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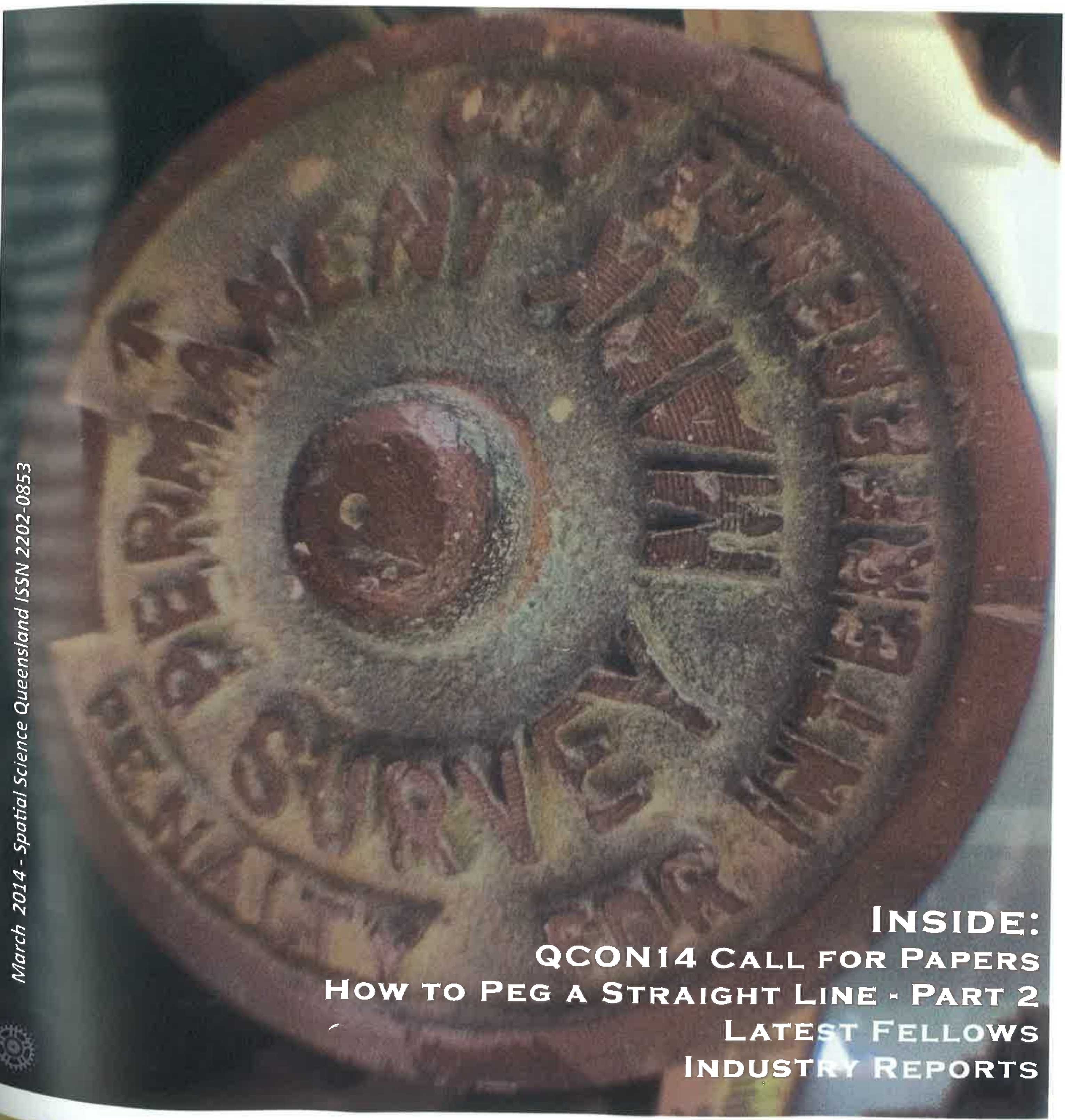

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COVER PHOTO BY INDUSTROPOP

HOW TO PEG A STRAIGHT LINE
PART 2
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The Solution

After you confirm with your client that it is a straight line on the ground that is required, you now decide to segment this line using appropriate geographic formula so you have equal ellipsoidal distances (rather than equal grid distances). Given the preceding information on geodetic lines we would be happy to follow the geodesic or either normal section in our example. We will use Vincenty's formula for following the straight line between the points on the ellipsoid (or ground in our example since we ignored the heights) (see Thomas & Featherstone, 2005 for a detailed vallidation). Vincenty's Formula are conveniently available to us in spreadsheet format from ICSM.

The first step is to enter the coordinates of Points 1 and 2 into Vincenty's Direct Formula as shown in Figure 6.

You now realise what the 'geodetic azimuths' from the inverse report in Figure 1 were referring to, even though they were not quoted to the same number of significant figures in that report. This provides a sense of comfort that you now understand what is going on, and perhaps enlightenment that this 'dark art' of geodesy may not be so daunting after all.

The next step is to use the azimuth from 1 to 2 and half the ellipsoidal distance (same thing as spheroidal distance) to calculate the centre point of your line (we will call this Point 4). And of course, being a thorough surveyor, you will also check this calculation back from Point 2 using the azimuth from 2 to 1. These calculations are shown in Figure 7.

You will note, as expected, that the coordinates are the same in both directions, and the reverse azimuths at the centre point are, allowing for a little rounding, 180° different meaning that they are tangential to the arc – again as expected.

To demonstrate how different this is from your earlier assumed centre point, the coordinates of this centre point (Point 4) converted using Refrearn's Formula, are E 749387.288 N 7066412.593 whereas the earlier coordinates of the centre point (Point 3) were E 749386.155 N 7066429.664. This means if you had pegged the first point without checking you would have placed the mark a little over 17 metres out of position – a good thing you are thorough.

Ellipsoid	GRS80					
Station 1	Point 1			Station 2	Point 2	
Latitude (φ_1)		-27°	00' 00.0000"	Latitude (φ_2)	-26°	00' 00.0000"
Longitude (λ_1)		155°	00' 00.0000"	Longitude (λ_2)	156°	00' 00.0000"
Spheroidal Dist. (S)	149041.3245					
Azimuth 1-2 (α_{12})		42°	12' 14.25840883"		User input	
Azimuth 2-1(α_{21})		221°	45' 27.85206163"		Result	

Figure 6 – Azimuths and distance between Point 1 and 2 from Vincenty

Ellipsoid	GRS80				
Station 1	Point 1		Station 2	Centre Point	
Latitude (φ_1)	-27°	00' 00.0000"	Azimuth (α_{12})	042°	12' 14.25840883"
Longitude (λ_1)	155°	00' 00.0000"	Ellipsoidal Dist (s)	74,520.66225	
Latitude (φ_2)	-26°	30' 03.21624 "	Reverse Azimuth (α_{21})	221°	58' 40.550415020"
Longitude (λ_2)	155°	30' 07.79004 "			

Ellipsoid	GRS80				
Station 1	Point 2		Station 2	Centre Point	
Latitude (φ_1)	-26°	00' 00.0000"	Azimuth (α_{12})	221°	45' 27.85206163"
Longitude (λ_1)	156°	00' 00.0000"	Ellipsoidal Dist (s)	74,520.66225	
Latitude (φ_2)	-26°	30' 03.21624 "	Reverse Azimuth (α_{21})	41°	58' 40.550415471"
Longitude (λ_2)	155°	30' 07.79004 "			

Figure 7 – Coordinate of centre of line from Points 1 and 2 using Vincenty

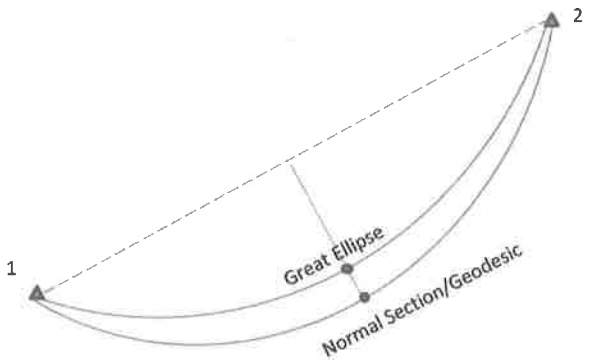

Figure 8 – Straight Line on CAD, Straight Line on the Ellipsoid, and Great Ellipse

If you now take the coordinates of Point 4 and plot them on your CAD package along with Points 1 and 2, you will see they are not in a straight line. Even though Point 4 is in a straight line between Points 1 and 2 on the ellipsoid/ground, it plots 12.492 metres off the straight line on CAD. The line bows away from the central meridian – in fact in general all such straight lines on the ellipsoid (normal section/geodesic) will bow away from the central meridian. It is now a simple matter to use Vincenty's Formula to calculate any other intermediate points at whatever interval you like to set out the straight line in the field for your client. Start with the coordinate of Point 1, input the forward azimuth (42°12'14".25840883) and then input different distances to segment the line as appropriate (and of course check the calculations in the reverse direction from Point 2) - problem solved. (If you want to follow through with these calculations yourself, three points, converted to grid coordinates, are provided in Table 3.)

I'm sure you were paying attention during the earlier discussion on great elliptic arcs. These will always be closer to the equator than the normal sections and this is obvious when you consider that the plane forming the great ellipse goes through the centre of the ellipsoid and the plane that forms the normal section contains the normal at the point (refer back to Figures 4 and 5). In our example the great ellipse is 11.71 metres off the straight line in CAD or 0.782 closer to the equator than the normal section.

Figure 8 is a summary of what we have discovered so far and this sort of diagram should be familiar if you have looked at the GDA Technical Manual lately (Intergovernmental Committee on Surveying and Mapping (ICSM), 2013).

Expanded Cadastral Context

It is interesting to consider how this line may be represented if it were a cadastral boundary instead of a conveyor belt. It would seem logical for the cadastral boundary to follow one of the normal sections (or geodesic for practical purposes). This is often done by segmenting the normal section/geodesic into 5km chords; the reason being that at 5km the arc-to-chord corrections are normally 2".5 or less and therefore plane bearings can be used for cadastral plan bearings. This has been done for the first three chords on the line in our example to see how good an approximation of the normal section curve this will provide. Starting at Point 1, Vincenty's Formula was used on an azimuth of 42°12'14".25840883 for ellipsoidal distances of 5, 10 and 15km (2.5km was also used to calculate an offset to the first chord). Note this is replicating calculations you would have made to peg the line anyway. These were

Table 3 –MGA94 Coordinates of Chord Points and Chord Bearings and Distances

		Grid Bearing Fwd and Rev	Grid/Plane Distance	Plane Bearing	**Ellipsoidal Distance**	**Rounded Bearing**
Point 1	E698454.234					
	N7011991.862					
		43°06'44".07				
			5000.473	43°06'42".2	**5000.0**	**43°06'40"**
		223°06'40".35				
5km	E701871.673					
	N7015642.319					
		43°06'40".31				
			5000.559	43°06'38".4	**5000.0**	**43°06'40"**
		223°06'36".53				
10Km	E705289.104					
	N7019292.902					
		43°06'36".55				
			5000.646	43°06'34".6	**5000.0**	**43°06'35"**
		223°06'32".70				
15Km	E708706.527					
	N7022943.611					

then converted to grid coordinates using Redfearn's Formula. From these coordinates grid bearings and distances, and plane bearings and distances were calculated and the bearings were rounded to the nearest 5" to show what might be quoted for each chord on a cadastral plan (refer to Table 3).

As discussed earlier, for most practical purposes the grid distance can be assumed the same as the plane distance and our calculations demonstrate this (as well as providing a check on our calculations). Each chord is 5km ellipsoidal distance. The grid distances reflect the line scale factor and vary for each chord line as expected. Remember the assumption in our example that, to simplify matters, all points are on the ellipsoid and therefore in our case the ellipsoidal distance will be the same as the ground distance (remember this is not the case in general though). In general we would expect the ground distance to vary for each chord due to different heights. What this means is the bold bearings and distances on the right hand side of Table 3 reflect the cadastral bearings rounded to the nearest 5" and the corresponding distances.

The rounded bearings are a little unsatisfying and somewhat confusing. The chord deflection is 3.8" but this is not obvious when the bearings are rounded. If long lines are to be depicted as normal sections/geodesics segmented into 5km chords, then we need to find a better way of showing the necessary details on a cadastral plan.

Calculations of the 2.5km chord reveal that the chord is 11mm offset from the normal section curve at that point (maximum separation). Decisions might also have to be had regarding whether or not 5km is the appropriate chord spacing. In this example 5km was used to carry out an investigation and the offsets and rounding are, of course, specific only to this case. Perhaps a broader examination needs to be undertaken into the survey plan guidelines and methods of depicting long line cadastral boundaries of this nature.

Lease Context

As a final discussion point, we will now briefly consider a different case of a lease bounded by constant latitudes and longitudes.

Lines of constant longitude (called meridians) are simply an extension of the example above and would be treated in the same manner. Unless the line was exactly on the central meridian of an MGA zone, then the line would bow away from the central meridian when plotted on the UTM/MGA projection. If the longitude of the line is on the central meridian the line would be a straight line coincident with the central meridian with forward and reverse azimuths of 0° and 180°. Regardless, the treatment described in the example above (using Vincenty's Formula) would still result in appropriate calculation of points along this line.

Lines of constant latitude (called small circles) are a different proposition though (we will assume we are not on the equator). The lines of constant latitude (except at the equator) will not contain the normal at the points and therefore they will not represent anything like a normal section (see Figure 9). Since the line of constant latitude is not a normal section, it needs to be treated differently.

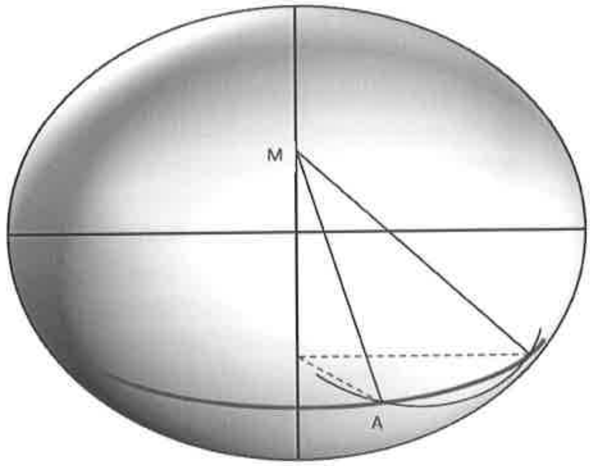

Figure 9 – 3D View of Small Circle at Point A

Let's look at a boundary that has to follow a latitude of -27° and extends from longitude 150° to 156° (from one side of MGA zone 56 to the other). If you put these two points into your CAD package and drew a straight line between them, you would be following a line of constant Easting, and not a line of constant latitude. If you pegged the centre point in this manner you would be about 3.5 km off line!

To peg the line of constant latitude in this case, instead of starting at one point and following a forward azimuth, you could simply use the constant latitude and varying longitudes to peg out the line (convert these to grid coordinates using Redfearn's Formula if you have to). If you decided to adopt the method in our earlier example you would still arrive at the correct answer though – you would just have to be careful of what azimuth you used (it will not be 90° as you may think – it will be more like 91°21'46".676...).

Because the terminal points are the same distance from the central meridian, both points have the same Northing on the UTM projection. This means the normal section/geodesic will follow a line of constant Northing and plot almost exactly as a straight line on the map projection. Note though that this is not following the line of constant latitude – we will look at that soon. I mentioned earlier that in general straight lines on the ellipsoid (normal section/geodesic) will bow away from the central meridian. There will be almost no bowing in this current situation though, and this is intuitive since the normal section line is at right angles to the central meridian (constant Northing) it is not possible to bow away from it. If there is no bowing, then you would expect the arc-to-chord correction to be zero - to help convince you, refer to Figure 10. The grid coordinates in this calculation are simply ϕ -27° λ 150° and ϕ -27° λ 156° converted using Redfearn's Forumla from ICSM spread-sheets.

Grid Bearing and Ellipsoidal Distance from Grid Coordinates — MGA

	Name	East (E)	North (N)	Zone
From (1)	101	202 273.9130	7 010 024.0332	56
To (2)	102	797 726.0870	7 010 024.0332	56
Ellipsoidal Distance (s)	595473.157			
Plane Distance (L)	595452.174			
Grid Bearing (β_1)	90°	00'	00.0000000000"	
Grid Bearing (β_2)	270°	00'	00.0000000000"	
Arc to Chord correction (δ_1)	0.00"			
Arc to Chord correction (δ_2)	0.00"			
Line scale factor (K)	0.999 964 76			

KEY: User input / Result

Figure 10 – Zero Arc-to-chord Correction for Line of Constant Northing

The line of constant latitude is an interesting one since it will plot as an arc and this may be counter intuitive for some. To help explain the situation, refer to Figure 11.

The distance between the intersection of the line of constant latitude with the central meridian and the intersection of the line of constant Northing and the central meridian is 3540.711 metres in this example and this provides some scale to the amount of bowing of this arc.

Lessons Learned and Conclusion

This discussion will now be rounded out with some key lessons learnt and some take-home messages.

· If you are dealing with long lines, it is important to agree with your client exactly what is meant if they ask you to set out a straight line. Does this mean a straight line on the ground, on the ellipsoid/globe ,or on the CAD design package?

· When dealing with long lines many of the assumptions we make for shorter lines are not valid and we need to treat them as geodetic problems rather than 'flat-earth society' problems.

· Calculations on the ellipsoid (using Vincenty's Formula) have been preferred in this paper since grid calculations can become a little unstable over long lines. In our earlier example you can input the coordinates of Point 1, set the forward azimuth and a distance of 39983770.016 (circumference of the earth at that point) and you will follow the normal section completely around the ellipsoid (the line is not repeatable though and you can refer to some of the quoted reference for adequate discussion on this). Try doing this with the ICSM GRIDCALC spread-sheet or similar formula though!

· We need a rigorous and repeatable method of depicting long lines on cadastral survey plans.

· Geodesy may not be the 'dark art' you may have first thought – it is just possible it has a useful place given today's reliance on technology such as GNSS, which is three dimensional and geodetic by nature.

It is hoped this paper has provided a useful refresher for some, and perhaps an eye opener for others. The context of long survey lines is a practical one and this has provided a simple platform to present several geodetic concepts that are becoming more important and relevant given recent advances in technology and computing power. Finally, it is reiterated that: a) the examples here have ignored the heights of points to simplify matters; and b) this paper is not meant to be definitive, rather it is designed to be read in conjunction with, and supplemented by, recognised reference works.

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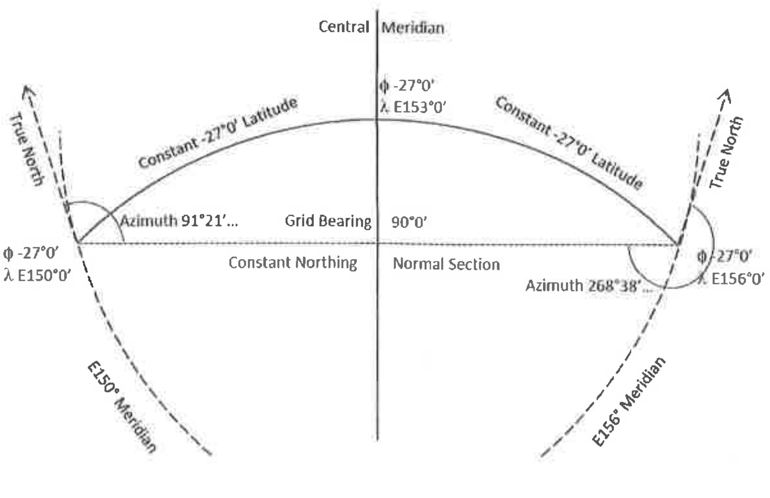

Figure 11 – Comparing a Line of Constant Latitude and Normal Section