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Tunnel Reinforcement Optimization for Nonlinear Material

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Abstract

In this paper, topology optimization is applied in optimizing tunnel reinforcement. Nonlinear behavior of geotechnical material is considered to illustrate the practical material behavior under working condition. The adjoint method is used to derive the nonlinear sensitivities. A revised bi-directional evolutionary optimization (BESO) is used to maximize the structural stiffness of reinforced tunnel with a prescribed volume of reinforcement. The developed BESO method is illustrated in a simple example of tunnel reinforcement design to verify the proposed approach.

Keywords: Topology optimization, tunnel reinforcement, BESO method, nonlinear material.

Introduction

Tunnel reinforcement is used to stabilize the opening and reduce deformation at the tunnel face. Due to complexity of geotechnical modeling, obtaining a reasonable design methodology is not very easy. Currently, tunnel reinforcement design is mainly relied on past experience and empirical recommendations. Design of tunnel reinforcement could be converted to optimizing the reinforcing material layouts in the design domain. Topology optimization can be used to deal with this type of problems and result in more efficient reinforcement design.

Topology optimization for continuum structures involves searching for an optimal layout of related members in a design domain under certain objective functions with defined constraints. A series of research were initially conducted to investigate applications of powerful topology optimization method on tunnel reinforcement design (Yin *et al*. 2000, Yin and Yang 2000a, Yin and Yang 2000b). Yin *et al.* (2000) initiated by applying the homogenization method in which every element in the design domain is assumed as a square cell made of original rock surrounded by reinforced rock. Linear elastic isotropic material behavior has been used in their model and the external work along the tunnel wall has been minimized under a prescribed reinforcement volume. Yin and Yang (2000a) conducted further research on optimizing tunnel support in various layered geological structure conditions. The Solid Isotropic Material with Penalization (SIMP) method was employed with a power-law interpolation to determine the optimum distribution of reinforcement density in the design domain. All of these layered geotechnical materials were regarded as working in the elastic regime. Another severe issue in tunnel reinforcement design, namely tunnel and sidewall heave caused by swelling or squeezing rock, was addressed in their continuous research by Yin and Yang (2000b). Liu *et al.* (2008) tackled a similar problem by another approach. A Fixed-Grid Bidirectional Evolutionary Structural Optimization (FG BESO), which was thought to overcome mesh-dependent zigzag of boundary problem in intermediate and final results, was proposed by Liu *et al.* (2008). A simultaneous optimization of shape and distributed reinforcement of an underground excavation for elastic material was also explored by Ghabraie (2010) using Bidirectional Evolutionary Structural Optimization (BESO) method.

These studies have opened a new trend in topology optimization applications on shape and reinforcement design of underground excavation. However, most of previous researches merely considered homogeneous, linear elastic material model, except for the nonlinear material model concerned for tunnel shape optimization by Ghabraie (2009). Although linear elastic material model is still commonly used in geomechanics, these results should be considered as the first step to move forward to more sophisticated nonlinear material models.

In this paper, application of topology optimization is investigated in finding optimal tunnel reinforcement design considering nonlinear material behavior. An effective optimization method, BESO, is employed. Firstly, sensitivity analysis for nonlinear material is derived based on which switching process is performed between original and reinforced elements. An elastic perfectly-plastic Mohr Coulomb model is utilized for both original and reinforced material. In order to overcome numerical instabilities, filtering and averaging techniques are also considered (Huang and Xie, 2007). For illustration, a simple example of optimizing tunnel reinforcement distribution is presented.

Optimization statement and sensitivity analysis

The investigated reinforcement design is aimed at minimizing a functional of the tunnel displacement under a predefined volume of reinforcement material. The external work along the tunnel wall is chosen as a proper objective function for measuring tunnel deformation. With linear material models, this objective function is equivalent to the mean compliance which has been widely used in linear structural topology optimization (Chu et al. 1996).

The optimization problem can be stated as:

$$\min W = \int \mathbf{f} \mathrm{d}\mathbf{u} = \lim_{n \to \infty} \left[\frac{1}{2} \sum_{i=1}^{n} (\mathbf{u}_i^T - \mathbf{u}_{i-1}^T)(\mathbf{f}_i + \mathbf{f}_{i-1}) \right] \tag{1}$$

subject to:

$$V_R = \sum_{e=1}^{M} V_e x_e$$

$$x_e \in \{0, 1\}$$

where W is the total external work, $\mathbf{u}$ is the displacement vector, $\mathbf{f}$ is the external force vector, n is the number of iterations in solving the non-linear equilibrium equations, V_e is the volume of element e, V_R is the prescribed reinforced volume, and M is the total number of elements in the design domain. x_e is the design variable of element e. $x_e = 1$ means that element e is reinforced and $x_e = 0$ means element e is not reinforced. Equilibrium requires the residual force vector to be eliminated, i.e

$$\mathbf{R} = \mathbf{f} - \mathbf{f}^{int} = 0 \tag{2}$$

where $\mathbf{f}^{int}$ is the internal force vector. The internal force vector can be expressed as:

$$\mathbf{f}^{int} = \sum_{e=1}^{M} \int_e \mathbf{C}_e \mathbf{B} \boldsymbol{\sigma} \mathrm{d}v = \sum_{e=1}^{M} \int_e \mathbf{C}_e \mathbf{B} \mathbf{D}_e \boldsymbol{\varepsilon} \mathrm{d}v \tag{3}$$

where $\mathbf{C}_e$ is the matrix to transform local force vector of element to global force vector, and $\mathbf{D}_e$ is the matrix defining the stress-strain relationship. The sensitivity of the objective function due to a change in variable x is:

$$\begin{aligned} \frac{dW}{dx} = \lim_{n\to\infty} \Bigg[& \frac{1}{2} \sum_{i=1}^{n} (\mathbf{u}_i^T - \mathbf{u}_{i-1}^T) \left(\frac{\mathrm{d}\mathbf{f}_i}{\mathrm{d}x} + \frac{\mathrm{d}\mathbf{f}_{i-1}}{\mathrm{d}x} \right) \\ & + \frac{1}{2} \sum_{i=1}^{n} \left(\frac{\mathrm{d}\mathbf{u}_i^T}{\mathrm{d}x} - \frac{\mathrm{d}\mathbf{u}_{i-1}^T}{\mathrm{d}x} \right) (\mathbf{f}_i + \mathbf{f}_{i-1}) \Bigg] \end{aligned} \tag{4}$$

Note that the second sum vanishes because at any point either the external force is zero or the displacement is fixed.

The adjoint method is applied by adding an adjoint term to the objective function (1):

$$W = \lim_{n\to\infty} \left[\frac{1}{2} \sum_{i=1}^{n} (\mathbf{u}_i^T - \mathbf{u}_{i-1}^T)(\mathbf{f}_i + \mathbf{f}_{i-1}) - \boldsymbol{\lambda}_i^T (\mathbf{R}_i + \mathbf{R}_{i-1}) \right] \tag{5}$$

Differentiating Eq. (5) and using Eq. (2) we obtain

$$\begin{aligned} \frac{dW}{dx} = \lim_{n\to\infty} \frac{1}{2} \sum_{i=1}^{n} \Bigg[& (\mathbf{u}_i^T - \mathbf{u}_{i-1}^T) \left(\frac{\partial \mathbf{f}_i}{\partial x} + \frac{\partial \mathbf{f}_{i-1}}{\partial x} \right) \\ & - \boldsymbol{\lambda}_i^T \left(\frac{\partial \mathbf{f}_i}{\partial x} - \frac{\partial \mathbf{f}_i^{int}}{\partial x} + \frac{\partial \mathbf{f}_{i-1}}{\partial x} - \frac{\partial \mathbf{f}_{i-1}^{int}}{\partial x} \right) \Bigg] \end{aligned} \tag{6}$$

In order to eliminate the unknown part $\frac{\partial \mathbf{f}_i}{\partial x} + \frac{\partial \mathbf{f}_{i-1}}{\partial x}$, $\boldsymbol{\lambda}_i$ is selected as

$$\boldsymbol{\lambda}_i = \mathbf{u}_i - \mathbf{u}_{i-1} \tag{7}$$

Substitute Eq. (7) into Eq. (6), the sensitivity of objective function is:

$$\frac{dW}{dx}=\lim_{n\to\infty}\frac{1}{2}\sum_{i=1}^{n}(\mathbf{u}_i^T-\mathbf{u}_{i-1}^T)\left(\frac{\partial\mathbf{f}_i^{int}}{\partial x}+\frac{\partial\mathbf{f}_{i-1}^{int}}{\partial x}\right) \tag{8}$$

In order to calculate the sensitivity of the internal force vectors, we use the material interpolation scheme suggested by Stolpe and Svanberg (2001) to evaluate the matrix $\mathbf{D}_e$ in terms of the design variables as

$$\mathbf{D}_e(x_e)=\mathbf{D}_e^2+\frac{x_e}{1+q(1-x_e)}(\mathbf{D}_e^1-\mathbf{D}_e^2) \tag{9}$$

where $\mathbf{D}_e$ is the stress-strain matrix of the e-th element, $\mathbf{D}_e^1$, $\mathbf{D}_e^2$ are the matrices of reinforced elements (material 1) and unreinforced elements (material 2). x_e is the design variable of the e-th element, $\mathbf{D}_e=\mathbf{D}_e^2$ at $x_e=0$ and $\mathbf{D}_e=\mathbf{D}_e^1$ at $x_e=1$. q is a penalty factor. A value of $q=0$ results in a linear interpolation and a positive value of q penalizes the intermediate values of design variables.

Differentiating Eq. (9), yields

$$\frac{\partial\mathbf{D}_e}{\partial x_e}=\frac{(\mathbf{D}_e^1-\mathbf{D}_e^2)(1+q)}{[1+q(1-x_e)]^2} \tag{10}$$

From Eq. (3) and Eq. (10), we have:

$$\begin{aligned}\frac{\partial\mathbf{f}^{int}}{\partial x_e}&=\sum_{e=1}^{M}\int_e \mathbf{C}_e\mathbf{B}\frac{\partial\mathbf{D}_e}{\partial x_e}\varepsilon dv\\&=\frac{(1+q)}{[1+q(1-x_e)]^2}\sum_{e=1}^{M}\int_e \mathbf{C}_e\mathbf{B}(\mathbf{D}_e^1-\mathbf{D}_\mathbf{e}^2)\boldsymbol{\varepsilon}\mathrm{d}v\\&=\frac{(1+q)}{[1+q(1-x_e)]^2}\left(\mathbf{f}_e^{int1}-\mathbf{f}_e^{int2}\right)\end{aligned} \tag{11}$$

Substitute Eq. (11) into Eq. (8), we have:

$$\begin{aligned}\alpha=\frac{dW}{dx_e}&=\frac{(1+q)}{[1+q(1-x_e)]^2}\lim_{n\to\infty}\frac{1}{2}\sum_{i=1}^{n}(\mathbf{u}_i^T-\mathbf{u}_{i-1}^T)\big(\mathbf{f}_i^{int1}-\mathbf{f}_i^{int2}+\mathbf{f}_{i-1}^{int1}\\&\quad-\mathbf{f}_{i-1}^{int2}\big)\\&=\frac{(1+q)}{[1+q(1-x_e)]^2}\left[\lim_{n\to\infty}\frac{1}{2}\sum_{i=1}^{n}(\mathbf{u}_i^T-\mathbf{u}_{i-1}^T)\big(\mathbf{f}_i^{int1}+\mathbf{f}_{i-1}^{int1}\big)\right.\\&\quad\left.-\lim_{n\to\infty}\frac{1}{2}\sum_{i=1}^{n}(\mathbf{u}_i^T-\mathbf{u}_{i-1}^T)\big(\mathbf{f}_i^{int2}+\mathbf{f}_{i-1}^{int2}\big)\right]\\&=\frac{(1+q)}{[1+q(1-x_e)]^2}(E_n^1-E_n^2)\end{aligned} \tag{12}$$

where E_n^1 and E_n^2 are the total strain energy of elements made of material 1 and 2, respectively. The sensitivity number (α) is a direct measure of variation of objective function. It can be clearly seen that the variation of objective function due to switching two materials is directly related to values of elemental total strain energy itself and independent of size of displacement increments.

The utilized optimization method requires a switching procedure of material between original rock and reinforced rock; hence, the element itself can be concerned as the design variable during optimization process.

Numerical calculation of sensitivity numbers

Elastic perfectly plastic Mohr-Coulomb model are employed for both original (material 2) and reinforced material (material 1). Two cases need to be considered as follows:

Case 1: the element is made of material 1 ($x_e = 1$), Eq. (12) becomes:

$$\alpha = (1+q)\left[E_n^1 - \lim_{n\to\infty}\frac{1}{2}\sum_{i=1}^{n}(\boldsymbol{\varepsilon}_i^T - \boldsymbol{\varepsilon}_{i-1}^T)(\boldsymbol{\sigma}_i^2 + \boldsymbol{\sigma}_{i-1}^2)V_e\right] \tag{13}$$

If $\varepsilon_n < \varepsilon_y^2$, with ε_y^2 being the yield strain of material 2, the element assumed to be made of material 2 is in its elastic region and Eq. (13) takes the form:

$$\alpha = (1+q)\left(E_n^1 - \frac{1}{2}V_e\boldsymbol{\varepsilon}_n^T\boldsymbol{\sigma}_n^2\right) \tag{14}$$

Otherwise, if $\varepsilon_n > \varepsilon_y^2$ that element behavior is elastic perfectly plastic and Eq. (13) takes the form:

$$\alpha = (1+q)\left\{E_n^1 - \frac{1}{2}V_e\left[\boldsymbol{\varepsilon}_y^{2T}\boldsymbol{\sigma}_y^2 + \left(\boldsymbol{\varepsilon}_n^T - \boldsymbol{\varepsilon}_y^{2T}\right)\left(\boldsymbol{\sigma}_n^2 + \boldsymbol{\sigma}_y^2\right)\right]\right\} \tag{15}$$

Or

$$\alpha = (1+q)\left\{E_n^1 - \frac{1}{2}V_e\left[\boldsymbol{\varepsilon}_n^T\left(\boldsymbol{\sigma}_n^2 + \boldsymbol{\sigma}_y^2\right) - \boldsymbol{\varepsilon}_y^{2T}\boldsymbol{\sigma}_n^2\right]\right\} \tag{16}$$

Case 2: the element is made of material 2 ($x_e = 0$), Eq. (12) becomes:

$$\alpha = \frac{1}{1+q}\left[\lim_{n\to\infty}\frac{1}{2}\sum_{i=1}^{n}(\boldsymbol{\varepsilon}_i^T - \boldsymbol{\varepsilon}_{i-1}^T)(\boldsymbol{\sigma}_i^1 + \boldsymbol{\sigma}_{i-1}^1)V_e - E_n^2\right] \tag{17}$$

If $\varepsilon_n < \varepsilon_y^1$, with ε_y^1 being the yield strain of material 1, the element assumed to be made of material 1 is in its elastic region and thus Eq. (17) takes the form:

$$\alpha = \frac{1}{1+q}\left(\frac{1}{2}V_e\boldsymbol{\varepsilon}_n^T\boldsymbol{\sigma}_n^1 - E_n^2\right) \tag{18}$$

Otherwise, if $\varepsilon_n > \varepsilon_y^1$, that element behavior is elastic perfectly plastic and Eq.(17) takes the form:

$$\alpha = \frac{1}{1+q}\left\{\frac{1}{2}V_e\left[\boldsymbol{\varepsilon}_y^{1T}\boldsymbol{\sigma}_y^1 + \left(\boldsymbol{\varepsilon}_n^T - \boldsymbol{\varepsilon}_y^{1T}\right)\left(\boldsymbol{\sigma}_n^1 + \boldsymbol{\sigma}_y^1\right)\right] - E_n^2\right\} \tag{19}$$

Or

$$\alpha = \frac{1}{1+q}\left\{\frac{1}{2}V_e\left[\boldsymbol{\varepsilon}_n^T\left(\boldsymbol{\sigma}_n^1 + \boldsymbol{\sigma}_y^1\right) - \boldsymbol{\varepsilon}_y^{1T}\boldsymbol{\sigma}_n^1\right] - E_n^2\right\} \tag{20}$$

where: σ_y^1, ε_y^1, σ_y^2, ε_y^2 are the yield stresses and yield strains of material 1 and material 2, respectively.

BESO procedure:
Based on the derived sensitivity numbers, BESO procedure repeatedly switches elements between the two phases of material: original rock (weak element) and reinforced rock (strong element). It has been realized that the BESO optimization method is prone to numerical instabilities due to mesh dependency and formation of checkerboard patterns (Sigmund and Petersson, 1998). In order to overcome these deficiencies, a linear filtering technique (Huang and Xie, 2007) is utilized here. The filtered sensitivity number for an element can be calculated as:

$$\hat{\alpha}_i = \frac{\sum_{j=1}^{M}\omega(r_{ij})\alpha_i}{\sum_{j=1}^{M}\omega(r_{ij})} \tag{21}$$

where *M* is the total number of elements, and $\omega(r_{ij})$ is a weight factor given as:

$$\omega(r_{ij}) = \max\{r - r_{ij};\ 0\} \tag{22}$$

where r_{ij} is the distance from centers of elements i and j, and *r* is the (predefined) filter radius.

Huang and Xie (2007) suggested averaging the sensitivity numbers in consecutive iterations to enhance the convergence properties of the method. The averaging scheme is presented as:

$$\alpha_i = \frac{\alpha_i^k + \alpha_i^{k-1}}{2} \tag{23}$$

Where *k* is the current iteration number. Then the value of $\alpha_i^k = \alpha_i$ would be used for the next iteration.

Example and Discussion

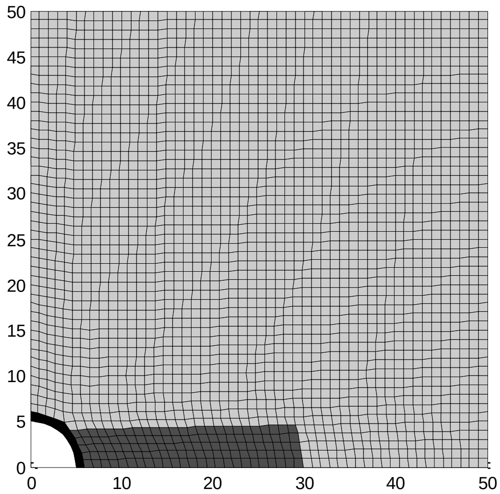

Figure 1. Initial guess design

To verify the approach it is used to optimize the reinforcement design of a circular tunnel under hydrostatic *in situ* stress conditions. Volume of reinforced material is predefined as 5 percent of the design domain area. A plane strain circular tunnel under geostatic condition is considered. The design domain is taken as a square of side length 10*h* (*h* is the tunnel diameter and *h* = 10m) The area around the tunnel wall is restricted to non-designed elements illustrated by black colored elements. An initial support design (the dark grey area) is also assumed as sketched in Fig. 1**.** Due to symmetry condition, only a quarter of design domain is modeled in finite element analysis with suitable symmetric constraints. Rock mass around the tunnel is assumed to be homogeneous. Other considered engineering properties of original rock and reinforced rock are shown in Table 1.

Table 1: Material properties

Material properties	**Original rock**	**Reinforced rock**
Young modulus (GPa)	0.1	0.3
Poisson’s ratio	0.3	0.3
Friction angle (^{0})	27	32
Dilation angle (^{0})	0	0
Cohesion (MPa)	0.1	0.3

The optimum reinforcement distribution achieved is shown in Fig. 2. As expected the tunnel reinforcement is distributed evenly around the tunnel wall in hydrostatic stress condition. The objective function variations are also shown in Fig. 2. It can be seen that the objective function reduces gradually with smooth changes and converges to a minimum after a few iterations.

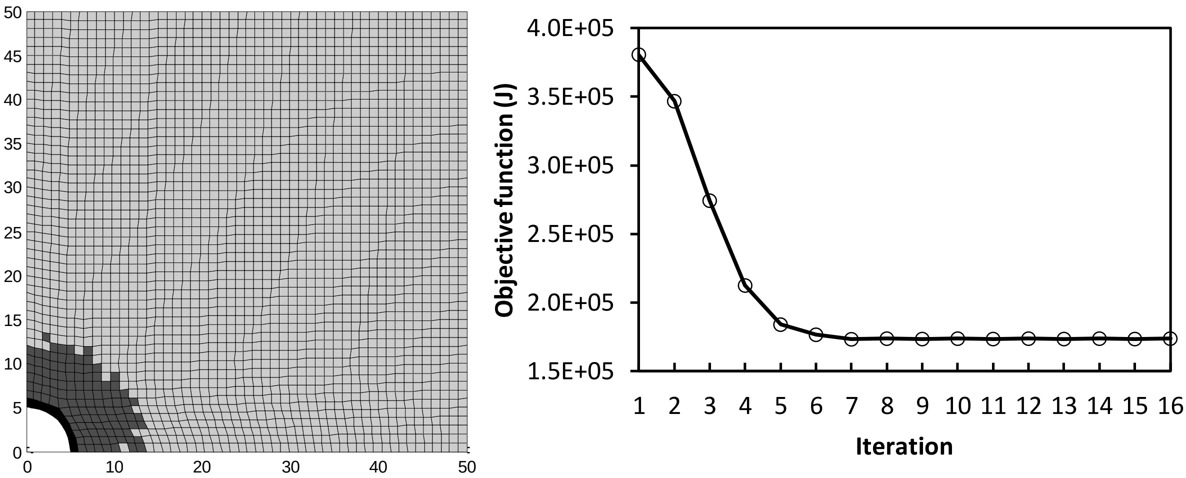

Figure 2. Reinforcement distribution and variation of objective function

Conclusion

A newly derived sensitivity analysis for reinforcement design in BESO method with nonlinear materials has been presented. In this approach, elastic perfectly plastic Mohr-Coulomb model has been chosen to consider material behavior. The results of a simple example involving optimal reinforcement distribution around a circular tunnel have been illustrated. It is shown that application of topology optimization in tunnel support design considering nonlinear material behavior is achievable with relatively simple numerical methods. More complicated geotechnical properties and geological conditions should be considered in future research to obtain more realistic material behavior.

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