorf*, 1956, p. 214).

4.1 Chapter introduction

This chapter recounts the synthesis and articulation of existing literature and research in which informed the battery of open-ended questions that helped collect and interpret data. The chapter, sequentially, recounts:

1. The objectives and methodologies employed to synthesise existing literature and research in the study (Gharajedaghi & Ackoff, 1985; Kant, 1788; Onwuegbuzie, Leech, & Collins, 2012; Patton, 1990).
2. The meaning and significance of the terms mathematics, language, adult-CALD bilingual learners, culture, Australian, and education as elements of the research problem and questions.
3. The taxonomy of existing knowledge that emerged from the synthesis and articulated into the battery of open-ended questions. The taxonomy details the contribution of the different research domains in questioning and interpreting the research problem.

The position of the synthesis of existing literature and research in the study is depicted in Figure 4.1.

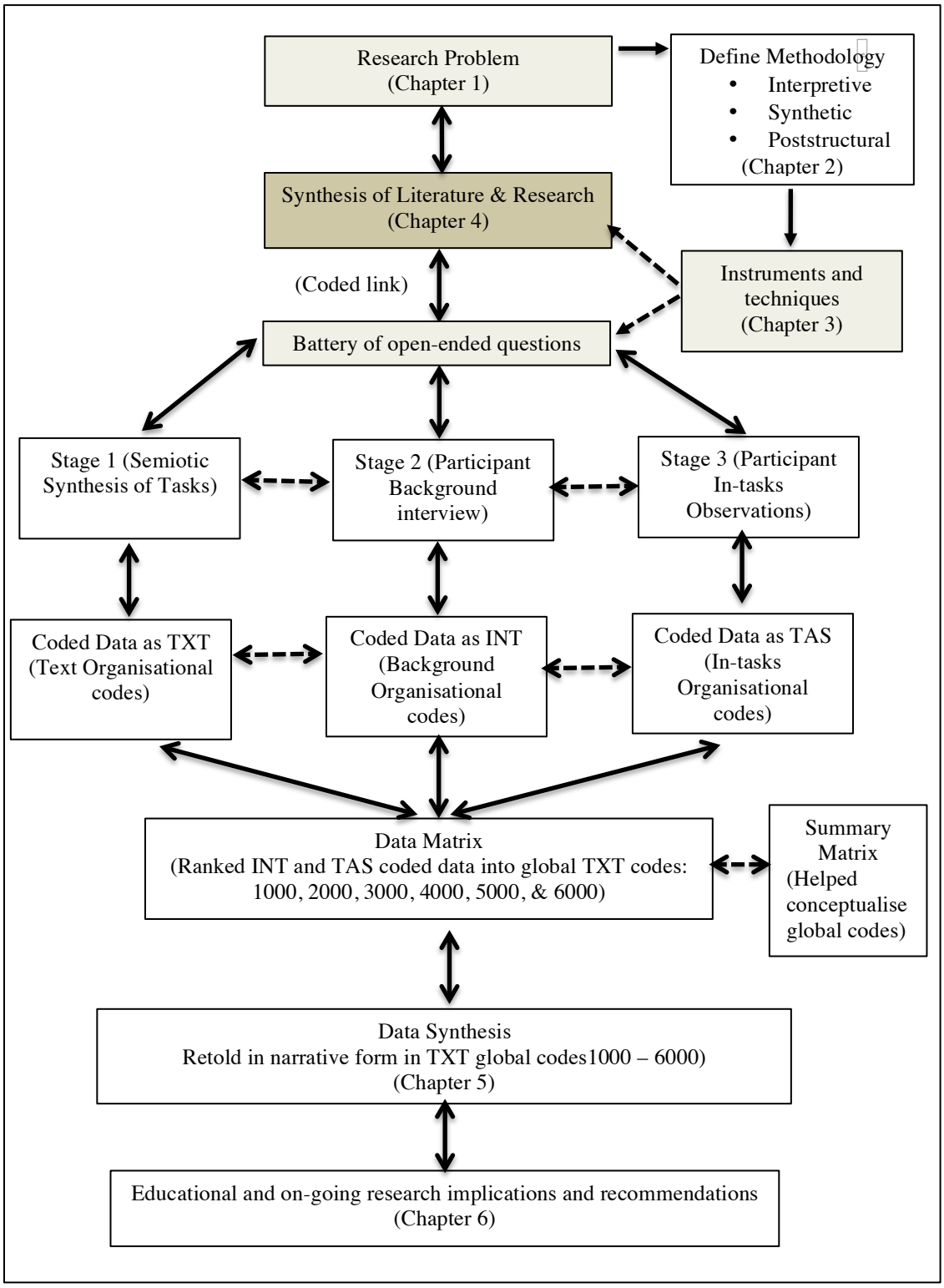


Figure 4.1: The position of the synthesis and articulation of existing literature and research in the study.

4.2 Literature review methodology

The terms mathematics, language, adult-CALD bilingual learners, culture, Australian, and education were identified as elements of the study that helped, firstly, interpret the constructs of the research problem and, secondly, search for relevant literature and research (Section 2.2). The elements generated key words that helped synthesise (search, interpret, and classify) existing knowledge in the literature and research review. The approach saved time in, respectively, defining the parameters that electronically searched for relevant literature, and classified existing knowledge. The resulting taxonomy articulated into the battery of open-ended questions that helped collect and interpret data (Section 3.2).

L and S science and language (Bloomfield, 1939) were employed to synthesise existing knowledge (Section 2.6). For example, cognitive science questioned the participants' thinking behaviours in M2 (Krummheuer, Leuzinger-Bohleber, Muller-Kirchof, Munz, & Vogel, 2013; Gillard, Van Dooren, Schaeken, & Verschaffel, 2009). Cultural science questioned the participants' social behaviours and experiences in M2 (Hamers & Blanc, 2000; Hersh, 1994; Trinick, Meaney, & Fairhall, 2014). Merging the two languages and sciences into battery questions interpreted the joint cross-cultural and cognitive behaviours that affected decoding and learning M2 (Whorf, 1956). As Bloomfield (1939) stated, "Linguistics is the chief contributor to semiotics. Among the special branches of science, it intervenes between biology, on the one hand, and ethnology, sociology, and psychology, on the other: it stands between physical and cultural anthropology" (p. 55).

The following section defines the L and S procedures and methodologies employed to synthesise existing knowledge from different research domains.

4.2.1 Definitions

An initial review of existing literature and research identified that the elements making up the research problem and questions were inadequately defined, and a more precise synthesis was required to interpret their meanings and relationships (Section 3.2). For example, as Hamers and Blanc (2000) stated, when interpreting language "it must be kept in mind that one aspect of language behaviour, for example interpersonal features, cannot be explained if other dimensions, e.g., intergroup relations, are ignored" (p. 3).

The synthesis interpreted and defined language, for example, as a construct from a broad range of theoretical and contextual perspectives to generate a comprehensive taxonomy (Onwuegbuzie, Leech, & Collins, 2012) of existing knowledge. The approach enabled the synthesis to sequentially, as Onwuegbuzie, Leech, and Collins (2012) stated:

> Obtain a complete picture of "what has been conducted before, the inferences that have emerged, the inter-relationships of these inferences, the validity of these inferences, the theoretical and practical implications stemming from these inferences, and the important gaps in the literature" (Onwuegbuzie et al., 2010, p. 179), as well as positions them "to select the most appropriate methodologies for their studies by allowing them to identify the strengths and weaknesses of approaches used in previous studies". (p. 4)

The taxonomy of existing literature and research enhanced the study's capacity to interpret and define the elements of the research problem, and articulate the knowledge into methodologically driven micro-data collection questions (Section 2.6.2).

4.2.2 Range and context

There was a distinct lack of literature and research in certain areas of the research problem that required inferences to be drawn from other areas of research (Onwuegbuzie, Leech, & Collins, 2012). For example, mathematics was extensively studied and, respectively, linked to culture (Barton, 2009; Hersh, 1994; Trinick, Meaney, & Fairhall, 2014; Whorf, 1956), language (Bloomfield, 1939; Halliday, 2006; O'Halloran, 2005), and cognition (Gillard, Van Dooren, Schaeken, & Verschaffel, 2009; Shreyar, Zolkower, & Perez, 2010). However, very little was known about learning M2 as an adult-CALD learner experience (AIR, 2006; COAG, 2008). This issue was managed by drawing inferences from research conducted on, for example, BL children (Farrugia, 2003; O'Neill, 2009; Parvanehnezhad & Clarkson, 2008) and monolingual-English adults (Halliday, 2006) to question adult-CALD BL learning behaviours in the battery of open-ended questions.

The meaning of mathematics was relatively well defined in a cultural context by researchers, for example, Barton, (2009), Ernest (2006; 2012), Hersh (1994), and Whorf (1956). Search in this area ceased when, for example, additional reading

stopped producing new insights and lines of questioning for the battery of open-ended questions. The inherent lack of literature and research associated with interpreting, for example, BL behaviours in mathematics generated a different kind of issue that had to be managed. The search for relevant literature and research in this area was conducted more systematically. For example, the terms and identifiers adult, CALD, BL, and learner were combined in different sequences with mathematics, culture, Australian, and education to widen the depth of the electronic searches that were made. This helped identify relevant questions and contexts that would have otherwise been overlooked by a less systematic approach.

The findings helped balance the type of questions generated in the battery of open-ended questions. The battery, however, was not concerned with verifying existing knowledge in this context (Sections 2.4 & 3.2). Instead it focused on identifying and contrasting the different theoretical propositions that could be employed to interpret the research problem.

4.2.3 Quality

Quality emerged as an issue, because of the problem associated with referencing existing knowledge through synthesis. Secondary sources were employed, for example, to cite no longer accessible original texts, however, these sources also articulated their own interpretations of the original text. For example, Kant maxims (1788) defined the ethical and synthetic logics employed in this study (Sections 2.4 & 2.10). However, it was not sure how accurately Kant's (1788) 18th century German was transposed into the English version cited in this study.

The synthesis managed this issue by, respectively, interpreting the original and secondary sources in the historical, linguistic, and cultural contexts they were written and translated in. The approach generated a more balanced L and S interpretation of the text. For example, Whorf (1956) was often cited as proposing culture and language moulds thoughts (Hamers & Blanc, 2000, p. 94). However, Whorf (1956, p. 94) employed 1950s American Standard English as his C, L, and S medium to interpret and articulate this proposition. Conceptually, other languages would have, from Whorf's (1956) own theoretical perspective, interpreted Hopi L and S behaviours differently.

The synthesis was mindful of the implications language as a medium had on articulating knowledge (McLuhan, 2001). Employing 21st century Australian Standard English to interpret Kant's (1788) 18th century German manuscript and Whorf's (1956) mid-20th century American Standard English was subjective in terms of the meanings that were transferable between the two languages. As a result, the proposed taxonomy cited the C, L, and S contexts and mediums that were employed to articulate this knowledge. This generated a more balanced interpretation of existing knowledge and the inferences that were drawn.

The study's synthetic epistemology also found it inappropriate to attribute knowledge to a particular author(s). For example, Halliday's (2006) SFL model is cited in the literature to identify that M2 meanings are defined by the form and function of the sign system (Kaartinen & Latomaa, 2011) (Section 4.4.2). However, Halliday's (2006) model reflected Bloomfield (1936) and de Saussure's (1910) earlier structuralist proposition that meanings are defined by the structure of the sign system. Referencing Halliday (2006) as the source of SFL paints a convenient but incomplete picture of the development of this knowledge.

The synthesis interpreted and referenced authorship in two ways in this context. Firstly, authorship identified the application of theory and knowledge rather than its source. For example, Halliday (2006) was referenced in the context where he employed functionalist and structuralist theories to define the M2 sign system and not, necessarily, the source of that knowledge. Secondly, the study's poststructuralist epistemology challenged the notion of a single authority over knowledge. The terms authorship and authority generated connotations of ownership that were, in part, epistemologically incongruent when knowledge emerged over time through cultural evolution (Whorf, 1956). The study's poststructuralist approach interpreted knowledge through its application, context, and historical development, rather than authority and ownership (Section 4.4.3). As Kant (1788) stated, "scholarship is actually only the embodiment of historical science" (p. 177).

The poststructuralist approach identified the hidden relationships, meanings, and differences (Miller, Whalley, & Stronach, 2005, p. 313) that emerged through the synthesis of existing literature and research. Existing knowledge was accounted for in qualitative terms, and not statistically to identify its popularity and authority. In this context less popular knowledge was just as important for conceptualising the

research problem. Outlined knowledge was not critiqued in this context, but instead merged it to interpret and define the research problem holistically.

4.2.4 Procedures and techniques

MacInnes's (2011, p. 137) Framework for Conceptual Thinking in Marketing was adapted to help synthesise and conceptualise existing literature and research in taxonomic form. The framework, while situated in the Marketing research domain, provides a set of methodologically driven questions that also help interpret, classify, and conceptualise knowledge in other research domains. The term conceptualisation in this context, as MacInnes (2011, p. 136) stated, takes on a discovery role to "envision (identify or revise)", "explicate (delineate or summarise)", and "relate (differentiate or integrate)" existing knowledge.

Table 4.1 depicts the adapted version of MacInnes's (2011, p. 137) framework employed to synthesise the interpretation of literature and research. Additional columns were added to question the individual elements, constructs, and contexts that constituted the research problem. The findings were then interpreted and articulated from a domain perspective into the battery of open-ended questions.

Table 4-1: Framework for conceptualising existing literature and research.

Questions/contexts	Language	Culture	Mathematics	Adult-CALD	Education	Australian
1. What are the constructs being addressed?						
2. What are the divisions and disciplines that affect the findings? 3. What is the research domain that governs the study? 4. What constitutes science? 5. What procedures are followed or recommended for conducting research?						
6. What are theoretical relationships being claimed?						

-Adapted from MacInnes (2011, p.137) Framework for Conceptual Thinking in Marketing.

4.3 Elements

This section recounts the broader constructs that emerged and defined the elements {language, culture, mathematics, adult, education, and Australian} in the research problem. The following section 4.4 recounts the taxonomy that emerged from their deeper synthesis (Onwuegbuzie, Leech, & Collins, 2012), coding, and articulation into the battery of open-ended questions. The taxonomy, for example, more specifically interpreted language from different theoretical and domain perspectives to question it as a: mental behaviour in the Cognitive Research domain (Ramachandran, 2011); grammar in Structural Linguistics (Halliday, 2006; Bloomfield, 1939); and, as identity in Cultural Research (Tabouret-Keller, 2000). Mathematics, for example, was interpreted and questioned as: language learning in education (Barton, 2009; Halliday, 2006; O'Halloran, 2005); semiosis in Structural Linguistics (Bloomfield, 1939); and, logics and pure reasoning in Pure Mathematics (Bloomfield, 1939; Devlin, 2000; Khait, 2005; Russell, 1912). This broad range of knowledge was then articulated a posteriori (Kant, 1788) into battery questions, without being analytically tied to proving or disproving the questions as propositions (Sections 2.4 & 3.2). The ensuing section identifies the broader meanings that emerged to first conceptualise the elements and constructs defining the research problem.

4.3.1 Language

The term language articulates different meanings in literature and research. It defines, for example, the sounds, words, and symbols humans employ to communicate (Fromkin, Blair, & Collins, 1999, p. 2). It identifies the different types of linguistic knowledge people need to survive (Milroy & Milroy, 1998; Fishman, 1980) and interpret knowledge in a particular society (Whorf, 1956). As Crystal (1992) stated:

> We look back at the thoughts of our predecessors, and find we can see only as far as language lets us see. We look forward in time, and we can plan only through language. We look outward in space, and send symbols of communication along with our spacecraft, to explain who we are, in case there is anyone there who wants to know. (p. 1)

The term semantics identifies the hidden meanings that emerge and define language use in different areas of society (Fromkin, Blair, & Collins, 1999). Language in this context describes the distinct discourses people employ to communicate in, for example: everyday English language, machine language, second language, sign language, and mathematical language.

English language

Crystal (1992, p. 285) identified more than 20,000 languages and dialects being spoken throughout the world, however, only 4,500 have managed to survive and are considered living languages in the context they are actively used. Of these, approximately 180 languages have official status in government and education. Crystal (1992, p. 358) added, English is spoken, as an official language by over 1 billion people in over 45 different countries. In this context, English holds power and prestige over many other languages (Baker, 2011; Fishman, 1980). Bialystock (2001) identified that the English language also creates barriers in this context, because it excludes non-English speaking minorities from certain communities having access to education, government, business, and law. Hornberger (2003) added education systems should be more supportive of CALD and indigenous learner needs in this area.

Second language

Second language describes the knowledge and processes involved in acquiring an additional language (Ellis, 2001, p. 11). The term bilingualism defines second-language learning behaviours through constructs, such as: simultaneous and consecutive language acquisition; community and formal language learning; first and second language similarities; code switching; and cognitive and learner difference (Baker, 2011; Ellis, 2001; Hamers & Blanc, 2000). Factors such as age, prior education, and learner aptitude describe why some people fail to learn an additional language successfully (Cummins, 2000; Ellis, 2001; Hamers & Blanc, 2000). The meaning and significance of these factors is synthesised in greater depth in the taxonomy that emerged.

4.3.2 Culture

The Macquarie Essential Dictionary (2000) identified that culture represents the skills, arts, beliefs, and customs that generations of people pass on to one another. In this context culture defines, as Wardaugh (2002) stated, "the know-how that a person must possess to get through the task of daily living" (p. 219). Cultural knowledge signifies the sociolinguistic competence a person also needs to function in different social contexts (Holmes, 2001, p. 370). This applies, for example, to being able to apply different registers of English at work, school, or in mathematical applications.

4.3.3 Mathematics

The meaning and significance of mathematics varied significantly in literature and research. Russell (1912) interpreted mathematics as logic and reasoning, Devlin (2000) as human discourse, Ernest (2006, 2010) semiotics, and Barton (2009) as language and culture. Merging the different interpretations through synthesis questioned, for example, how people enact M2 through cognition, language, and cultural practices.

Mathematics as a construct evolved over time and, in part, differentiated English and Western cultures (Barton, 2009; Khait, 2005). Khait (2005, p. 141) defined this evolution through schools of thought. The Logistic School interpreted mathematics as logical, deduction, and pure reasoning. The Formalist School interpreted mathematics more pragmatically through axiomatic rules and codes. The approach freed mathematics from the contradictions that challenged the Logistic School (Section 4.4.4). Intuitionism interpreted mathematics as a social and philosophical construct, subject to human interpretation and variances.

In an applied context Devlin (2000, p. 1) defined mathematics as the discourse that interprets and solves problems through recognising patterns such as motion, chance, and shape, in phenomena. M2 is different because it transposes everyday language into complex entities, grammars, and discourses (Halliday, 2006; O'Halloran, 2005; Presmeg, 2014; Sfard, 2005). M2 is also difficult to learn in this context without formal instruction (Halliday, 2006; O'Halloran, 2005; Sfard, 2005). The term numeracy emerged and defines the application of mathematics in solving everyday real-life problems (COAG, 2008, p. xi; Tout & Motteram, 2006). Innumeracy

(COAG, 2008) describes people who lack skills in solving everyday mathematical problems through language.

4.3.4 Education

Education emerged and defines the contents and processes that are "worthwhile" learning in a society (Higginbotham, 1990, p. 42). It also identifies the different philosophical choices stakeholders make in education (Lloyd, 1990). For example, teaching mathematics as an everyday construct versus pure mathematics (COAG, 2008; Tout & Motteram, 2006). There are different theories that governed how language and mathematics should be taught in this context. Social-based constructivists' theories believe learning is scaffolded through prior educational experiences (Section 2.4) (Piaget, 1972; Tout & Motteram, 2006). Alternatively, interpretive theories believe knowledge emerges through the exploration and conceptualisation of new meanings and relationships (Section 2.4) (Ernest, 2012). The functionalist perspectives believe learning emerges through reconciling the form and function of the sign system (Halliday, 2006).

4.3.5 Australian

The term Australian emerged, in the main, as an adjective to define constructs, such as, language, culture, history, and education. The term, however, was not applied to distinguish Australian mathematics, despite the significance and effects factors, for example, Australian English, culture, and education, have on shaping discourses and meanings-making processes in M2.

The term Australian education also describes the approaches employed to teach mathematics and language. Australian language education is criticised in this context, because educational practices often fails to carryout what stakeholder such as government believe is important in terms of learning first and additional languages (Lo Bianco & Slaughter, 2009). Multilingual countries such as Canada and Switzerland are more progressive in achieving the benefits expected from a bilingual education (Baker, 2011; Lo Bianco & Freebody, 2001). As Lo Bianco and Slaughter (2009) stated, "Australia has an impressive record of policy development and program innovation in second language education, but a relatively poor record for consistency of application and maintenance of effort" (p. 6).

4.3.6 Adult

The term adult also emerged as an adjective to describe constructs such as maturity, behaviour, language, and education. Different educational theories articulate how adult learners should be taught mathematics and language in this context. Very little, however, is known about teaching M2 as an adult-CALD BL construct.

4.4 Taxonomy of existing knowledge

The taxonomy of existing knowledge helped synthesise more precisely the elements and constructs that defined the research problem. As Onwuegbuzie, Leech, and Collins (2012) stated:

> As is the case for a domain analysis, taxonomic analysis leads to further structural questions. After these questions are answered, the reviewer can refine the taxonomy and use it in the report (i.e., literature review section) to help the reader understand the phenomenon of interest. (p. 20)

The taxonomy, respectively:

1. Interpreted more precisely the elements and constructs that made up the research problem.
2. Classified existing theory, science, and language to articulate data collection questions.
3. Helped envision new relationships.
4. Generated a framework for systematically questioning and interpreting data and new relationships.
5. Created a framework for building and/or modifying existing knowledge.

Two issues, however, challenged the design stage of the taxonomy. Firstly, there were no taxonomic frameworks found in the literature review that suited the specificities of the study. The design was, therefore, conceptualised and developed, in the main, from the ground up. Secondly, relevant literature and research were interdisciplinary in nature, and this made it difficult to classify individual publications from a domain perspective. As a result, the taxonomy interpreted and classified the contents of the publication from a domain perspective rather than the publication as a whole. For example, Steinbring's (2006) article titled *What makes a*

sign a mathematical sign - an epistemological perspective on mathematical interaction was cited in the *Journal of Educational Studies in Mathematics*. The contents, however, addressed a broad range of inter-connected theories and knowledge that could have, equally, been published in, for example, Philosophy, Mathematics, or Linguistics publications. The contents and theoretical elements that made up the article were, therefore, synthesised and classified into different domain perspectives. The decision, however, was a qualitative choice based on the relevance of the content, and not a statistical one that reflected the type of journal it was published in. The approach generated a more "comprehensive" synthesis (Onwuegbuzie, Leech, & Collins, 2012, p. 4) of existing literature and research and the inferences that could be drawn from it.

The Australian Standard Research Classification List (ABS, 1998) was employed to, firstly, interpret and classify the contents of relevant publications from a domain perspective and, secondly, identify where the findings from the proposed study would contribute its own knowledge. There were 24 divisions, 139 disciplines, and 898 research domains listed on the ABS Classification (ABS, 1998) to choose from. Table 4.2 identifies the respective disciplines, divisions, and domains that helped synthesise and classify existing literature and research in the taxonomy. The findings, contents, and implications are recounted in the following section.

Table 4-2: Framework for synthesising and classifying existing knowledge.

ABS Divisions	**Disciplines**	**Domains/remarks**
Philosophy and religion	Philosophy	Philosophy of Language. (Philosophy of Mathematics not specifically categorised by ABS) Philosophy of Education (Sometimes cited under Education)
Mathematical Science	Mathematics	Mathematical Logic (ML not specifically categorised by ABS).
Behaviour and cognition	Psychology	Sense and Perception. Learning Memory, Cognition and Language. Biological Neuropsychological.
	Linguistics	Applied Linguistics and Educational Linguistics. Discourses and Pragmatics. Sociolinguistics. Language in Time and Space. Linguistic Structures.
Language and culture	Language Studies	English. English as an Additional/Second Language.

	Cultural Studies	Multicultural, intercultural and cross-cultural.
Education	Education Studies	Educational Psychology Sociology of Education
	Curriculum Studies	English Education Mathematics Education
	Professional Development	Adult Education

-Adapted from the Australian Standard Research Classification List (ABS, 1998)

4.4.1 Philosophy

A small body of literature and research interpreted the ontological (human existence), epistemological (knowledge and truth), axiological (values), and ethical factors that affected the interpretation of the research problem and behaviours in it. A synthesis of these interpretations follows.

Philosophy of language and mathematic

Duval (2006) believed knowledge is articulated differently in M2. Unlike the physical sciences, for example, Chemistry, Astronomy, and Biology which employ instruments to interpret meanings, Mathematics is only accessible through interpreting its highly abstract sign system (Duval, 2006). Steinbring (2006) identified that mathematical signs also preserve its knowledge in the same way.

De Cruz (2006) identified mathematical knowledge was interpreted historically in two alternative ways as part of human existence. The Platonist view believes mathematical knowledge exists as an independent entity that is separate to the human mind. Alternatively, the physical existence view believes mathematical knowledge is created and exists because of the human mind. De Cruz (2006) added that the constructionist theories of learning emerged from the physical existence view, because they believe mathematical knowledge is scaffolded in the mind through a person's cultural and humanistic experiences. In this context, mathematics is empirical and testable, because it does not rely on Platonists' metaphysical explanations to prove knowledge. Barton (2009, pp. 69-70) identified, that the Platonic view does not fit well with the notion that mathematical knowledge is created and learnt through language, culture, and social interaction.

Alternative theories on learning, for example, mentalist, structuralism, functionalism, and pragmatism, define how ML is acquired as a humanistic experience (Section 2.4). Mentalists believe humans have an innate biological capacity for learning a language (Chomsky, 1972; Ellis, 2001) such as mathematics. Structuralists believe language is acquired through learning the sign system that is used to communicate (Bloomfield, 1939; de Saussure, 1910). For example, the meaning and significance of symbol '3' is defined by its position relative to other signs in M2.

Functionalists emerged from structuralist theories by adding that meanings are more specifically interpreted through the form and function of the sign system (Halliday, 2006). For example, the symbol '3' is defined by the function it performs when adding numbers, for example, in '3 + 3 = 6'. Halliday (2006) identified language also has meta-functions that express ideational, interpersonal, and textual meanings. Section 4.4.2 synthesises in greater detail the meaning and significance of the form and function of the M2 sign system.

The pragmatists believe M2 is less predictable than the structuralist and functionalist theories suggest (Peirce, 1931). Pierce (1931) identified that M2 is also defined by the social and cultural conventions which encode meanings in the sign system. The number and symbol '13', for example, is interpreted as 'bad luck' in certain social contexts, and it is not be arbitrarily replaced by other numbers and signs for instance '7'.

Philosophical beliefs affect the way human behaviours are also questioned and interpreted in the M2 sign system (Ernest, 2012). A post-positivist approach interprets human behaviour objectively through empirical and quantitative observations (Section 2.4). Interpretive approaches challenge the post-positivist approach by identifying the subjective and qualitative factors that also affect human behaviour in M2. Pragmatist approaches believe constructs such as language and mathematics are best interpreted through the semiotic contexts that affect human behaviour in the sign system (Section, 2.4). As Geertz (1973) stated:

> In short, we need to look for systematic relationships among diverse phenomena, not for substantive identities among similar ones. And to do that with any effectiveness, we need to replace the "stratigraphie" conception of

the relations between the various aspects of human existence with a synthetic one. (p. 8)

Philosophy of education

Howe and Moses (1999) identified that education often takes on an advocacy role, in that it advances a moral-perspective and deals with vulnerable student populations. Foundational literacy and numeracy education seeks to improve the social and economic well being of minority groups (Black, 2002; Perkins, 2009). Nonetheless, foundational literacy and numeracy education is, in part, unethical, because it often leads learners to believe participation generates employment and economic opportunities when it does not (Black, 2002). Black (2002) identified that the economic climate, for example, is more significant in generating job opportunities than suggested in existing literature and research. Apathy and disillusionment emerges when learners do not find a job after attending foundational LLN programs (Black, 2002; Griffin, 2014; Miralles, 2004).

Some researchers believe education should be more inclusive of CALD learners' needs (Lo Bianco & Slaughter, 2009; Tochon, 2014) when teaching, for example, mathematics, as an additional language construct. Prins and Uljin (1998) proposed removing vague and indirect English from mathematical text, and replacing it with native languages to help indigenous minorities learn how to do Western forms of mathematics.

4.4.2 Mathematics

A significant body of literature and research interpreted the meaning and significance of the mathematics and mathematical language in the research problem.

Mathematical science

Mathematical science defined the theory and application of mathematical knowledge in solving problems (Devlin, 2000). L and S theory helps interpret the meaning and significance of the sign system in articulating this knowledge (Peirce, 1931; Bloomfield, 1939). As Broomfield (1939) stated, "in practice we labor under a load of traditional and popular misconception about language, a great deal of doubt, error,

and dispute will be avoided if mathematicians and logicians acquire enough linguistics to remove these conceptions" (p. 56).

Devlin (2000, p. 17) identified that M2 emerged through different stages of historical and linguistic development. In early human history ML identified the tokens that were used to count objects in society, for example, 'sheep'. Symbols replaced tokens over time and numbers articulated cardinal and ordinal relationships. For example, Roman numerals such as 'III' and 'IV' were used to count quantities of 'sheep' and their flock sizes in Roman society (Devlin, 2000). The symbols allowed Romans to conceptualise objects and relationships in abstract ways without physically counting them.

Mathematics became more sophisticated and intricate in its form and function over time. Distinctive periods identified the development of, for example, the geometry of the Greeks between 500BC to 300AD, and Newton and Leibniz's 17th century calculus. The latter period saw the removal of visual objects such as human eyes and body parts, from the text to articulate perspectives and dimensions in M2 (Devlin, 2000; O'Halloran, 2005, 2010). The humanistic symbols were replaced with more sophisticated signs and entities such as equations and formulas, to articulate meanings (O'Halloran, 2010). As Devlin (2000) stated, the question of "what is mathematics" is now answered by "what mathematicians do" (p. 3): that is, interpret and define "patterns" (p. 9) in phenomena. Specialist areas of mathematics interpret, for example, patterns in probability, motion, and human behaviour (Devlin, 2000).

Mathematical language

A person's language and culture affects how they interpret and articulate meanings in mathematics (Whorf, 1956). Barton (2009, p. 35) identified navigating in western culture, for example, often conceptualises travel as a static and linear relationship between two geographic points. Alternatively, Pacific Islanders have more dynamic ways, for instance sensing patterns in waves and currents to interpret navigation between two points.

The English version of ML also emerged from the lexicon and grammar of its everyday language (Bloomfield, 1939; Halliday, 2006). As Halliday (2006) stated, "even the most abstract kind of semiosis, like that of mathematics, is still parasitic on

natural language" (p. 119). Halliday (2006) defined the transformation of everyday English into M2 as follows:

1. Mathematical and scientific language altered the lexicon and syntax of everyday English into new ways of construing knowledge. For example, the clause 'he departed' changes to 'his departure' and 'departure' to articulate the action of leaving (Halliday, 2006, p. 117). The metaphor altered the adverbial clause into a, respective, adjective of description and noun entity (Section 2.6.2).
2. The semiosis, as Halliday (2006) identified, creates another plane of semiotic realitty, whereby theories and ideas are construed through more delicate and complex semiotic representations. The M2 sign system enables the human mind, from a physical existence perspective (Section 4.4.1), to conceptualise objects through new and more abstracted ways of thinking. As Halliday (2006) stated, for example, "thing *a* undergoes process *b*, in manner *c* to the extent that in manner *x* person *w* does action *y* to thing *a*" (p. 35).
3. Mathematical and scientific language generates distinctive types of taxonomies to interpret and classify objects (Section 2.6.2). As Halliday (2006) stated, for example, "taxonomies use a 'classifier + thing' (for example): heat + resistance, to create a new structure 'heat resistance'" (p. 39).
4. M2, however, differs from other forms of scientific language, because it tends to '"stretch the grammar" (Halliday, 2006, p. 167) and "cluster together" (Halliday, 2006, p. 178) more in its syntax and register. The symbol 'π', for example, signifies complex meanings, relationships, and thinking behaviours that are mathematical in nature.
5. M2 is also distinguished by its distinctive "diatypic register" (Halliday, 2006, p. 140). The register is defined by its:
    a. Field –what was happening, for example: a mathematical proposition.
    b. Tenor – who was communicating, for example: mathematics as a specialist-to-specialist discourse.
    c. Mode – how meanings were exchanged, for example: mathematical grammar, graphs, and images to communicate meanings precisely in mathematics.

6. M2 also performs metafunctions to "enlarge" meanings (Halliday, 2006, p. 26). Metafunctions articulate:
    a. Interpersonal meanings – to create and/or maintain social relationships.
    b. Ideational meanings – to construct and construe experiences, for example: asking and answering 'wh' questions.
    c. Experiential meanings – to expressing experiences.
    d. Textual meanings – to organise the lexicon and grammar to communicate a particular purpose, for example: mathematics as a specialist discourse.
7. M2 is problematic to learn, because it makes semantic leaps that are difficult to interpret without formal instruction (Halliday, 2006, p. 176). For example, graphic and algebraic registers are employed in parallel as different constructs to articulate similar data in M2 (O'Halloran, 2005).

Shreyar, Zolkower, and Perez (2010) applied Hallidays' (2006) model to study the discourse and learning behaviours of children in M2. The study identified that children often interpreted ideational meanings incorrectly, because they did not know how to ask appropriate 'wh' type questions when learning M2. The authors identified, firstly, the discourse could be interpreted statistically to interpret human behaviour (Bortolussi & Dixon, 2003) and, secondly, the problem can be addressed by teaching suitable questioning and discourse techniques (Kaartinen & Latomaa, 2011).

Schleppegrell (2007) and O'Halloran (2005) contended the challenge faced when teaching and learning mathematics in this context is to understand how its sign system functions. M2 employs a combination of dense noun phrases, visual displays, symbolisms, and discourses that are difficult to interpret without formal instruction (Halliday, 2006; O'Halloran, 2005, 2010; Presmeg, 2006, 2014; Schleppegrell, 2007). As O'Halloran (2005) stated:

> The meaning of the written symbolic mathematics is not equivalent to the spoken language because symbolic mathematics draws upon spatiality and unique grammatical strategies (e.g. ellipsis of operations, rules of order and brackets) to encode meaning. Quite simply, talking mathematics is not the same as doing mathematics. (p. 20)

The meanings and significance of M2 as a sign system were interpreted from a semiotic perspective in two ways. De Saussure's (1910) structuralist perspective sees meanings emerging through a dyadic relationship, where the signifier represents an object, and the signified the meaning that is generated. The signifier is arbitrary in this context, because it can be replaced by different representations to signify the same meanings, for example: '3', 'III', and 'three' to signify the quantity three. Peirce's (1931) pragmatist perspective sees signs having a third-tier layer of meanings that are also defined by the contexts in which they are interpreted. For example, the sign '13' signifies 'bad luck' in certain C, L, and S contexts. This makes the M2 sign system less arbitrary than de Saussure's (1910) dyadic model suggests. Peirce's (1931) third-tier meanings are problematic to decode and learn without C, L, and S experience. The phrase 'C, L, and S experience' signifies the knowledge needed to decode the sign system in different contexts. As Ernest (2006) stated:

> The term semiotic system is here used to comprise three necessary components. First, there is a set of signs, each of which might possibly be uttered, spoken, written, drawn, or encoded electronically. Second, there is a set of rules of sign production, for producing or uttering both atomic (single) and molecular (compound[5]) signs. Third, there is a set of relationships between the signs and their meanings embodied in an under-lying meaning structure. (p. 70)

Ernest (2006, p. 71) stated that the sign system was, in part, also learnt by "novice" learners through everyday "case law" experiences. In this context, repetition and practice help memorise the grammars and lexicon needed to decode the sign system without fully understanding its underpinning logics. Ernest (2006) identified case law experience is employed to introduce learners to complex rules and thinking behaviours in M2 such as axiomatic and deductive logics. Axiomatic codes identified the rules of Associativity ($(a + b) + c = a + (b + c) \cdot (a \times b) \times c = a \times (b \times c)$), Commutativity ($a + b = b + a \cdot a \times b = b \times a$), and Distributivity (of $\times$ over $+$: $a \times (b + c) = a \times b + a \times c \cdot (b + c) \times a = b \times a + c \times a$) that encoded the sign system. In this context, axiomatic codes are learnt through repetition and everyday discourse, and then through planned instruction involving a deeper and more sophisticated

[5] Brackets and words cited in original quotation

understanding of the semiosis.

O'Halloran (2005, p. 6) identified that learning M2 included learning:

1. The intra-semiosis processes that transpose meanings within a semiotic register.
2. The inter-semiosis processes that transpose language, images and symbolic resources across different registers.
3. The inter-modality processes that transpose overlapping modalities such as visual, aural, haptic, and other sense modalities, when reading and learning M2 texts.

The term modality is applied in two interrelated ways in this context. Firstly, it identifies the human senses, for example, visual, tactile, and auditory, used by readers to interpret the sign system (Ernest, 2010; O'Halloran, 2005). Secondly, it identifies the reality, truth, and value people attach to learning the sign system (Barthes, 1974; Chandler, 1994). As Chandler (1994) stated, "the extent to which a text may be perceived as 'real' depends in part on the medium employed. Writing, for instance, generally has a lower modality than film and television" (p. 109).

The sign system is untruthful and mythical on occasions when a reader's cultural and social background precludes them from interpreting its values (Barthes, 1974). For example, a person's cultural background might prohibit their gender from, for example, learning and participating in financial mathematics (Appendix E: Task 15). Myths and ideologies reflect the third-tier meanings generated by the sign system (Barthes, 1974; Chandler, 1994; Peirce, 1931). As Ernest (2006) stated, the study of semiotics is "concerned with patterns of sign use and sign production, including individual creativity in sign use, and the underlying social rules and contexts of sign use" (p. 67).

The synthesis of semiotic-related literature and research identified that the following C, L, and S conventions were significant in encoding meanings in the M2 sign system through form, function, and contexts. De Saussure (1910) and Peirce (1931) identified that there are paradigmatic and syntagmatic conventions that encode meanings and processes in M2. Syntagmatic meanings emerge by changing the position of the sign in the syntax, and paradigmatic by changing the way the sign looks. Syntagmatic codes articulate new meanings but not necessarily new

paradigms. A minus sign '-' placed in front of '5' articulates a new paradigm for '5' (-5), and '5' transposed in the clause '5(3 + 4)' to '3(5+ 4)' articulates a new syntagmatic meaning for '5' but not a new paradigm. De Saussure's (1910) structuralist interpretation believes the signifier is arbitrary in this context, because meanings emerge through the position of the sign in relation to other signs in the system (Chandler, 1994, p. 141). Peirce (1931) pragmatist interpretation believes the signified/interpretant articulates a third level meaning (Chandler, 1994, p. 63). This makes M2 signs less arbitrary than de Saussure (1910) dyadic model suggests. For example, the sign '-' placed in front of '-5' is not arbitrarily swapped with '+' to articulate the same meaning. This transformation involves recoding the sense generated by the sign (Jakobson & Halle, 1956).

Peirce (1931) identified how the sign system articulates meanings through three types of sign representations: iconic, indexical, and symbolic. Iconic signs figuratively represent the objects being communicated. For example, a pie chart represents the processes of cutting up a whole into parts. Indexical signs maintain a physical and visual connection to their meanings. For example, maps and graphs resemble the locations and data they represent. Symbolic signs are metaphorically disconnected from their literal sense. For example, numerical symbols are physically disconnected from the objects they signify.

Jakobson and Halle (1956) identified that conventions such as binary oppositions (big/tall, large/small) and markedness (happy/un happy) encode distinctive meanings in the sign system. The sign '-' in the equation '3^{-2}' marks the meaning and process needed to decode the signs '2' and '3'. The equation '3^{-2}' is decoded in contrast to '3^{2}', and has an equivalent symbolic meaning to '9' and'1/9'.

The M2 sign system also has "agentic" characteristics (Ernest, 2006, p. 69) that encode its own goals, reactions, and cognitive properties (Leslie, 1993, p. 2; Tylen, 2007). Leslie (1993) stated, "Agency begins with the idea that Agents are a class of objects possessing sets of causal properties that distinguish them from other physical objects" (p. 1). This affects how people decode the sign system. Ernest (2006) stated that interpreting the M2 sign system from "a semiotic perspective transcends the traditional subjective-objective dichotomy", because signs are an "agentic act" that affect human behaviour (p. 68). Ernest's (2006) proposition is, however, highly theoretical and more applied research is required to interpret its implication on

decoding and learning M2 in a cross-C, L, and S context.

M2 meanings and processes, however, are not universally shared across languages and culture and, therefore, mathematical ideas are subject to different forms of representations and interpretations (Barton, 2009; Whorf, 1956). Aberrant decoding behaviours emerge when the sign system is decoded differently than the system and narrator intended (Jakobson & Halle, 1956).

4.4.3 Behaviour, cognition, and linguistics

Literature and research from the Behavioural, Cognitive, and Linguistic disciplines interpret psychological and linguistics behaviours in the M2 sign system. Psychology interprets behaviours through the Sense and Perception, Learning Memory, Cognition and Language, and Neuropsychology domains. Linguistics interprets behaviours through Applied and Educational Linguistics, Discourse and Pragmatics, Sociolinguistics, Language in Time and Space, and Structural Linguistics domains.

Sense and perception

Sense and perception helps interpret the modalities people employ to decode and learn M2 (Ernest, 2010; Thibault, 2011). Arzarello, Robutti, and Bazzini (2005), Ernest (2010), and Thom and Roth (2011) identified that children employ distinctive physical behaviours like pointing, acting, and gesturing, to learn M2 through semiotic interaction. Physical connections are also affected by the mediums employed to encode and decode meanings (Chandler, 1994; Martin, 2011; Whorf, 1956). Anecdotally, solving problems in M2 through analogue mediums such as pen and paper, generate a different physical and mental connection to the sign system than digital mediums, such as, computers and calculators. Similarly, Barton (2009) commented that Pacific Islanders employ tides and currents rather than compasses to navigate. These types of senses and perceptions are passed down through the C, L, and S practices of the community (Barton, 2009; Krummheuer, Leuzinger-Bohleber, Muller-Kirchof, Munz, & Vogel, 2013; Whorf, 1956). As Whorf (1956) stated:

> Newtonian space, time, and matter are no intuitions. They are recepts from culture and language. That is where Newton got them. Our objectified view of time is, however, favorable to historicity and to everything connected with the keeping of records, while the Hopi view is unfavorable thereto. (p. 153)

Learning memory

The Cognitive research domain interprets the effects factors such as long and short-term memories have on human behaviour and cognitive performance (Hunt, 2005, p. 3) in M2. This body of research often correlates intelligence and cognitive performances as de facto relationships (Hunt, 2005, p. 5), because quicker responses are associated with more intelligent behaviour. Intelligence in this context identifies a person's ability to draw on different types of memory and cognitive behaviours to solve complex problems (Sternberg & Pretz, 2005, p. x). Reber (1984, p. 249) identified that the term memory generally interprets three types of human behaviour:

1. The mental functions and processes involved in retaining information about events, images, and ideas.
2. The hypothesised storage system that exists in the mind/brain that holds the information.
3. And/or the information that is retained.

Hunt (2005, p. 6) identified memory also takes on different forms to help solve problems, for example:

1. Crystallised when new problems are solved via old solutions and experiences.
2. Fluid when new solutions are conceptualised to solve new problems.
3. Spatial-visual when responses are heightened through extra sense and perception, as displayed in sports and musical skills.

Heuristics defines, as Raab and Gigerenzer (2005) stated, the "mental device that can solve a class problem in situations with limited knowledge and time. Models describe not IQs but mechanisms and models of understanding and problem solving" (p. 188). Heuristic behaviour identifies the ecological and social clues people employ to interpret and solve problems quickly. Alternative, analytical thinking employs deeper methodical processes to solve problems (Gillard, Van Dooren, Schaeken, & Verschaffel, 2009; Raab & Gigerenzer, 2005). Analytical thinking describes, for example, the inductive and deductive processes that are often taught to solve problems in M2 (Gillard, Van Dooren, Schaeken, & Verschaffel, 2009). Reber (1984) defined induction as the process where "general principles are inferred from within specific cases" (p. 352) and deduction the process which "begins with a set of assumptions and draw conclusions and theorems to generalise meanings that affect particular circumstances" (p. 178). Problems emerge when learners are unable to

distinguish the deductive and inductive thinking behaviours that are needed to solve certain classes of problems in M2 (Morris, 2002).

Todd (1999) identified that people employ different kinds of heuristics such as, take the best, most popular, and/or quickest solution, to solve problems quickly. Gillard, Van Dooren, Schaeken, and Verschaffel (2009) believed intuitions are important in this context, because they identify the reasons why people choose certain solutions for instance self-evidence and personal experience rather than analytical justification. For example, choosing the answer '43' in the multiple-choices question in '4 x 3 = ?' (Appendix E: Task 11), because the numbers '4' and '3' appear more frequently in the text than the analytical response '12' (Section 5.3.9).

There was, however, very little literature and research found in this area that helps interpret heuristics and analytic behaviours when solving problems in M2 in a cross-C, L, and S context. Psychometric testing can be employed, for example, to observe and measure the speed and processes people employ to solve problems (Hunt, 2005). Poor performances emerge when people fail to retain, access, and/or apply appropriate memory behaviour (Hunt, 2005). There are different kinds of memory-related behaviours and problems that emerge and can be observed in this context, for example: short, long, working, biological, and intellectual memory behaviour and problems (Hunt, 2005, p. 3).

The relationship between memory and cognitive behaviour is, however, extensively debated in terms of how it is constructed, firstly, in the mind (Hambrick, Kane, & Engle, 2005; Ramachandran, 2011) and, secondly, in a cross-C, L, and S context (van de Vijver & Matsumotoa , 2011). Cognition and intelligence reflects what a person's culture thinks is important to remember (Armour-Thomas, 2002; Whorf, 1956), and this requires a different type of interpretation that is often overlooked in cross-C and L studies (van de Vijver & Matsumotoa , 2011). Crystallised, fluid, and spatial-visual intelligence are valued differently in this context across cultures and languages.

Cognition and language

M2 employs highly symbolic metaphoric language to represent and solve complex problems (Duval, 2006; Halliday, 2006; O'Halloran, 2010). The symbols '$\sum$' and 'σ', for example, emerged metaphorically from the 18th letter of the Greek alphabet sigma to signify processes for interpreting sums and standard deviations in M2. Duval (2006) identified that the M2 sign system undergoes distinctive treatments and conversions in this context that create a cognitive paradox for learners. Duval (2006) stated, "treatments are the transformations of representations that occur in the same register" and "conversions are transformations of representation that consist of changing a register without changing the objects being denoted" (p. 111). The former identifies a syntactic change of meaning in the register, for example: transposing the symbols '0', '2', and '4' in the equation '0.20 + 0.25 = 0.45'. The latter identifies a paradigmatic change for the same symbols, for example: decimals into fractions in the equation '0.20 + 0.25 = 9/20'. The paradigmatic transformation of symbols from decimals into fractions creates a cognitive paradox, because it employs different symbolic representations and thinking to articulate similar meanings (Duval, 2006). Treatments and conversions are also difficult to interpret in real-life applications. For example, cutting a whole apple in half in most cases is interpreted as 2 halves, however, cutting a rug in half creates 2 rugs (Barnett & Cici, 2005, p. 213).

Metaphoric processes are difficult to describe, because they involve interpreting what goes on in an abstract way in someone else's mind. As Ramachandran (2011) stated, "we don't have the foggiest idea of how metaphors work or how they are represented in the brain" (p. 97). Ramachandran (2011) claimed the metaphor might, therefore, be better interpreted through the physical and neurological behaviours people display when, for example, processing metaphoric language such as M2. In this context Ramachandran (2011, p. 97) believed synesthesiacs sometimes respond to colours as a metaphor for conceptualising and processing numbers. Aberrant neurological behaviours, such as synaesthesia and aphasia, identify the dysfunctional behaviours that emerge and affect decoding and learning a language (Jakobson & Halle, 1956; Ramachandran, 2011).

Whorf (1956) identified that metaphoric processes also reflect the linguistic and cultural practices of the community. In this context Whorf (1956) defined M2 in the following way:

> In our language, that is SAE (Standard American English), plurality and cardinal numbers are applied in two ways: to real plurals and imaginary plurals. Or exactly if less tersely: perceptible spatial aggregates and metaphorical aggregates. We say 'ten men' and also 'ten days.' Ten men either are or could be objectively perceived as ten, ten in one group perception -ten men on a street corner, for instance. But ten days' cannot be objectively experienced. We experience only one day, today; the other nine (or even all ten) are something conjured up from memory or imagination. If 'ten days' be regarded as a group it must be an "imaginary", mentally constructed group. Whence comes this mental pattern?
>
> Just as in the case of the fire-causing errors, from the fact that our language confuses the two different situations, has but one pattern for, both. When we speak of 'ten steps forward, ten strokes on a bell,' 'or any similarly described cyclic sequence, "times" of any sort, we are doing the same thing as with 'days.' CYCLICITY brings the response of imaginary plurals. But a likeness of cyclicity to aggregates is not unmistakably given by experience prior to language, or it would be found in all languages, and it is not". (p. 139)

Neuropsychology

The neuropsychological domain interprets more specifically the role and function of the brain when learning a language such as M2. Two critical observations emerged in this context to help conceptualise the research problem.

Firstly, humans are born with a biological capacity to learn a complex language (Chomsky, 1972; Fromkin, Blair, & Collins, 1999), for example, mathematics. Piaget (1972) identified that this capacity also emerges through different stages of biological and neurological development. Abstract and metaphoric thinking, for example, do not fully develop until puberty (Piaget, 1972). Puberty as a construct is also often cited as a milestone in which the capacity to learn an additional language starts to decline with age (Munnich & Landau, 2010).

Secondly, a person's neurological capacity to learn a language is also affected by environment factors such as education and culture (Slobin, 1997; Vygotsky, 2011). Munnich and Landau (2010) identified, for example, that while the capacity to learn spatial and temporal references in L2 decreases with age, the time and type of L2 immersion a learner receives also affects the rate of decline. Immersion identifies the timing, quantity, and quality of C, L, and S experience received when learning a language (Hamers & Blanc, 2000). Immersion that involves, for example, formal education and related cultural experiences enhances learning an additional language (Baker, 2011; Hamers & Blanc, 2000; Lo Bianco & Freebody, 2001; Munnich & Landau, 2010; Tochon, 2014).

Extreme examples of feral children who missed out on learning a first language during critical stages of neurological and social development never fully recover to acquire language successfully later in life. The capacities of CALD BL learners who missed out on formal education due to, for example, extended stays in refugee camps and gender inequalities, anecdotally, display similar problems when learning L2 and M2. The synapses that transmit neural activity in the brain enlarge and develop differently through formal education (Neubauer & Fink 2005; Ramachandran, 2011). These experiences also appear to enhance the capacity of people to think, learn, and solve complex problems through language (Neaubauer & Fink, 2005; Ramachandran, 2011) such as M2. Smaller synapses also require more neural energy to process the same types of thoughts (Neaubauer & Fink, 2005). A distinctive link is made between the development of a person's brain and the ability and neural energy required to learn an additional language. Very little research has been conducted in this area within the context of the research problem.

Applied and educational Linguistics

Linguistics interprets human behaviour through language use. Crystal (1992, p. 412) identified that the specialist areas of linguistics include: Psycho and Neurolinguistics, which studies cognitive and neurological behaviour; Sociolinguistics, which studies social behaviour; and, Educational Linguistics, which studies the application of linguistic theory and methods in education. Applied Linguistics is concerned with the application of different linguistic theory and methodologies to help resolve language-learning problems (Crystal, 1992; Ivanic & Tseng, 2005).

There was, however, a distinct lack of research in this area of the research problem, and what research exists was mostly conducted on children and non-CALD adults. The next subsection synthesises the educational implications and inferences (Onwuegbuzie, Leech, & Collins, 2012) that emerged from this associated research. The findings helped question and interpret the adult-CALD BL behaviours and experiences that affected decoding and learning M2.

Gender

Girls, generally, performed poorer than boys when learning mathematics as a subject at school (Rothman & McMillan, 2003). The probable causes of this problem were interpreted in two ways. Firstly, girls because of neurological and cognitive differences relate less well to learning M2 as a technical discourse than boys (Neaubauer & Fink, 2005). Secondly, environmental factors such as cultural and social experiences of girls affect them differently when learning mathematics (Rothman & McMillan, 2003). It was not clear, however, if or which factor affects girls more when learning M2 in a cross-C, L, and S context. Poorer socio-economic backgrounds, for example, detracted from both boys and girls learning M2 (Rothman & McMillan, 2003). There was no research sighted in this area in the context of adult-CALD BL female learners in M2 (Section 2.3).

Age

An extensive body of research identified an age-related hypothesis that believes younger learners have a biological advantage in learning an additional language (Ellis, 2001; Hamers & Blanc, 2000). However, there was no research sighted in the context of an age-related hypothesis also applying to learning M2 in a cross-C, L, and S context. Anecdotally, mathematics involves learning highly abstracted metaphoric language that is not, theoretically, fully conceptualised in the brain by learners until post-puberty (Piaget, 1972). Furthermore, adult-CALD learners might be more motivated than younger learners in learning L2/M2 to help settle in a new country. Both factors negated the potential negative effects that an age-related hypothesis had on adult-CALD BL learners learning M2.

Learner motivation, self-esteem and self-concepts

Child (2004) stated, "a working definition of motivation would be that it consist of internal process and external incentives which spurs us on to satisfy some need" (p. 226). Child (2004) also identified that internal processes are difficult to interpret, because it involves some guessing and theorising how these factors affect learning in the mind. The explanations include, for example, interpreting highly abstract behaviours such as learner anxiety, attitude, self-esteem, self-concepts, and instrumental and intrinsic motivations that affect learning an additional language (Baker, 2011; Gardner, Masgoret, Tennant, & Mihic, 2004).

Very little research found identified if adult-CALD BL learners have any additional anxieties about learning M2 over, for example, L1, M1, and L2. Furthermore, the negative factors that affect monolingual English learners in a classroom might not apply in adult-CALD BL learning environments. Anecdotally, ESL classrooms display greater capacity and empathy in addressing learners' needs, despite the regular criticisms about mainstream VET classrooms (Fitzsimons & Godden, 2000).

Learner aptitude and intellect

The effect aptitude and intellect have on learning an additional language is debated in literature and research (Ellis, 2001; Gardner, Masgoret, Tennant, & Mihic, 2004). Furthermore, very little is known about how these factors particularly affect learning M2 in an adult-CALD BL context. This makes it difficult to interpret what constitutes intelligent behaviour and a positive aptitude when learning M2.

Some adult-CALD BL learners might, for example, grasp and solve problems more quickly in M2, because M1 and M2 share signs and meanings. Others learners might find the M2 sign system incongruent because of the cross-C, L, and S differences that exist (Barton, 2009; Whorf, 1956). For example, some cultural groups might exclude learners because of their gender and ethnicity from learning certain types of mathematics such as money and finance (Appendix E: Task 15). This makes M2 cognitively and ideologically incongruent for some learners to decode and learn.

Socio-economic backgrounds

People from CALD backgrounds are statistically linked (Section 1.1) to unemployment, poverty, illiteracy, and innumeracy (ABS, 2006a; ABS, 2006b; ABS, 2013; Looney, 2008; Shomos & Forbes, 2014). However, the cause and effect of this relationship is complex to interpret. Illiteracy and innumeracy, for example, might be a product rather than the cause of poverty and unemployment in CALD populations (Black, 2002). Improving foundational LLN skills does not transpose precisely into generating employment and wealth opportunities for CALD learners (Black, 2002). Furthermore, not all CALD learners come from lower socio-economic backgrounds, as many skilled migrants relocate to Australia for humanitarian and professional reasons. It takes time for these migrants to establish networks and pathways that lead to settlement and employment opportunities (Miralles, 2004). The issue of unemployment occurs regardless of migrant learner's literacy and numeracy skills.

In summary, learner differences in the Linguistics domain identifies why some adult-CALD BL learners have problems learning M2 as a new language. More research is needed to interpret how these factors specifically affect decoding and learning M2.

Discourses and pragmatics

Some researchers interpret learner problems through discourse behaviours in M2 (Sfard, 2005). Discourse analysis identifies, for example, the problem of prolonging a naturally spoken conversation in a particular language (Crystal, 1992). M2 discourses often appear vague and indirect in this context (Rowland, 1999). According to Rowland (1999), however, primary school children employ distinctive types of vague hedges, for example, 'I think', to exchange, learn, and prolong discourses in M2. Pietarinin (2006) identified that these types of discourse are strategic in nature, because they establish the linguistic rules employed to learn in classroom situations. These rules, however, are not universally shared across languages and cultures (Barton, 2009; Bloomfield, 1939; O'Neill, 2009; Whorf, 1956). Idiomatic phrases such as 'I think', are problematic, because some cultures might not allow learners to debate mathematical ideas as liberally suggested in the aforementioned studies.

A few researchers interpreted the code-switching behaviours of CALD BL children when learning M2. Hammers and Blanc (2000) defined code-switching as a "bilingual communication strategy consisting of an alternative use of two language in the same utterance" (p. 309). The small body of research in this area generated the following observations:

1. Parvanehnezhad and Clarkson (2008) studied the code-switching strategies of a group of Iranian BL children, and found the more proficient L1 and L2 students are also more strategic and successful in deploying code-switching techniques when learning M2 (Section, 2.3).
2. Farrugia (2003) identified that M2 symbols and concepts are often first introduced in L1 and M1 words to learn a new idea, and then M2 terminology is introduced to interpret more complex mathematical problems. M1 and M2 symbols are arbitrary in this context (de Saussure, 1910), because the symbolic representations can be swapped, for example, between M1 in Maltese and M2 in English to articulate meanings and processes.
3. Duyck and Brysbaert (2008) studied the eye movement of a group of Dutch multilingual learners and found that multilingual learners could transpose mathematical meanings backwards and forwards across three different languages: Dutch, English, and German. However, the participants were quicker at recognising and transposing Dutch to German symbols because of their linguistic similarities.

Sociolinguistics

Tabouret-Keller (2000, p. 315) identified that language functioned as an external behaviour for defining oneself as part of a group. Language acts such as dialects, slangs, and idiomatic expressions, also emerge and distinguish the community's behaviours (Tabouret-Keller, 2000). Whorf (1956) believed language acts also reflect the cultural needs of the community. M2 reflects the cultural knowledge and discourses needed to survive in an English speaking community. M2 generates two types of problems for adult-CALD BL learners in this context. Firstly, M2 represents discourses and language acts that many monolingual English speakers also find difficult to learn (Halliday, 2006; Sfard, 2005). Secondly, M2 discourses and language acts are difficult to learn because of cross-C, L, and S differences (Solano-

Flores, Barnett-Clarke, & Kachchaf, 2013). Little or no research was sighted in the context of adult-CALD BL learning M2 as a two-tiered linguistic act and problem.

Language in space and time

The capacity of CALD BL learners to decode and learn spatial and temporal references in L2 and M2 was more extensively studied (Barton, 2009; Danzinger, 2010; Feist, 2008; Feist & Gentner, 2007; Hickman & Hendriks, 2010; O'Neill, 2009). This body of research identifies how spatial references and logics that define L2 and M2 are difficult to reconcile in a cross-C, L, and S context (O'Neill, 2009). The prepositions 'in' and 'into' define containment relationship between two or more objects in L2 and M2, for example: 'the milk is in the jug', 'the bird is in the sky', 'just in time', and 'three goes into six twice'. Munnich and Landau (2010, p. 34) identified that four prepositional logics helped interpret symbolic relationships in L2:

1. Containment - the bird is in the tree.
2. Contact - the bird is on the branch.
3. Proximal - the bird is next to the tree.
4. Distal - the bird is to the north of the tree.

These logics are at odds with cultures and languages that do not see the 'sky', 'time', or 'numbers' as containers in which things might symbolically be placed into, as you would with a 'jug' (O'Neill, 2009; Whorf, 1956).

Prepositional logics are also defined by the functional relationships people attach between objects (Carlson, Reiger, Lopez, & Corrigan, 2006). For example, Carlson, Regier, Lopez, and Corrigan (2006) found participants in their study, given a choice, preferred to position the symbol for a toothpaste over the bristle end of a toothbrush, because of their functional relationship. Feist and Genter (2007) asserted, however, that people also draw on their cultural and linguistic experiences to interpret these types of symbolic relationships. The form and function of a toothbrush and the mathematical sign system are not universally shared across communities (Whorf, 1956).

Linguistic structures

Ernest (2006) identified that the M2 sign system has agentic characteristics that bind and shape the thinking and physical behaviours of readers. Leslie (1993, p. 4) stated agents as having the following physical and cognitive properties:

1. Mechanical: Agents have mechanical properties that mere physical objects do not have. The main difference in terms of having an internal and renewable source of "energy" or FORCE, versus not possessing such a source and thus having to rely on external sources.
2. Actional: Agents do not simply move and take part in events. Agents act in pursuit of goals and re-act to the environment as a result of perceiving. Mere physical objects do not act in pursuit of goals and do not perceive their environment. Further, the acting and re-acting Agent can come together with another Agent and inter-act.
3. Cognitive: The behaviour of Agents is determined by cognitive properties, e.g., holding a certain attitude to the truth of a proposition. Mere physical objects do not have cognitive properties. (p. 4)

Tylen (2007) applied Leslie's (1993) interpretation of agency to study the form and function of the sign system and its effect on human behaviour. Readers in this context respond to signs by enacting them through physical responses, like pointing and gesturing in M2 (Ernest, 2010; Thom & Roth, 2011). The effect agency has on decoding and learning M2 was synthesised as follows:

1. Vectors define the direction and magnitude that mathematical meanings and respective thinking behaviours change at the "rank" and "clause" level in the sign system (Halliday, 2006, pp. 62-63). Different registers, for example, algebra, tables, and graphs, generate different types of thinking behaviours (O'Halloran, 2005).
2. Translational and punctuated equilibriums identify points where meanings are (re)-packaged in the text to signal the different types of thinking behaviours that are needed (Halliday, 2006, pp. 27-28). For example, the equations '0.20 + 0.25 = 0.45 = 9/20 = 1/5 + 1/4' employs decimals and fractions to articulate similar meanings, but different types of thinking behaviour (Duval, 2001, p.

108). The equal sign '=' identifies points where thinking processes changes at a clause level.

3. Terms such as energy and force emerge to describe how the sign system functions, marks, and contrasts thinking behaviours in the M2 sign system (Ernest, 2006). The sign '-', for example, marks a paradigmatic change in thinking for the sign '2' to '-2'. The signs '-' and '2' are also elastic, because at a clause level they are employed to generate different types of meanings and thinking behaviours, for example: '-3^{-2}'. Meanings and thinking behaviours emerge by 'compounding' signs into molecular structures (Ernest, 2006, p. 69).
4. Meanings emerge, as Halliday (2006) stated, through "chains of reasoning" that articulate new "lines of argument leading on from one step to the next" (p. 61). Semiotic synergies exist at molecular level, because M2 seeks to employ less signs to articulate meanings (Bloomfield, 1939), for example: '$E= MC^2$'.
5. C, L, and S conventions, however, encode different types of signs, meanings, and behaviours in the text (Peirce, 1931; Jakobson & Halle, 1956; Tylen, 2007; Eco, 1981).

The text can, therefore, be employed to observe and interpret the physical and mental behaviours of readers (Eco, 1981; Ernest, 2010; Holland, 1990), and where aberrant decoding behaviours emerge (Jakobson & Halle, 1956; Durst-Andersen, 2011). Holland (1990) identified that behaviours can be interpreted through, for example:

- The thoughts and feelings that emerge via slips of the tongue, speech acts, and free associations when reading the text.
- The accounts of childhood and human behaviour, defences and adaptations, and imagery evoked by the text.
- The role of identity and how it emerges as themes and variations by the histories within the text.

The respective contexts foregrounded the types of questions that were needed to interpret the agentic characteristics of the M2 text and the respective cross-C, L, and S behaviours and problems in it. As Eco (1981) stated: "reading, however, no longer refers to problems of the critical interpretation or less refined hermeneutic: rather, it is concerned with the formidable question of the reader's responses as possibility built into the textual strategy" (p. 35).

4.4.4 Language and culture

The meaning and significance of language and culture was interpreted in this study through two associated disciplines: Language Studies and Cultural Studies. Language Studies was, respectively, interpreted through English (Crystal, 1996; Fishman, 1980; Milroy & Milroy, 1998), and English as an Additional/Second Language (Risager, 2006; Tabouret-Keller, 2000). Cultural Studies was interpreted through Multicultural, Intercultural, and Cross-cultural Research domains (Dockery, 2009; Geertz, 1973; Milroy & Milroy, 1998; Risager, 2006; Rowland, 1999; Schutz, 1932; van de Vijver & Matsumotoa , 2011; Whorf, 1956).

Language studies: English and English as an additional/second language

Whorf (1956) identified language functions as a medium for interpreting and conveying meanings in the community. The medium, as Chandler (1994) stated, defines "such broad categories as speech and writing or print and broadcasting or relate to specific technical forms within the media of mass communication or the media of interpersonal communication" (p. 515).

The medium also generates a message in itself that reflects the C, L, and S conventions of the community. As McCluhan (2001) stated, "in a culture like ours, long accustomed to splitting and dividing all things as a means of control, it is sometimes a bit of a shock to be reminded that, in operational and practical fact, the medium is the message" (p. 1).

The English language also functions as a medium for encoding and decoding meanings and processes in the M2 sign system (Bloomfield, 1939; Whorf, 1956). For example, the time reference '1:30' is expressed in English as 'half-past-one', and/or 'one-thirty'. Italian adds a conjunction 'e' (and) to the syntax to express the same time reference'un-ora-e-messa' (1-hour-and-half). Adding the conjunction 'e' (and) to a time reference does not transpose precisely back into English for interpreting time, for example: 'one-and-a-half-hour' instead of 'a-half-past-one'.

The English language has distinctive linguistic structures and associated thinking processes that are not universally shared with other languages and cultures in mathematics (Bloomfield, 1939; Whorf, 1956). The commutative law, for example, breaks down in this context (Barton, 2009, p. 50), because the M2 sign system

assumes a degree of objectivity and neutrality in the medium and thinking behaviours that are employed to interpret and articulate meanings (Sections 4.4.2 & 5.3.7). For example, 5 x 6 cows is axiomatically equivalent to 6 x 5 cows, however, 6 lots of 5 cows might signify greater wealth and value than 5 lots of 6 cows in certain communities. As Whorf (1956) stated:

> We are inclined to think of language simply as a technique of expression, and not to realize that language first of all is a classification and arrangement of the stream of sensory experience which results in a certain world-order, a certain segment of the world that is easily expressible by the type of symbolic means that language employs. In other words, language does in a cruder but also in a broader and versatile way the same thing that science does. We have just seen how the Hopi language maps out a certain terrain of what might be termed primitive physics. (p. 55)

Multicultural, intercultural and cross-cultural studies

Risager (2006, p. 6) identified that a general consensus exists in literature and research that language and culture are inseparable as constructs, because language reflects the cultural needs of the community. Risager (2006) added that language also functions as a classifier and marker of a person's cultural identity in this context. Variations in dialects and slangs, for example, identify a person's cultural and linguistic background, and psychological behaviour (Hamers & Blanc, 2000; Holmes, 2001).

A person's identity and behaviour also changes through new C and L experiences (Hamers & Blanc, 2000). Indigenous languages, cultures, and behaviours, for example, disappear, because more dominant languages and cultures replace old ones so the community can survive (Hornberger & Skilton-Sylvester, 2003). Migrants also disassociate themselves from home languages and cultures, because of the adoption of new C and L practices (Baker, 2011). These experiences change and challenge the psychological state of additional/second language learners, by creating new cultural identity and ways of thinking through language (Hamers & Blanc, 2000).

It is difficult to interpret exactly how M2 might change and challenge an adult-CALD BL learner's identity and psychological state. M2, for example, might share signs and meanings with M1, and this makes the cultural and psychological transition

less problematic. Alternatively, M2 might employ different signs and processes than M1, and this creates a new C, L, and S experience that challenges and changes learners (Hamers & Blanc, 2000; Whorf, 1956). Armour-Thomas (2002) identified the following questions should, therefore, be asked when interpreting cross-C and L behaviours:

1. Does intelligence have the same meaning between groups of people?
2. Are there stereotypical beliefs about intelligence?
3. Are cultural attributes comparable?
4. Are the conventions of discourse comparable?
5. Do racial/ethnic groups have comparable familiarity within their symbolic systems?

4.4.5 Education

Three disciplines helped interpret education as a construct in the research problem: Educational Studies, Curriculum Studies, and Professional Development. Educational Studies interprets the problem through two associated research domains: Psychology and Sociology of Education. The psychological implications for education are also synthesised through Behavioural and Cognitive domains because of their cross-disciplinary relationship (Section 4.4.3). Specific educational implications are, in the main, interpreted through the Education domains. Curricula implications emerge through Mathematics and English Education. Professional Development emerges through Adult Education Practices.

The majority of research identified in this area was conducted on children (Duval, 2006; Gillard, Van Dooren, Schaeken, & Verschaffel, 2009; Morris, 2002; O'Neill, 2009; Piaget, 1972; Vygotsky, 2011), and very little has been published on adult learners (Balatti, Black, & Falk, 2006; Duffin & Simpson, 2000; Falk & Millar, 2001). The specific study of adult-CALD BL in M2 was negligible in this area (AIR, 2006; COAG, 2008). The ensuing sections synthesise the educational implications and inferences (Onwuegbuzie, Leech, & Collins, 2012) that emerged from this small body of knowledge.

Sociology of education

CALD learners over-represent the number of illiterate and innumerate people in Australian society (ABS, 2006b; ABS, 1998; Balatti, Black, & Falk, 2006; Looney, 2008; Perkins, 2009). As a result, Federal and State funding focuses on building foundational LLN capacities in this area (Black, 2002; DEEWR, 2014; Griffin, 2014; Miralles, 2004; NCVER, 2006). Foundational LLN education also generates social-psychological benefits, such as: building learner self-esteem and social cohesion in certain communities (Papen, 2005). LLN funding, however, does not transpose precisely into creating employment and wealth opportunities in CALD minority groups (Section 4.4.1) (Black, 2002).

Skovsmose (2001) identified there are cognitive advantages associated with learning mathematics as a subject. For example, mathematics creates:

1. A space for analysing hypothetical scenarios.
2. A means for investigating and comparing those scenarios.
3. A means for interpreting the implications those scenarios might have on a society, for example, choosing the best and worst situations and solutions.

Fitzsimons (2002) identified that mathematics education is implemented in different ways in Australian society. Each context signalled a different reason for teaching and learning M2 as a construct, for example:

1. A science: Pure and Applied Mathematics.
2. A cognitive tool for solving everyday problems: keeping time and paying bills.
3. An aesthetic activity: solving mathematical problems for leisure and entertainment.

High levels of innumeracy in CALD minorities groups restrict them from having access to the social and personal benefits of being numerate in M2 (ABS, 2006a).

English and mathematics education

The term numeracy is used interchangeably with foundational mathematics to identify the application of mathematics in solving everyday problems (Tout & Motteram, 2006; COAG, 2008). Numeracy is, however, a relatively new educational construct (Tout & Motteram, 2006) and, as a result, requires further

conceptualisation and development as a specialist area of mathematics (AIR, 2006; COAG, 2008). Anecdotally, it is difficult to distinguish mathematics and numeracy as constructs when numeracy tasks are also symbolically sophisticated, and Pure and Applied Mathematics are lexically rich to decode (Appendix E: Tasks 7, 10, & 15). This makes it difficult to distinguish the parameters of numeracy as a specialist area in mathematics education.

Adult numeracy education is often subsumed into foundational language and literacy curricula, such as: the Certificate in Spoken and Written English (AMES, 2013) employed in adult migrant education programs; and, the Certificate in General Education for Adults (DEECD, 2012) employed in mainstream adults Commonwealth funded LLN programs. Problems emerge in both areas of education, because teachers lack formal capacities to teach mathematics at a foundation or advanced level (COAG, 2008; Trinick, Meaney, & Fairhall, 2014). It was not a curricula requirement, for example, to have formal training in mathematics to teach it as a subject in the Certificate of Written and Spoken English (AMES, 2013) and Certificate in General Education for Adults (DEECD, 2012).

Adult-CALD BL learners bring with them a range of experiences in mathematics that are difficult to interpret and manage in the classroom. Anecdotally, teachers might have a less formal understanding of mathematical concepts compared to their adult-CALD students. An unsophisticated discourse in M2, for example, does not mean adult-CALD learners have an unsophisticated understanding of mathematical concepts (Section 5.4).

Adult education and professional development

There are different reasons why teachers have problems teaching M2 in a cross-C, L, and S context (Trinick, Meaney, & Fairhall, 2014). Issues included, for example, a lack of formal training and experiences in teaching M2 as a numeracy construct (AIR, 2006; COAG, 2008), and an inability to reconcile CALD-learner competencies, behaviours, and needs in M2.

Little or no literature identified best practices for teaching M2 as an adult construct in a cross-C, L, and S context. Anecdotally, teachers in this area draw on a combination of language and literacy teaching experiences, and quasi literature and research to manage the problem. The problem also requires knowledge of the form

and function of the M2 sign system and the learning behaviours within it (Duval, 2006; Ernest, 2006; Presmeg, 2006; Schleppegrell, 2007).

4.5 What science governed existing literature and research?

Reber (1984, pp. 439 & 670) identified that the term science reflected three interrelated meanings in existing literature and research:

1. It defined the name of the discipline that focuses on the derivation of basic principles and laws, for example: Philosophy, Mathematics, Psychology, Linguistics, Language Studies, Cultural Studies, and Education.
2. It defined the procedures that are employed to resolve problems through different research domains. Procedures most often:
    a. Identify a researchable problem.
    b. State the problem that is tied to existing theory and/or empirical fact.
    c. Articulate a testable hypothesis or proposition to resolve the problem.
    d. Gather data that accepts or rejects the hypothesis or proposition.
    e. Modify existing knowledge to accommodate new findings.
3. It defined the body of knowledge that emerges from the application of scientific procedures.

The Philosophy domains, in the main, employ Logics (Russell, 1912) and Linguistics (Peirce, 1931; Bloomfield, 1939; Barton, 2009) to interpret the constructs of the research problem. Logics lent to interpreting the research problem through the form and function of the M2 sign system (Bloomfield, 1939; de Saussure, 1910; Halliday, 2006). This approach omitted the role humans also play in generating meanings. Linguistics lent to interpreting the research problem through the pragmatic and humanistic behaviours that also affect people in the sign system (Jakobson & Halle, 1956; Peirce, 1931). Merging the two approaches generated a more holistic interpretation of the research problem.

The Cognitive and Applied Linguistics research domains most often interpret language behaviour through empirical evidence and case study. Hypothesis testing is often used to test, interpret, and articulate theory. Procedures from the Cognitive and Linguistics domain are used in other research domains, for example, Education (O'Neill, 2009) and Cultural Studies (Risager, 2006) that interpret human behaviour in language learning situations.

Culture and Language Studies is also, mostly, case-based and empirical. Human behaviour is interpreted and defined more subjectively in this context (Blumer, 1969; Geertz, 1973; Schutz, 1932). Interpretive and qualitative research techniques lend more to the studying of human behaviours in a cross-C, L, and S context (Section 2.4).

The Education domains often employ mixed qualitative and quantitative methods to study and articulate educational solutions (Creswell, 2003). Both qualitative and quantitative approaches are criticised, because they fail to follow established scientific procedures (Onwuegbuzie & Daniel, 2003). This made it difficult to interpret the validity and meaningfulness of the findings that emerge. The synthesis found philosophical beliefs and scientific principles are rarely made explicit in publications. This also made it difficult to interpret the implications of the findings and recommendations.

The synthesis was cognisant of the divergent scientific procedures, language, and problems that emerge in conducting a cross-C, L, and S study. The procedures developed in this study were, therefore, defined, coded, and linked to identify philosophical, scientific, and linguistic underpinnings (Section 3.2).

4.6 Conceptual relationships

The terms language, culture, mathematics, adult-CALD BL learner, education, and Australian helped conceptualise the constructs of the research problem. The term conceptualise, as MacInnes (2011, p. 136) stated, helped "envision", "explicate", "relate", and "debate" the potential relationships, causes, and effects of the problem. For example, the effectiveness of the Australian BL education system was defined, in part, through its ability to address linguistic, cultural, and educational policy. However, as Lo Bianco and Slaughter (2009) stated:

> While there appears to be public appreciation of the importance of second languages, there is less appreciation of the degree of institutional commitment, levels of funding and provider change required to achieve effective language knowledge through formal education. (p. 6)

The elements also functioned as umbrella terms to conceptualise the micro relationships that exist in the research problem. Table 4.3 summarises the micro

relationships identified in the synthesis of existing literature and research. These findings enabled the synthesis to explicate, compare, and contrast in tabular form the potential relationships, causes, and effects that exist. These findings helped conceptualise the form and function of the battery of open-ended questions in collecting and interpreting data. For example, the meaning and significance of the English language as medium for decoding and learning M2 was, respectively, questioned and synthesised from a language, cultural, mathematics, cognitive and educational perspective (Section 5.3.9).

Table 4-3: Framework for conceptualising potential micro relationships

Constructs	Potential micro relationships
• Language	L and S structures, functions, social, cultural, **English**, power, identity, education, second language, bilingualism, diversity, scientific, mathematics, metaphors, behavioural, cognitive.
• Culture	Customs, beliefs, knowledge, identity, mathematics, diversity, language, behaviour, **English**.
• Australian	Identity, culture, diversity, **English**, economic imperatives, nationalism, education, non-Australian.
• Adults	Age, different to children, maturity, experience, education, cognitive, CALD.
• Mathematics	Patterns, shapes, education, social, power, identity, cognitive, formal, semiotic, language, registers, ethno, metaphoric, **English**, foundational, knowledge, non-mathematics.
• Education	Literacy, numeracy, **English**, children, adult, mathematics, foundational, organisations, ideologies, BL, VET, second language, CALD.

4.7 Chapter summary and implications for data collection

The synthesis of existing literature and research generated a comprehensive interpretation of existing knowledge, and the inferences that could be applied to study the research problem (Onwuegbuzie, Leech, & Collins, 2012, p. 4). The procedures employed to synthesise and articulate this knowledge into data collection in the battery of open-ended questions in this study emerged as follows:

1. The elements and wording of the research problem and questions were used to electronically search for relevant literature and research.
2. Individual readings were summarised into notes and reflections on the research problem.

3. MacInnes's (2011) Framework of Conceptual Thinking and The Australian Standard Research Classification List (ABS, 1998) were employed to help, firstly, interpret and classify the contents of the readings into a research domain and taxonomic perspective and, secondly, identify where the findings from the data collection might contribute its own knowledge (Table 4.2). The six research divisions identified in the synthesis of existing literature and research were (Sections 4.4.1 - 4.4.5):
    a. Philosophy
    b. Mathematics
    c. Behaviour, Cognition, and Linguistics
    d. Language and Cultural Studies
    e. Education
4. An annotated summary of theoretical ideas and potential questions for collecting and interpreting data were drafted from the synthesis of existing literature and research for the three stages of data collection (Appendix C). For example, the following data collection questions emerged within the Mathematics domain from interpreting Devlin (2000, p. 1) statement that "Mathematics is the science of patterns":
    a. Stage 1 (Semiotic Synthesis of Mathematical Tasks). *Does the* (M2) *sign system signify* (have) *distinguishing* (L and S) *structural patterns?*
    b. Stage 2 (Participant Background Interviews). *What is mathematics?*
    c. Stage 3 (Participant In-tasks Observations). *Can you see patterns in the way the symbols are used in this task?*
5. The battery of open-ended and micro questions (Appendix D) refined and organised the first draft questions into data collection questions. For example, first draft question 6 was linked with first draft question 31 to generate Stage 1 battery Question 18 (*Does the sign system signify systemic, random, or irrational patterns – draw a flow chart identifying process flow?)*. Draft question 6 transposed directly into battery Question 30 for Stage 2 and Question 22 for Stage 3.
6. Battery questions were coded and linked to their theoretical underpinning and the data that emerged. Existing knowledge was

coded a posteriori (Kant, 1788) in this context to synthesise the research problem, without being tied to proving existing theory.

The synthesis of literature and research generated a holistic interpretation of the research problem and associated data collection questions. The findings and implications emerging from the data collection stages are synthesised in the subsequent chapter.

Chapter 5: Data Evaluation, Synthesis, and Findings

> What emerges is a picture of mathematics as a plaited braid of many strands that merge and split, fold back and tangle – but a braid in which there is no 'one way' unless you are looking from inside one of the strands.
>
> (Barton, *The language of mathematics: telling mathematical tales*, 2009, p. 137).

5.1 Chapter introduction

Data were synthesised from three different stages of observations to generate a holistic interpretation of the research problem (Gharajedaghi & Ackoff, 1985; Patton, 1990). Stage 1 data collection conducted a semiotic synthesis on the mathematical tasks that were employed to observe the participants' in-tasks decoding and learning behaviours in Stage 3. Stage 2 collected data on the participants' background that affected decoding and learning M2. The data observations were summarised into a Data Matrix (Appendix F) and Summary Matrix (Appendix G) to interpret global themes and resolutions for the research problem and questions. The findings that emerged in the Data Matrices are recounted in narrative form (Section 3.8) through the six global data themes that head the sections of this chapter:

1. TXT1000: Philosophical codes that are problematic to decode in M2.
2. TXT2000: Structural codes that are problematic to decode in M2.
3. TXT3000: BL decoding behaviours that emerge and affect how M2 is decoded and learnt.
4. TXT4000: BL humanistic behaviours that emerge and affect how M2 is decoded and learnt.
5. TXT5000: BL cultural behaviours that emerge and affect how M2 is decoded and learnt.
6. TXT6000: BL educational experiences that emerge and affect how M2 is decoded and learnt.

The position of the data synthesis and narrative in the study is highlighted in Figure 5.1.

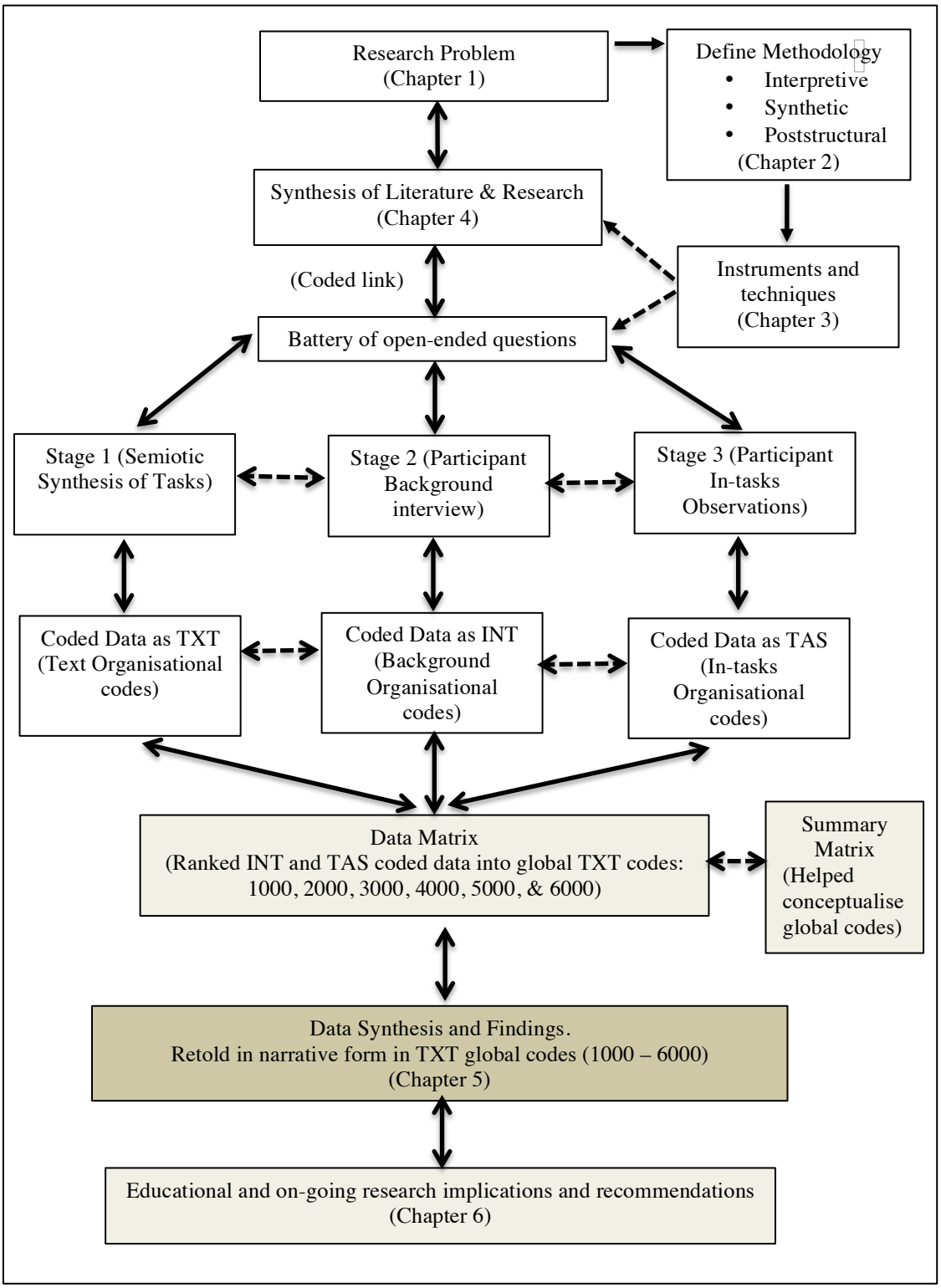


Figure 5.1: Position of data synthesis in the study

The findings that emerged from the data synthesis addressed the research problem and questions as follows:

Question 1: *What are the linguistic and semiotic structures in mathematical language that create problems for adult-CALD bilingual learners when learning mathematics in M2*?

Philosophical and structural codes define the C, L, and S conventions that encode and distinguish the M2 sign system. The codes require distinctive L and S competencies to decode and learn successfully. Adult-CALD BL learners also bring with them distinctive decoding, humanistic, physical, cultural, and educational behaviours and experiences that create problems when decoding and learning M2 in a cross-C, L, and S context.

Question 2: *How can knowledge of adult-CALD bilingual learner related problems in linguistic and semiotic structures in mathematics be used to enhance mathematics education in Australian classrooms?*

Knowledge of adult-CALD BL learner problems in M2 enhance the capacity of teachers and learners to:

a. Interpret the C, L, and S conventions that encode the M2 sign system.
b. Identify the competencies needed to decode and learn M2.
c. Reconcile and manage the BL behaviours and experiences that emerge and create problems when decoding and learning M2 in a cross-C, L, and context.

Question 3: *How can case-based interpretive research methodology deepen our understanding of adult-CALD bilingual learners who are engaging in linguistic and semiotic structures in mathematics as an additional language?*

Case-based interpretive research methodology:

a. Enhances the teacher and researcher's capacity to observe and interpret L and S structures, and BL decoding and learning behaviours that create problems when teaching and learning M2. An alternative

post-positivist approach lacks the same level of breadth and sophistication in interpreting a complex language-learning problem.

b. Enables the knowledge gained from studying L and S structures, and BL decoding and learning behaviours in M2 to articulate structural and language-based teaching solutions.

c. Generates research techniques that are transferable to other areas of education that have complex language-learning problems to resolve.

Chapter 6 will recount in greater depth the educational, theoretical, and ongoing research implications that emerged from the data synthesis.

5.2 TXT1000: Philosophical codes that are problematic to decode in M2

Philosophical codes identify the third-tier (Peirce, 1931) ideological meanings, values, and beliefs that encode the M2 sign system. Philosophical codes are problematic to decode and learn, because their meanings are hidden and difficult to interpret in a cross-C, L, and S context. Ambiguities emerged when, for example:

1. Participant 4 stated the term marital status in Task 4, Figure 5.2, identified "*de facto*" relationships that were difficult to interpret in a cross-C, L, and S context.

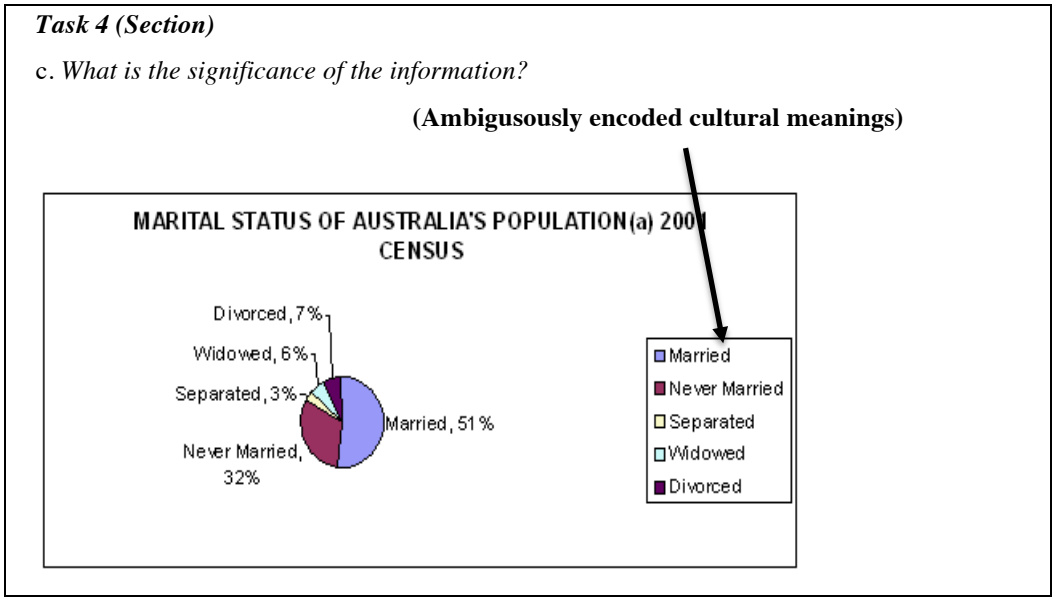


Figure 5.2: Example of ambiguously encoded cultural and social meanings in M2 text

2. Task 15, Figure 5.3, signalled that maximising profit and wealth was ideologically desirable, however the cultural and political backgrounds of

some of the participants excluded females from learning and participating in this type of mathematics.

Task 15 (Section)

Below is a table that charts the share price for company ABC over 10 weeks of trading on the stock exchange.

Draw the information on a graph paper plotting the price on the vertical axis and the respective weeks on the horizontal axis.

(Ideologically encoded)

a. Explain the significance of the line $dy/dx = 10 - 2x$ in determining when to buy and sell shares?

Figure 5.3: Example of ideologically encoded maximising profit in M2 text

3. Task 14, Figure 5.4, signalled the ideal way to travel between two points was in a straight line, however, some cultures employ more dynamic and non-linear ways to interpret travel (Barton, 2009; Whorf, 1956). Task 14 was interpreted non-linearly when, for example, it was envisaged dangerous to walk through a park late at night.

Task 14 (Section)

There are two ways you can travel from your house to a supermarket S. You can travel 60m straight down to the end of the street and turn left 90 degrees and walk another straight 80m to the supermarket. Or, you can travel diagonally from your house across a park straight to the same supermarket.

How much distance is it less to travel across the park compared to going around the around by two road?

Figure 5.4: Example of ideologically encoded linear travel in M2 text

Adjectival terms such as western and English defined the ideological conventions that encode M2. Maximising profit in Task 15 and minimising travel time in Task 14 was not universally understood in the respective participants' backgrounds. As Barton (2009) stated, "the point is that we choose what to make into our system. We

experience aspects of our quantitative (relational and spatial) world and then create systems to handle them, to explain them, to communicate them" (p. 78).

Nonetheless, the participants also found M2 shared ideological meanings in M1. The participants did not find the construct of time, distance, or supermarket problematic to conceptualise in Task 14. Alternatively, it was the more formulaic and symbolic representations that were new and difficult, for example, for Participant 4 and 5, to interpret. Shared ideological meanings made the tasks easier to decode and learn in a cross-C, L, and S context.

The meaning and significance of philosophical codes was synthesised through the observations that were made in the three stages of data collection. Stage 1 marked the L and S structures (Jakobson & Halle, 1956) that were envisioned ideologically difficult to decode and learn in a cross-C, L, and S context. Stage 2 identified the participants' BL experiences that affected the interpretation of philosophical codes in M2. Stage 3 identified the in-tasks L and S structures and BL behaviours that created problems when decoding and learning philosophical meanings. The three stages generated a synergetic interpretation of the philosophical conventions that encode and create problems in M2.

The term BL identified that the participants had "access to two or more linguistic codes" to interpret meanings (Hamers & Blanc, 2000, p. 368). Participant 2, for example, identified the symbol '六' in Task 3 signified the quantity 'six' in Chinese. This made the symbols '六' and '6' arbitrary representations (de Saussure, 1910), because Participant 2 could employ either symbol to signify the quantity 'six'. Philosophical codes, however, made the M2 sign system less arbitrary to interpret in this context. The constructs of linearity and profit relied on the participants understanding their underpinning meanings and implications in the tasks. However, as Whorf (1956) and Barton (2009) identified, not all cultures and languages universally share these types of meanings and processes. Axiomatically, '3 x 5' generated the same meaning as '5 x 3'. However, the commutativity law broke down in this context, because '5 cows in 3 containments' could be valued differently than '5 containments with 3 cows' in different communities (Barton, 2009).

The participants made real-life connections that helped interpret philosophical meanings in the tasks. The terms speed and money in Tasks 10 and 15, for example, were interpreted through every-day experiences and conversations. However, some

of the participants' backgrounds prohibited females from driving cars and learning finance in this context. The study for ethical reasons did not disclose the effect the participants' gender and culture had on learning M2, other than synthesise the factors significant in a global context in shaping decoding and learning behaviours. The participants believed activities such as building and finance were predominantly a male function in their community. The participants' personal experiences in, for example, family, relationships, food, and camp-life affected their interpretation of the meanings in Tasks 4 and 8, Figure 5.5. These experiences were identified through the discourse the participants displayed in the respective task activities.

Task 8 (Section)

A camp organiser calculates that if he **(Male)** *has 250 people in the camp and that there are 875 bowls of rice that can be shared equally, each person receives 3 and a half bowls of rice.*

(A potentially lived experience)

Look at organiser's calculations below and explain how the organiser calculated the 3 and a half bowls of rice. Write next to the organiser's calculations your explanations on how the problem was solved.

Figure 5.5: Example of personal experiences interpreted in M2 text

Meanings also appeared mythical (Barthes, 1974) in certain contexts. Task 15 signalled that maximising wealth was truthful and necessary, however, the social, religious, and economic backgrounds of some participants precluded certain members of their community from conceptualising and accumulating wealth in this context. The participants also displayed different beliefs about the truthfulness of the M2 sign system. Participants 1 and 2 believed statistics such as Tasks 4 and 5 were used for political reasons to tell lies in their home countries. Alternatively, Participant 3 was less sceptical and stated that mathematics (logics) is "always truthful" and "unquestionable".

Settlement experiences changed the perceptions and ideological connections the participants made in the M2 sign system. Participant 1 stated they were not motivated to learn M1 at school, however, circumstances had changed and they were keen to learn M2 to help fulfil their settlement goals. The instrumental reasons the participants displayed for learning M2 included: finding a more meaningful job; having social and financial independence; and, helping to solve everyday problems such as shopping and travel.

Finding 1

Philosophical codes (TXT1000) encode ideological meanings such as profit, gender, and linear relationships in M2. They are implicit in the text, and difficult to interpret without C, L, and S experience. Meanings have third-tier ideological connotations that are not shared universally with other cultures and languages.

Philosophical codes identify distinct focuses for teaching M2 in a cross-C, L, and S context. Ideological meanings can be more explicitly articulated in the text, and learning activities can focus on decoding and learning the philosophical conventions that encode M2. The participants found ideological meanings difficult to interpret without support in this context. The text, therefore, can be modified to articulate the epistemological, ontological, and ideological conventions that encode the sign system.

5.3 TXT2000: Structural codes that are problematic to decode in M2

Structural codes define, as Eco (1981, p. 37) identified, the contextual fate in which the text bounds and shapes the decoding behaviours of its readers. The term decoding emerged and identifies the mental and physical processes employed to interpret the sign system (Jakobson & Halle, 1956). M2 is "functional" and "systemic" in this context, because meanings and processes are decoded, in part, through the form and function of the sign system (Halliday, 2006, p. 87; Jakobson & Halle, 1956). For example, at a molecular level the symbols '-', '2', and '3' in Task 6 cluster into entities such as '3^{-2}', to symbolically solve complex problems (Bloomfield, 1939; Halliday, 2006). The terms sign and symbols emerge interchangeably in this context, because symbols function as signs in M2 (Peirce, 1931).

Adjectival terms, for example, complex, intricate, agentic, functional, and delicate help define the structural characteristics of the M2 sign system. The term structural in this context identifies how meanings and processes are, in part, decoded through the form and function of the sign system (Bloomfield, 1939; de Saussure, 1910; Halliday, 2006). The symbol 'h', for example, was interpreted by the participants in Task 13 through its physical characteristics and its position in the equation '$h^2 = 4^2 + 3^2$'.

The sign system, as a result, generated abstract meanings and thinking behaviours for the participants. The symbol 'S' in Task 10 enabled the participants to interpret the construct speed and acceleration without, necessarily, driving a car or drawing an image to interpret its meanings (Devlin, 2000). However, the symbol 'S' could also be used to signify the 'price of shares' in Task 15 and associated 'danger' as a third-tier meaning (Peirce, 1931) in Task 10. Third-tier constructs emerged through the pragmatic meanings the participants attached to the sign system (Section 2.7)

The participants, in the main, interpreted the tasks through the forms and functions of the sign system. The formula '$m^2 = l \times w$' in Task 11 made sense to the participants, because they recognised M2 employed distinctive symbolisms and logics to solve both real and fictitious problems in this context. Everyday English was also used to interpret meanings. However, M2 is not unique in the sense that other sign systems such as film, art, music, narrative, and dreams, also employ everyday language to interpret and explain complex symbolic meanings (Freud, 1911; Holland, 1990). Everyday language was more likely communicated and understood by the participants in the tasks. Nonetheless, Participant 5 also drew the dimensions of the fictitious room in Task 11 to conceptualise its meanings. The participants also preferred, in the main, to interpret and answer the Tasks through its more symbolic language and structure than everyday language. Participant 3 stated that they found everyday "Aussie" words difficult in this context to understand in M2.

The participants all preferred to answer Task 10, Figure 5.6, with the formula in the box, because it appeared less ambiguous to decode than the wordy text. Participant 2 also answered the task with a formula despite identifying it was written incorrectly. The formula was inadvertently written incorrectly by the Author, and it should have read 'S= 16.7A'. Participant 2 believed the formula was, nonetheless, still more 'truthful' in its symbolic form than its associated text.

Task 10.

Mathematical equations and formulas can be used to help solve mathematical problems without actually physically doing things. For example, from experience you may know that a pattern exists when you press the foot on the accelerator in a car because not pressing on the accelerator the car will not move, pressing the accelerator 3 cm the car will travel at 50km/hr., and pressing at 6 cm the car will travel at a 100km/hr.

An equation/formula can be derived from this pattern that calculates HOW fast a car would travel when pressing on the accelerator at other measurements.

(Emphasised sign)

$S=20A$

Where: S = speed, A = centimetres pressed on the accelerator

a. How fast would the car travel if you press the accelerator 8cm on the accelerator?
b. If you were traveling 30 km/hr., how much would the accelerator be pressed in cm?

Figure 5.6: Example of a wordy introduction and emphasised signs in M2 text.

The participants' decoding behaviours were interpreted through their discourses and physical responses, such as: body language, questions, and speech acts in the text (Eco, 1981; Ernest, 2010). For example, the multiple-choice structures in Tasks 13 and 14 prompted the participants to respond axiomatically first, and then through discussing, contrasting, and marking the optional signs in the tasks. The in-tasks behaviours coupled with the observations made in Stage 1 identified the L and S structures that encoded and created problems in the tasks.

Nine codes emerged as findings from the synthesis and identified the structural conventions that encode meanings in the sign system through its L and S forms and functions. The codes also identified the L and S competencies the participants needed to decode and learn the tasks. The study, however, did not compare differences between M1 and M2 in this context, other than identify that M1 varied, firstly, in many ways to M2 and, secondly, between participants. However, as Bloomfield (1939) stated:

> It is enough for us to know that nearly all the structural features of our language which we are inclined to accept as universal – features such as the actor-action sentence, the elaborate part-of-speech system, or the special infliction of our nouns and verbs – are peculiarities of the Indo-European family of languages and are by no means universal in human speech. (p. 3)

Finding 2

Structural codes (TXT2000) encoded meanings through the form and function of the sign system. Structural codes are, in part, functional and systemic and, therefore, more predictable to decode than Philosophical codes (TXT1000).

The following sub-sections summarise the nine structural codes that emerged from the synthesis and defined the L and S form and function of the M2 sign system. The findings stated at the end of the codes are interpreted as sub-findings to Finding 2 to address the research problem and questions.

5.3.1 Paradigmatic and syntagmatic codes

Paradigmatic conventions encode meaning by changing the way the sign looks, and syntagmatic conventions by repositioning the sign in the syntax (de Saussure, 1910). The minus '-' sign positioned in front of '-5' signifies a new paradigm for '5', and '5' transposed from '5(3 + 4)' to '3(5+ 4)' identifies a syntagmatic transformation. Paradigmatic and syntagmatic processes can occur at the same time in the text at a clause level. For example, the '4' transposed from outside to inside the brackets in Task 8, Figure 5.7, generated a syntagmatic transformation, and '4' transposed from divisions into fractions notation generated a paradigmatic transformation for '4'.

Task 8

Match the equations in the first table with their equivalent forms in the second table and then write them with their answers in the third table.

3 × 6	a(b + c)	2 ÷ 4	6 (4)	4 (2 + a)	4a ÷ 2a

(A paradigmatic transformation for '4') **(A syntagmatic transformation for '4)**

$\frac{4}{2}$	$^2/_4$	3(6)	ab + ac	4 (6)	4a + 4(2)

Figure 5.7: Example of a paradigmatic and syntagmatic transformations in M2 text

Paradigmatic and syntagmatic codes make the sign system elastic and fluid, because signs are transposed in and out of different positions to articulate meanings. For example, '3', '2', and '-' articulate meanings at an atomic level, and at molecular level when clustered into different structures such as '3^{-2}' in Task 6.

Finding 2-1

Paradigmatic and syntagmatic codes generate distinctive meanings and processes in M2. They are problematic to decode and learn when their meanings, form, and function differ in a cross-C, L, and S context in M1 and M2 for learners.

5.3.2 Iconic, symbolic, and indexical codes

Iconic, symbolic, and indexical codes identify the connections (Peirce, 1931, p. 155) the participants made in the sign system and the meanings that were interpreted. A relational meaning between the sign and the meaning signified an iconic representation. A physical connection made to the sign signified an indexical sign. A meaning that was abstracted and represented in the mind was a symbolic sign (Peirce, 1931). The diagram in Task 15, Figure 5.8, signified an iconic representation of the share price, predictably, going up and down in the market place. The figure displayed in Task 14 was both an indexical and iconic representation of life and travel in an Australian urban landscape.

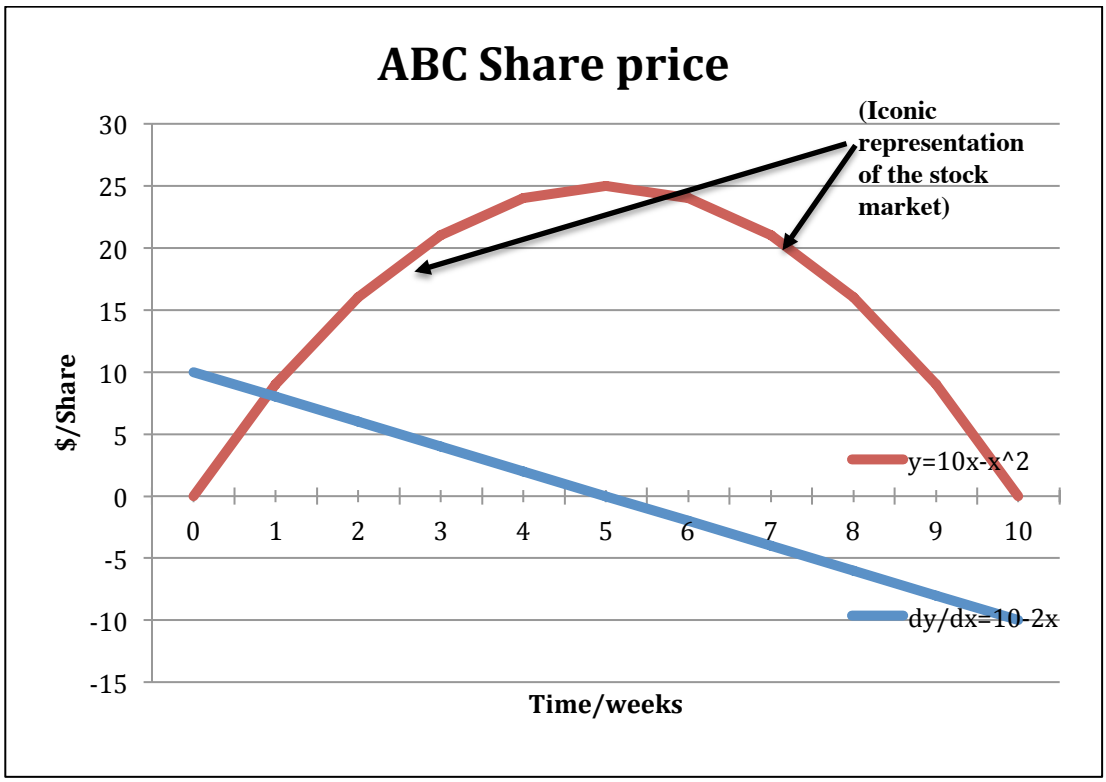


Figure 5.8: Example of an Iconic sign of share prices in M2 text

The tasks required a combination of iconic, symbolic, and indexical decoding experiences to interpret their meanings, forms, and functions. Iconic, symbolic, and indexical signs were, however, systemic and functional in M2, because they enabled the participants to interpret and solve different types of problems. For example, the pie chart in Task 4 enabled the participants to interpret the statistical meanings as parts of the whole. The indexical signs found in historical texts such as eyes and the body that encode human perspective were, however, replaced with more sophisticated symbolic signs, like formulas, tables, and figures (Devlin, 2000; O'Halloran, 2005). This made it conceptually harder to interpret human relationships and perspectives in the tasks.

Finding 2-2

Iconic, symbolic, and indexical signs encode distinctive relational meanings and processes in the M2 sign system. They are problematic to decode and learn when their meanings, form, and function vary in a cross C, L, and S context.

5.3.3 Opposition, marking, and contrasting codes

The M2 sign system encodes meanings and processes by opposing, marking, and contrasting signs (Jakobson & Halle, 1956). The signs '+' and '-' mark the meanings of other signs by adding and subtracting their meanings at a molecular level (Ernest, 2006). The sign '-' in equation '3^{-2}' in Task 6, Figure 5.9, articulated a distinctive molecular meaning that had an equivalent meaning '1/9'. '1/9' was also interpreted and decoded in opposition to '3^{2}'.

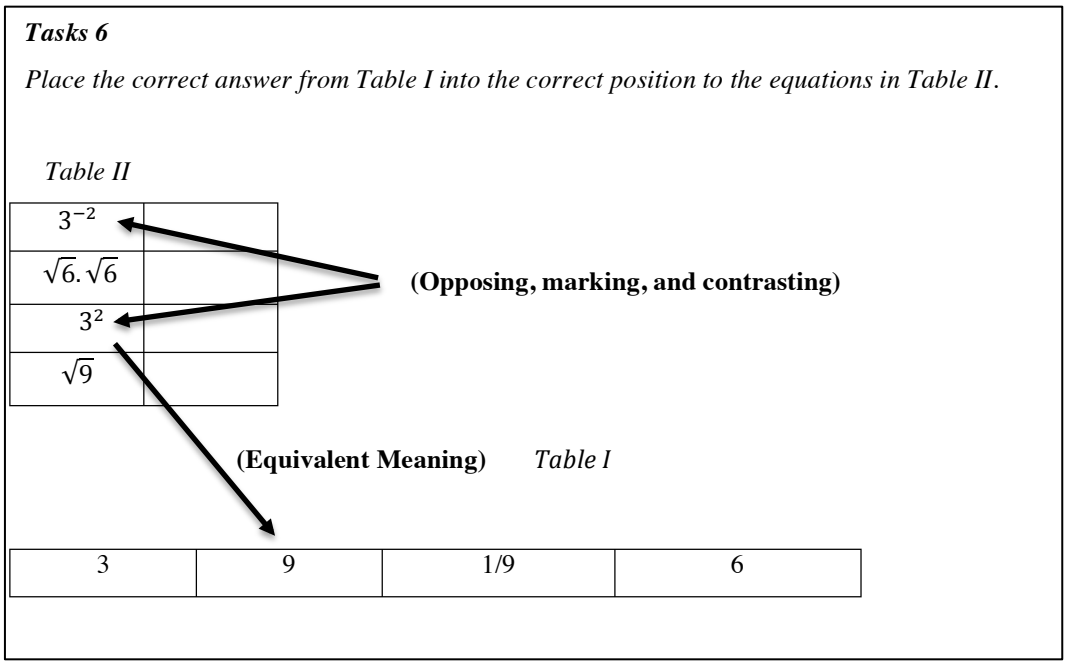


Figure 5.9: Example of opposition, marking, and contrasting of signs in M2 text

Everyday language also encodes meanings and processes through opposition, marking, and contrasting signs in M2 text. The prefixes 'im', 'il', and 'un' generated opposite meanings such as 'impossible, 'unlikely', and 'illogical', by opposing, marking, and contrasting the words used in the in-tasks interviews. The participants demonstrated problems in this area when they were also unable to interpret their meaning, form, and function in associated discourse.

Finding 2-3

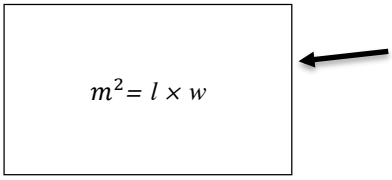
Opposition, marking, and contrasting signs encode distinctive meanings and processes in M2. Their forms and functions are problematic to decode and learn when they differ in a cross C, L, and S context

5.3.4 Physical codes: shape, position, energy, motion, force, synergy, timing, and aesthetic characteristics

The physical and aesthetic characteristics of the M2 sign system encode distinctive meanings and processes. Figure 5.10 identifies that the formula in Task 11 was located as a predicate proposition after the antecedent sentence: *The formula to calculate the square area of a room is*. However, the participants responded to the formula first, because its image appeared to be more significant. In this context, the participants believed the formula should be decoded before its wordy antecedent proposition (Kant, 1788).

Task 11

The formula to calculate the square area of a room is



(Predicate read before antecedent proposition)

Where: m^2= square meters, l= length of the room in meters, w= width in meters

Circle the correct answer:

Figure 5.10: Example of physical codes in M2

The M2 sign system displayed energy and force (Leslie, 1993; Tylen, 2007) in shaping the thinking behaviours of the participants in the text. Algebraic signs, for example, expanded and contracted into more sophisticated and delicate propositions

and ways of thinking (Bloomfield, 1939; Halliday, 2006). The term "delicacy" (Halliday, 2006, p. 114) identified that the M2 sign system economises the number and types of signs it needs to articulate meanings (Bloomfield, 1939; Peirce, 1931). Semiotic synergies and thinking processes existed in the formulas in Tasks 10, 11, and 12, because the tasks economised the number of signs they needed to interpret and solve problems.

The terms inertia, motion, synergy, and force emerged and described the physical characteristics of the M2 sign system. Force identifies the intensity and direction in which the sign system generates meanings and thinking behaviours. Question marks and equal signs, for example, signal points where a change in thinking occurs.

Finding 2-4

Physical codes encode the M2 sign system and they are problematic to decode and learn when their meanings, form, and function vary in a cross C, L, and S context. The M2 sign system also has distinctive aesthetic patterns that encode meanings making processes. The text is decoded from the left and top down in English, whereas in other languages, for example, Farsi and Arabic, the constructs are decoded from the right to left. Learner competencies identify the skills needed to decode the M2 sign system through its shape, position, energy, motion, force, synergy, timing, and aesthetic characteristics.

5.3.5 Agentic codes

The M2 sign system encodes meanings and processes through its self-determining goals and cognitive properties (Ernest, 2006; Leslie, 1993, p. 2; Tylen, 2007). The terms autonomy, force, and energy emerged and defined the agentic characteristics of the M2 sign system, and its effect on the decoding and learning behaviours in the tasks.

Autonomy means that the M2 sign system exists, in part, independently of the participant's mind and experience (Section 4.4.1). Force identifies the degree in

which the sign system, respectively, bounds and shapes decoding and thinking behaviours. Energy refers to the synergetic characteristics of the M2 sign system. Tasks 13, Figure 5.11, employed semiotic entities such as formulas and equations, to encode complex thinking behaviours in this context.

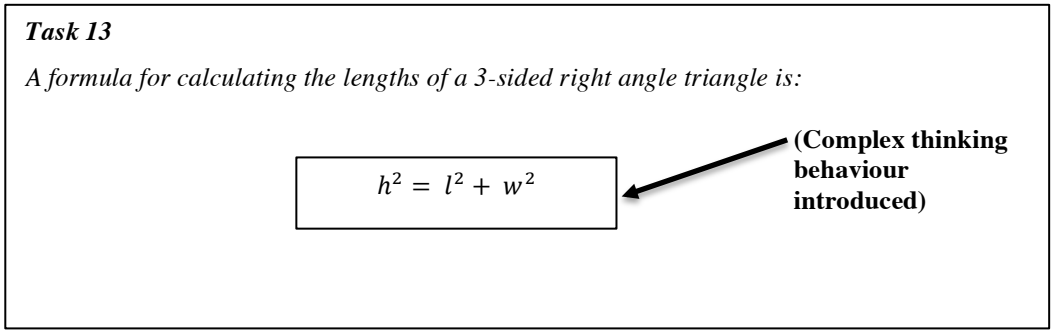


Figure 5.11: Example of agentic coding in M2 text

Finding 2-5

Agentic codes bind and shape decoding and learning behaviours through the sign system's kinetic properties: autonomy, force, and energy. Problems emerge in M2 when BL learners are unable to reconcile their meanings, form, and function in a cross C, L, and S context.

5.3.6 Metaphoric codes

Metaphors encode meanings and processes through signs that disconnect the sign system from its literal sense (Halliday, 2006). Their meanings and significance of metaphoric codes emerge in M2 in three ways:

1. Metaphors, as Chandler (1994) stated, involved "one signified acting as a signifier referring to a rather different signified" (p. 516). For example, the term big smoke functions as a metaphor for describing the city. In this context, as Holland (1990) stated, "consciousness is continually being imagined (imaged, in-formed) by the metaphors in the very text it is writing or reading" (p. 43). The sign 'Σ' represents a complex metaphor for signifying the summation and addition of numbers. M2, however, also employs different types of signs to articulate the same metaphoric meanings.

For example, the symbols ‘six’, ‘VI”, and ‘6’ are interchangeably employed to signify the same ordinal relationships and quantity for ‘six’.

2. Metaphors emerge at different levels of stratification in a language, however, M2 tends to cluster and employ fewer signs to articulate the same meanings (Bloomfield, 1939; Halliday, 2006). The M2 sign system also systematises the metaphoric processes that it employs to solve problems. The algebraic formula in Tasks 15 systematised and economised the number of tokens and types of signs (Peirce, 1931) it needed to solve a complex financial problem.
3. Metaphoric codes function across different registers in M2 (O'Halloran, 2005). For example, algebraic, graphic, and tabular representations were used as different metaphors to articulate the same data in Tasks 15.

Metaphors are ambiguous to interpret without C, L, and S experience. The term big smoke signifies a connection between pollution, smoke, and a city, that requires the decoder to understand the context and contents of the expression. Similarly, the pie chart in Task 4, Figure 5.2, functioned as a metaphor for expressing the characteristics of marital status. Participant 4 found the pie chart ambiguous to interpret in this context, because de facto relationships were not clearly articulated. The Author stated to Participant 4, “perhaps de facto type meanings were hidden in the percentage ‘%’ sign”, and they should be interpreted “as part of the whole marriage system”?

The participants found the symbolic representations of the tasks, for example, equations, charts, and formulas, more predictable and less ambiguous to interpret than its everyday language. As, Participant 3 stated “Aussie language” has hidden meanings that only “Australians understood”, and “this was harder to learn than mathematics”. Participant 3 interpreted the symbols ‘$\phi\phi$■■■’ in Task 1, easily as a metaphor for ‘23’, because the connecting logics and meanings were more explicit in the text.

Equivalent and equilibrium signs identify points where the metaphoric representations changed horizontally and vertically in the text to articulate the same meanings. Figure 5.12 illustrates the points where the equivalent '⇔' and equilibrium '=' signs in Task 15 employed different representations to articulate meanings in an algebraic process. This made the sign system more explicit and predictable to interpret and explain to the participants. The tasks were, however, harder to interpret horizontally across the different registers such as tables, charts, and algebraic representations (O'Halloran, 2005). In this context, the equivalent and equilibrium signs '⇔' and '=' in Task 15 were replaced by less sophisticated wording and associated discourse such as "a way of representing the data".

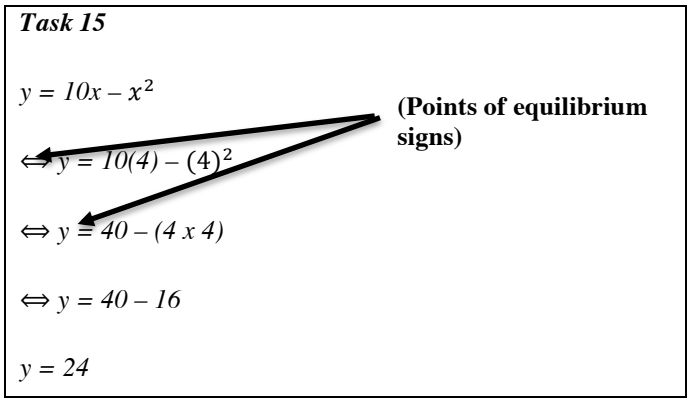


Figure 5.12: Example of mathematical metaphors read vertically through points of equilibrium.

Participants 1 and 2 stated that the equivalent sign '⇔' was also an unnecessary inclusion in the equation, because '*y =*' meant the same thing to them as '⇔' to them. Nonetheless, its inclusion did not detract the participants from interpreting the equation. Participants 3, 4, and 5 found the arrows positioned at the ends of sign '⇔' helped identify the equation could be read backwards and forwards in this context.

Finding 2-6

Metaphoric processes encode meanings and process that disconnect the sign system from their literal sense (Halliday, 2006). This makes the M2 sign system difficult to decode and learn without C, L, and S experience. However, equivalency and equilibrium signs such as '⇔' and '=', make metaphoric processes in M2 more explicit and predictable to interpret than everyday English.

5.3.7 Axiomatic and logic codes

Axiomatic processes and logics encode meanings through mathematical propositions. Axiomatic codes included rules of:

- Associativity: ((a + b) + c = a + (b + c), and (ab) x c = a (bc))
- Commutativity: (a + b = b + a, and ab = ba)
- Distributivity of the function x: (a (b + c) = ab + ac, and (b + c)a = ba + c x a).

The participants interpreted axiomatic and logic codes in the tasks, in part, because M2 and M1 shared these meanings. Participants 1 and 2 also answered tasks successfully, because they were able to remember and transpose these meanings more effectively than the other participants. Participants 3 and 4 stated that they had learnt theses rules in M1 before, but had "forgotten" and now "needed help to remember". Alternatively, according to Participant 5, these types of meanings were new to them.

Some tasks appeared more efficient in explaining and transposing axiomatic and logic codes from M1 into M2. The participants found the wordy tasks such as Task 7 and 8, Figure 5.13, confusing to decode in this context. Alternatively, Task 9, Figure 5.14, was more explicit in prompting and recalling relevant meanings and processes.

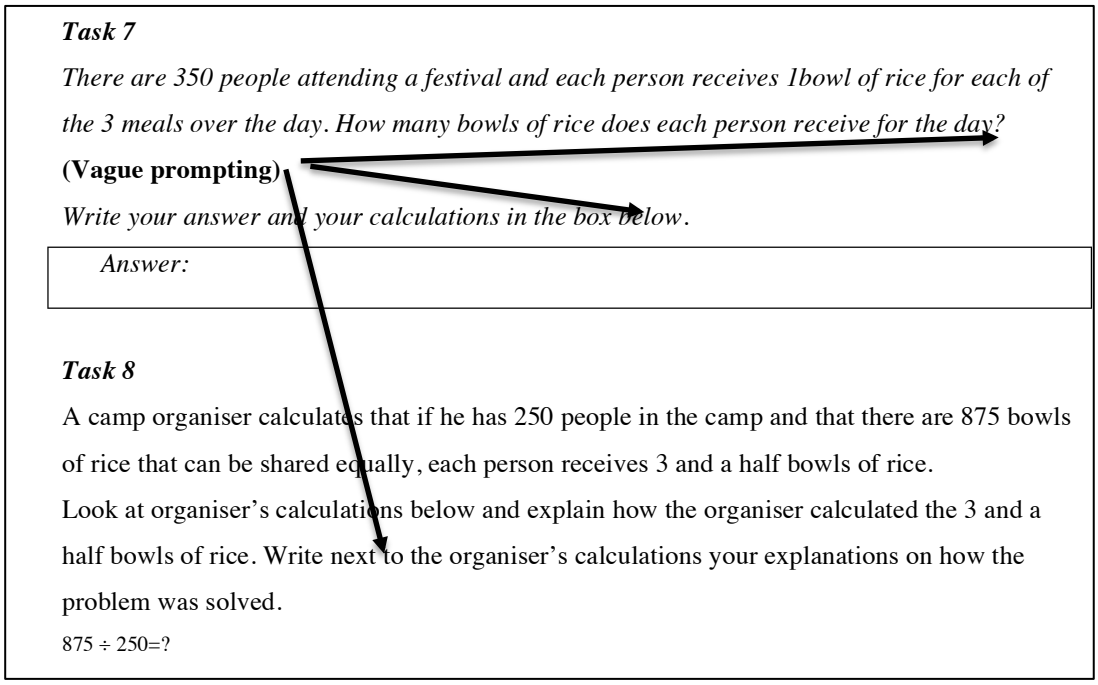


Figure 5.13: Example of vague prompting of mathematical propositions

Tasks 9

Match the equations in the first table with their equivalent forms in the second table and then write them with their answers in the third table. **(Direct prompting)**

3 × 6	*a(b + c)*	*2 ÷ 4*	*6 (4)*	*4 (2 + a)*	*4a ÷ 2a*

$\frac{4}{2}$	$^2/_4$	*3(6)*	*ab + ac*	*4 (6)*	*4a + 4(2)*

24		
8 + 4a		
2		
18		
(ab + ac)		
.5		

Figure 5.14: Example of direct prompting of mathematical propositions

Finding 2-7

Axiomatic and logic codes encode distinctive propositional meanings and processes in M2. The codes are interpreted, in part, through coordinate L and S experiences in M1. Problems emerge when BL learners are unable to remember and transpose shared meanings.

5.3.8 Discursive codes

According to Crystal (1992) the term discourse described "a continuous stretch of (especially spoken) language longer than a sentence" (p. 451). The participants found discourses in M2 problematic to maintain, firstly, as a continuous stretch of spoken language and, secondly, as a form of written text. Task 10 in Figure 5.6, above, was particularly confusing for the participants to maintain as a form of spoken and written discourse.

The study did not explore the difference in discourse between L1/M1 and L2/M2, other than synthesis the fact that participants found L2/M2 different and difficult to

maintain without support. Participants 4 and 5 stated that it was "more difficult to explain" in English what they knew about the task than it was to "solve" it. It was not clear, however, if the participants had similar problems sustaining discourses in L1/M1.

The synthesis employed Four Conversational Maxims from the Broader Cooperative Principle (Crystal, 1992; Fromkin, Blair, & Collins, 1999, p. 186) to interpret why the participants and text failed to maintain appropriate discourses in M2. The four maxims identified:

1. 'Quantity' created problems when the participants and the text said too much or not enough for the requirement of the discourse. For example, the participants found the beginning of Task 10, Figure 5.6, said too much.
2. 'Irrelevance' created problems when the text and participant discourse were disconnected from its context. Part of the text in Task 7, Figure 5.13, was irrelevant and disconnected. The participants engaged in irrelevant discourses when they used incorrect words to interpret and explain meanings. Participant 3 interpreted and transposed words incorrectly using their smart phone.
3. 'Manner' created problems when the participants and text engaged in disorderly and obscure discourse. The beginning of Task 10, Figure 5.6, was disorderly and obscure for the participants to interpret without support. The participants engaged in disorderly discourse when they chose inappropriate words and phrases to communicate.
4. 'Quality' created problems when the participants and text made unsupported claims. Participant 2 identified the formula in the box in Task 10 was inconsistent, however, they still used it in their spoken and written discourse to answer the tasks.

Finding 2-8

M2 has distinctive discourse codes that encode the sign system. Discourses in M2 are problematic to decode, learn, and sustain without cross C, L, and S support and experience

.

5.3.9 Medium

Chandler (1994) stated that the medium is:

> A transparent vehicle of representation by readers of tasks composed within it, but the medium used may itself contribute to meaning: a hand-written letter and a word-processed circular could carry the same verbal task but generate different connotations. Signs and codes are always anchored in the material form of a medium - each of which has its own constraints and affordances. *(*p. 515)

The participants employed different mediums, for example, technology, text, and the English language, to decode the tasks. English language was, however, particularly difficult to negotiate as a medium in a cross-C, L, S context. Participant 3 stated, for example, M2 was "tricky" because speakers often used "Aussie" sounding words (slangs) to explain meanings. These meanings also varied at a morphemic level for the participants. Participant 2 believed the word 'apple' in Tasks 3 was not a number, however adding 's' turned it into a plural form, 'apples', which to them was more like a number.

Participant 2's interpretation of plural signs was, however, based on their knowledge of English morphemic structures, and it was unsure if L1/M1 generated the same sorts of inflexions. Italian, for example, encodes 'masculine' and 'feminie' vowel sounds {a, e, i, o} to end of words to define their grammar. English does not employ the same morphemic meaning, and learners from an Italian background might inappropriately respond to M2 in Italian grammar, if they were unaware of the difference.

The participants identified two levels of problems when decoding and learning the tasks in this context. Firstly, M1 and M2 might employ different signs to articulate the same meanings. Participant 2 identified the dot '.' and comma ',' were used in opposite ways in M1 and M2, for example: $1.000.000.00 instead of $1,000,000.00. Secondly, the M2 might employ different processes to articulate meanings. The first issue identified a representative-related decoding and learning problem, and the second a process-related learning problem. Participants 3, 4, and 5 were multilingual and negotiated the problems from multiple C, L, and S perspectives.

Finding 2-9

The medium binds and shapes the way philosophical and structural codes are encoded and interpreted in M2. In this context the medium constrains and affords (Chandler, 1994) the meanings and behaviours that are employed to encode and decode the system. English is particularly ambiguous and problematic as a medium for decoding M2 in a cross C, L, and S context.

Structural codes identify the conventions and competencies needed to decode and learn the M2 sign system through form and function. They also identify problem areas when M2 is decoded differently in a cross-C, L, and S context from a structural perspective. The nine structural codes identified in the synthesis are:

1. Paradigmatic and syntagmatic codes
2. Iconic, symbolic, and indexical codes
3. Opposition, marking, and contrasting codes
4. Physical codes (shape, position, energy, motion, force, synergy, timing, and aesthetic relationships)
5. Agentic codes
6. Metaphoric codes
7. Logic and axiomatic codes
8. Discursive codes
9. Mediums

The following sections recount the participants' BL behaviours and experiences emerging from the data observations that affect decoding and learning philosophical and structural codes in M2. The findings identify adult-CALD BL learners also bring with them distinctive C, L, and S behaviours and experiences that create problems when decoding and learning M2 in a cross-C, L, and S context. Four types of BL behaviours and experiences emerged as findings:

1. TXT3000: BL decoding behaviours
2. TXT4000: BL humanistic behaviours
3. TXT5000: BL cultural behaviours
4. TXT6000: BL educational experiences

5.4 TXT3000: BL decoding behaviours that emerge and affect how M2 is decoded and learnt.

The participants employed distinctive BL decoding behaviours to decode the tasks. At an individual level, Participant 2 spent significant time re-reading the text backwards and forwards to check if they were interpreting the meanings appropriately in English. Alternatively, Participant 3 used the dictionary on their 'Smart Phone' to help clarify meanings, and Participant 5 redrew figures and rewrote sections of texts to interpret the tasks as a new L and S experience. At a group level, Participants 3, 4, and 5 spent more time than Participants 1 and 2 questioning and comparing differences between M1 and M2. The former behaviours coincided, in part, with Participants 3, 4, and 5 being multilingual, and Participants 1 and 2 receiving more formal education in L1 and M1. Shared behaviours included reading and repeating the text, interviewer conversations, and instructions aloud in English.

The terms independent and accuracy emerged and defined the participants' C, L, and S competencies in the tasks. Independent behaviour identified that the participants decoded the tasks without support. Participants 1 and 2 were more independent in responding to the text than the other participants. Nonetheless, Participants 1 and 2 still needed support to interpret everyday word meanings. Participants 5 found it difficult to interpret both everyday words and the more formulaic meanings.

Accuracy identified the quality of the L and S response by the participants. Incorrect multiple-choice answers were, in part, L and S accurate in that the participants recognised they had to respond to tasks by '*circling the correct answer'*. Participant 5 was, in part, inaccurate when he/she wrote correct answers outside the boxes and spaces provided by the text. It was not clear why Participant 5 responded in this way, other than to observe they tried but did not respond to the textual cues provided.

The text identified, as Jakobson and Halle (1956) stated, the "breakdown in communication" and "the nature and structure of the particular mode of communication that has ceased to function" (p. 55). The participants' decoding and

learning behaviours and problems were interpreted in this context through their physical responses and behaviours in the text (Jakobson & Halle, 1956; Eco, 1981; Ramachandran, 2011).

The participants decoded the tasks through a combination of coordinate and compound BL experiences. Coordinate experiences identified the tasks were decoded, in part, as a shared C, L, and S experiences in M1 and M2. Compounded experiences required learning M2 as a new code (Hamers & Blanc, 2000). Tasks 14 and 15 generated a new compound BL experience for Participants 4 and 5, because they articulated a new way for conceptualising and interpreting travel and finance.

Compound and coordinate BL experiences identify the different types of C, L, and S support that are needed to teach and learn M2 in this context. Compound learning experiences focus on learning M2 as a new L and S code, and coordinate learning focus on, for example, remembering and transposing shared meanings and processes. As a coordinate BL experience Participant 4 stated, they "read these types of tasks many years ago" in M1, but "needed help in remembering" and transposing these meanings. Participant 3 stated that, while they were taught to read from right to left in M1, it "didn't matter" in the tasks by reading from the left, because "it meant the same thing in mathematics". Participant 3 identified M2 processes were arbitrary in this context, because these processes could also be code-switched and read from different directions by BL learners. Code switching confirmed that the participants employed two or more linguistic codes (Hamers & Blanc, 2000) and processes to decode the tasks.

Participants 3, 4, and 5 engaged in more noticeable code-switching behaviours than Participants 1 and 2 such as identifying, comparing, and questioning differences between M1 and M2 to answer the tasks. It was unclear, however, if it was Participants 3, 4, and 5's less formal educations (Parvanehnezhad & Clarkson, 2008; Cummins, 2003) or their multilingual backgrounds that affected their code-switching behaviours more in this context. The study envisioned it was most likely a combination of both experiences that generated the participants' distinctive behaviours in M2. The study interpreted multilingualism in this context as an added dimension of bilingualism, in that it involved learning an additional L and S code on top of what has already been learnt (Hamers & Blanc, 2000). Hammers and Blanc (2000) defined bilingualism as "the (psychological) state of an individual or

community characterised by the simultaneous presence of two languages" (p. 368).

Hammers and Blanc's (2000, p. 26) Model of the Psychological Dimensions of Bilinguality helped synthesise and classify the different types of BL immersion experiences that affected the participants decoding and learning behaviours in the tasks. The data observations identified the distinctive BL immersion experiences that create problems when decoding and learning M2 in a cross-C, L, and S context.

1. Age and length of immersion.

   Participants 1 and 2 were immersed in M2 as a coordinate BL learning experience in M1 from a younger age and for a longer period of time than the other participants. Participants 1 and 2 were able to answer the tasks independently and accurately, because M1 and M2 also shared meanings and processes. Alternatively, Participant 5 stated, they learnt mathematics more through their migrant education experiences in Australia than they did in their home country in L1/M1. This made Participant 5's late immersion in M2 more of a compound learning experience.

2. Type of immersion.

   The participants' immersion in M2 was affected by different kinds of environmental experiences, for example: education, culture, play, work, and settlement. Participant 3 learnt, in part, to count quantity and volume as a construct by playing cultural games as a child that involved physically swapping different size stones. Alternatively, Participant 1 learnt mathematical meanings more through technology by playing computer games.

   These experiences affected how the participants, respectively, decoded and learnt M2 as a physical and technological activity. Participant 4 stated they had to work and support their family before they were able to attend formal ESL classes in Australia. This experience made the work environment an important immersion experience for Participant 4 to learn M2.

3. Intensity and balance.

   Participants 1 and 2, respectively, learnt more M2 through their formal L and

S experience in M1. Participants 3 and 4 learnt more M2, in part, through learning L2 in their home countries. This enabled Participants 1 and 2 to interpret the more formulaic structures, and Participants 3 and 4 the everyday words more competently in the tasks. The participants all stated, however, they would have liked to learn more M2 in their English migrant language classes in Australia. The classes skewed the participants towards learning L2 over M2 as an L and S construct.

4. Attitude towards learning.

   The participants all stated they wanted to learn more M2 to help fulfil their short- and long-term settlement goals. The goals included finding a more meaningful job, accessing further education, and integrating into a new community [6].

The aforementioned dimensions of bilinguality (Hamers & Blanc, 2000) helped interpret the effect factors such as age, gender, settlement, politics, education, work, and perceptions, had on the participants decoding and learning M2 in a cross-C, L, and S context. Political unrest, for example, affected the length and depth of immersion some participants had in learning M1 and M2 as formal coordinate BL experience. As Participant 3 stated, it "depends on the (whose) brain, or how your brain is (affected by) politics and war". The participants identified, however, their migrant language-learning classes helped address these types of learner problems. According to Participant 5 these educational experiences made them feel more confident as a person and learner in their new community.

The participants' bilingual experiences affected how they engaged in M2 as discourse. Participant 4 was more competent in making general conversation, because that person was immersed in L2 for a longer period of time than the other participants. Participant 4, however, found it difficult to engage in more formal mathematical discourses. Alternatively, Participant 2 found it difficult to engage in M2 as a spoken everyday discourse. This did not mean Participant 2 had an unsophisticated understanding of mathematical concepts.

[6] Last contact - Participants 1, 2, and 5 enrolled in higher education, and Participants 3 and 4 started full-time work.

The participants' bilinguality affected how they engaged in M2 as a heuristic activity. Raab and Gigerenzer (2005) stated that heuristics behaviours "solve a class of problem in situations with limited knowledge and time. Models describe not IQs but mechanisms and models of understanding and problem solving" (p. 188). Participant 2 responded quickly and accurately to Tasks 4 and 5 without fully understanding, for example, what the term significance meant in the question: *what is the significance of the information?* The sign system was arbitrary in this context (de Saussure, 1910), because Participant 2 interpreted unknown signs quickly through their experiences in decoding L and S structures in M1 and M2 as coordinate experience. The meaning of the word *significance* was interpreted through its position between the two other words that were more familiar in M2: *what (significance) information?*

Some participants also responded incorrectly by selecting signs that looked familiar in the text. Participants 4 and 5 incorrectly selected *'a = 43'* in Task 11, because the numbers were sighted in the question: *what is* m^2 *of a room that is 4 meters long by 3 meters wide*? The same participants responded correctly with *'d = 12'*, however, when asked to explain their choice orally. Participants 4 and 5, coincidently, also preferred to read the tasks aloud during the in-tasks interviews.

The participants needed different levels and types of discourse support to interpret the tasks. Participants 1 and 2 required help to interpret the tasks in everyday English, and Participants 3 and 4 needed more support to interpret and articulate mathematical concepts. Alternatively, Participant 5 found it difficult to engage in both types of discourses. In-task support included being relevant and precise when communicating in M2 (Section 5.3.8).

The participants' competencies in M2, however, also changed quickly through new settlement and educational experiences. Competencies described, at best, the participants' decoding and learning behaviours at a particular point in time in L2 and M2, rather than as a static characteristic or problem. Participant 2 stated that they initially felt "unhappy" about the amount of L2 they learnt in Australia, however they were "surprised and motivated" by the amount they actually learnt in a short period of time.

The type and level of support given in the interviews also articulated quickly into more sophisticated discourse. The tasks fulfilled two functions in this context. Firstly, they identified the L and S competencies needed to interpret M2 as a discourse. Secondly, they identified the problems that emerged when M2 is decoded as a discourse in a cross-C, L, and S context. Combining the two sets of observations identify strategies such as glossaries and guidelines, for teaching and learning M2 as discourse (Section 5.3.8).

Finding 3

BL decoding behaviours (TXT3000) identify the distinctive decoding behaviours BL learners employ to decode and learn M2. Reading aloud, underlining new words, having C, L, and S hesitations, questioning M2 as a language, and code switching between M1 and M2 identify distinctive types of BL decoding behaviours in M2. These behaviours are also affected, for example, by the learner's age, gender, and political, educational, economic, and settlement experiences. Problems emerge when M2 BL decoding behaviours and experiences are incongruent for decoding and learning M2 in a cross C, L, and S context.

The following sections reconcile in greater depth the humanistic, cultural, and educational behaviours and experiences that affected decoding and learning M2. Their findings articulated different types of educational strategies to address the research problem.

5.5 TXT4000: BL humanistic behaviours that emerge and affect how M2 is decoded and learnt

BL humanistic behaviours define the effect physiological and emotional factors, for instance, human perceptions, modality (speaking, listening, reading, and writing), and gender, have on decoding and learning M2. These behaviours are biological in nature and, therefore, potentially shared with monolingual learners: however, their cause and effects emerge differently in a BL context. The participants employed

distinctive visual, oral, and tactile interactions to interpret the tasks as a BL experience, such as: repeating conversations, reading aloud, redrawing images, and transposing meanings into M1/L1 first to interpret meanings in M2/L2.

The participants' humanistic behaviours were shaped, in part, through their childhood learning experiences. Participants 1 and 2 were raised in a technologically rich environment, and this affected how they decoded and learnt M2 through technology. Participant 1 stated, for example, computers now "did complicated mathematical jobs" they "no longer have to do" mentally in their "job". Alternatively, Participant 4 was exposed to less technology, and this affected how they perceived and interacted in M2 through technological mediums such as computers. Some cultural experiences also excluded girls because of their gender from going to school and learning mathematics through text and technology. Missing out on this type of cognitive and neurological development is envisaged to detract from learning more complex M2 (Section 4.4.3).

Gender differences were interpreted in two ways in M2. Firstly, males might be neurologically more suited to decoding and learning M2 as a technical discourse (Neaubauer & Fink, 2005). Secondly, mathematical tasks that involve activities like finance and building, might be envisaged a male function in some cultures. These experiences make certain aspects of M2 cognitively and ideologically incongruent for females to decode and learn. Nonetheless, gender-related learning problems are complex to interpret because there are other factors that emerge and affect behaviours in M2. The female participants in this study, for example, did not display problems interpreting the tasks as a technical or gender-related discourse. The other humanistic factors that emerged and affected participants' decoding and learning behaviours in the tasks were synthesised as follows.

The tasks were physically and mentally demanding for the participants to decode, and the less competent participants appeared to be more fatigued in decoding the more demanding tasks. This behaviour reflected the capacity of the participant to think and solve complex problems in M2. A link was made, in part, between the potential development of the brain and, respectively, the neural energy that is required to process these types of thoughts (Neaubauer & Fink, 2005). Short 1-3 minute breaks emerged every 15-20 minutes during the in-tasks interviews to help the participants manage this issue. The interludes engaged in more general

discussions about the effect the participants' backgrounds and perceptions had on learning M2.

Different types of perceptions affected the participant behaviours in the tasks. Participants 3 and 4 believed they needed more help remembering mathematics in M1, before they answered questions in M2. Conversely, Participant 5 believed M2 was more about learning M2 as a new linguistic experience. The participants were affected by the views they had on how their age and educational experiences also affected learning M2. Participant 3 believed that some people were naturally "better at learning mathematics", and others were "too old" or lacked "education" to learn it properly. Participant 2 found it challenging to attend English language classes, because they believed other students did not share the same "respect" (values) in the classroom. Participant 2 was also initially cautious about participating in the one-on-one in-tasks interviews and answering the tasks questions correctly. However, they quickly overcome their nervousness and contributed to meaningful discussions, because of the way the interviews were conducted.

Factors such as maturity, family commitments, and settlement experiences affected the perceptions the participants had on learning M2 in an additional language context. The participants believed M2 was important for fulfilling their short- and long-term settlement goals, such as generating financial and job opportunities. These perceptions helped overcome some of the negative experiences and anxieties that affect younger CALD learners when learning an additional language (Gardner, Masgoret, Tennant, & Mihic, 2004). Participant 3 stated they also enjoyed learning M2 for recreational reasons that involved playing games and solving problems with numbers. Coincidentally, Participant 3 was younger and had fewer evident family commitments than the other participants.

Teaching strategies can help reconcile the BL humanistic behaviours that emerge and create problems when decoding and learning M2. The foundational physical and cognitive connections learners make in M2 can be studied and enhanced through re-enacting body movement and human interaction (Provinca Autonomata di Trento, 1999) in the sign system (Ernest, 2010). These learning behaviours reflect the meanings that are generated through, for example, body gestures and physical interactions in the sign system (Ernest, 2010; Thibault, 2011). The behaviours can also be modified through the mediums, for example, colour, rhyme, and tactile

interactions that are employed to learn M2 (Ernest, 2010; Thom & Roth, 2011). As Ernest (2010) stated:

> The resultant semiotic tools that develop incorporate spoken language, but also the full panoply of signs. Meanings are expressed through multimodal sets of signs including verbal sounds and spoken words, bodily gestures, arrangements of material objects, various markings, icons, pictures, written language and symbolic text. (p. 103)

The interviewer recalled learning space, value, and number by playing with a multi-colour-coded building block system as a child. The semiotic construct replicated in Figure 5.15 may similarly be explored to teach M2 to BL learners who have missed out on learning foundational mathematics. Blocks are, respectively, swapped through different visual and tactile interactions to learn different semiotic relationships in M2. Participant 3 engaged in a similar learning experience by playing cultural games that involved swapping different size stones. On the other hand, Participants 1 and 2 learnt to count numbers, place, and value more through technology in M1. Participants 4 and 5 missed out on this type of technological experience and, as a result, require a simpler approach in learning M2 as a technological interaction.

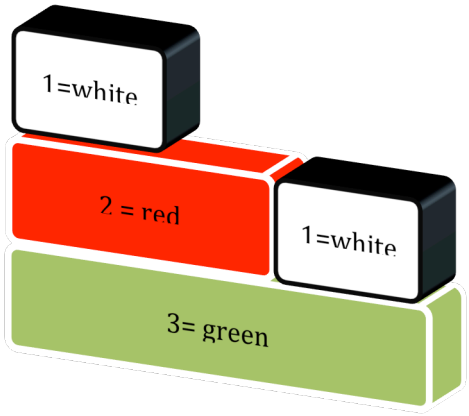


Figure 5.15: Example of colour-coded building block system for learning foundational semiotic meanings in M2.

Reconciling and managing the timing and quantity of M2 learnt in a session will help manage learner fatigue. The participants who are less competent displayed greater fatigue. Nonetheless, not all the tasks were equally demanding. For example, Participant 4 found the more symbolic tasks difficult to focus on for long periods of time. The more competent Participants 1 and 2 found everyday English more mentally demanding to negotiate in this context.

Humanistic behaviours identify, for example, the perceptual, modality (speaking, listening, reading, and writing) and gender-related factors that affect decoding and learning M2. Educational activities can reconcile problems in this area by enhancing the semiotic resources and teaching strategies that are employed in M2. Teaching strategies can also focus on learners achieving settlement.

Finding 4

BL humanistic decoding behaviours (TXT4000) identify the humanistic behaviours and experiences that affect decoding and learning M2 in a cross C, L, and S context. They identify the effects, for example, emotions, perceptions, modality (speaking, listening, reading, and writing), gender, and memory have on decoding and learning M2. Childhood and memory-related experiences also emerge and affect how BL learners, for example, interpret foundational meanings such as value, place, and numbers, in M2. M2 is difficult to decode and learn when learners are excluded because of their gender and ethnicity, for example, from learning foundational mathematics.

5.6 TXT5000: BL cultural behaviours that emerge and affect how M2 is decoded and learnt

The participants' decoding and learning behaviours in M2 were affected by their cultural experiences. Firstly, the tasks generated new ways of encoding and conceptualising cultural meaning for the participants. For example, Participants 3, 4, and 5 found Tasks 14 and 15 articulated a new way for conceptualising travel and finance. Secondly, the participants brought with them their own cultural experiences and behaviours that affected their interpretation of the tasks. Participants 4 and 5's interpretation and discourses on marriage and alcohol in Tasks 4 and 5, in part, reflected their cultural beliefs and experiences. Coincidentally, Participants 1, 2, and 3 were younger and less concerned about discussing these types of meanings in the tasks.

Ideological meanings are problematic to interpret in a cross-C, L, and S context (Sections 5.2 & 5.3). The text was open, in part, to different interpretations, because constructs such as alcohol, marriage, and profit, were not articulated precisely in the tasks. The terms speed and acceleration, for example, were not envisaged to signify danger in the design stage of Tasks 10, however, the respective interpretations emerged in the discussions.

The participants decoding and learning behaviours were also affected by their religious beliefs. The interviews did not question the participants directly about their religious beliefs, other than synthesise that this factor affected the deeper metaphysical meanings some of the participants attached to the tasks. The Author identified, for example, in his own learning experiences in a Roman Catholic School during 1960s and 1970s that he was required to head each page of his mathematics exercise book with a symbolic homage to God. This type of symbolism shaped, in part, how the Author conceptualised M2 as a metaphysical construct. The absence of religious signs in the text might similarly appear metaphysically incongruent to certain learners. The term metaphysical signifies the nature of the mind and the spiritual connections the decoder makes to the sign system: that is, what is real, believable, and transcendentally justifiable in learning (Chandler, 1994; Derrida, 1967; Kant, 1788; Peirce, 1931).

The participants believed learning M2 was an emancipative experience, because it helped create economic, social, and cultural opportunities. However, these outcomes were also achieved at a social and cultural cost. Participant 1 stated, they had foregone a significant amount of their "leisure and social time" to learn L2 as an adult. The capacity of the participants to maintain home languages and cultures was also challenged in this context. According to Participant 2, learning a home language and culture offered "another extra (economic and cultural) chance for migrant children", however they were difficult to maintain, because "people did not speak and practice them outside the home". Participant 3 believed it was not so important for migrant children to maintain home languages and culture, because they were also not good at them and preferred to speak English at home anyway. Participant 5 stated that despite being isolated from their home community, they were happy speaking English and M2 at home with their children.

The participants saw learning M2 as an additive C, L and S experience rather than one that detracted from maintaining their home language and culture (Baker, 2011; Hamers & Blanc, 2000; Tochon, 2014). In this context the participants believed M2 enhanced their linguistic, intellectual, and cultural capacities in their new community like solving everyday problems and accessing educational opportunities. These observations confirmed that M2 emerges through adaptation and accommodation rather than the redundancy of prior C, L and S knowledge (Piaget, 1970, cited in Hamers and Blanc, 2000, p. 16). As Tochon (2014) stated:

> All cultures and subcultures have their own way of interpreting human experience. Reading and language learning can be enmeshed into a new type of endeavor characterized by the conquest of the soul and the realization of what is at the root of humanity in terms of it commonalities, allowing shared and peaceful understanding. Reading becomes a method of transpersonal development through dialogue with the Other as being epistemologically different. Cultures are dynamic and adaptive, which situates learning not as the acquisition of a fixed embodiment of knowledge with its traditions, but as the entry into an intercultural dialogue that stimulates the transformation of cultures towards a deeper cosmopolitan understanding. (p. 295)

Participant 5 noted that learning L2 and M2 changed them for better as a person, because it generated a new way of thinking and communicating in a new community. There was, however, a limited window of opportunity in which the participants might see themselves positively in this context (Black, 2002). The potential to find meaningful employment and mainstream education opportunities is problematic and statistically challenging for adult-CALD BL learners (Section 1.1).

Finding 5

BL cultural behaviours (TXT5000) affect decoding and learning M2. M2 generates new cultural experiences that challenge and change, for example, BL learners' psychological behaviours and perceptions. Constructs such as linear relationships, gender neutrality, and maximising profit, in M2 are not universally shared across cultures and languages. BL cultural experiences also identify learners disassociating themselves from home languages and culture when learning M2 as a new cross C, L, and S experience.

5.7 TXT6000: BL educational experiences that emerge and affect how M2 is decoded and learnt

The participants' decoding and learning behaviours were influenced by their educational experiences. Some BL experiences lent more to decoding and learning the task as M2 than others. Participants 1 and 2 were more competent in M2, in part, because, firstly, M1 and M2 shared meanings and processes and, secondly, they had more formal educations in M1. This experience helped decode the more sophisticated mathematical structures in the tasks. Alternatively, while Participants 3 and 4 had less formal educations, their longer immersion in L2 helped them decode everyday words. The task required competencies in both everyday and formal mathematical discourses to decode and learn successfully.

The participants' language classes were also skewed towards learning M2 as an informal everyday discourse. The participants identified the formal mathematics addressed in the tasks was rarely taught in their ESL classrooms and, respectively, they wanted "to learn more". Two educational issues emerged and affected the research problem in this context. Firstly, existing curricula might inadequately address the participants' needs in M2 (AIR, 2006; COAG, 2008). Secondly, teachers might lack skills in this area of their education (AIR, 2006; COAG, 2008). A more

precise understanding of L and S structures, and BL decoding and learning behaviours would address curricular and teacher expertise in M2.

The synthesis identified structural and language-based teaching strategies are needed to teach and learn M2 in a cross-C, L, and S context. Examples that emerged from the in-tasks observations included the participants:

1. Learning to identify and decode the L and S processes that were implicit and difficult to interpret in the text.
2. Reconciling the conversational maxims that were needed to articulate meanings.
3. Reconciling the strategies needed to trigger, recall, and transpose meanings across M1 and M2 as two semi-related L and S codes. Glossaries and textual clues helped prompt memory behaviours in this context.

The participants also decoded and learnt M2 without, necessarily, understanding its underpinning logics and structural conventions. Participants 4 and 5, for example, quickly responded to axiomatic propositions by repeating and rewriting them without fully understanding their logics. Nonetheless, while it was possible for Participants 4 and 5 to learn M2 through repetition, the more complex structures found in Tasks 13, 14, and 15 required a more conscious explanation and understanding of the coding conventions involved (Sections 2.3 & 4.3.1). As Ernest (2006) stated:

> The rules of sign production are in most cases implicit, and are acquired by 'case law' by novice users of a semiotic system. Once a semiotic system is fully developed historically or mathematically the rules might be made more explicit. (p. 71)

Finding 6

BL educational experiences (TXT6000) affect learning M2 in a cross C, L, and S context. Some BL educational experiences lend more to decoding and learning M2 than others. For example, M2 is easier to decode and learn when it shares signs, processes, and learning experiences with M1. However, cross C, L, and S differences make meanings difficult to transpose without formal instruction. Existing curricula and teacher skills are, in the main, inadequate in this area to help resolve the problem of teaching and learning M2.

5.8 Chapter summary and findings

The study was undertaken to help resolve the *problem of teaching and researching adult-CALD BL learners in M2*. The problem was interpreted in two ways. Firstly, linguistic and semiotic science envisaged the research problem as a sign and language-learning problem (Duval, 2006; Ernest, 2006; Halliday, 2006; O'Halloran, 2005; Presmeg, 2006; Schleppegrell, 2007). Secondly, interpretive and synthetic logics were employed to collect and articulate qualitative data into a holistic interpretation of the research problem and develop potential solutions (Gharajedaghi & Ackoff, 1985; Geertz, 1973; Patton, 1990; Schutz, 1932).

Adopting interpretive logics lent to exploring and collecting data that were difficult to interpret in a cross-C, L, and S context (Blumer, 1969; Geertz, 1973; Schutz, 1932; Schwandt, 1994). This meant watching, listening, asking, recording, and examining (Schwandt, 1994, p. 119) the symbolic interactions of BL learners in the M2 sign system (Blumer, 1969; Geertz, 1973; Jakobson & Halle, 1956). Including a poststructuralist open-ended approach helped question the hidden, unpredictable, expressive, and non-functional relationships that emerged and created problems (Miller, Whalley, & Stronach, 2005).

Adopting synthetic logics lent to interpreting the unknown factors and relationships that lay outside the boundaries and assumptions made in existing theory (Kant, 1788). An analytic approach would have, theoretically, bounded the number of propositions employed to interpret the research problem (Gharajedaghi & Ackoff, 1985; Patton, 1990). Synthetic logics interpreted and merged the research problem from multiple theoretical perspectives (Gharajedaghi & Ackoff, 1985; Patton, 1990). A synthetic approach was also employed to interpret and articulate existing knowledge (Onwuegbuzie, Leech, & Collins, 2012) into the battery of open-ended questions that helped collect and interpret data.

Three research questions emerged from the preliminary review of existing literature and research to define the focus of the data collection and synthesis. The first two questions addressed the linguistic, semiotic, and educational implications of the research problem, and the third the validity and reliability of the case-based interpretive research design in addressing these questions. The respective research questions asked:

1. *What are the linguistic and semiotic structures in mathematical language that create problems for adult-CALD bilingual learners when learning mathematics in English as an additional language?*

2. *How can knowledge of adult-CALD bilingual learner-related problems in linguistic and semiotic structures in mathematics be used to enhance mathematics education in Australian classrooms?*

3. *How can case-based interpretive research methodology deepen our understanding of adult-CALD bilingual learners who are engaging in linguistic and semiotic structures in mathematics as an additional language?*

The following data collection instruments and procedures were developed to interpret and answer the research problem and questions:

1. The elements of the research problems were defined and articulated into the vectors for conducting a literature and research review.
2. Existing knowledge was synthesised and articulated into a coded battery of opened-ended questions that helped collect and interpret data. Coding

maintained links between the existing research domains and the findings that emerged. Data collection questions were, however, coded a posteriori to interpret data inductively from the observations that emerged.

3. Data were collected in three stages:
    i. Stage 1 conducted a semiotic synthesis on the tasks that were employed to observe and interpret participants' behaviours in Stage 3.
    ii. Stage 2 conducted background interviews to identify the participants' BL experiences that affected decoding and learning M2.
    iii. Stage 3 conducted in-tasks observations on participants decoding and learning M2. Interviews were digitally recorded and written up as text.
4. Data from Stages 1, 2, and 3 were synthesised, coded, and summarised into three-tier observations, interpretations, and implications data sets.
5. These data sets were then transposed onto a multidimensional Data and Summary Matrix for further synthesis and global coding. This enabled the study to rank and pivot the coded data sets from different theoretical perspectives to interpret the research problem.
6. The findings of the data synthesises generated a holistic interpretation and answers to the research problem and questions.

The findings of the data synthesis articulated the following answers to the research problem and questions:

Research Question 1:

L and S structures and BL decoding and learning behaviours in M2 are defined by the following relationship:

M2 is encoded through two types of coding conventions that create problems when decoded and learnt in a cross-C, L, and S context.

1. TXT 1000: Philosophical codes that encode ideologies, values, and cultural meanings.
2. TXT 2000: Structural codes that encode meanings through the form and function of the sign system. Nine types of structural codes were identified through the data synthesis:
    2.1. Paradigmatic and syntagmatic codes

2.2. Iconic, symbolic, and indexical codes
2.3. Opposition, marking, and contrasting codes
2.4. Physical codes (shape, position, energy, motion, force, synergy, timing, and aesthetic relationships)
2.5. Agentic codes
2.6. Metaphoric codes
2.7. Axiomatic and logic codes
2.8. Discursive codes
2.9. Mediums

Adult-CALD BL learners also bring with them C, L, and S behaviours and experiences that create problems when decoding and learning M2. Problems develop when BL decoding behaviours and experiences are incongruent for decoding and learning M2 in a cross-C, L, and S context. Four types of BL behaviours and experiences were identified as problems when decoding and learning L and S structures in M2:

1. TXT3000: BL decoding behaviours
2. TXT4000: BL humanistic behaviours
3. TXT5000: BL cultural behaviours
4. TXT6000: BL educational experiences

Research Question 2:

Knowledge of L and S structures and adult-CALD BL learner problems in M2 enhances the capacity of teachers and researchers to interpret:

1. The C, L, and S conventions that encode and define M2 as a sign system.
2. The BL behaviours and experiences emerging and creating problems in M2 when decoded and learnt in a cross-C, L, and S context.
3. The competencies and strategies needed to teach and learn M2 as a sign and language-learning problem.

Research Question 3:

The case-based interpretive and synthetic methodologies employed in this study generated comprehensive and holistic answers to Research Questions 1 and 2. The approach enhances teacher and researcher capacities to:

1. Reconcile the L and S structures in M2 that are otherwise difficult to interpret through other research methods.
2. Articulate curriculum and teaching practices that more precisely address the research problem and questions as a structural and language-based learning problem. The research techniques are also recommended for other areas of education that have complex language-learning problems to resolve.

The findings of the synthesis articulate structural and language-based competencies and strategies for teaching and learning M2. Specifically, the findings identify the philosophical and structural conventions, and BL behaviours and experiences that emerge and create problems when decoding and learning M2 in a cross-C, L, and S context. In this context the BL participants in this study brought with them their own distinctive decoding, humanistic, cultural, and educational behaviours and experiences that influenced how they decoded and learnt M2 at a particular point in time.

The participants interpreted the tasks through a combination of coordinate and compound BL experiences. Coordinate experiences (Hamers & Blanc, 2000, p. 39) contend that M2 shares meanings and processes with M1. Compound experiences (Hamers & Blanc, 2000, p. 39) assert that M2 is decoded and learnt as a new L and S code. The alternative experiences identify different focuses for teaching M2. Coordinate experiences focus on remembering, transposing, and code switching signs across two L and S codes. Compound experiences focus on learning M2 as a new sign system.

Competency is interpreted in two ways. Firstly, it identifies the accuracy in which the M2 sign system is decoded from the L and S perspective. For example, it is just as important from this perspective to interpret the meaning, form, and function of multiple-choice structures, as it is to choose the correct answer. Problems emerged when the participants misinterpreted the form and function of the sign system.

Secondly, competency identifies how independently the learner decodes and learns the sign system. Problems emerged when the participants were unable to decode and learn the sign system as autonomous learners. Merging the two aforementioned dimensions create the level and type of BL support needed to decode and learn M2.

The text fulfils two functions in this context. Firstly, it identifies the philosophical and structural conventions that encode the M2 sign system. Secondly, the text reflects the type of semiotic resources (O'Halloran, 2010) that are employed to teach and assess competencies in M2. Synthesising the two functions identifies the form and function of the text in, respectively, assessing and modifying decoding and learning behaviours (Eco, 1981; Ernest, 2010; Jakobson & Halle, 1956). The participants believed their educational experiences were inadequate in this area to address their short- and long-term settlement goals.

Existing curricula and teacher capacities can be enhanced through a more precise understanding of M2 text and BL learner behaviours in it. The final chapter which follows defines in greater depth the educational and future research implications and recommendations that emerged from these findings.

Chapter 6: Implications, Recommendations, and Conclusions

> The addressee of a coded message is supposed to be in possession of the code and through it he interprets the message. Unlike this decoder, the cryptanalyst comes into possession of a message with no prior knowledge of the underlying code and must break this code through dexterous manipulations of the message. A native speaker responds to any text in his language as a regular decoder, whereas a stranger, unfamiliar with the language, faces the same text as a cryptanalyst.
>
> (Jakobson & Halle, *Fundamentals of language,* 1956, p. 57)

6.1 Chapter introduction

This chapter defines the educational and theoretical on-going research implications that emerged from the data synthesis. The findings advocate structural and language-based teaching strategies, and interpretive synthetic research techniques to address the research problem. The findings also articulate knowledge for other areas of education that have complex language-learning problems to resolve.

The position of the implications, recommendations, and conclusions that emerged from the study is depicted in Figure 6.1.

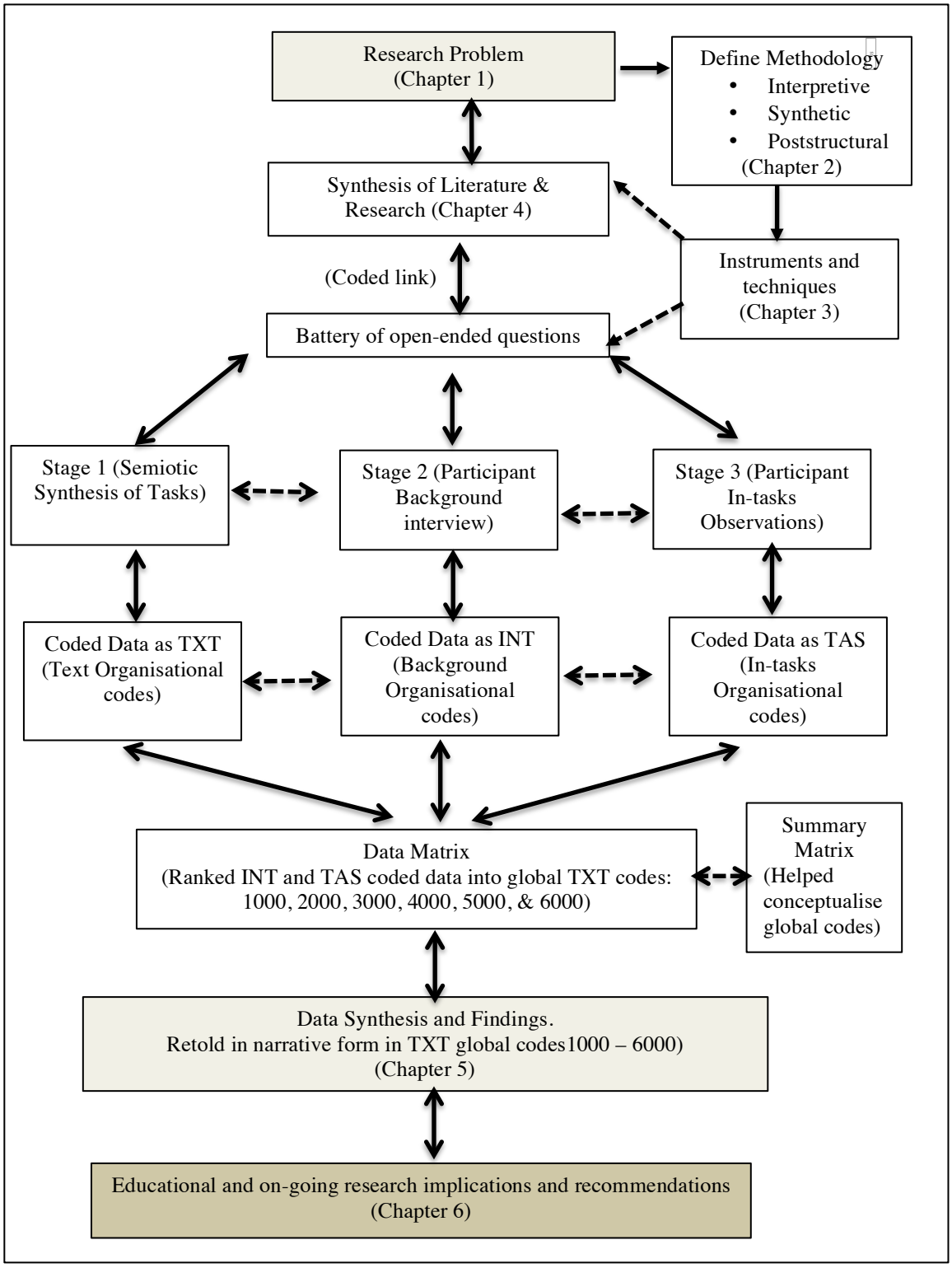


Figure 6.1: The position of the research implication, recommendations, and conclusion in the study.

6.2 Research problem

The terms mathematics, language, adult-CALD bilingual learners, culture, Australian, and education helped interpret the constructs of the research problem. However, the terms were imprecisely and inconsistently defined in existing literature and research, and therefore fail to articulate a holistic solution to the problem without further study. Interpretive logics were, therefore, employed to explore their meanings and relationships more deeply (Geertz, 1973; Schutz, 1932; Schwandt, 1994). Synthetic logics helped to interpret and define the unknown relationships that emerged outside the assumptions of existing theory (Kant, 1788). An alternative analytic approach would have interpreted the constructs a priori rather than open-endedly to identify new relationships. Poststructuralist open-ended rather than closed questioning techniques were employed to explore hidden relationships and meanings.

A preliminary review of existing literature and research identified that little was known about resolving the research problem or how it should be studied. Linguistic and semiotic science and language assisted studying the research problem as a sign and language-learning issue (Bloomfield, 1939; Duval, 2006; Ernest, 2006; Halliday, 2006; O'Halloran, 2005; Peirce, 1931; Presmeg, 2006). M2 in this context is defined by the philosophical and structural conventions that encode the sign system (Bloomfield, 1939; Halliday, 2006; Peirce, 1931). BL learners also bring with them decoding, humanistic, cultural, and educational behaviours and experiences that create problems when decoding and learning M2 in a cross-C, L, and S context. The misalignment identifies the structural and language-based teaching strategies needed to address the problem.

6.3 Research questions

BL learners employ distinctive decoding and learning behaviours that distinguish them from monolingual English learners in the M2 sign system. The term sign system identifies the L and S conventions that encode M2, and decoding the behaviours learners adopt to decipher the system (Jakobson & Halle, 1956). The synthesis interpreted and defined M2 structures and BL decoding behaviours in the following context to answer the research problem and questions.

6.3.1 One

What are the linguistic and semiotic structures in mathematical language that create problems for adult-CALD bilingual learners when learning mathematics in English as an additional language?

Philosophical codes (TXT1000) encoded ideological meanings such as profit, gender, and linear relationships, in the M2 sign system. Philosophical codes are implicit in the text and difficult to interpret without C, L, and S experience. Meanings emerge pragmatically (Peirce, 1931) in this context, because ideological meanings are not shared universally with other cultures and languages (Whorf, 1956; Barton, 2009).

Structural codes (TXT2000) encoded meanings through the form and function of the sign system. Structural codes are functional and systemic (Halliday, 2006) and therefore more predictable to decode than *Philosophical codes (TXT1000)*. The terms sign and symbol emerge as being interchangeable, because symbols function as signs in the M2 sign system (Peirce, 1931). This makes M2 symbols arbitrary, in part, because they can be swapped with other symbolic representations to articulate the same meaning (de Saussure, 1910). The symbol 'S' in Task 10 was arbitrary, because it could equally denote and be replaced with other symbolic representations to signify constructs such as 'shares' in Task 15. Nonetheless, mathematical symbols also attract third-tier meanings in certain social contexts. These meanings are not arbitrarily replaced by other sign representations (Peirce, 1931), for example 'S' signifying danger in a car and stock market crash in Tasks 10 and 15.

Structural codes transpose the M2 sign system into a highly objectified and nominalised language that is difficult to decode and learn without formal instruction (Halliday, 2006; Sfard, 2005). Problems emerge when both monolingual and BL learners employ aberrant decoding behaviours to decipher the sign system (Jakobson & Halle, 1956). The term aberrant means that the sign system is decoded differently than the system and encoder intends (Jakobson & Halle, 1956). Breakdowns for decoders occur at a tactile, auditory, perceptual, and textual level (Eco, 1981; Jakobson & Halle, 1956). The M2 sign system generates distinctive kinds of breakdown points when decoded in a cross-C, L, and S context.

Nine structural codes emerge from this research and define the C, L, and S conventions that encode and create problems in M2 when decoded through its forms and functions. The nine structural codes observed in the synthesis are summarised and coded as follows:

1. *Paradigmatic codes* encode meanings by changing the way the sign looks, and *syntagmatic codes* through repositioning the sign in the syntax. Paradigmatic and syntagmatic codes are also confusing because they can function simultaneously with signs in M2.
2. *Iconic, symbolic, and indexical codes* encode meanings by changing the significance of the sign in the sign system (Peirce, 1931). The symbols 'Ψ', '$\sum$', and 'Π' are, for example, employed as symbolic signs to solve complex problems in M2. The signs are also an iconic representation of M2, because they represent and distinguish M2 from other sign systems.
3. *Opposition, marking, and contrasting codes* encode meanings by establishing polarity in the sign system (Jakobson & Halle, 1956, p. 4). The atomic signs, for example, '-', differentiate the meaning and thinking behaviour required to interpret the signs '2' and '3' in complex molecular structures such as '3^{-2}' and '1/9'.
4. *Physical codes* encode meanings through their physical characteristics and attributes. Physical codes define the effect that, for example, shape, energy, motion, force, synergy, timing, and the aesthetic characteristics of the sign, have on encoding meanings. The M2 sign system displays energy and force when it transposes and expands in its capacity to signify and solve problems mathematically. The process articulates the M2 sign system into more complex and delicate ways of thinking scientifically (Halliday, 2006).
5. *Agentic codes* encode the M2 sign system by defining the mental and physical behaviours that are required to decode the system. The term agency identifies that the sign system has its own goals, reactions, and cognitive properties that encode behaviours (Leslie, 1993; Tylen, 2007). The agentic characteristic of the M2 sign system is difficult to negotiate without formal instruction (Ernest, 2006).
6. *Metaphoric codes* encode the sign system by changing the way signs articulates metaphoric meanings (Halliday, 2006). Metaphors in everyday language are literally disconnected from their adverbial meanings (Halliday,

2006). Mathematical signs such as tables, figures, and algebraic equations, also function as metaphors in M2, however their forms and functions are, in part, more predictable to decode than everyday language. Equilibrium and equivalency signs such as '=" and '⇔', identify points where metaphoric transformations occur in M2 for instance in algebraic representations. Similarly, tables and charts are employed as metaphors for interpreting the same data pattern across different registers.

7. *Logic and axiomatic codes* encode meanings through mathematical propositions and laws, such as: associativity, commutativity, and distributivity. Logic and axiomatic codes encode assumptions that are sometimes difficult to interpret in real-life contexts (Barton, 2009). '5 cats in 3 rooms' and '5 rooms with 3 cats' is axiomatically equivalent, however, the commutativity law breaks down when the two constructs are valued differently in the community. Logic and axiomatic codes are not universally shared across cultures and languages (Barton, 2009; Whorf, 1956).
8. *Discursive codes* encode the rules for communicating meanings as a continuous form of text and speech in M2. Four conversational maxims, quantity, relevance, manner, and quality (Fromkin, Blair, & Collins, 1999, p. 187) help define appropriate discourse behaviours in M2. These rules are difficult to sustain without instruction.
9. The *medium* binds and shapes the way philosophical and structural codes are encoded and interpreted in M2. In this context the medium constrains and affords (Chandler, 1994) the meanings and behaviours employed to encode and decode the system (McLuhan, 2001). English is particularly ambiguous and problematic as a medium for decoding M2 in a cross-C C, L, and S context.

BL learners bring with them behaviours and experiences that affect how they decode and learn M2 as a sign system. Four types of BL behaviours and experiences emerged from this research synthesis to define this behaviour. Their meaning and significance are summarised and coded as follows:

1. *TXT3000BL decoding behaviours* define the distinctive types of decoding behaviours BL learners employ to decode and learn M2. Reading aloud, underlining new words, having C, L, and S hesitations, questioning M2 as a

language, and code switching between M1 and M2 identify distinctive types of BL decoding behaviours in M2. The behaviours are also affected by the learner's age, gender, and political, educational, economic, and settlement experiences. Settlement experiences, for example, shape how BL learners envision themselves as learners in M2. Problems emerge because M2 is inadequately taught in the classroom and cannot address these types of issues. Problems also develop because BL competencies are difficult to assess in a cross-C, L, and S context. Decoding and learning behaviours change quickly through new settlement and immersion experiences.

2. *TXT4000 BL humanistic decoding behaviours* define the more humanistic behaviours and experiences that affect decoding and learning M2. They identify the effects of behaviours such as emotions, perceptions, modality (speaking, listening, reading, and writing), gender, and memory have on decoding and learning M2 in a cross-C, L, and S context. Childhood experiences emerge and affect how BL learners, for example, interpret foundational meanings like value, place, and numbers, in M2. M2 is difficult to decode and learn when learners in the past were excluded due to their gender and ethnicity from, for example, learning foundational mathematics.

   Humanistic experiences also identify memory-related problems that affect decoding and learning M2 from M1 experiences. Two types of memory problems emerge in this context. Firstly, BL learners have problems recalling shared meanings and processes in M1. Secondly, BL learners have issues transposing shared meanings and process when decoding and learning M2. The first identifies content and the second a process-related memory problem. Both confirm different foci for teaching and learning M2 as a memory-related learning behaviour.

   Humanistic behaviours also identify problems associated with learner motivations and perceptions in M2. Positive motivations contribute to learning M2 as an additional language construct (Gardner, Masgoret, Tennant, & Mihic, 2004). Anecdotally, BL motivations wane over time due to poor classroom experiences, and outcomes that do not lead to employment opportunities (Black, 2002).

3. *TXT5000 BL cultural behaviours* define the cultural experiences that affect decoding and learning M2. M2 generates new cultural experiences that challenge and change BL learners' psychological states (Hamers & Blanc, 2000). Constructs, for example linear relationships, gender neutrality, and maximising profit, in M2 are not universally shared across cultures and languages (Barton, 2009; Whorf, 1956). These experiences are also associated with BL learners disassociating themselves from home languages and culture.

4. *TXT6000 BL Educational experiences* define the educational experiences that affect learning M2 in a cross-C, L, and S context. Some BL experiences lend more to decoding and learning M2 than others. For example, M2 is easier to decode and learn when it shares signs and processes with M1. However, cross-C, L, and S differences make these meanings difficult to transpose without formal instruction. Existing curricula and teacher capacities or skills are, in the main, inadequate in this area of education.

Philosophical (TXT1000) and structural (TXT2000) conventions encode distinctive ideological values, beliefs, and meaning making process in the M2 sign system. Adult BL learners bring with them distinctive decoding (TXT3000), humanistic (TXT4000), cultural (TXT5000), and educational (TXT6000) behaviours and experiences that affect how they decode and learn M2 as a sign system. BL problems emerged because of cross-C, L, and S differences, and the misalignment of decoding and learning behaviours and experiences when decoding and learning L and S structures in M2.

6.3.2 Two

How can knowledge of adult-CALD bilingual-related problems when decoding linguistic and semiotic structures in mathematics be used to enhance mathematics education in Australian classrooms?

Knowledge of Research Question 1 defines the coding conventions and BL decoding and learning behaviours that emerge and create problems when teaching and learning M2 in a cross-C, L, and S context. The knowledge articulates structural and language-based teaching strategies so that the problem can be addressed.

Respectively, the text, and BL decoding and learning behaviours can be modified, for example, in the following ways to enhance M2 education:

1. Making *philosophical codes (TXT1000)* more explicit in the text for BL learners. Activities should reconcile the ambiguous ideological meanings that are difficult to interpret in a cross-C, L, and S context. The role of ideologies needs to be explored and made more explicit. Activities should focus on making deeper real-life connections for BL learners such as integrating learning M2 to help achieve settlement goals.
2. Learning M2 through the *structural codes (TXT2000)* that define the form and function of the sign system. Analogically, teaching M2 is envisioned as being similar to writing English subtitles in foreign language movies: terminology needs to accurately reflect the logics, sequences, subplots, and actions that define the underpinning story. Examples of structural and language-based teaching activities include:
    a. Distinguishing the *paradigmatic* and *syntagmatic* conventions that encode symbolic transformations in M2. For example, the '5' prefixed with a minus sign '-' in '-5 + 5 = 0' generates a paradigmatic change of thinking, which is different to a syntactic transformation of '5(3 + 4) = (5 x 3) + (5 x 4)'. The treatment of the '5' is ambiguous to interpret (Duval, 2001) unless the decoder distinguishes the paradigmatic and syntagmatic processes involved.
    b. Enhancing the *discourse* strategies employed to articulate M2 as a continuous form of discourse. The Cooperative Principle Model of quantity, relevance, manner, and quality (Fromkin, Blair, & Collins, 1999, p. 187) can be employed to interpret and enhance discourse behaviours in both text and speech.
    c. Learning to decode *axiomatic propositions and logics* through the form and function of the M2 sign system. Linguistic repetition (Ernest, 2006) can scaffold an initial immersion into mathematical discourse, for example: repeating in an oral and textual exercise '5 cats <u>in</u> 3 rooms is the same as 5 rooms <u>with</u> 3 cats'. The preposition is underlined in this context to signify the learner reconciling and distinguishing the symbolic logics employed to decode the task (O'Neill, 2009). As Bloomfield (1939) wrote:

> Within mathematical discourse there is probably no linguistic difference between applied mathematics – that is, calculations that form part of scientific discourse – and pure mathematics, where calculation is made for its own interest, with arbitrary axioms replacing the observational data and the hypothesis of scientific procedure (p. 43). In all this development we have not left the domain of language (p. 44).

d. Interpreting the effect the *medium* has on, respectively, constraining and affording (Chandler, 1994) decoding and learning behaviours in M2. Mediums such as text, technology, colour, and music, can be explored to enhance learners' cross-C, L, and S engagement in the sign system.
e. Structural and language-based learning activities can be integrated to generate a holistic learning experience. *Logic and axiomatic codes* can be scaffolded through repetition/case law experience (Ernest, 2006, p. 72) and *discourses* practices that employ maxims of quantity, relevance, manner, and quality.

BL learners bring with them decoding and learning behaviours that also affect teaching and learning M2. The learning activities should focus on, for example:

1. Enhancing *TXT3000 BL decoding behaviours* in M2. Resource such as glossaries and in-text explanations to help interpret vague and ambiguous meanings. Using aural and tactile activities will enhance the physical connections (Ernest, 2006) BL learners make in the sign system. Alternative mediums and modalities such as film and colour can also be explored to enhance in-tasks decoding and learning behaviours.
2. *TXT4000 Humanistic connections* can be enhanced through more focused learning activities. Colour coding can be explored to enhance the semiotic and cognitive connections (Ramachandran, 2011) BL learners make in the sign system. Fatigue can be managed through better timing and sequencing of learning activities. Learning can also make better connections to achieving settlement goals such as enhancing employability skills and financial literacy. Memory-related behaviour should be more precisely aligned to decoding M2 to help BL learners reconcile and transpose meanings across two or more linguistic codes.

3. *TXT5000BL cultural experiences* can be enhanced through a more precise understanding of the cultural problems that emerge and challenge learners. Cultural immersion should be managed through the adaption and accommodation of new meanings, rather than making old ones feel redundant to learners (Baker, 2011; Tochon, 2014).
4. *TXT6000BL educational experiences* should focus on learning the C, L, and S contents, processes, and contexts that are important for BL learners decoding M2. Activities should jointly enhance the physical, cultural, cognitive, and semiotic connections (Thibault, 2011) BL learners make in the sign system. Cultural meanings in M2 should be learnt as targeted experience when living in a new community (Tochon, 2014).

Philosophical and structural codes define the C, L, and S conventions that encode and distinguish the M2 sign system. BL behaviours and experiences define the cross-C, L, and S factors that emerge and create problems when decoding and learning M2. Combining the two sets of observations holistically enhances the competencies and strategies needed to teach, learn, and assess M2 in a cross-C, L, and S context.

6.3.3 Three

How can an interpretive and case-based research design deepen our understanding of adult-CALD bilingual learners who are having problems decoding linguistic and semiotic structures in mathematics as additional language?

There was a distinct lack of research in the problem area, and case-based interpretive research design coupled with synthetic logics and thematic coding lent to studying the complex research problem in a cross-C, L, and S context (Section 2.4). Synthetic logics contributed to interpreting the relationships that emerge outside the assumptions made in existing theory. Case study techniques help synthesise data that are important but empirically underrepresented in individual and group observations (Section 2.4). Thematic coding enhances the legitimacy of the findings that emerged (Section 3.4).

A case-based interpretive synthetic research design generates a more precise and comprehensive interpretation of the M2 sign system and decoding and learning behaviours in it. An alternative analytical approach would have interpreted the complex language-learning problem by simplifying its elements into constructs,

whereas a synthesis identifies the common truths and causes that lead to effects (The Shorter Oxford English Dictionary, 1990). The study's research design synthesised data from multiple theoretical perspectives and contexts to articulate a holistic solution to research problem.

Interpretive and synthetic logics also help interpret the operational and ethical threats that challenge a cross-C, L, and S study (Section 2.10). Factors such as cross-C, L, and S communication and ethics are difficult to interpret via analytic a priori propositions (Kant, 1788). A synthetic approach merges a posteriori the complex array of situated cross-C, L, and S behaviours and contexts into a more precise and ethical collection of data (Kant, 1788) (Section 2.10).

Thematic coding techniques coupled with a case-based interpretive synthetic research design enhance the validity, reliability, and generalisability of the findings that emerge (Section 2.12). The study was cognisant of the criticisms leveraged against both quantitative and qualitative research designs, and thematic coding maintained important links between the logics that underpinned the data collection stages and the subsequently articulated findings. Checklists can be employed to, respectively, critique the alignment of the data collection instruments (Braun & Clarke, 2006), and make recommendations for ongoing improvement in the research design (Section 2.12).

Data are interpreted and coded thematically as signs in this context to articulate holistic solutions. As Schutz (1932) stated:

> It should be noted that not all evidences are signs, but all signs are evidences. For an evidence to be a sign, it must be capable of becoming an element in a sign system with the status of coordinating scheme. This qualification is lacking in some evidence. A tool, for instance, although it is an evidence of what went on in the mind of its maker, is surely no sign. However, under "evidences" we mean to include not only equipment that has been produced by a manufacturing process, but judgment that has been produced by thought, or the message content which has been produced by an act of communication. (p. 43)

The findings that emerge through a coupled case-based interpretive synthetic research design and thematic coding techniques generate teaching strategies that are more cognisant of decoding and learning problems in the M2 sign system (Section 2.11).

6.4 Implications for theory

Interpretive and synthetic logics were employed, firstly, because not much was known about the causes and effects of the research problem and, secondly, new theory was needed to articulate solutions. A post-positivist analytic approach would have encoded data collection questions a priori and, therefore, not allowed observations to emerge open-endedly to generate new theory (Cleve, 2003). Figure 6.2 summarises the theoretical implications emerging from the interpretive synthetic a posteriori line of questioning.

Theoretical implications

Research problem = M2 coding conventions + BL decoding problems in M2 + existing research techniques employed to interpret the problem.

1. *Coding conventions = the TXT 1000 (Philosophical codes) + TXT 2000 (Structural codes) that emerge and make M2 different and problematic to decode.*
2. *BL decoding problems = the TXT 3000 (BL decoding behaviours) + TXT4000 (BL human and physical behaviours) + TXT5000 (BL cultural experiences) + TXT6000 (BL educational experiences).*
3. *Techniques = analytic post-positivist research approaches.*

Proposition 1:

Move BL learner decoding and learning competencies closer to that required by the M2 sign system. Competencies in M2 are enhanced through structural and language-based learning strategies.

Proposition 2:

Employ case-based interpretive synthetic research design and thematic coding techniques to interpret the complex language-learning problem. The procedures are recommended for areas of education that have complex language-learning problems to resolve.

Figure 6.2: Theoretical implications

The theoretical implications identify that *the problem of teaching and researching adult-CALD BL learners M2 (Mathematics in English as an Additional Language)* is a product of (1) the coding conventions and BL decoding and learning behaviours and experiences that emerge and create problems, and (2) the research techniques that are employed to interpret the problem. The theory advocates (1) structural and language-based teaching strategies, and (2) case-based interpretive synthetic research design coupled with thematic coding techniques to address the research problem.

The theoretical implications articulate two types of conceptual knowledge (MacInnes, 2011):

1. New ways of conceptualising cross-C, L, and S decoding and learning behaviours in M2. This knowledge emerges through case-based interpretive synthetic research techniques.
2. New ways of conceptualising teaching and learning M2 in a cross-C, L, and S context. This knowledge emerges through structural and language-based teaching strategies.

As Geertz (1973) stated:

> In short, we need to look for systematic relationships among diverse phenomena, not for substantive identities among similar ones. And to do that with any effectiveness, we need to replace the "stratigraphie" conception of the relations between the various aspects of human existence with a synthetic one; that is, one in which biological, psychological, sociological, and cultural factors can be treated as variables within unitary systems of analysis. (p. 8)

6.5 Implications for policy and practice

It is envisaged that structural and language-based teaching strategies will affect educational policy and practice in three ways. Firstly, the strategies articulate knowledge that helps teach and assess competencies in M2. The strategies, respectively, identify:

1. Knowledge for decoding and learning M2 as a sign system. Philosophical codes identify the ideological conventions that encode meanings, and

structural codes the meanings and processes that are encoded through the form and functions of the sign system.

2. Criteria for assessing BL competencies and needs in M2. BL decoding behaviours and experiences identify problem areas for teaching and assessing M2 in a cross-C, L, and S context.

Secondly, structural and language-based teaching strategies articulate curricular and policy solutions. Adult-CALD minorities over represent, respectively, illiterate, innumerate, and unemployed people in Australian society (ABS, 2006b; ABS, 2011; NCVER, 2006; Perkins, 2009). The problem emerges, in part, because of poor CALD BL learning experience in M2. The proposed strategies identify areas for curricular and teacher development, by focusing on teaching M2 as sign and language-learning problem. Structural and language-based teaching strategies are envisaged for a standalone mathematics course or blended into courses that involve teaching M2 as an integrated outcome.

Thirdly, changes to structural and language-based teaching strategies are envisioned for other areas of education where complex language-learning problems have to be solved. The structural and language-based teaching strategies articulated in this research identify techniques for interpreting, for example:

1. BL learning behaviours and problems in other areas of education.
2. Monolingual English learning behaviours and problems that emerge in M2 because of difficulties in decoding and learning the language.

6.7 Limitations

The synthesis approached the research problem from the bottom up rather than the top down to make recommendations. The bottom-up approach scaffolds funding, curricular, and teacher development through better understanding the text and BL decoding and learning behaviours in it (Section 6.5). A top-down approach questions the research problem by interpreting the influence, for example, funding, curriculum, and professional development have on the problem. The top-down approach, however, overlooks the microelements that affect the problem and potential solutions.

The elements that made up the research problem (mathematics, language, adult-CALD bilingual learners, culture, Australian, and education) defined the factors and relationships that needed to be explored in greater depth. The elements were, respectively, employed to search, synthesise, and articulate existing literature and research into the battery of open-ended questions that served to collect and interpret data. The approach generated a holistic interpretation of the constructs that made up the research problem and potential solutions (Gharajedaghi & Ackoff, 1985; Patton, 1990). An alternative analytic vector would have narrowed the number of questions, data sets, and solutions that emerged to interpret the problem. For example, interpreting the effect culture had on learning M2 without reconciling the broader effect factors, such as the agentic characteristics of the sign system and the medium also have on culture when decoding and learning M2.

L and S language and science, and interpretive and synthetic logics articulated the research problem as a sign and language-learning problem. The approach, however, generated two methodological issues that needed to be reconciled. Firstly, the approach generated a large set of quantity non-discrete qualitative data that was complex to synthesise. Secondly, a synthetic approach made it challenging to explain datum within the set when it was merged with other data. Anecdotally, Quantum Mechanics faces a similar dilemma in that the broader the picture being observed and explained the harder it is to describe individual physical phenomena (Heisenberg, Crull, & Bacciagaluppi, 2011). An analytic approach would have, for example, generated more precise individual observations by limiting the number of theoretical propositions that are employed to interpret and explain data relationships. An analytic approach, however, also emerges at an epistemological cost, because less attention is paid to merging, contrasting, and comparing the data observations across the big picture (Sections 2.4 & 2.6).

Coding and digital technology addressed the aforementioned problems of complexity and uncertainty in the synthesis. The technique enabled the synthesis to transpose analogic signs into digital reproductions for interpreting the research problem. The coding and conversion of analogic datum into digital signs emerged as follows:

1. Synthetic logics (Kant, 1788) were adopted because little was known about the research problem and how to best study it (Section 2.4).

2. Existing knowledge was applied a posteriori to interpret and define unknown relationships and meanings (Kant, 1788).
3. Datum was interpreted as an analogic sign, and its meanings and significances were coded in the context of the whole sign system (Jakobson & Halle, 1956; Peirce, 1931; Schutz, 1932). As Chandler (1994) stated:

> Analogical signs (such as paintings in a gallery or gestures in face-to-face interaction) are signs in a form in which they are perceived as involving graded relationships on a continuum rather than as discrete units (in contrast to digital signs). (p. 460)

4. Thematic coding, digital technology, and data management software transposed analogic data into digital reproductions. As Chandler (1994) commented: "digital technology can transform analogical signs into digital reproductions, which may be perceptually indistinguishable from the 'originals'" (p. 460).
5. The problems of complexity and uncertainty were reconciled, because at any point along the continuum digital data can be interpreted in detail as individual signs and/or transposed in the context of the whole sign system. This, respectively, generated a more precise and comprehensive interpretation of the large qualitative data set that was collected to define the research problem.

The time and resources available delineated the breath and depth of the findings that emerged. As a result, follow-up questions need further observation, such as: interpreting the effect modalities, for example, film, art, and music have on decoding and learning M2; and studying the effect that structural and language-based teaching strategies have in a classroom situation. The one-on-one interviews were conducted to interpret individual behaviours and experiences that were difficult to observe in public places such as classrooms. However, it is envisaged BL learners also behave differently when there are other learners around them. The study was unable to observe these types of contexts and behaviours precisely without further investigation. The unanswered questions foreground the following questions for future research:

1. What are the effects of structural and language-based teaching strategies on teaching M2 in a classroom situation?

2. What mediums and modalities can be employed to enhance structural and language-based learning in M2?

6.8 Recommendations for future research

The data collection instruments and techniques allow for new observations and scenarios to be studied and added to what has already been learnt. Two checklists (Section 2.11) were developed to help critique and make recommendations for improving future research in this area. The observations and recommendations that emerged are summarised as follows.

The Data Matrix ranked and pivoted coded data themes from different subordinate and superordinate positions to interpret relationships. However, ranking transposed qualitative data into ordinal relationships that were quantitative in nature. The outcomes generated an epistemological dilemma, because ranking was a theoretical and not a quantitative choice. Figure 6.3 summarises the technique and dilemmas that emerged from ranking and pivoting qualitative data sets from three different stages of data collection.

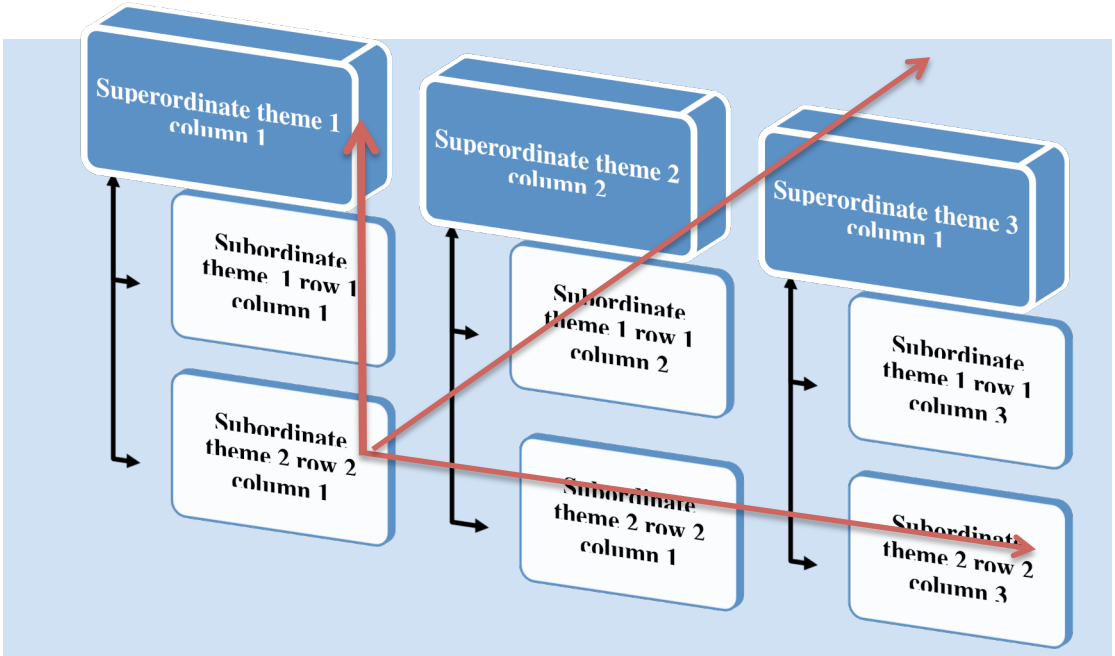


Figure 6.3: Multidimensional data array for ranking and pivoting qualitative data sets.

The Data Matrix (Appendix F) generated a multidimensional view of the research problem that allowed data sets to be ranked and pivoted (up, down, and across in three directions) into different superordinate and subordinate relationships to interpret the problem. The choices were, however, theoretical and not quantitative. The study interpreted the matrix's final position as a TXT (Text) superordinate relationship and problem. INT (Background Experiences) and TAS (In-tasks Data Observations) form subordinate data relationships that can also be articulated into superordinate propositions. Clarifying and enhancing the capacity of the multidimensional data array to interpret and synthesise qualitative data in this context into ordinal and pseudo-quantitative relationships identified an area for ongoing research and development.

Interpretive synthetic logics employed logics such as if, then, and only if, propositions that can articulate data in the Data Matrix. The complex task was, however, performed manually with the help of Excel spreadsheets, and logic-based software would have enhanced the speed and accuracy in which the observations emerged. Suitable software was not found and further technological developments are required for this area of research.

The research problem was interpreted through linguistic and semiotic science and language to articulate structural and language-based teaching strategies. The research design was, therefore, as much a linguistic and semiotic choice as it was an epistemological and methodological one. The form and function of language and semiotics in shaping the research design and the findings that emerge remains a critical area for future investigation. As Bloomfield (1939) remarked:

> Popularly and even to a larger extent, academically, we are not accustomed to observing language and its effects; these effects are generally explained instead by postulations of "mental" factors. In the cosmos, language produces human society, a structure more complex than the individual, related to him somewhat as the many-celled organism is related to the single cell. (p. 55)

The study's linguistic, semiotic, interpretive, and synthetic vector enhances the interpretation and resolution of complex language-learning problems. The strategies and procedures developed in this study are, therefore, recommended for other areas

of education that have complex language-learning problems to resolve. The procedural recommendations include:

1. Defining the elements that make up the research problem and questions. The elements help search, synthesise, and articulate the review of existing literature into a battery of open-ended questions that help collect and interpret data. Coding the data collection questions meant maintaining links between the research domain and theory that generated the data collection questions, and the findings emerging from the study. A synthetic and thematic approach is recommended to code data collection questions a posterior to interpret unknown factors and relationships in the research problem.
2. Interpreting datum as signs, because their meanings are defined by their relationships with other signs in a sign system (Schutz, 1932). Datum are interpreted through three levels of significance (Peirce, 1931):
    a. Observations to define the physical manifestation of datum.
    b. Interpretations to define the sense generated by the datum.
    c. Implications to define the meanings that emerge in the context of the whole sign system. Datum can have multiple-layered meanings that are metaphorically disconnected from their physical representation (Halliday, 2006). However, the meanings are not infinite either, as they are encoded and interpreted through C, L, and S conventions (Eco, 1981; Schutz, 1932).
3. Employ Linguistics and Semiotic science and language to interpret and define the relationships and meanings in the research problem.
    a. Linguistics and semiotic science can be employed to interpreting behaviours associated with sign-related discourse (Bloomfield, 1939; Jakobson & Halle, 1956).
    b. Learner problems emerge because of their failure to decode and learn the sign system (Jakobson & Halle, 1956). Linguistics and Semiotic science identifies:
        i. The L and S competencies needed to decode and learn a complex sign system such as M2 (Ernest, 2006, 2010; O'Halloran, 2005, 2010).

ii. The aberrant decoding and learning behaviours that emerge and affect human behaviour in the sign system (Jackobson & Halle, 1956).
iii. Teaching strategies that are more cognisant of the sign system and learner decoding behaviours and problems in it.

4. Develop checklists to interpret and enhance the alignment of the data collection techniques that are employed to interpret and articulate findings (Braun & Clarke, 2006). Such a critique will help manage and define the study's legitimacy as science.

6.9 Summary and conclusions

The research problem emerged because (1) little was known *about the problem of teaching M2 in an adult-CALD BL cross-C, L, and S context*, and (2) less was known *about the logics, science, and language that should be employed to study the problem*. Case-based interpretive synthetic logics, and linguistic and semiotic science and language were employed to interpret and resolve the complex language-learning problem. The findings define the philosophical and structural conventions, and BL decoding and learning behaviours that emerge and create problems when decoding and learning M2 in a cross-C, L, and S context.

The study interpreted the M2 sign system through, firstly, its form and functions and, secondly, the social contexts that shape adult-CALD BL human behaviour it. A poststructuralist social semiotic approach allowed for third-tier meanings and social contexts to be interpreted for resolving the *problem of teaching and researching adult-CALD bilingual learners when decoding and learning M2*. The procedures are recommended for other areas of education that also have complex language-learning problems to resolve.

The thesis advocates *structural and language-based teaching solutions and case-based interpretive research techniques to address the research problem*. It is envisaged the implications will improve policy and practice from the ground up through future curricular and teacher development in this area. This will assist adult-BL CALD learners in successfully decoding and learning the M2 sign system in a cross-C, L, and S context.

Bibliography

ABS. (2006a). *Adult Literacy and Life Skills Survey: summary results*. From ABS: http://www.abs.gov.au

ABS. (2006b). *Australian Bureau of Statistics: 2006 Population Census Data*. From ABS: http://www.abs.gov.au

ABS. (2011). *Australian Bureau of Statistics: 2011 Population Census Data*. From ABS: http://www.abs.gov.au

ABS. (1998). *Australian Standard Research Classification (ASRC), 1998*. From Australian Bureau of Statistics: http://http://www.abs.gov.au/ausstats/abs

ABS. (2013). *Programme for the International Assessment of Adult Competencies, Australia, 2011-12*. From ABS: http://www.abs.gov.au

AIR. (2006). *A review of the literature in adult numeracy: research and conceptual Issues*. From The American Institute for Research: http:///www.air.org

AMES. (2013). *Certificate in Written and Spoken English*. Adult Migrant Education Services NSW.

Armour-Thomas, E. (2002). *Assessment of psychometric intelligence for racial and ethnic minorities: some unanswered questions*. From Sage online reference: http:///www.knowledge.sagepub.com

Arzarello, O., Robutti, O., & Bazzini, L. (2005). Acting is learning: focus on the construction of mathematical concepts. *Cambridge Journal of Education , 35* (1), 55-67.

Attride-Stirling, J. (2001). Thematic networks: an analytic tool for qualitative research. *Qualitative Research , 1* (3), 385-405.

Bailystock, E. (2001). *Bilingualism in development: language, literacy and cognition*. Cambridge: Cambridge University Press.

Baker, C. (2011). *Foundations of bilingual education and bilingualism*. Clevedon, UK: Multilingual Matters.

Balatti, J., Black, S., & Falk, I. (2006). *Reframing adult literacy and numeracy course outcomes: a social capital perspective support document*. From National Centre for Vocational Educational Research: http://www.ncver.edu.au

Barnett, S., & Cici, S. (2005). The role of transferable knowledge in intelligence. In R. Sternberg, & J. Pretz (Eds.), *Cognition and intelligence: identifying the mechanisms of the mind*. Cambridge: Cambridge University Press.

Barthes, S. (1974). S/Z. (R. Miller, Trans.) Malden, MA: Blackwell. From http://vogmae.net.au/intmedia/pubs/BarthesExtract.pdf

Barton, B. (2009). *The language of mathematics: telling mathematical tales*. New York: Springer.

Bernardo, A., & Calleja, M. (2005). The effects of stating problems in bilingual students' first and second languages on solving mathematical word problems . *The Journal of Genetic Psychology , 166* (1), 117-128.

Black, S. (2002). *Whose economic wellbeing? A challenge to dominant discourses on the relationship between literacy and numeracy skills and (un) employment*. From National Centre for Vocational Educational Research: http://www.ncver.edu.au

Bloomfield, L. (1939). *Linguistic aspects of science* (Vol. 1). Chicago: University of Chicago Press.

Blumer, H. (1969). *Symbolic interactionism: perspective and method*. Englewood Cliffs, NJ: Prentice Hall.

Bortolussi, M., & Dixon, P. (2003). *Psychonarrantology: foundations for the empirical study of literary response*. New York: Cambridge University Press.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology , 3*, 77-101.

Brown, T. (1997). *Mathematics education and language: interpreting hermeneutics and post-structuralist*. Hingham, MA: Kluwer Academic Publishers.

Carlson, L., Reiger, T., Lopez, W., & Corrigan, B. (2006). Attention unites form and function in spatial language. *Spatial Cognition and Computation , 6* (4), 2965-308.

Chandler, D. (1994). *Semiotics for beginners*. From Semiotics for beginners: http://www.aber.ac.uk/media/Documents/ S4B/

Child, D. (2004). *Psychology and the teacher*. New York: Continuum International.

Chomsky, N. (1972). *Language and mind*. New York: Harcourt Brace Jovanovich.

Cleve, J. (2003). Immanuel Kant (1724-1804). In J. Dancy, & E. Sosa, *Blackwell companion to philosophy: a companion to epistemology*. Malden, MA, USA: Blackwell Publishing.

COAG. (2008). *National Numeracy Review Report*. From Council of Australian Governments Human Capital Working Group: http://www.coag.gov.au

Cohen, L. (2007). *Research methods in education*. Florence, KNT: Routledge.

Condelli, L., Wrigley, H., Yoon, W., Cronen, S., & Seburn, M. (2003). *What works study for adult ESL literacy students*. From American Institute of Research: http://www.air.org/

Creswell, J. (2003). *Research design: qualitative, quantitative, and mixed methods approaches*. Thousand Oaks, California, USA: Sage Publishing.

Crystal, D. (1992). *The Cambridge Encyclopaedia of Language*. Cambridge: Cambridge University Press.

Crystal, D. (1996). *The Cambridge Encyclopaedia of the English Language*. Cambridge: Cambridge University Press.

Cummins, J. (2003). BICS and CALP: origins and rationale for the distinction. In *Sociolinguistics: essential readings*. Oxford: Cambridge University Press.

Cummins, J. (2000). *Language, power and pedagogy: bilingual children in the crossfire*. Cleverdon, GBR: Multilingual Matters Limited.

Danzinger, E. (2010). Dexis, gesture, and cognition in spatial frame of reference typology. *Studies in Language , 34* (1), 167-185.

de Cruz, H. (2006). Towards a Darwinian approach to mathematics. *Foundations of Science , 11*, 157-196.

de Freitas, E., & Zolkover, B. (2011). Developing teacher capacity to explore non-routine problems through a focus on the social semiotics of mathematics classroom discourse. *Research in Mathematics Education , 13* (3), 229-247.

de Saussure, F. (1910). *Third course of lectures on general linguistics*. From Marxists.org: http://www.marxists.org/reference/subject/philosophy/index.htm

DEECD. (2012). *Certificate in General Education for Adults*. Melbourne, VIC: Department of Education and Early Childhood Development.

DEEWR. (2014). *Language, literacy and numeracy program 2010-2013 overview paper*. From Australian Commonwealth Government: http://www.llnp.deewr.gov.au

Denzin, N., & Lincoln, Y. (1994). Introduction: entering the field of qualitative research. In N. Denzin, & Y. Lincoln (Eds.), *Handbook of Qualitative Research*. London, GBR.

Derrida, J. (1967). *Writing and difference*. (A. Bass, Trans.) London: Routledge.

Devlin, K. (2000). *The language of mathematics: making the invisible visible*. New York: W.H. Freeman and Company.

Dockens, W. (2008). The purse and the butterfly: a mathematical psychology approach to the genome in the new reality. *Journal of Organisational Transformation and Social Change , 5* (1), 67-102.

Dockery. (2009). *Cultural dimensions of Indigenous participation in education and training*. From National Centre for Vocational Educational Research: http://www.ncver.edu.au

Doyle, S. C. (1928). *The adventures of Sherlock Holmes: A scandal in Bohemia*. London: John Murray.

Duffin, J., & Simpson, A. (2000). Understanding their thinking: the tension between the cognitive and the affective. In D. Coben (Ed.), *Perspectives on adults learning mathematics: research and practice*. Hingham, MA: Kluwer Academic Publishers.

Durst-Andersen, P. (2011). *Linguistic supertypes: a cognitive-semiotic theory of human communication*. (P. Cobbey, & K. Kull, Eds.) NY: DeGruyterMouton.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics , 61*, 102-131.

Duyck, W., & Brysbaert, M. (2008). Semantic access in number word translation: the role of crosslingual lexical similarity. *Experimental Psychology , 55* (2), 102-112.

Eco, U. (1981). The theory of signs and the role of the reader. *The Bulletin of Midwest Modern Language Association , 14* (1), 35-45.

Ellis, R. (2001). *The study of second language acquisition*. Oxford, GBR: Oxford University Press.

Ernest, P. (2006). A semiotic perspective of mathematical activity: the case of number. *Educational Studies in Mathematics , 61*, pp. 61-67.

Ernest, P. (2010). Mathematics and metaphor: a response to Elizabeth Mowat & Brent Davis . *Complicity: An International Journal of Complexity and Education , 7* (1), 98-104.

Ernest, P. (2012). What is 'first philosophy' in mathematics education? *For the Learning of Mathematics , 32* (3), 8-14.

Falk, I., & Millar, P. (2001). *Review of research: literacy and numeracy in vocational education and training* . From NCVER: http://www.ncver.edu.au

Farrugia, M. (2003). From one language to another: a semiotic interpretation of the translation of mathematical word. *Research in mathematics education , 5* (1), 31-44.

Feist, M. (2008). Space between languages. *Cognitive Science , 32*, 1177-1199.

Feist, M., & Gentner, D. (2007). Spatial language influences memory for spatial scenes. *Memory and Cognition , 35* (2), 283-296.

Fishman, J. (1980). The Whorfian hypothesis: varieties of valuation, confirmation and disconfirmation. *International Journal of the Sociology of Language , 26*, 25-40.

Fitzsimons, G. (2002). *What counts as mathematics*. Hingham, MA: Kluwer Academic Publishers.

Fitzsimons, G., & Godden, G. (2000). Review of research on adult learning mathematics. In *Perspectives on adults learning mathematics: research and practice*. Hingham, MA: Kluwer Academic Publishing.

Freud, S. (1911). *The interpretation of dreams* . From Project Gutenberg: http://www.Gutenberg.us

Fromkin, V., Blair, D., & Collins, P. (1999). *An introduction to language*. Southbank, VIC: Thomson.

Gardner, R., Masgoret, A., Tennant, T., & Mihic, L. (2004). Integrative motivation: changes during a year-long intermediate-level language course. *Language Learning , 54* (1), 1-34.

Geertz, C. (1973). *The interpretation of cultures*. New York: Basic Books.

Gerring, J. (2007). *Case study research: principles and practices*. Cambridge: Cambridge University Press.

Gharajedaghi, J., & Ackoff, R. (1985). Towards systemic education of systems scientist. *Systems Research , 2* (1), 21-27.

Gigerenzer, G., & Goldstein, D. (1999). Betting on one good reason: take the best and its relatives. In G. Gigerenzer, & P. Todd (Eds.), *Simple heuristics that makes us smart*. New York: Oxford University Press.

Gillard, E., Van Dooren, W., Schaeken, W., & Verschaffel, L. (2009). Dual processes in the psychology of mathematics education and cognitive psychology. *Human Development , 52*, pp. 95-108.

Griffin, T. (2014). *Access and participation in tertiary education*. From Research, National Centre for Vocational Education: http://www.ncver.edu.edu

Guba, E., & Lincoln, Y. (1994). Competing pradigms in qualitataive research. In N. Denzin, & Y. Lincoln (Eds.), *Handbook of qualitative research*. London.

Guest, G., MacQueen, K., & Namey, E. (2012). *Applied thematic analysis*. Thousand Oaks, CAL: Sage Publications.

Halliday, M. (2006). *The language of science* (Vol. 5). (J. Webster, Ed.) London: Continuum.

Hambrick, D., Kane, M., & Engle, R. (2005). . The role of working memory in high-level cognition: domain-specific versus domain-general perspectives. In R. Sternberg, & J. Pretz (Eds.), *Cognition and intelligence: identifying the mechanisms of the mind*. Cambridge: Cambridge University Press.

Hamers, J. F., & Blanc, M. H. (2000). *Bilinguality and bilingualism*. Cambridge: Cambridge University Press.

Heisenberg, W., Crull, E., & Bacciagaluppi, G. (2011). *Translation of: W. Heisenberg, 'Ist eine deterministische Erg ̈anzung der Quantenmechanik m ̈oglich?'*. From PhilSci-Archive: http://philsci-archive.pitt.edu/8590/

Hersh, R. (1994). Fresh breezes in the philosophy of mathematics. In P. Ernest (Ed.), *Mathematics, education and philosophy: an international perspective*. London: Routledge Falmer.

Heywood, D., & Stronarch, I. (2005). Philosophy and hermeneutics. In B. Somekh, & C. Lewin (Eds.), *Research methods in social science*. London: Sage Publications.

Hickman, M., & Hendriks, H. (2010). Typological constraints on the acquisition of spatial language in French and English. *Cognitive Linguistics , 21* (2), 189-215.

Higginbotham, P. (1990). Aims of education. In D. Lloyd (Ed.), *Philosophy and the teacher*. London: Routledge.

Holland, N. (1990). *Guide to psychoanalytical psychology and literature-and-psychology*. Oxford: Oxford University Press.

Holmes, J. (2001). *An introduction to sociolinguistics*. London: Longman.

Hornberger, N. (2003). Continua of biliteracy: an ecological framework for educational policy, research, and practice in multilingual settings. (C. Baker, & N. Hornberger, Eds.) *Bilingual Education and Bilingualism , 41*, 61-63.

Hornberger, N., & Skilton-Sylvester, E. (2003). Revisting the continua of biliteracy: international and critical perspective. In N. Hornberger (Ed.), *Continua of biliteracy: an ecological framework for educational policy, research, and practice in multilingual settings*. Cleverdon, GBR: Multilingual Matters.

Howe, K., & Moses, M. (1999). Ethics in educational research. *A Review of Research and Education , 24* (2), 21-59.

Hunt, E. (2005). Information processing and intelligence: where we are and where we are going. In R. Sternberg, & J. Pretz (Eds.), *Cognition and intelligence: identifying the mechanisms of the mind*.

Ivanic, R., & Tseng, M. (2005). *Understanding the relationship between learning and teaching: an analysis of the contribution of applied linguist*. From National Research and Development Centre for Adult Literacy and Numeracy.

Jakobson, R., & Halle, M. (1956). *Fundamentals of language*. The Hague, Netherlands: Mouton & Co.

Johnson, R., & Onwuegbuzie, A. (2004). Mixed methods research: a research paradigm whose time has come. *Educational Researcher , 33* (7), 14-26.

Kaartinen, S., & Latomaa, T. (2011). The role of sign vehicles in mediating teachers mathematical problem solving. In S. Hamel (Ed.), *Semiotics: theory and applications* (pp. 234-243). NY: Nova Science Publishers.

Kant, I. (1788). The critique of practical reason. (P. McPherson-Rudsill, Trans.) From http://kantwesly.com/Kan/t

Kess, J. (1992). *Psycholinguistics: Psychology, linguistics, and the study of natural language*. Philadelphia: John Benjamins Publishing Company.

Khait, A. (2005). The definition of mathematics: philosophical and pedagogical aspects. *Science and Education , 14* (2), 137-159.

Krummheuer, G., Leuzinger-Bohleber, M., Muller-Kirchof, M., Munz, M., & Vogel, G. (2013). Explaining the mathematical creativity of a young boy: an interdisciplinary venture between mathematics education and psychoanalysis. *EducationaL Studies in Mathematics , 84*, 183-199.

Leron, U., & Hazzan, O. (2006). The rationality debate: application of cognitive psychology to mathematics education. *Educational Studies in Mathematics , 62*, 105-126.

Leslie, A. (1993, Nov). *A theory of agency*. From Centre for Cognitive Science Rutgers University.

Levinson, M. (2004). Navigating without fixed points: the perils of open-ended research. In P. Coombes, M. Danaher, & P. Danaher, *Strategic uncertainties: ethics, politics and risks in contemporary education research*. Flaxton, AU: Post Pressed.

Lloyd, D. (1990). Introduction. In *Philosophy and the teacher*. London: Routledge.

Lo Bianco, J., & Freebody, P. (2001). *Australian literacies: Informing national policy on literacy education* (2nd ed.). Melbourne: Language Australia Ltd.

Lo Bianco, J., & Slaughter, Y. (2009). *Second languages and Australian schooling*. (C. Glascodine, Ed.) Camberwell, VIC: ACER Press.

Looney, J. (2008). *Teaching, Learning and Assessment for Adults: Improving Foundation Skills*. From OECD Publications: Centre for Educational Research and Innovation.

Loue, S. (2002). *Textbook of research ethics: theory and practice*. Hingham, USA: Kluwer Academic Publications.

MacInnes, D. (2011). A framework for conceptual contributions in marketing. *Journal of Marketing , 75*, 136-154.

Martin, J. (2011). Multimodal semiotics: theoretical challenges. In S. Dreyfus, S. Hood, & M. Stenglin (Eds.), *Semiotic margins: meanings in multimodalities* (pp. 243-265). London: Continuum.

McDougall, J. (2004). Research is not a dirty word: the risks involved in trying to overcome teacher/theorist divide. In P. Coombes, M. Danaher, & P. Danaher (Eds.), *Strategic uncertainties: ethics, politics and risk in contemporary educational research*. Flaxton, QLD: Post Pressed.

McLuhan, M. (2001). *Understanding media: the extensions of man*. Florence, KY: Routledge.

Miller, L., Whalley, J., & Stronach, I. (2005). From structuralism to poststructuralism. In B. Somekh, & C. Lewin (Eds.), *Research methods in the social sciences*. London: Sage.

Milroy, J., & Milroy, L. (1998). *Authority in language: investigating standard English*. London: Routledge.

Miralles, J. (2004). *A fair go: factors impacting on vocational education and training participation and completion in selected ethnic communities* . From National Centre for Vocational Educational Research: http://www.ncver.edu.au

Morris, A. (2002). Mathematical reasoning: adults' ability to make inductive-deductive distinction. *Cognition and Instruction , 20* (1), 79-118.

Munnich, E., & Landau, B. (2010). Developmental decline in the acquisition of spatial language. *Language Learning and Development , 6*, 32-59.

NCVER. (2006). *Breakfast briefings: reading between the lines; summing up adult literacy and numeracy research*. From National Centre for Vocational Educational Research: http://www.ncver.edu.au

NCVER. (2011). *Building the foundations: outcomes from the adult language, literacy and numeracy search conference*. From National Centre for Vocational Educational Research: http://www.ncver.edu.au

Neaubauer, A., & Fink, A. (2005). Basic information processing and the psychophysiology of intelligence. In *Cognition and intelligence: identifying the mechanisms of mind*. Cambridge: Cambridge University Press.

O'Halloran, K. (2005). *Mathematical discourses: language, symbolism and visual images*. London: Continuum.

O'Halloran, K. (2010). The semantic hyperspace: accumulatng knowledge across semiotic resources and modalities. In F. Christie, & K. Maton (Eds.), *Disciplinarity: functional linguistics and sociological perspectives*. London: Continuum.

O'Neill, S. (2009). *Relationship concepts: language, thinking and pedagogy and the spatial metaphor of time*. Brisbane: Post Pressed.

Onwuegbuzie, A., & Daniel, L. (2003). Typology of analytical and interpretational errors in quantitative and qualitative educational research. *Current Issues in Education , 6* (2), 2003.

Onwuegbuzie, A., & Johnson, B. (2006). The validity issue in mixed research. *Research in the Schools , 13* (1), 48-63.

Onwuegbuzie, A., Leech, N., & Collins, K. (2012). Qualitative analysis techniques for the review of the literature. *The Qualitative Report , 17* (56), 1-28.

Papen, U. (2005). *Adult literacy as social practices: more than skills*. New York: Routledge.

Parvanehnezhad, Z., & Clarkson, P. (2008). Iranian bilingual students reported use of language switching when doing matematics. *Mathematics Education Research Journal , 20* (1), 52-81.

Patton, M. (1990). *Qualitative evaluation and research methods*. London: Sage.

Peirce, C. S. (1931). *Collected papers of Charles Sanders Peirce*. (C. Hartshorne, & P. Weiss, Eds.) From Philpapers: http;//www.philpapers.org/rec/HARTCP

Perkins, K. (2009). *Adult literacy and numeracy: research and future strategy*. From National Centre for Vocational Educational Research: http://www.ncver.edu.au

Piaget, J. (1972). Intellectual evolution from adolescence to adulthood. *Human Development , 15*, 1-12.

Pietarinen, A. (2006). The evolution of semantics and language-games for meanings. *International Studies , 7* (1), 79-104.

Piper, H., & Simons, H. (2005). Ethical responsibility in social research. In B. Somekh, & C. Lewin (Eds.), *Research methods in the social sciences*. London: Sage Publications.

Presmeg, N. (2014). Contemplating visualization as an epistemological learning tool in mathematics. *ZDM Mathematics Education , 46*, 151-157.

Presmeg, N. (2006). Semiotics and the "connections" standards: Significance of semiotics for mathematics teachers. *Educational Studies in Mathematics , 61*, 163-182.

Preston, N. (1996). *Understanding ethics*. Sydney: Federation Press.

Prins, D., & Ulijin, J. (1998). Linguistic and cultural factors in the readability of mathematics texts: the Ehorfian hypothesis revisited with evidence from South African context. *Journal of Research in Reading , 21* (2), 139-159.

Provinca Autonomata di Trento. (1999). *Il filo di Arianna: a cura del centro studi Maria Silvia Rapizza*. Trento, ITA: Castel Ivano.

Raab, M., & Gigerenzer, G. (2005). Intelligence as heuristics. In R. Sternberg, & J. Pretz (Eds.), *Cognition and intelligence: Identifying the mechanisms of the mind*. Cambridge: Cambridge University Press.

Ramachandran, V. (2011). *The tell-tale brain: unlocking the mystery of human nature*. London: William Heinemann.

Reber, A. (1984). *The Penguin dictionary of psychology*. London: Allen Lane.

Risager, K. (2006). *Language and culture: global flows and local complexity*. Clevedon, GBR: Multilingual Matters.

Robinson-Pant, A. (2005). *Cross cultural perspectives in educational research*. Berkshire, GBR: McGraw-Hill Education.

Rothman, S., & McMillan, J. (2003). *Longitudinal Surveys of Australian Youth: Research report 36. Influences on achievement in literacy and numeracy*. From ACER: http://www.acer.edu.au

Rowland, T. (1999). *Pragmatics of mathematics education: vagueness in mathematical discourses*. Florence, KY: Routledge.

Russell, B. (1912). *The problem of philosophy*. New York, NY: Henry Holt and Company .

Saldana, J. (2009). *The coding manual for qualitative researchers*. Thousand Oaks, California: Sage.

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly , 23*, 139-159.

Schutz, A. (1932). The phenomenology of the social world. In C. Calhourn, J. Gerteis, J. Moody, S. Pfaff, I. Virk, G. Walsh, & F. Lehnert (Eds.), *Contemporary sociological theory*. Evanston, IL: Northwestern University Press.

Schwandt, T. (1994). Constructivist, interpretivist, approaches to human inquiry. In N. Denzin, & Y. Lincoln (Eds.), *Handbook of qualitative research*. London: Sage Publications.

Sfard, A. (2005). What changes when learning goes to school: the communicational version, the case of mathematics. *European Journal of School Psychology , 3* (1), 393-326.

Shaw, R. (1912). *Pygmalion* . From Project Guttenberg: https://www.gutenberg.org/

Shomos, A., & Forbes, M. (2014). Literacy and numeracy skills and labour market outcomes in Australia. Canberra, ACT, AUS.

Shreyar, S., Zolkower, B., & Perez, S. (2010). Thinking aloud together: A teacher's semiotic mediation of a whole-class conversation about percents. *Educational Studies in Mathematics , 73* (21), pp. 21-53.

Skovsmose, O. (2001). Mathematics in action: a challenge for social theorising. In E. Simmit, & B. Davis (Ed.), *Canadian Mathematical Study Group Proceedings. 25*. Alberta: CMESG.

Slobin, D. (1997). The universal, the typological, and the particular in acquisition. In D. Slobin (Ed.), *The crosslinguistic study of language acquisition*. Mahwah, NJ: Lawrence Erlbaum Associates.

Solano-Flores, G. (2014). Probabilistic approaches to examining linguistic features of test items and their effect on the performance of English language learners. *Applied Measurement in Mathematics , 27*, 236-247.

Solano-Flores, G., Barnett-Clarke, C., & Kachchaf, R. (2013). Semiotic structure and meaning making: the performance of English language learners on mathematics tests. *Educational Assessment , 18*, 147-161.

Somekh, B., & Lewin, C. (Eds.). (2005). *Research Methods in the social sciences*. London: Sage.

Sommerville, M. (2006). *The Ethical Imagination: journeys of the Human Spiri*. Toronto: Anasi Press.

Stake, R. (1994). Case studies. In N. Denzin, & Y. Lincoln (Eds.), *Handbook of qualitative research*. Thousand Oaks, CLF: Sage Publications.

Stapar, S. (2014). Language, Literacy and numeracy - the foundations of Australia's future. Melbourne, VIC, AUS.

Steinbring, H. (2006). What makes a sign a mathematical sign? - An epistemological perspective on mathematical interaction. *Educational Studies in Mathematics , 61*, 133-162.

Sternberg, R., & Pretz, J. (2005). Cognition and intelligence: identifying the mechanisms of the mind. In Preface, R. Sternberg, & J. Pretz (Eds.), *Cognition and intelligence: identifying the mechanisms of the mind*. Cambridge: Cambridge University Press.

Stiles, W. (2009). Logical operations in theory-building case studies. *Pragmatic Case Studies in Psychotherapy , 5* (3), 9-22.

Tabouret-Keller, A. (2000). Language and identity. In F. Coulmas (Ed.), *The handbook of sociolinguistics*. Oxford, UK: Blackwell Publishing.

The Shorter Oxford English Dictionary. (1990). *The Shorter Oxford English Dictionary on Historical Principles*. Oxford: Claredon Press.

Thibault, P. (2011). Languaging behaviour as catalytic process: steps towards a theory of living language (Part I & II). *The Public Journal of Semiotics , III* (2), 2-151.

Thom, J., & Roth, W. (2011). Radical embodiment and semiotics: toward a theory of mathematics in the flesh. *Educational Studies in Mathematics , 77*, 267-284.

Tochon, F. (2014). *Help them learn a language deeply*. Blue Mounds, WSC: Deep University Press.

Todd, P. (1999). Inferences heuristics versus complex decision machines. *Minds and Machines , 9*, 461-477.

Tout, D., & Motteram, G. (2006). *Functional numeracy in context*. Camberwell, VIC: ACER Press.

Trinick, T., Meaney, T., & Fairhall, U. (2014). Teachers learning the registers of mathematics and mathematics education in another language: an exploratory study. *ZDM Mathematics Education , 46*, 953-965.

Tylen, K. (2007, Spring). When agents become expressive: a theory of semiotic agency. (L. Andreassen, L. Brandt, & J. Vang, Eds.) *Cognitive Semiotics* , 84-101.

van de Vijver , F., & Matsumotoa , D. (2011). Introduction to the methodological issues associated with cross-cultural research. In F. van de Vijver , & D. Matsumotoa (Eds.), *Cross-cultural research methods in psychology*. Cambridge: Cambridge University Press.

van de Vijver, F., & Lueng, K. (2011). Equivalences and bias: a review of concepts, models, and data analytical procedures. In D. Matsumotoa, & F. van de Vijver, (Eds.), *Cross-cultural research methods in psychology*. Cambridge: Cambridge University Press.

Vile, A. (2009). *What can semiotics offer mathematics education?* From BSRLM: http://www.bsrlm.org.uk

Vygotsky, L. (2011). The dynamics of the schoolchild's mental development in relation to teaching and learning . *Journal of Cognitive Education and Psychology , 10*, 198-211.

Walsham, G. (1995). Interpretive case studies in IS reserach: nature and method. *European Journal of Information System , 4*, 74-81.

Wardaugh, R. (2002). *An introduction to sociolinguistics*. Oxford, UK: Blackwell Publishing.

Whorf, B. (1956). *Language, thought, and reality: selected writings of Benjamin Lee Whorf*. (Y. Carroll, Ed.) New York: The Technology Press of MIT and John Wiley & Sons, Inc.

Yin, R. (1994). *Case study research: design and methods*. London: Sage Publications.

Appendix A: Ethical Clearance

University of Southern Queensland

TOOWOOMBA QUEENSLAND 4350 CRICOS: QLD 00244B NSW 02225M

AUSTRALIA

TELEPHONE +61 7 4631 2300

www.usq.edu.au

OFFICE OF RESEARCH AND HIGHER DEGREES
Ethics Committee Support Officer
PHONE (07) 4631 2690 | FAX (07) 4631 1995
EMAIL ethics@usq.edu.au

Wednesday, 27 June 2012

Anthony Rigoni
Email: [redacted]

CC: Shirley O'Neill (supervisor)

Dear Anthony

The Chair of the USQ Fast Track Human Research Ethics Committee (FTHREC) recently reviewed your responses to the FTHREC's conditions placed upon the ethical approval for the below project. Your proposal now meets the requirements of the *National Statement on Ethical Conduct in Human Research (2007)* and full ethics approval has been granted.

Project Title	An analysis of linguistic and semiotic factors that create problems for adult-CALD learners when learning mathematics in English as a second language.
Approval no.	H12REA126
Expiry date	30.12.2013
FTHREC Decision	**Approved**

The standard conditions of this approval are:

(a) conduct the project strictly in accordance with the proposal submitted and granted ethics approval, including any amendments made to the proposal required by the HREC
(b) advise (email: ethics@usq.edu.au) immediately of any complaints or other issues in relation to the project which may warrant review of the ethical approval of the project
(c) make submission for approval of amendments to the approved project before implementing such changes
(d) provide a 'progress report' for every year of approval
(e) provide a 'final report' when the project is complete
(f) advise in writing if the project has been discontinued.

For (c) to (e) forms are available on the USQ ethics website: http://www.usq.edu.au/research/ethicsbio/human

Please note that failure to comply with the conditions of approval and the *National Statement (2007)* may result in withdrawal of approval for the project.

You may now commence your project. I wish you all the best for the conduct of the project.

Ethics Committee Support Officer
Office of Research and Higher Degrees

Appendix B: Participant Information and Consent Form

USQ
USTRALIA

niversity of Southern Queensland

The University of Southern Queensland

Proposed Research Participant Information Sheet

HREC Approval Number: H12REA126

Full Project Title: ***A case-study analysis of linguistic and semiotic factors that create problems for adult-CALD learners when learning mathematics in English as a second language*****.**

I would like to invite you to take part in this research as a participant in my **Doctor of Education** research**.**

Principal Researcher: Anthony Rigoni

1. <u>Procedures</u>

This research seeks to conduct a case-study analysis on adults who have had the experience of learning mathematics and numeracy in English as a second language. The objective of the project is to identifying and analyse the factors in language that create barriers for many adult-CALD (Culturally and Linguistically Diverse) learners when learning mathematics and numeracy in a second language. This research is seen as important because it wishes to help improve teaching and learning practices in the critical area of adult literacy and numeracy education in VET.

The actual research will ask individual participants to participate in an in-depth interview and case study that seeks to identify how learners engage in different types of mathematical language. The objective of this research is not to test participants to see how 'good' they are in doing mathematics or English but, instead, to survey, identify, and analyse how individual participants actually engage in different aspects of mathematics when it has been acquired in English in a second language context.

The interviews are to be conducted in private and with total anonymity. Interviews will involve participants making themselves available for **3** sessions (3-5 hours each) over a period of 8 weeks. The sessions will be broken into three stages of participation: an initial survey, participating in a reading/ writing task, and an instruction session that will focus on teaching the participant how they can improve their ability to do mathematics and numeracy.

Participants will also be asked to provide some background data, such as their age, gender, county-of-origin, and education. However, personally identifiable data will not be used or published.

A total of 5-10 participants are to be interviewed within the case studies. Other participants may include people you know in your school or participants from other schools you do not know. Your personal identity and data will not be shared with other participants. Furthermore, your **personal identity, name, location, and your school name and location will be withheld from publication. A generic reference and name of 'Participant A' and 'A large regional TAFE provider' will be used instead of identifiable names.**

Audio recordings are also planned for the interviews to help the researcher in note taking and transcribing recordings into texts. Extracts of the recordings may also be sited in the research. However: a participant's personal identity will not be identifiable; participants may opt not to have recordings made; audio type files that identify a person's voice will be kept in secure storage until files have been converted into texts and all personally identifiable information is removed. Audio type files will be deleted at the conclusion of the study.

Data collection will also require the participant to respond to some mathematical tasks in writing. This data may also be sited in the research. However, personally identifiable information will not identifiable in these transcripts.

2. <u>Voluntary Participation</u>

Participation is entirely voluntary. **If you do not wish to take part you are not obliged to do so**. If you decide to take part and later change your mind, you are free to withdraw from the project at any stage. Any information already obtained from you will be destroyed. You will also be given the opportunity to provide feedback at the end of each survey session so that you may add, delete, amend any response you give. **However, once the surveys have been finalised, your participation has been coded and you are not personally identifiable within the data analysis and the research, and a milestone date is reached where the research may be published you will not be able to withdraw your data.**

If you wish, you may provide the researcher with your contact details and you will be informed when the research milestone for publication date is reached. This feedback will also identify where you may first site the research if it is published.

Your decision whether to take part or not to take part, or to take part and then withdraw, will not affect your relationship with the University of Southern Queensland or the xxxxxxx Institute of TAFE.

Please notify the researcher ASAP if you decide to withdraw from this project.

Should you have any queries regarding the progress or conduct of this research, you can contact the principal researcher:

Anthony Rigoni
University of Southern Queensland
Candidate for Doctor Of Education
Contact Details:
xxxxxxxxxx
If you have any ethical concerns with how the research is being conducted or any queries about your rights as a participant please feel free to contact the University of Southern Queensland Ethics Officer on the following details.

Ethics and Research Integrity Officer
Office of Research and Higher Degrees
University of Southern Queensland
West Street, Toowoomba 4350
Ph: +61 7 4631 2690
Email: ***<u>ethics@usq.edu.au</u>***

The University of Southern Queensland

Consent Form

HREC Approval Number: **H12REA126**

TO: ***[state who this Consent Form is going to, eg. Participants, Organisations, Parents/Guardians etc.]***

Full Project Title: *An analysis of linguistic factors that create problems for adult-CALD learners when learning mathematics in English as a second language.*

Researcher: Anthony Rigoni

- I have read the Participant Information Sheet and the nature and purpose of the research project has been explained to me. I understand and agree to take part.

- I understand the purpose of the research project and my involvement in it.

- I understand that I may withdraw from the research project at any stage and that this will not affect my status now or in the future.

- I confirm that I am over 18 years of age.

- I understand that while information gained during the study may be published, I will not be identified and my personal results will remain confidential. *If other arrangements have been agreed in relation to identification of research participants this point will require amendment to accurately reflect those arrangements.*

- I understand that I will be audio taped during the study and this file may be converted to text. The use of this file is however made conditionally as per information supplied in the Proposed Research Participant Information Sheet.

Participants under the age of 18 normally require parental or guardian consent to be involved in research. The consent form should allow for those under the age of 18 to agree to their involvement and for a parent to give consent. Copy and paste another signature field if necessary.

Name of participant…….....

Signed……**Date**……

If you have any ethical concerns with how the research is being conducted or any queries about your rights as a participant please feel free to contact the University of Southern Queensland Ethics Officer on the following details.

Ethics and Research Integrity Officer
Office of Research and Higher Degrees
University of Southern Queensland
West Street, Toowoomba 4350
Ph: +61 7 4631 2690 Email: ***ethics@usq.edu.au***

Appendix C: Annotated Summary of Theoretical Ideas into First Draft Data Collection Questions (sample).

(Pages 1, 2, and 9 of 24)

Key:

- Research focus <u>underlined</u>.
- **Theory/concept in bold.**
- Questions generated for Stage 1 (semiotic synthesis task) in plain type.
  Questions generated for Stage 2 (Open-ended Interviews) and Stage 3 (In tasks interviews and observations) in italics.
- *104 concepts investigated*

<u>Mathematics</u>

1. **There are different types of mathematics. For example, numeracy is different from mathematics (COAG, 2008).** Why choose this text? Why is this text mathematics? Does the sign system signify a differential linguistic register (Halliday, 2006)? Is the sign system different from everyday language?
2. **Mathematics is heterogeneous, consisting of many types, threads and parts (Barton, 2009).** Does the sign system signify heterogeneous ideas and meanings? *Can you give examples of different types of mathematics?*
3. **Mathematics is a socially derived and has different social applications (Brown, 1997; Halliday, 2006; Hersch, 1994; Ernest, 2006; COAG, 2008).** Who would value this text (Chandler, 1993)? *Do you talk much using numbers? How often do you use mathematical symbols and figures during the day?*
4. **Mathematical language and everyday language are different because mathematical grammar is highly 'packed' and 'dense' (Halliday, 2006; O'Halloran, 2005).** How does the signs in this text distinguish it from everyday language? *Is it harder to learn mathematical symbols and numbers than everyday English?*
5. **Mathematics is just 'gossip' (Barton, 2009; Devlin, 2001).** Does this text talk about something that can be said in everyday language? *Do you use numbers to talk and explain things in your first language? Do you use numbers to explain things English?*

(Page 1 of 24)

6. **Mathematics is the science of 'patterns' (Devlin, 2000).** Does the sign system signify distinguishing structural patterns? *What is mathematics? Can you see patterns in the way the symbols are used in this task?*
7. **Mathematics uses a variety of different types of semiotic mediums, including: images, graphs, geometrical diagrams, and symbolic representations (Schleppegrell, 2007).** What are the physical characteristics of the signs used in the task? *What types of symbol and figures do you think are used to do mathematics in this task? Does the task use symbols that are not mathematics?*
8. **Einstein is reported saying (Dockens, 2008) his 'theory of relativity' was derived from his senses and experiences of the world and this shaped how abstract thoughts, such as mathematics, are generated.** Are the signs derivatives of real life experiences? *Do you use fingers or other things to help you use numbers?*
9. **No kind of mathematical processing can be performed without a semiotic system of representation (Duval, 2006).** Are there computational processes that occur without signs? *Do you prefer to use numbers that are written in full words (nine) or as symbols (9)? Can you use numbers without using symbols?*
10. **A person maybe better in doing one type of mathematics (e.g. geometry) than another (e.g. calculus).** Does the sign system signify identifiable genres or registers in mathematics? *Can you give examples of different types of mathematics? Are you good at using numbers in some ways and not others?*
11. **Mathematics requires specialised forms thinking that occur in abstract, and this is different from using everyday language (Halliday, 2006).** Does the sign system signify links to real life? Does the sign system signify processes that require abstract thinking that are different from other forms of written texts? *Can you identify what the symbols and figures are asking you to do next?*

Culture

12. **Some languages and cultures have different ways of conceptualising mathematics. For example, spatial and temporal reasoning varies greatly between languages (Whorf, 1956; Barton, 2009).** Does the sign system signify cultural assumptions? *Do you think the way we use numbers and mathematics in Australia is different to the way you use them in your home country? Does the task look different to the way you may have seen it written in your first language?*

51. **Peirce (2010) believed mathematical signs are like other socially constructed sign systems (e.g., everyday language) and have three differentials: a representament (the form the sign takes); an interpretant (the sense made of the sign); and an object (what the sign refers to or means at a semantic level).** Do signs signify both denotational and connotational meanings (Barthes, 1974) within the system? Does the signs system signify ideological codes

and idiolects? *Can you tell what the symbol or figure means? Does the text remind you of any anything else in real life?*

52. **The sign may be iconic, indexical, and symbolic (Pierce, 2010; Ernest, 2006). Mathematical signs are most often perceived as 'symbolic', however, they can be iconic (by representing something it signifies, e.g., the Roman numeral III) and indexical (signs that look like what they signify, e.g., maps).** Are the signs symbolic, iconic, or indexical forms? Would symbolic, iconic, or indexical forms of signs signify different codes for reading them? *How real is the symbol or figure? Do some signs make more sense to you than others? Are any of these symbols or figures easier to understand and follow than others?*
53. **Mathematical language has distinctive (dense) attributive phrases, specialised conjunctions, and processes of identity and equality that are different from everyday language (Schleppegrell, 2007). The processes that require the reader to read mathematical texts differently from everyday language, however, are not always made explicit by the sign system.** Does the sign system signify a distinctive lexicon or grammatical patterns? Does the sign system signify tokens and types distinctions as patterns? Does the signs system signify the reading style needed for encoding and decoding the signs? *Which symbols and figures are complex (hard to read and understand) and which ones are not?*
54. **Whorf (1956) hypothesises that a person's culture affects they way they interpret sign meanings.** Who would value this text (Chandler, 1993)? Does the sign system signify culturally or linguistically assumed meanings? *Do you find any of the symbols and figures in this task a strange way of using numbers?*
55. **Ferdinand de Saussure's (1910) believed signs and their values are arbitrary because there is no physical connection between how the sign is represented and the meanings that are signified.** Do signs have arbitrary meanings, or are there ideologies and pragmatics values signified within the sign system (Pierce, 2010)? *What do you think would happen if you swapped one symbol or figure for another?*

(Page 9/21)

Appendix D: Battery of Open-ended and Micro Question

Task 1: Semiotic Analysis of Tasks				
No.	**Domain**	**Focus RQ**	**Concepts theory**	**Questions**
1	Philosophical	Mathematics	1	Why choose this text (Chandler, 1993)?
2	Philosophical	Mathematics	103	Why is this text mathematics?
3	Philosophical	Mathematics	8	Are the signs derivatives of real life experiences?
4	Philosophical	Language	67	Does the sign system signify modality values that are fictional (Chandler, 1993)?
5	Philosophical	Mathematics	2	Does the sign system signify heterogeneous ideas and meanings?
6	Philosophical	Culture	18	Does the sign system signify different epistemologies?
7	Philosophical	Language	51	Are there ideological codes or idiolects present in the sign system?
8	Philosophical	Language	54	Who would value this text (Chandler, 1993; Whorf, 1956)?
9	Ling Structures	Language	73	Does the sign system have signs that are shared with other mediums, for example, sciences (Halliday, 2006), everyday language, and filmic codes (where genre, camerawork, editing and manipulation of time and sequences occur (Chandler, 1993)?
10	Ling Structures	Education	83	Does the sign system signify independence and exclusivity to everyday discourses which exist outside its sign system?
11	Ling Structures	Language	53	Does the sign system signify a distinctive lexicon or grammatical patterns?
12	Ling Structures	Education	86	Does the sign system signify signs in L1 that can be swapped into L2 to generate the same meanings?
13	Ling Structures	Language	60	Does the sign system signify paradigmatic or syntagmatic structures?
14	Ling Structures	Language	52	Do signs signify denotational and connotational meanings (Barthes, 1974?).
15	Ling Structures	Language	50	Do the signs signify processes that define entities?
16	Ling Structures	Language	55	Do signs have arbitrary meanings, or are there ideologies and pragmatics values signified within the sign system (Pierce, 2010)?
17	Ling Structures	Language	44	Do the signs signify non-abstract meanings – see indexical signs?
18	Ling Structures	Adult	31	Does the sign system signify systemic, random, or irrational patterns– draw a flow chart identifying process flow?

19	Ling Structures	Language	63	Does the sign system signify forms of articulation that are metaphoric, metonymic (Jakobson, 1956), or analogic in nature?
20	Ling Structures	Language	63	Are there 'myths' articulated within the order of sign signification (Barthes, 1974?)?
21	Ling Structures	Language	57	Does the sign system signify field, tenor, and mode to express meanings?
22	Ling Structures	Language	57	Does the sign system signify interpersonal, ideational, and textual meanings that are systemic?
23	Ling Structures	Language	65	Does the sign system signify clusters of signs (e.g., type/token distinctions) that affect how meanings are generated as process?
24	Ling Structures	Language	61	Does the sign system require conflicting, contradicting, or contrasting elements to signify meanings?
25	Ling Structures	Language	62	Do the signs signify opposition or markedness to generate meanings?
26	Ling Structures	Language	71	Does the sign system signify it has ‘agency’ (energy and force; goals and reaction; and cognitive properties (Leslie, 1993))?
27	Ling Structures	Language	70	Do the signs signify physical characteristics?
28	Ling Structures	Language	72	Does the sign system signify vectors in which the sign creates force and direction in meaning making?
29	Ling Structures	Language	72	Does the sign system signify vectors that reversible or are there redundancies in which the signs cannot be ‘unpacked’ (Halliday, 2006)?
30	Ling Structures	Language	72	Do states of equilibrium exist within the sign system where meanings do not require additional information or signs to signify meaning?
31	Ling Structures	Language	72	Does the sign system signify elasticity within signs so they can be swapped ‘paradigmatically’ or ‘syntagmatically’ (Morris, 1946)?
32	Ling Structures	Language	72	Do synergies exist in the sign system in which articulation signify exponentially more complex meaning making processes?
33	Ling Structures	Language	72	Is the sign system fluid in signs being able to fulfil different roles in the text?
34	Ling Structures	Language	72	Does the sign system signify signs that are magnetic or polar?
35	Ling Structures	Language	72	Can derivatives of signs be condensed into one symbol that will represent several thoughts and ideas into one (Freud)?
36	Ling Structures	Education	74	What code does the sign system signify in order to be read?
37	Ling Structures	Language	65	Does the sign system signify that codes used to encode and decode the signs are different?
38	Ling Structures	Education	104	Does the sign system signify that everyday language is required to read the signs?
39	Ling Structures	Education	105	Does the sign system signify that a specific mathematic register is required to read the signs?
40	Ling Structures	Mathematics	9	Do computational processes occur without signs signifying what the processes are?
41	Ling Structures	Education	81	Does the sign medium shape the contents and how the perceptual habits of the reader are to be modified within the texts (Marshall McLuhan, 1911-1980 cited Chandler 1993)?

42	Cognitive	Mathematics	11	Does the sign system signify links to real life?
43	Cognitive	Language	44	Can the meanings signified by signs be generated by tools and instruments?
44	Cognitive	Adult	36	Does the sign system signify formal and/or ecological queues in which to read the signs?
45	Cognitive	Mathematics	11	Does the sign system signify processes that require abstract thinking that are different to other forms of written texts?
46	Cognitive	Language	46	Do the signs signify metaphors, metonyms, or analogies, or do they have their own identity that can no longer be 'unpacked' (Halliday, 2006) as a metaphor?
47	Cognitive	Adult	34	Does the sign systems signify the forms of logic (inductive, deductive, metaphoric, analogic, heuristic) needed to be able to read the signs -use flow charts?
48	Cognitive	Language	45	Does the sign system signify orientation, structural, temporal, and functional type metaphors?
49	Cognitive	Language	58	Does sign system signify 'chains of reasoning' that leap between the sign structures and semiotic resources (O'Halloran, 2005; 2010) - use flow charts?
50	Cognitive	Language	49	Does the sign system signify it is systemic for expressing new meanings at a metafunctional level, or is meaning making bounded by the sign system?
51	Cognitive	Language	59	Do the signs signify it has a space for readers to think about the processes?
52	Cognitive	Adult	29	Does the sign system signify what the cognitive requirement such as memory processing that is needed to read the signs?
53	Cognitive	Education	78	Does the sign system signify specialised thinking (inductive, deductive, abduction (Jakobson, 1956), heuristic (Raab & Gigerenzer, 2005; Todd, 1999) case law experiences (Barton, 2009)) is needed to read the texts -use flow charts?
54	Cognitive	Education	97	Does the sign system signify readers are able to use the sign system to encode and decode from either experience, infer relationships between two or more concepts within and outside the system; and, correlate and infer any rules to the new situation (Spearman, 1932 cited in Sternberg, R., & Pretz, J, 2005, p. vii)?
55	Cognitive	Language	47	Does the sign system signify assumptions about the reader being able to distinguish metaphoric meanings within sign?
56	Cognitive	Culture	15	Does the sign system signify change may occur in the way the reader thinks?
57	Cognitive	Education	99	Does the sign system signify the reader can think creatively?
58	Cognitive	Education	96	Does the sign system signify an ideal or preferred role of the reader by the narrator?
59	Cognitive	Adult	37	Does the sign system signify abstract concepts that may preclude some readers from reading because of their lack of maturity (Piaget, 2007)?
60	Cognitive	Australian	24	Ceteris paribus, does the sign system signify it favours one type of learner over another?
61	Biological	Education	90	Does the sign system signify meanings being made within body movements or physical gestures?

62	Biological	Adult	40	Does the sign system signify it is a gender?
63	Biological	Education	92	Does the sign system signify non-textual type modalities such as speaking are needed to generate meanings that exclude other languages and cultures from participating?
64	Biological	Adult	40	Does the sign system favour being read by one gender?
65	Biological	Adult	41	Does the sign system signify connotations that may be motivational for some learners and not others?
66	Biological	Education	99	Does the sign system signify that readers will enjoy the task?
67	Biological	Education	100	Does the sign system signify that positive motivation towards learning mathematics is a prerequisite to being able to read the signs?
68	Biological	Education	99	Does the sign system signify potential for readers to be stimulated to solve puzzles or win games?
69	Cultural	Language	66	Does the sign system signify assumed social and cultural conventions as codes (Derrida, 1967)?
70	Cultural	Language	69	Does the sign system signify a culturally assumed knowledge, which also signals how the text should be read?
71	Cultural	Culture	17	Does the sign system signify differential cultural epistemologies?
72	Cultural	Culture	20	Does the sign system signify CALD-related factors are excluded from the text?
73	Cultural	Culture	21	Does the sign system signify cultural and psychological conflicting ideas to some CALD readers?
74	Cultural	Australian	25	Does being able to read (encode and decode) the sign system change the reader culturally?
75	Cultural	Culture	23	Does the sign system signify connotations of national identity and social cohesion?
76	Cultural	Culture	22	Does knowledge of the mathematical sign system signify power?
77	Cultural	Education	76	Does the sign system signify they create social barriers (cognitive, linguistic, economic-cultural) that may restrict some people from being able to read (encode and decode) the signs?
78	Cultural	Education	103	Does the sign system signify readers being able to build reader self-esteem and social cohesion (Papen, 2005)?
79	Educational	Education	79	Does the sign system signify grammatical patterns that have distinctive ‘registers’ (Halliday, 2006) that need be learnt to enable reading?
80	Educational	Education	82	Does the sign system signify associative/communicative/distributive type laws governing mathematical sign processes can only be learnt by ‘case law’?
81	Educational	Education	93	Does the sign system signify discourses are vague, functional, or strategic?
82	Educational	Education	88	Does the sign system signify it detracts from easy reading (encoding and decoding)?
83	Educational	Education	94	Does the sign system signify the signs need to be transposed or reframed for readers to be able to read the signs?
84	Educational	Education	85	Does the sign system signify the bilingual readers needs to be academically proficient in L2 first in order to be able to be read the sign system?
85	Educational	Australian	26	Does the sign system signify equal opportunity for everyone to be able to read mathematics?

86	Educational	Education	80	Does the sign system signify a form of symbolic capital that can be acquired through education?
87	Educational	Education	77	Does the sign system signify pedagogical practices can be taught to enable encoding a decoding of the sign system?
88	Educational	Education	94	Does the sign system signify that more than one participant (teacher/learner) is needed to read the text?
89	Educational	Education	95	Does the sign system signify that the teacher has an ideal cognitive role in the text?
90	Educational	Education	95	Does the sign system signify the narrator has a cognitive role within the text?
91	Educational	Education	84	Does the sign system signify learning to read the sign system is exclusive to formal educational environments, or can the system be the acquired in other socialised learning environments (Vygotsky, ?)?
92	Educational	Education	102	Does the sign system signify social opportunities being created through education of the sign system?

Task 2: Survey and open-ended questions				
Sequence	**Domain**	**Focus RQ**	**Concept/theory**	**Questions+ who/what/how/where/when/why**
Backgrounds				
1	Cultural	Education	102	How long have you been in Australia?
2	Cultural	Education	100	Background data: do you have children- what is you age group- what job do you do?
3	Edu. English	Education	85	How many other languages can you speak, read or write?
Experiences in education, language, and mathematics				
4	Edu. English	Adult	38	How old where you when you started learning English - how did you find the experience?
5	Cultural	Education	102	Did/do you go to English classes as much as you could – how often?
6	Edu. English	Education	98	Can you tell me what your English classes have been like in Australia?
7	Edu. English	Education	103	Do/did you feel happy attending English classes?
8	Edu. English	Education	98	Did your teachers explain things so you could understand and learn things easier?

9	Edu. English	Education	83	Do/did you find it hard to learn everyday English?
10	Edu. English	Education	92	When learning English in Australia did you talk much about what you where learning to other students in the classroom - Did it help?
11	Edu. English	Education	92	When you where learning English in Australia did you talk much about what you where learning to people outside the classroom – did it help?
12	Edu. English	Education	92	Do you like to talk to others who speak your first language in English or your first language?
13	Edu. English	Education	85	Are you good in reading and writing in your first language? What type of writing do/did you do in your first language?
14	Edu. English	Education	97	Do you read much?
15	Edu. English	Education	85	What type of writing do you do in English?
16	Edu. Maths	Culture	21	Did you study mathematics at school?
17	Edu. Maths	Adult	40	Did you like mathematics at school?
18	Edu. Maths	Education	77	What type of mathematics did you learn in your English classes?
19	Edu. Maths	Education	103	Did/do you feel happy with the amount of mathematics you learnt in English in these classes?
20	Cultural	Education	100	Is there any one good at home in doing mathematics?
Perceptions of language, education, and culture in Australian society				
21	Sociological	Education	76	Would being good in English make a difference to getting a good job and making money?
22	Sociological	Culture	23	Do you think Australia would be a better place to live if everyone was able to speak English well?
23	Sociological	Education	76	Would being good in mathematics make a difference to having a good job and making money?
24	Sociological	Culture	23	Do you think Australia would be a better place to live if everyone knew how to do mathematics well?
25	Sociological	Language	71	Does knowing how to use numbers help you in real life?
26	Edu. English	Australian	24	Do you think it is harder for children from migrant families to do well at school (than children from ‘Australian’ families)?
27	Edu. Maths	Australian	24	Do you think it is harder for children from migrant families to learn mathematics (than children from ‘Australian’ families)?

28	Edu. Maths	Australian	27	Do you think it is easier for Australian born adults to learn mathematics than migrants?
29	Cultural	Culture	13	Do you think it is important to remember your language and culture?
Perceptions of mathematics				
30	Mathematical	Mathematics	6	What is mathematics?
31	Mathematical	Mathematics	2	Can you give examples of different types of mathematics?
32	Mathematical	Mathematics	3	How often do you use mathematical symbols and figures during the day?
Experiences in learning mathematics				
33	Edu. Maths	Education	85	What types of mathematics or numbers did/do you learn in your first language?
34	Edu. Maths	Education	80	Are you good at doing mathematics in your first language?
35	Edu. Maths	Education	80	Are you good at doing mathematics in English?
36	Edu. Maths	Education	84	Did you learn to use numbers outside the classroom?
37	Edu. Maths	Adult	38	How was your experience when learning how to use numbers in English for the first time?
38	Edu. Maths	Education	82	Did you often make mistakes in learning how to use numbers and mathematical symbols in English?
39	Edu. Maths	Education	84	Did you learn to use more complex mathematics outside the classroom, for example, at work?
40	Edu. Maths	Education	92	Did to talk much to others when learning how to use numbers and doing things mathematically in the English?
41	Edu. Maths	Education	84	Did/does anyone in your family or friends help you to learn or use numbers?
42	Edu. Maths	Language	58	Do you think that everyday language is harder to learn than mathematics?
43	Edu. Maths	Adult	41	Do you like or dislike how you were taught to use numbers in English?
44	Edu. Maths	Adult	36	If you had a chance, would you like to learn more about mathematics in English?
45	Edu. Maths	Education	77	What type of teaching and learning would you prefer to use to learn mathematics in English?
46	Edu. Maths	Education	84	Do you teach others in you family how to use numbers?
Structural perceptions of mathematics				
47	Ling. Structural	Language	66	Do you believe that films are more truthful than things you read?
48	Ling. Structural	Language	67	Is mathematics about doing things that are real or not real?

49	Ling. Structural	Mathematics	4	Is it harder to learn mathematical symbols and numbers than everyday English?
50	Ling. Structural	Language	42	Do you think there are similarities between learning a new language and learning mathematics in a first language?
51	Ling. Structural	Culture	12	Do you think the way we use numbers and mathematical symbols in Australia is different to the way you use them in your home country?
Cognitive related habits and perceptions in reading mathematics				
52	Perceptual	Mathematics	8	Do you use fingers or other things to help you use numbers?
53	Cognitive	Education	90	Do you close you eyes and imagine how to work out problems with numbers in your head?
54	Cognitive	Education	91	Do you write things down in numbers so you can remember things?
55	Cognitive	Mathematics	9	Do you prefer to use numbers that are written in full words (nine) or as symbols (9)?
56	Cognitive	Language	44	Do you find you have to use a calculator all the time when using numbers (quantities, space, number)?
57	Cognitive	Mathematics	9	Can you use numbers without using symbols?
58	Cognitive	Mathematics	3	Do you talk much using numbers?
59	Cognitive	Mathematics	5	Do you use numbers to explain things in English?
60	Cognitive	Mathematics	5	Do you use numbers to talk and explain things in your first language?
61	Cognitive	Adult	30	How were you taught to remember (use) numbers - was this hard for you?
62	Cognitive	Education	88	Were you confused when learning mathematics in your first language?
63	Cognitive	Adult	29	How do you try to remember how to do things that use numbers in your first language?
64	Cognitive	Language	59	Is remembering how to use numbers different to remembering everyday English language?
65	Cognitive	Mathematics	10	Are you good at using numbers in some ways and not others?
Behavioural perceptions of age, thinking, and adulthood				
66	Biological	Education	99	What do you do during your spare time (paint, cook, bush walk, do word or number puzzles)?
67	Biological	Education	99	Do you watch much TV - what sorts of programs do you like to watch?

68	Biological	Education	99	Do you like solving puzzles? Do like doing things that involve numbers (games, lotto)?
69	Biological	Education	99	Do you like playing games (e.g., cards)?
70	Biological	Education	100	Do you like working with numbers?
71	Biological	Language	48	Do you like painting or writing stories?
72	Biological	Language	48	Do like to do things differently from the way you are usually told how to do things, e.g., changing cooking recipes?
73	Biological	Education	100	Do you like mathematics?
74	Biological	Adult	37	As you are get older is it easier for you to learn to use numbers in English?
75	Biological	Adult	28	In the classroom what is the difference between being an adult and not being an adult?
76	Biological	Adult	37	As you get older is it harder for you to learn English?
77	Cognitive	Culture	15	Has your way of thinking and doing things changed since you have been in Australia?
78	Biological	Language	48	Have you changed (creativity) since you started getting older?

Task 3: Tasks questions	Domain	Focus RQ	Concept theory	Questions
Warm up questions				
1	Mathematical	Language	68	If 3 X 5 = 15 and 5 X 3= 15, is 3 cats in 5 rooms the same thing as having 5 cats in 3 rooms?
2	Edu. Maths	Education	89	Do/did you use people who speak your first language to help explain how to read and use numbers - was it helpful, confusing, or wrong?
3	Cognitive	Language	72	What do you do when you are unsure of how and when to use numbers?
4	Biological	Education	101	What was your experience like in learning mathematics at school?
5	Biological	Language	71	Are you happy when you use numbers to solve everyday problems?
In task questions				
Thinking with signs				
6	Cognitive	Education	88	Do you understand what the task is asking you to do?
7	Cognitive	Education	88	What part of is the task is easier to read than others- mark?
8	Cognitive	Language	72	Are there parts within the task where you understood enough that you could jump to the next stage without reading too deeply (mark)?
9	Cognitive	Education	96	Can you tell by the way task is written how you are expected to read and use the symbols and figures?
10	Cognitive	Language	57	Do you think the position of the symbols and figures in the text changes the way you do the task?
11	Cognitive	Education	97	Can you explain how and why you went for that answer -clues?
12	Cognitive	Education	96	Can you tell by the way the task is written how you are expected to think?
13	Cognitive	Language	70	Do you create a picture or image in your head that helps you to do these tasks?
14	Cognitive	Adult	30	Did you need more time to do this task – how/why?
15	Cognitive	Education	105	Have you forgotten any thing that may help you to do the tasks (formulas, signs, equations)?
Sign meanings				
16	Ling. Structural	Education	78	Do you know what that symbol and figure mean, and what you are supposed to do with them-are they important?
17	Ling. Structural	Education	75	Do you find these symbols and the way they are used easy to read and understand - which ones are not?
18	Ling. Structural	Language	45	How is one symbol or figure related to the one next to, above, or below it?
19	Ling. Structural	Language	72	Can some symbols be swapped within the task and still makes sense?
20	Ling. Structural	Education	97	Is there another way of using the symbol or figure used in this task as information?
21	Ling. Structural	Adult	32	Can you see other things (symbols) you need to think about?
22	Ling. Structural	Mathematics	6	Can you see patterns in the way the symbols are used in this task?

23	Ling. Structural	Language	59	Can you mark the symbols that require you to thinking more than the others?
24	Ling. Structural	Language	43	Can that symbol be used somewhere else in the task?
25	Ling. Structural	Language	55	What do you think would happen if you swapped one symbol or figure for another?
26	Ling. Structural	Language	72	Is there any symbol or figure that looks useless or you don't need to read and answer the question?
27	Ling. Structural	Language	72	Are there any stages in the task where new meanings are easier to understand than before?
Sign flows				
28	Ling. Structural	Language	63	Does it make sense if you go (read) backwards or jump between the tasks?
29	Ling. Structural	Language	65	Do you find any of the symbols and figures or parts of this task more important than others?
30	Ling. Structural	Education	97	Why is this symbol or figure placed there?
31	Ling. Structural	Mathematics	11	Can you identify what the symbols and figures are asking you to do next?
32	Ling. Structural	Language	60	Can you explain by looking at the symbols and figures what is happening in the task?
33	Ling. Structural	Language	58	Can you say show how you moved from reading one symbol and figure to another by drawing a line between them?
34	Ling. Structural	Language	72	How did you do that (moving between symbols and figures)?
35	Ling. Structural	Language	64	Does it make sense to you jump between different parts of the task (graphs, symbols, linguistic forms)?
36	Ling. Structural	Language	58	Can you put a question mark next to the symbol you are not sure what it means or what you are suppose to do next?
37	Ling. Structural	Language	62	Are any of the symbols or figures confusing as to what you are suppose to do with them?
Sign origin				
38	Ling. Structural	Mathematics	7	What types of symbols or figures do you think are used to do mathematics in this task?
39	Ling. Structural	Language	73	Have you seen any of these symbols and figures used in this text elsewhere, like on TV or in the newspaper?
40	Ling. Structural	Mathematics	7	Does the task use symbols that are not mathematics?
41	Ling. Structural	Adult	31	Does the symbol or figure make sense or do they look strange?
42	Ling. Structural	Language	61	Are their any symbols, drawing, or words missing that would make more sense to include in the tasks?
43	Ling. Structural	Culture	12	Does the task look different to the way you may have seen it written in your first language?
44	Ling. Structural	Language	56	Where do you think the symbols and figures used in this task originally come from?

Post tasks questions				
45	Sociological	Culture	21	Do you find anything useful about this task you could use in real life?
46	Edu. Maths	Education	89	Would you change the way the task is written so it would make more sense to you?
47	Edu. Maths	Education	95	How would you like to be taught how to do the task?
48	Edu. Maths	Education	77	Have you been taught to do any of these tasks before?
49	Biological	Education	100	What do think of when you see these tasks?
50	Ling. Structural	Language	72	Are their parts where you had to stop and think a bit longer (identify with a mark)?
51	Perception	Adult	34	Do you think this task is pretending to be something that is not (really) real?
52	Ling. Structural	Education	83	Would you prefer to see a lot more words used to explain these tasks or not?
53	Biological	Education	99	Which one of these tasks is more pleasant to look at or do?
54	Biological	Adult	28	Do you think an adult could better answer the task than a child?
55	Cognitive	Education	101	Do you feel like talking more about these tasks?
56	Biological	Education	101	Would you like me to explain in greater depth how to get the answers to these tasks?
57	Cultural	Education	95	Can you describe what sort of person you think wrote these tasks?
58	Edu. Maths	Education	81	Would you like to learn more about using the symbols and figures in the tasks?
59	Biological	Education	103	Do you feel happy sitting here and doing these tasks?
Observational questions				
Obs.1	O- Cognitive	Culture	19	In task observation - how, when, and what type of participant questions occur?
Obs.2	O-Structural	Education	92	In task observation - does the sign system signify modalities such as speaking for the participant to generate meanings?
Obs.3	O-Structural	Education	99	In task observation- reactions to solving and engaging in tasks as problems: responses, explanations, and sign manipulation?
Obs.4	O-Structural	Education	105	In task observations - how participants respond to task?
Obs.5	O-Cognitive	Adult	32	In tasks observation - how quickly or methodological the participant performed the task?
Obs.6	O-Cognitive	Adult	36	In task observation - how participants respond and reading semiotic queues?
Obs.7	O-Biological	Adult	40	In task observation- biological variances within task: male/females/age/CALD within task?
Obs.8	O-Biological	Education	91	In task observation – record the physical aspects of participation within the sign system (e.g., paper usage, figures, drawings, body movements)?
Obs.9	O-Biological	Education	99	In task observation - does the participant display discomfort or positive engagement - this task will require some stimulation, e.g., feeding explanations and information and recording responses?

Obs.10	O-Biological	Education	101	In task observation - does the sign system signify anxiety by the reader towards the signs that precludes the system being read?
Obs.11	O-Structural	Adult	31	In task observation - hesitation and fluency?
Obs.12	O-Structural	Education	87	In task observations - how language strategies such as code switching occurs within the sign system occurs?

Micro Questions	Domain	Focus	Concept	Main Questions/Subordinate Questions
69 Examples 1: Semiotic Analysis of tasks	**66**	**Cultural**	**Language**	**66/Questions/Does the sign system signify assumed social and cultural conventions as codes (Derrida, 1967)**
What- examples?				What are the codes that are assumed as conventions within the sign system?
How				How are conventions signified?
When				Does the sign system signify patterns in which more or less meanings are culturally dependent?
Where				Where within the sign system are social and cultural conventions assumed as codes?
Why				Why are there social and cultural conventions in this task - what role do they perform - are they important?
Who				Who would think social and cultural conventions are or are not important?
28 Example 2: Survey questions.	27	Edu. Maths	Australian	27/Question/Do you think it is easier for Australian born adults to learn mathematics than migrants?
What- examples?				What type of mathematics are you referring to?
How				How do you think it may/may not be a problem?
When				How often do you see this type of problem?
Where				Can you give an example of where you may/may not of seen it as a problem?
Why				Why would it be easier/harder for them?
Who				Can you give an example of anyone you know?
31 Example 3: Task-related question.	11	Ling. Structural	Mathematics	11/Question-Can you identify what the symbols and figures are asking you to do next?
What- examples?				What is the symbol or figure asking you to do here?
How				Can you show me how you got that answer -draw a flow line/place a mark?
When				Can you tell me when the symbol or figure is asking you to that? -Or -When did you learn to do that?
Where				Can you show where the symbols or figure is leading to in the task?
Why				Why did that symbol get (signal) you to do that?
Who				The task is asking you to do something, can you tell me who might be the person behind the tasks- or- Who showed you how to do that?

Appendix E: Mathematical Tasks

1. Choose one of the numbers in the first table and write it in the space next to symbol in the second table that matches what that symbol means.

11	1	4	23	2	0

■■	
ϕ■	
■■■■	
∞	
$\phi\phi$■■■	
■	

2. Choose one of these symbols and complete the statements below.

"− ,> ,× ,÷ ,/ ,∪ ,≤ ,= ,−"

a) 000____00

b) 3____2 = 6

c) 000 ____000 = 0

d) 0000/00 ____ 00

e) 1____2 = .5

f) 000____00 = −0

g) 21____22

h) {000}____{0,00,000} = {0,00,000}

3. Tick ✔ which symbol is not a number.

1	六	Apples	A	π	$^{22}/_{7}$	200,000	Apple	+	Five

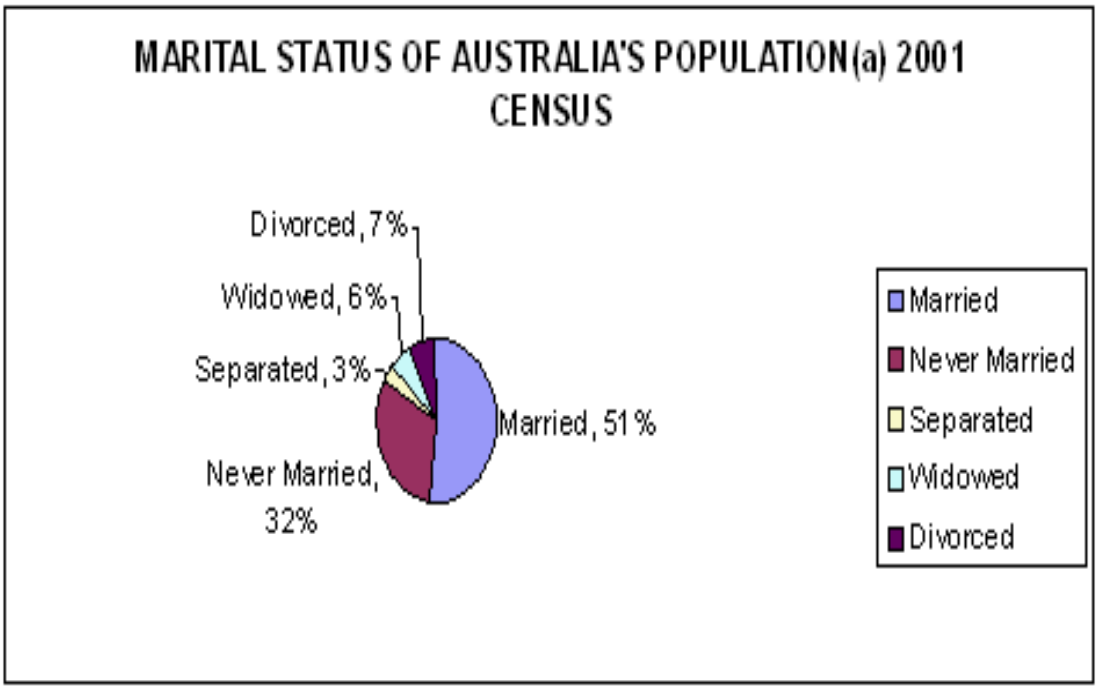
4. Below is a chart showing information about Australians. Answer the following questions:

a. What is the information about?

b. How is the information presented?

c. What is the significance of the information?

e. Are there any trends in the data?



Source: Australian Bureau of Statistics C10, census Data 2001. Publication 1331.0 Statistics: A Powerful Edge)

5.

PURE ALCOHOL AVAILABLE FOR CONSUMPTION

Year ended 30 June	Beer	Wine	Spirits	RD	Total
	Volume of pure alcohol ('000 litres)				
2005	75 075	57 275	19 667	15 338	167 355
2006	76 388	58 311	19 154	16 383	170 236
2007	76 849	62 263	19 355	18 123	177 590
2008	79 496	62 807	20 160	18 693	181 156

2009	81 148	65 600	22 865	13 056	182 669
2010	79 734	68 452	23 023	12 811	184 021

Per capita consumption of pure alcohol (b) (litres)

2005	4.62	3.53	1.21	0.94	10.31
2006	4.63	3.53	1.16	0.99	10.31
2007	4.63	3.70	1.15	1.08	10.57
2008	4.63	3.66	1.17	1.09	10.56
2009	4.62	3.73	1.30	0.74	10.40
2010	4.44	3.81	1.30	0.70	10.25

(a) Ready to Drink (pre-mixed) beverages.
(b) Liters per person aged 15 years and over.

Source: Australian Bureau of Statistics 4307.0.55.001: Apparent Consumption of Alcohol, Australia, 2009-10.

The table above shows information about Australians. Answer the following questions:

a. What is the information about?

b. How is the information presented?

c. What is the significance of the information?

e. Are there any trends?

6. Place the correct answer from Table I into the correct position to the equations in Table II.

Table II

3^{-2}	
$\sqrt{6}.\sqrt{6}$	
3^{2}	
$\sqrt{9}$	

Table I

3	9	1/9	6

7. There are 350 people attending a festival and each person receives 1bowl of rice for each of the 3 meals over the day. How many bowls of rice does each person receive for the day?

Write your answer and your calculations in the box below.

Answer:

8. A camp organiser calculates that if he has 250 people in the camp and that there are 875 bowls of rice that can be shared equally, each person receives 3 and a half bowls of rice.

Look at organiser's calculations below and explain how the organiser calculated the 3 and a half bowls of rice. Write next to the organiser's calculations your explanations on how the problem was solved.

875 ÷ 250=?

250 (1

500 (2

750 (3

1000 (4

1250 (5

```
      3.5
-----------
     875
   -  750 (3)
-----------
     125
-    125.0 (1/2)
------------
       0
```

9. Match the equations in the first table with their equivalent forms in the second table and then write them with their answers in the third table.

3×6	a(b + c)	$2 \div 4$	6 (4)	4 (2 + a)	$4a \div 2a$

$\frac{4}{2}$	$2/4$	3(6)	ab + ac	4 (6)	4a + 4(2)

24		
8 + 4a		
2		
18		
(ab + ac)		
.5		

10. Mathematical equations and formulas can be used to help solve mathematical problems without actually physically doing things. For example, from experience you may know that a pattern exists when you press the foot on the accelerator in a car because not pressing on the accelerator the car will not move, pressing the accelerator 3 cm the car will travel at 50km/hr., and pressing at 6 cm the car will travel at a 100km/hr.

An equation/formula can be derived from this pattern that calculates how fast a car would travel when pressing on the accelerator at other measurements.

S= 20A

Where: S = speed, A = centimetres pressed on the accelerator

c. How fast would the car travel if you press the accelerator 8cm on the accelerator?

d. If you were traveling 30 km/hr., how much would the accelerator be pressed in cm?

11. The formula to calculate the square area of a room is

$$m^2 = l \times w$$

Where: m^2= square meters, *l*= length of the room in meters, *w*= width in meters

Circle the correct answer:

1. What is $m^2$of a room that is 4 meters long by 3 meters wide?
   a. 43
   b. 34
   c. 21
   d. 12
   e. 23

2. How many tiles would you need to tile a room that is 4 × 3 meters if the tiles measure •5 × •5 of a meter?
   a. 43
   b. 48
   c. 84
   d. 34
   e. 12

12. The formula used to calculate the cubic volume of a six-sided crate is

$$v^3 = l \times w \times h$$

Where v^3 = cubic volume, *l* = length, *w* =widths, and *h* = height

Circle the correct answer:

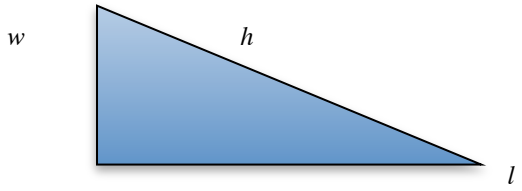
1. What is the cubic area (v^3) of a crate with the dimensions of *l* = 4, *w* =3, and *h* = 2?
   a. 24
   b. 42
   c. v^3. 23
   d. 34. v^3

2. How many boxes with the dimensions 1 meter long, 1.5 meter wide, and 1 meter high would fit in the crate?
   a. 12
   b. 21
   c. 16
   d. 15
   e. 25

13. A formula for calculating the lengths of a 3 sided right angle triangle is:

$$h^2 = l^2 + w^2$$

Where:



a. Given $l = 4$ and $w = 3$, explain how the formula is used to calculate $h = 5$

$\therefore h^2 = 4^2 + 3^2$

$\therefore h^2 = 16 + 9$

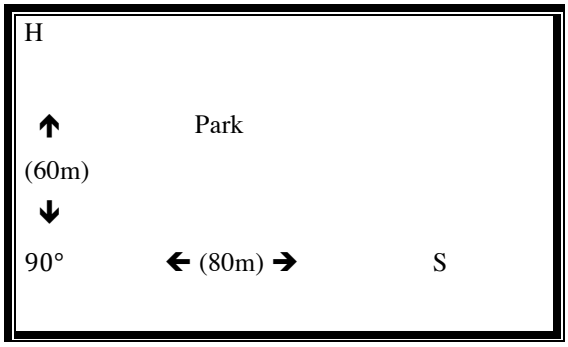
$\therefore h^2 = 25$

$\therefore h = \sqrt{25}$

$\therefore h = 5$

14. There are two ways you can travel from your house to a supermarket S. You can travel 60m straight down to the end of the street and turn left 90 degrees and walk another straight 80m to the supermarket. Or, you can travel diagonally from your house across a park straight to the same supermarket.

House



Supermarket

How much distance is it less to travel across the park compared to going around the around by two road?

a. 100
b. 40
c. 100
d. 20
e. 90

15. Below is a table that charts the share price for company ABC over 10 weeks of trading on the stock exchange.

Draw the information on a graph paper plotting the price on the vertical axis and the respective weeks on the horizontal axis.

Week	ABC Share Price	Rate of change in share price
0	$0	10
1	$9	8
2	$16	6
3	$21	4
4	$24	2
5	$25	0
6	$24	-2
7	$21	-4
8	$16	-6
9	$9	-8
10	$0	-10

A way of representing the data is presented in the chart below. Two lines are shown as equations within the line chart. The first line shows the price of shares as an equation $y = 10x - x^2$ where y= price and x=time.

For example, when $x = 4$

$y = 10x - x^2$

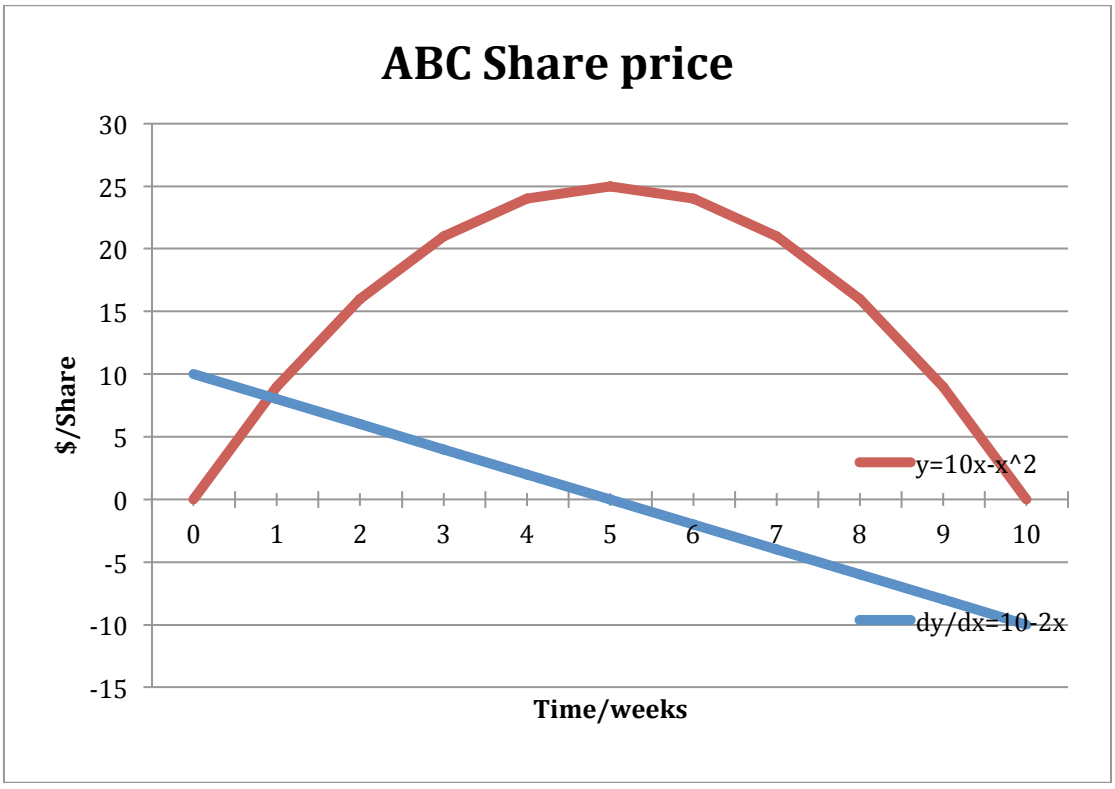
$\Leftrightarrow y = 10(4) - (4)^2$

$\Leftrightarrow y = 40 - (4 \times 4)$

$\Leftrightarrow y = 40 - 16$

$y = 24$

A second equation shows the rate of change as the share price is changes over time $dy/dx = 10\text{-}2x$ where dy/dx = is the rate of change in share price and x=time.



a. Explain the significance of the line $dy/dx = 10 - 2x$ in determining when to buy and sell shares?
b. When is the rate of change equal to the share price?
c. What happens to the share price in week 5?

Appendix F: Data Matrix (sample)

(Pages 1, 7, 36, 50, 63, & 64 of 64)

Observation	***Interpretation***	***Implication***
TXT1000 *PHILOSOPHICAL* CODES THAT AFFECT HOW L AND S STRUCTURES ARE ENCODED AND BECOME PROBLEMATIC TO DECODE IN M2.		
TXT1001 Why choose these tasks (see Chandler, 1993), and does this factor signal BL readers will have problems decoding M2?		
The content of the sign system is reconciled as follows: 1. The tasks are exemplars of the type of mathematical decoding that occurs around year 10 in an Australian high school. After year 10, mathematics is post compulsory. 2. The tasks are exemplars of the standard of knowledge needed to engage in VET type training. 3. The tasks provide insight into how L and S structures function and are potentially problematic to decode as M2, e.g.: • Tasks 1and 3 interpret decoding L and S signs at an abstracted level. • Task 2 interprets decoding L and S signs as logic. • Tasks 4 and 5 interpret decoding L and S structures as a graphical representation, and a chance/probability construct. • Task 6 interprets decoding L and S structures as indices across different S resources. • Task 7 interprets decoding ML in highly lexical forms. • Task 8 interprets decoding L and S structures in constructs of a third person. • Task 9 interprets decoding L and S structures in	The contents of the sign system and the questions asked of it elicit a predetermined type of response and interpretation. Reconciling this factor is important because it signals the data is, in part, is bounded by the L and S systems being studied. Further, readers have different experiences, and it is possible to misinterpret how L and S structures are problematic to decode in ML. BL readers also bring with them unique L and S experiences that add a level of complexity to interpreting the problem: e.g., ML may share L and S structures across languages and it is possible for BL readers to reconcile some M2 signs from M1 experiences and not others. Therefore, BL readers reconcile the M2 sign system differently to monolinguals because there are philosophical reasons why it may or may not be considered mathematical.	L and S structures are problematic to decode in M2 if BL readers are unable to reconcile M2 may have different philosophical meanings that affect how it is decoded as ML. An appropriate space is needed for BL readers to reconcile how philosophical implications affect decoding M2.

algebraic and tabular forms. • Task 10 interprets decoding formulas as ML. As Participant 2 stated later in an interview, 'it is like physics'. • Tasks 11, 12, 13, and 14 interpret complex and dense ML in applied contexts. Multiple-choices answers signal how problem solving occurs within L and S structures: it is of interest to interpret how readers reconcile answers that look similar. • Task 15 interprets decoding L and S structures when solving (potentially) real-life problems using a combination of calculus and graphical notations.		*(Page 1/64)*

Internals\\Background\\10. Text observation and interpretation> - § 1 reference coded [38.49% Coverage]

Observation	***Interpretation***	***Implication***
TXT2000 STRUCTURAL CODES THAT AFFECTED HOW L AND S STRUCTURES WERE ENCODED AND BECAME PROBLEMATIC TO DECODE IN M2.		
TXT2009 Does the sign system share meanings with other mediums, e.g., scientific language (see Halliday, 2006), everyday language, or filmic codes (e.g., genre, camerawork, editing, and manipulation of time and sequences (Chandler, 1993), and how does this factor create problems for BL readers when decoding M2?		
Observation	***Interpretation***	***Implication***
L and S structures appear unique in ML, however, the decoding behaviours may be shared with other mediums: e.g., questions and descriptions are decoded in everyday language as they do in disciplines such as business and science; and, the tasks can also be decoded sequential and erratically as they do in films where	The ability to decode signs in and across mediums (e.g., films to mathematic, and text) is challenging because it requires L and S experience to reconcile the same signs in different mediums. However, people have predispositions to thinking with signs (see spatial, temporal, functional, and logic	L and S structures are problematic to decode in M2 if BL readers are unable to reconcile signs from other mediums: e.g., knowledge of everyday language, literature, and films are important in reconciling M2.

plots disappear and reappear in different scenes.	differentials that exist within cultures and language) and this affects how signs are reconciled across different languages: e.g., it is not sure if Arabic share similar films genres as English and, therefore, the signs that make sense when decoded across different medium in English may not be the same in other languages and cultures.	
TXT2010 Does the sign system signal independence and exclusivity from everyday discourses, and how does this factor create problems for BL readers when decoding M2?		
Signs can be decoded at a molecular (signs joined with other signs to articulate meanings) and atomic level (signs signal meanings on their own). At an atomic level signs can be construed into everyday discourses, however, at a molecular level the sign system has distinctive patterns, preciseness, density, and logics, and this affects how meanings are reconciled.	The sign is fluid because it is interchanged and connected to other signs to generate complex and delicate meanings (Halliday, 2006): e.g., the '%' has a certain delicacy when attached to numbers (e.g., 2%), however, when attached unconventionally (e.g., hat%) the meaning is ambiguous. Nonetheless, everyday discourses help to reconcile and contextualise delicacies in ML.	L and S structures are problematic to decode in M2 if BL readers are unable to reconcile everyday discourses are needed to contextualise complex sign meanings in M2. *(Page 7/64)*
INT74005 Participant perceptions on changing with age that affect how M2 is decoded: Participant 5 believes BL children are able to take up education relatively easy (compared to adults) in a L2 and M2 contexts, however, personal experiences still shapes success.	A BL reader's age, thinking, and behaviours are reconciled from personal experiences.	L and S structures are problematic to decode in M2 if BL readers are unable to reconcile personal experiences in acquiring M2 as they get older. An appropriate space is needed to help manage negative personal perceptions. *(Page 36/64)*

<Internals\\Background\\8. Tasks Observation Summaries> - § 13 references coded [44.27% Coverage]

TAS101A1 Participant perceptions of ML that affect how (L and S structures) M2 is decoded (and becomes problematic): Participant 1had an insightful range of perceptions about ML that affected how they decoded M2. The perceptions included the belief that while ML signified quantities and processes, it also had ambiguities and redundancies.	BL readers bring with them different experiences and perceptions of ML that affect how they decoded it and generate meanings. Participant 1 decoded most tasks independently: however, sometimes they did not choose the correct answers. This may have been because they opted or could not to decode ambiguous signs as truthful statements. BL readers may have different ways of perceiving and decoding ambiguous signs, and they may avoid linking them to the main text they are confusing. Participant 1 employed different processes to decode ambiguous signs in the tasks.	L and S structures are problematic to decode in M2 if BL readers are unable to reconcile ambiguous structures and signs in M2. An appropriate space is needed for BL readers to interpret L and S structures in M2 so factors such as redundancies and ambiguities can be reconciled from a BL perspective.

Observation	*Interpretation*	*Implication*
INT75001 Participant perceptions on creativity that affect how M2 is decoded: Participant 1's self-perceptions are mixed as they appear contradictory (e.g., not liking change, not carrying about mathematics but nonetheless relocating to a new country and being good at decoding M1).	BL readers signal they have complex behaviours and backgrounds that affect how they decoded M2. Participant 1 manages and describes their self-perception within a combination of contexts that includes their cultural and linguistic experiences as well as the experience in participating in this interview. Each context affects how Participant 1 data is generated and interpreted.	L and S structures are problematic to decode in M2 if BL readers are adversely affected by negative self-perceptions (e.g., 'I am no good at maths', 'I cannot decoded English'). It is important to create appropriate spaces that are L and S suitable for decoding M2 in a positive way.

TAS107A3 Participant physical interaction and processes that affect how M2 is decoded: Participant 3 was focused and responded to the tasks by answering questions in writing and orally: however, writing full responses appeared challenging and time consuming for Participant 3. They also spoke quietly to themself while decoding some tasks.	BL readers may reconcile M tasks better as an oral activity than in writing. Participant 3 appeared more challenged to write full written responses than answering them orally in this interview.	L and S structures are problematic to decode in M2 if BL readers are unable to write in M2. An appropriate space is needed for BL readers to reconcile (enact and transpose) written and oral responses in M2.
TAS107A4 Participant physical interaction and processes that affect how M2 is decoded: Participant 4 was focused, but responded with significant hesitation when decoding (and writing) at a micro level until support was given.	Participant 4 signals the significance a space needed to reconcile signs and structures when decoding M2. Furthermore, Participant 4 signalled decoding M2 is physically demanding, and the energy needed is significantly more than M1. Participant 4 was focused but it is hard to perceive their energy being maintained indefinitely, and perhaps this is why not all the tasks were decoded within this interview.	L and S structures are problematic to decode are unable in M2 if BL readers due to fatigue are unable to maintain physical interaction when decoding M2. Decoding M2 is (more) physically demanding and an appropriate space (including extra time) is required. The construct of M competency is challenged when time is not appropriately factored into assessing BL readers. *(Page 50/64)*
Observation	*Interpretation*	*Implication*
TAS107A5 Participant physical interaction and processes that affect how M2 is decoded: Participant 5 was focused and they responded to the tasks by writing, speaking, and asking questions in a very pensive manner.	Participant 5 signals decoding M2 can be a physically and mentally demanding space. Managing and reconciling timing when decoding M2 will affect how accurately it is decoded.	L and S structures are problematic to decode if BL readers are unable to reconcile and manage the timing and physical energy required to decode M2. An appropriate space is required for BL readers to reconcile timing and energy issues when decoding M2.

TAS111A2 Participant displays of discomfort that affect how M2 is decoded: Participant 2 did not display discomfort and engaged in decoding the task in a positive manner - even if they didn't understand some words.	BL readers face different types of discomfort (e.g., emotional, physical, social, cultural) that can detract from decoding M2 accurately. Participant 2 displayed no discomfort and decoded the tasks accurately despite being challenged at time by ambiguous signs and meanings.	L and S structures are problematic in M2 if BL readers find it discomforting to decode M2. An appropriate space is needed to manage BL emotional, physical, social, and cultural needs when decoding ML in an M2 context.

TAS112A1 Participant displays of specific anxieties that affect how M2 is decoded: Participant 1 appeared happy and smiled a lot. Participant 1 was also happy when they remembered M things, or were able to identify inconsistencies in the sign system.	M2 challenge readers in different ways that include reconciling personal anxieties towards decoding M2. However, while anxieties towards decoding M2 are complex to interpret, bilingualism signals there is an additional level of complexity that need to be reconciled when interpreting ML in an M2 context: e.g., not all anxieties are necessarily bad for decoding M2.	L and S structures in M2 are problematic in M2 if BL readers are anxious and decoded M2 as a negative experiences. An appropriate space is needed for BL readers to avoid unnecessary negative anxiety when decoding M2.
TAS112A2 Participant displays of specific anxieties that affect how M2 is decoded: Initially, Participant 2 appeared a bit unsure of what they were expected to do within the interview, but once they decoded the introductory statements and completed the first Task they appeared relaxed and participated positively.	BL readers may initially show unique anxieties towards decoding M2: however, while Participant 2 was initially unsure what they were supposed to do, they soon relaxed and engaged in decoding M2 in a positive way.	L and S structures in M2 are problematic to decode in M2 if BL r anxieties cannot be managed to create positive outcomes when decoding M2. An appropriate space is needed to reconcile (transpose and manage) BL type anxieties towards decoding (acquiring) M2 as a positive experience.
INT31003 Context-related factors and beliefs that affect how M2 is decoded by BL readers: While Participant 3 had a deep and complex understanding of 'what is mathematics', they needed an online translator (via smart phone) to help explain it in L2.	BL readers are able to use technologies to help decoded M2. Participant 3 was able to use online technologies to decode (communicate) in L2 and M2, which was otherwise difficult to do.	L and S structures are problematic to decode in M2 if BL readers are unable to reconcile and manage technologies to decode M2. An appropriate space is needed to also reconcile M2 with technologies.

INT73004 Participant perceptions on changing while living in Australia that affect how M2 is decoded: Participant 4 had to work and could not attend language classes until after many years in Australia.	BL readers may not have opportunities to learn M2 formally at school. Participant 4 had to work and was unable to attend English classes until recently and this shaped how they are acquiring M2.	L and S structures are problematic to decode in M2 if BL readers do not have the opportunity to attend M2 language classes. An appropriate space is needed (IT, correspondence) for BL readers to acquire M2 in different environments such as home and at work.

Observation	*Interpretation*	*(Page 63/64)* *Implication*
INT73005 Participant perceptions on changing while living in Australia that affect how M2 is decoded: Participant 5 believes living in Australia has enhanced their educational opportunities and despite being a BL adult they see themself positively through education.	BL reader may see a window of opportunity in participating in education in a new L2/M2 context. Participant 5 sees engaging in L2/M2 as a positive educational experience.	L and S structures are problematic to decode in M2 if BL readers are unable to take up educational opportunities that help to decode M2. An appropriate space is needed to help BL readers to reconcile decoding M2 as part of creating and taking up opportunities.

INT75005 Participant perceptions on creativity that affect how M2 is decoded: Participant 5 appeared to enjoy the opportunity to participate in this interview.	BL readers are more productive when they enjoy participating in decoding and discussing how M2 is decoded.	L and S structures are problematic to decode in M2 if BL readers are unable to enjoy (gain) from decoding M2 at a personal level. An appropriate space is needed to help BL readers enjoy decoding M2 at the first level of introduction.

<Internals\\Background\\8. Tasks Observation Summaries> - § 1 reference coded [2.04% Coverage]

TAS102A1 Participant competency factors that affect how M2 is decoded: Participant 1 displayed a variety of competencies when decoding M2: e.g., Participant 1 accurately recalled M processes and signs, gave explanations about concepts and meanings, and decoded prepositional phrases (e.g. above, on, under) fluently. However, on some occasions Participant 1 displayed some inaccuracy when decoding M2: e.g., Participant 1 misread some linguistic terms (e.g., the meaning of 'significance'), spent some time recalling certain processes in M1 and transposing them into M2, and needed a moderate level of support in interpreting some tasks.	Different processes are required to decode ML from a BL perspective: e.g., M signs and meanings can be conceptualised (mentally and physically) in more than one way such as culture and language (Barton, 2009). Participant 1 displayed a variety of 'appropriate' BL decoding competencies: however, in some contexts they were also inaccurate. Nevertheless, from a BL perspective, competency in decoding M2 is influenced by a number of factors that include language balance.	L and S structures are problematic to decode in M2 if BL readers are unable to reconcile appropriate decoding processes to help decoded M2. An appropriate space is needed to reconcile M2 as a BL outcome. *(Page 64/64)*

Appendix G: Summary Data Matrix

Factors that emerge, affect, and create problems in decoding L and S structures in M2	Meaning and significance	Incongruence	Teaching Implications
Section 1: (TXT1000) philosophical codes	• Philosophical codes identify the ideological values and beliefs that encode meanings in ML. o They vary between languages and cultures. o Enable learners to think and interpret different human experiences through mathematical signs. • ML can be: o arbitrary in nature when it shares signs and meanings (de Saussure, 1910). o Non-arbitrary and 'pragmatic' (Pierce, 2010) when signs are not shared with other C, L, and S systems. Examples in ML: • Cultural conventions and ambiguities: ▪ Everyday idioms and words ▪ Straightness. ▪ Maximising. ▪ Failure in logic and axiomatic codes (5 cats in 3 rooms ≠ 5 rooms with 3).	• Competency in M1 lends to competency in M2, however philosophical codes are less likely shared between M1 and M2. • Philosophical codes challenge existing BL thinking and decoding behaviours. o How BL see themselves as learners. o Ideologically contentious in some contexts. ▪ Cultural ambiguities ▪ Gender role ▪ Makes M2 untruthful and not real.	• Imbed and focus on learning philosophical codes in M2 as a cross C, L, and S activity. o Different to teaching monolinguals. ▪ Different ambiguities and problems. o Important BL learners understand the meaning, structure and function philosophical codes when decoding M2. • Focus on: o Establishing epistemological, ontological, and ideological beliefs and values in M2. o Reconciling contentions issues like gender, class, ethnicity, and cultural meanings that may emerge. o Generating life connections and experiences.

	• Philosophical codes also bound how the study elicits (self-determining) questioning, collection, and interpretation of data and testing. • BL are different to monolingual learners. ○ Find ideological and pragmatic meanings ambiguous and pragmatic. ▪ Untruthful and mythical • In certain contexts ML can be a highly emotive learning experience. ○ See settlement goals. ▪ A new life. ▪ Instrumental reasons. ▪ New community.		○ Generating truthfulness and reality. ▪ Include settlement issues: how, why, and when M2 is acquired by learners. ○ Reconciling what happens when codes change thinking and decoding behaviours. • Competency means establishing a qualitative assessment of 'accuracy' and 'independent' decoding behaviours in M2. • Outcomes that focus on M2 are envisaged to enhance teaching M1 and L1 as well.

Section 2: (TXT2000) structural codes	• Structural codes generates meanings through the position and the relationship of the sign within L & S structures in ML. • ML is 'functional' and 'systemic' (Halliday, 2006). • M2 has distinctive functions and L & S structure that make it different to M1. o Everyday English language is ambiguous and vague. o Includes different symbolic language. • Humans use symbols to encode meanings, decoding, and thinking in film, art, music, narrative (Holland, 2007), and the dream (Freud, 1931). o The terms sign and symbol are used interchangeably in this context. • 9 types of structural codes emerge and affect how M2 is decoded as ML. o Paradigmatic and syntagmatic codes. o Iconic, symbolic, and indexical codes. o Opposition, marking, and contrasting codes. o Physical codes (the effect the shape, position, energy, motion, force, synergy, timing, and the aesthetic positioning and relationships of signs have on encoding meanings). o Agentic codes. o Metaphoric codes. o Logic and axiomatic codes.	• BL learners find M2 'tricky' to decode in a cross-C, L, and S context. o Different to monolinguals. • BL related-problem can be observed through reader pauses, hesitations, questions, gestures, body movements, and statements and then linking them to the C, L, and S contexts, contents, and processes that create problems in decoding M2. • Competency is rarely assessed in terms of accuracy and independence in decoding M2. o Helps describe the quality of responses given by BL in M2 (as competency). o Heuristic characteristic in M2 observed (Raab and Gigerenzer, 2005). • Important to reconcile the effect BL decoding experiences and behaviours have on decoding structural codes in M2. Depend on: o C, L and S experiences (Play, education, work, settlement). o Age, memory, and lack of practice. • The Medium is important because it ties the structural codes together. o Different mediums generate divergent decoding behaviours and meanings. o Consider English language as a medium. ▪ Creates a distinctive problem in decoding M2	• Imbed and focus on learning to decode M2 as structural codes. o Include subtitles aspect. • Reconcile code-switching and memory process that are BL in nature. o Focus on transposing and negotiating C, L and S knowledge backwards and forwards. • Reconcile and manage how BL decoding behaviours change due to the agentic nature of the sign system. o Reconcile structural codes that are contentious and personally challenging: e.g., gender and culture. o Make explicit hidden social and cultural meanings. • Reconcile the different visual, textual, auditory modalities used to learn M2. • Manage fatigue (the mental and physical demands) placed on BL learners.

	○ Discursive codes ○ The medium. • Competency in M2 includes the ability to decode different structural codes at the same time.	• Problems emerge in decoding L & S structures at an atomic versus cellular level. ○ Different thinking processes required at different levels of 'stratifications' (Halliday, 2006). • BL Learners rely significantly on recalling and transposing decoding M1 into M2. ○ Not sure how neatly prior M1 experiences transpose into decoding the M2. • BL decoding behaviours change constantly in M2. ○ Signs 'Agentic' in nature. ○ Memory-related processes are frustrating for BLs. • M2 codes share decoding behaviours, in part, with other modalities (e.g., films, art, music) ○ Not sure how neatly BL decoding experiences in modalities transpose into decoding M2. ○ Problem when M2 employs distinctive (sound, sight, and tactile behaviours) to decode structural codes. • English creates C, L and S mediums and resources that are difficult to reconcile for BL learners	

Section 3: (TXT3000) BL decoding behaviours	• BL decoding behaviours are shaped through (different) C, L and S experiences. • Decoding behaviours identify the current states of bilinguality (Hammers and Blanc, 2000) and states of immersion in M2. o Reflect different levels and types of C, L, and S balances and dominances in ML. • BL learners have distinctive and observable physical and mental decoding characteristics. o Underlining new words, decoding text aloud, and re-drawing and rewriting figures and formulas in English. o Spend considerable time and energy decoding M2. o Types of responses to questions. o Time and hesitation of questioning, thinking, discussion, and comparing meanings to M1. • BL learners acquire M2 as a 'mental tool' (Fitzsimons, 2002; Skovsmose, 1994, 2001) that help solve everyday problems and settlement goals. • Nonetheless, no two BL reader appears to have exactly the same type of mental and physical decoding behaviour in M2. o Factors such as age, politics, education, gender, settlement, and economics	• Transposing and decoding meanings through discourse codes develop differently for BL learners. o Identify three levels of discourse incongruences in M2: ▪ BL Learners may understand mathematical concepts, however have problems communicating them in English. ▪ BL learners may communicate M2 as a discourse in English, however may they have problems reconciling mathematical concepts. ▪ BL learners may find M2 (both) difficult to communicate and decode as mathematical concepts. • Study avoids using cognitive explanations to explain decoding behaviours in M2 . o Better explanations made by observing the physical. behaviours and C, L, S contexts. o Not having to guess what is going on inside BL mind. • Multilingual problems different but still BL in nature. o Coordinate and compounded BL experiences (Hammers and Blanc,	• Imbed and focus on enhancing BL decoding behaviours in M2. • BL readers are different because M2 is acquired in a cross-C, L and S activity. o An unsophisticated discourse in M2 does not mean a BL learners has an unsophisticated understanding of mathematical meanings. ▪ Include learning M2 as a discourse activity. • M2 tasks act (both) as semiotic artefacts and as 'semiotic resources' (O'Halloran, 2005, 2010) for learning ML a different way. o While related treat the two outcomes differently. ▪ An artefact it helps describe coding characteristics in M2. ▪ A semiotic resource plays an important role in modifying decoding behaviours. ▪ Semiotic resources are 'agentic' in nature ▪ Includes generating and reconciling new cultural, spatial, temporal, and

	affect the mental and physical characteristically of BL learners differently. ○ BL learners tend to believe settlement and educational opportunities in Australia help overcome previous (negative) political and economic experiences. • Multilingualism adds a degree of complexity in understanding BL decoding behaviour. ○ Interpret multilingual decoding behaviour as BL in nature, because it reflects different degrees of language balances and dominances that affect decoding M2.	2000) provides some explanations. • Less sign sharing experience coincides with BL learners acting less familiar, less accurate, and needing more support. • Identifies 4 dimensions of immersion in M2 ○ The age and length of immersion in M2. ○ The intensity and balance in which M2 is acquired. ○ The reader's attitude towards acquiring M2. . • Competency in M2 changes quickly through ongoing C, L and S immersion ○ As a result, M2 discourses and assessments in them are transitory and fluid in nature. • M2 discourses signal a different type BL incongruence and support is needed. ○ Some BL learners find some signs generate opposite meanings in M1. ○ Remembering how to decode and transpose M1 into M2 is particularly problematic. ○ Can be emotionally threatening. • Learning M2 challenges BL learners to reframe and re-scaffold their role as a reader and learners in ML.	functional meanings through M2. • Provide C, L, and S cues and clues that help decode signs as subtitles. ○ Include glossaries of terms and idioms. ▪ Include a phrasal and discourse aspect in glossary. ○ Textual clues help interpret ambiguous meanings. ▪ Text explanations • Early C, L, and S experience is envisaged to generate different aural, textual, and tactile experiences and meanings that need to be changed. ○ Use different type of modalities to help generate prepositional signs and logics. ○ Help ground M2 meanings into more familiar C, L, and S contexts and experiences. • Reconcile and manage the behaviours of BL learners in public spaces, such as class and work. ○ Often seen as threatening. • Maintain motivational experiences and engagement in M2. ○ Target achieving settlement goals as part of learning M2

Section 4: (TXT4000) BL humanistic behaviours	• BL human and physical behaviours identify the more humanistic factors that affect BL decoding behaviours in M2. o Reconciles, for example, the role of human emotions, perceptions, modality (speaking, listening, decoding, and writing), gender, and memory have in decoding and learning M2 in a cross-C, L and S activity. • Different childhood experiences are envisaged to shape BL human and physical decoding behaviours in M2. o Example include playing games that transpose into decoding place, value, and number as ML. o Monolingual and BL learners may share childhood experiences in ML, however BL experiences lend to different types of decoding behaviours. ▪ Example, gender, culture, politics, and game play may differ b/n cultures and languages. ▪ Technological environment. ▪ Grammars can also generate sounds that have gender meanings. • Includes describing BL human and physical behaviour as a 'memory recaller'. o BL learners need more 'time to practice	• Gender, culture, grammar, memory, perceptions, and motivation can create incongruent BL human and physical decoding behaviours in M2 o Generate fatigue. ▪ Extra effort in listening, speaking, decoding, writing, and thinking in M2. • Negative perceptions create incongruences. o Example, saying they are 'not good at mathematics' and are 'too old and lack education'. ▪ Anecdotally not true! o Poor experience in acquiring M2 in the classroom also detracts from learning. ▪ Tends to be true. o Sometimes emotionally challenging to attend English language classes when cultural beliefs clash.	• Imbed and focus on reconciling and managing appropriate BL human and physical decoding behaviours in M2. o Combining mathematics and language education can expedite the way BL learners acquire M2 as a human and physical behaviour. • Generate C, L, and S clues and cues that help BL learners make better connections conventions when learning M2. o Example, of body movements and decoding the Tasks by mimicking text sounds. • Some types of physical behaviours appear to lend more than others to decoding M2 as a foundational experience o Playing games and interacting in technologies, are examples. • Decoding mathematical signs (numbers) by colour, rhyme, and tactile interaction important when there is a distinct lack of foundational experience in decoding ML. o Articulate physical objects into abstracted signs. o Focus on bridging the gap that exist between decoding M2 as a coordinate and compound human

	remembering' mathematical processes in M1 before decoding M2. • BL reader perceptions play an important role in shaping biological behaviour. o Motivations. • Positive motivation lends to acquisition, and negative motivations detracts from learning (Gardner, Masgoret, Tennant, and Mihic (2004). • BL learners have strong views about how their age and migrant type experiences affects their abilities to decode and learn M2. o Maturity and settlement (necessity) reshape/shape behaviours.		and physical experience. o Includes making physical, tactile, and audible connections. • Focus on interacting more on remembering and transposing mathematics as a human and physical cross-C, L, and S experience. • Focus on emotional and perceptual behaviours. o Reconcile, for example, gender and culture when learning M2. ▪ However, can be difficult to interpret in a cross-C, L, and S activity. ▪ Not sure if, for example, the BL learners communicate (openly) about how gender and culture affects them in M2. • Reconcile and manage energy and fatigue when decoding M2. o Lower competencies usually mean higher the fatigue. o Fatigue affects the timing, quality, and quantity of M2 acquired. o Time and intensity. o Transpose mathematical meanings as BL memory type behaviour in M2.

Section 5: (TXT5000) BL cultural behaviours	• BL cultural behaviours identify the ideological and cultural experiences that affect BL decoding behaviours in M2. • M2 challenges and changes the cultural behaviours into thinking in an English, westernised, and Australian way in ML. o For example, statistics, graphs, and the letters (a, b, c, and d) in multiple-choices shape a particular type cultural behaviour. • M2 can share cultural meanings with M1, and this affects how neatly BL experiences are transposed into decoding M2. • However, cultural experiences are delicate and difficult to interpret. o BL learners tend to avoid asking metaphysical type questions that, for example, question the role of God in M2. o Ambiguous subjects such as alcohol are marriage are delicate subjects to reconcile in M2 as a cross-C, L, and S activity. o For ethical reasons it is difficult to question the cultural backgrounds of learners and its implications for decoding M2. • Settlement experiences generate new types of cultural behaviours in BL learners.	• Cultural codes create incongruences by the way they are decoded by BL learners. o Problems associated with decoding everyday expressions and idioms in M2. ▪ 'Tricky' unless explained properly in English. o Western epistemologies are ambiguous to interpret in M2. • BL learners usually find cultural concepts ambiguous to decode without support. o Example include, decoding constructs such as straightness, maximising, and minimising space, value, and money in M2. • While Barton (2009) and the Sapir-Whorf Hypothesis (1949, 1956) suggest incongruences exist because of the cultural the behaviours of learners in M2, it I not unsure if competencies are affected more by cultural backgrounds or the BL learners (lack of) educational experience in ML. o A combination of both factors need to be considered when teaching M2! • Cultural codes signal they also create social and economic inequities, because they preclude many BL learners from participating in employment and further training. o Teaching M2 to adult-CALD BL	• Imbed and focus on learning M2 as a cross-Cultural behaviour and experiences. • Focus on cultural immersion in M2, such as, connecting old and new cultural conventions. o Develop an independent understanding of the different types and levels of cultural meanings articulated in M2. o Forms part of a community language that includes, in part, learning everyday language and culture. • Focus on cultural incongruences that emerge. o Certain types of cultural conventions are more ambiguous and challenging to learn than others in a cross C, L, and S activity. o Reconcile the negative behaviours that emerge and affect decoding and acquiring M2 as a cultural behaviour. • Transposing and acquiring new cultural behaviours is physically and mentally demanding for the Participants as well, and this also requires education.

	○ Develop emancipative decoding behaviours that include acquiring M2 for gender, economic, and social reasons. ○ Settlement creates cultural challenges. ▪ BL learners set high expectations for themselves in education for settlement reasons. • Maintaining home language is important because it often seen as generating 'another chance' for migrant children', however their a disassociation occurs in maintaining traditional C, L, and S behaviours in L1/M1 and replacing it with L2/M2. ○ Migrant children prefer to speak English at home.	learners is important because they over represent lower socio-economic status in Australian society (ABS ALLS, 2006; Perkins, 2009).	• Acquiring M2 as a settlement goal is done at a cost. ○ Less leisure time and forgoing home culture and language • Adopt a constructivist approach to learning M2 by 'adaptation' and 'accommodation' (Piaget, 1970, cited in Hamers and Blanc, 2000, p. 16). ○ M2 as an additional rather than a superseding BL experience. • Educational experiences in M2, in part, act as a panacea that help overcome inequalities and barriers to learning and employment. • There is a window of opportunity and a time frame in which the BL learners are keen to participate in learning M2 and L2. After which it becomes more difficult to engage BL learners in M2 as a new cultural experience.

Section 6: (TXT6000) BL educational experiences	• BL educational experiences affect decoding behaviours in M2. o However, BL educational experiences vary more than they are necessarily similar. o M2 is acquired through different C, L, and S experiences, with different intensities over time. • Some educational experience lend more to decoding more mathematical signs than they do in everyday language. o Makes reconciling educational relationships difficult to generalise. o Explore educational experiences more through the C, L, and S contexts that shape educational experiences in M2. • Factors that shape educational experiences include: o Multiple-lingual experiences. o Different opportunities to attend formal English. o Current educational experiences. o Current migrant language class experiences. ▪ Favours decoding L2 over M2. Imbalance emerges. • Generally BL learners see themselves positively in education as part of settlement in Australia. o Believe M2 generates VET type	• ML requires a particular type of educational experiences to help decode the sign system (Halliday, 2006; O'Halloran, 2005; Sfard, 2005), o M2 requires an additional and different type of educational experiences to help decode ML in a cross-C C, L and S activity • Formal education can help expedite learning M2, however current educational experiences tend to create problems in their own rights. o May not address learning M2 as coding and BL decoding behaviour. o Ambiguous pragmatic and cultural signs not explained. o Imbalance in teaching L2 over M2. o Extensive resources and funding (AMEP and LLNP/SEE type programs) made available, but curriculum and teacher appear to lack a capacity to teach M2. • Teacher plays and important role as a medium when decoding and learning M2. o The medium is the message (McLuhan, 1964) • Current M2 education is discriminatory.	• Imbed learning M2 as an additional BL educational experiences, and focus on reconciling and managing the contexts that shape BL educational experiences in M2. o M2 as BL immersion. o Practice decoding the sign system as M2 o New words and sign meanings are important, however remembering and transposing mathematical knowledge between C, L, and S systems is also critical in learning M2. • Enhance the capacity of teachers to teach M2. • Gain a deeper understanding of the meaning and function of L and S structures in M2 and how they create problems. o Decoding philosophical and structural codes, for example, can make M2 easier to learn. • Focus on the role of the teacher as one of many mediums that are used to decode and learn M2. • Enhance the amount of M2 that is learnt over L2 in the classroom. • Scaffold M2 experiences through social interaction and relationships

	opportunities. ○ Problematic when current M2 educational experiences do not generate a learning opportunity.		(Vygotsky, 2011). ○ M2 not only acquired through formal classroom experiences. ○ Consider work and socialising applications and experiences. • Nonetheless, formal education grounds and expedites learning the more complex codes and conventions that encode M2 as a language. • Focus on learning M2 through 'case law' (Ernest, 2006) type experiences because it is possible to learn axiomatic, discursive, and grammatical codes without the reader being aware of how and why the codes function in M2. ○ Case experiences modify C, L and S behaviour through practice without the reader knowing why they necessarily doing things. ○ This is <u>not to say</u> learning M2 by deeper reasoning is unwarranted, however M2 can be acquired by repetition and immersion. • Imbed employment-related outcomes when learning M2. • Formal education in M2 can generate economic, social, aesthetic, cultural, and technological opportunities and benefits for BLs