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Solar Radiation and the UV index: An application of Numerical Integration, Trigonometric functions, Online Education and the Modelling Process

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Abstract

A short series of practical classroom mathematics activities employing the use of a large and publicly accessible scientific data set are presented for use by students in years 9 and 10. The activities introduce and build understanding of integral calculus and trigonometric functions through the presentation of practical problem solving that focuses on Public Health and developing a personal understanding of solar ultraviolet radiation and the UV Index. The classroom activities are presented using an approach that encourages self-discovery of mathematical concepts by application of mathematical modelling. These activities also develop critical thinking skills through application of data mining, data processing and presentation. They also emphasize the importance of drawing valid conclusions, concepts important to scientific research, statistics and epidemiology.

Key words: UV index; Online; Mathematics education; Mathematics modelling; Big data

Introduction

The application of mathematics in deriving, simulating and modelling real world phenomena is essential in developing an understanding of physical processes. These concepts are often exposed to students as part of many and various national curriculum studies in science but are not often presented in the mathematics classroom until the later years of schooling. It is important to recognize that learning mathematics need not be done in isolation or through the presentation of simplified abstract concepts or algorithms; it is a process in which the derivation of a solution is just as important as the solution itself (Lesh & Doerr, 2003). An approach that builds upon the knowledge and skills already developed by students that emphasizes how to build a working model for any practical situation will contribute significantly to strengthening meaningful mathematical understanding of existing concepts and assist in developing new ones (Novak, 2010).

We present a short series of practical activities to investigate whether an approach that combines practical modelling and problem solving related to an important scientific concept can work with a group of motivated middle school aged mathematics students. To do this, our mathematics student group is presented with the idea of a numeric index. Like other indices, the ultraviolet index (UVI) represents a more complex physical reality that is not easily understood by the general public. It is a simplification. Importantly, this common public health concept lends itself to open discussion of mathematical concepts developed during the middle years of schooling. To understand the physical processes that affect solar radiation we present our student group with the physical meaning of the UVI and introduce two important mathematical concepts, integration and trigonometry as tools that can be used to measure and model it. The emphasis here has been to not separate the physics from the mathematics but to present both concepts as an integral part of the mathematical modelling process. These ideas were presented as part of the University of Southern Queensland's *Mathematics Enrichment* program.

Making Models

The USQ *Mathematics Enrichment* program was used to facilitate the delivery of two practical problem solving exercises related to solar radiation and the UVI. The program is specifically designed to develop an interest in mathematics by students aged between 13 and 15 in years 9 and 10 from local and remote schools within the southern Queensland geographical region. The region of participating schools includes the city of Toowoomba

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(approximate population of 130 000), the Brisbane city satellite suburb of Springfield (approximate population of 50 000) and surrounding rural communities (up to 200 km from the Toowoomba university campus).

The *Mathematics Enrichment* program encourages students from participating schools with an interest in mathematics to visit the university Toowoomba or Springfield campuses up to six times throughout the academic year. Typically, groups of up to ten students per participating school attend scheduled *Mathematics Enrichment* activities with twenty to fifty students in one session. The program is co-educational, meaning student groups from participating schools are mixed into non-school specific student groups of up to five per group to solve applied mathematics problems presented by a visiting scholar or staff member of the university. Students unable to attend in person are often encouraged to join working groups via online internet communication and video link ups. Students attending one of the *Mathematics Enrichment* sessions were introduced to solar radiation stimulus and problem solving material over two, one hour Activity sessions. Each one hour Activity session was roughly divided between 30 minutes to introduce the topic and 30 minutes of student practice time. Sessions were held at the Toowoomba campus, repeated at the Springfield campus, and another was conducted in a local school (Table 1).

Table 1. UVI Mathematics Enrichment activities.

Session	Concept Discussion (30 minutes)	Student Practice (30 minutes)
Activity 1	Solar radiation science and measurement	1. Measuring solar spectra integrals
Activity 2	Trigonometric modelling and the UVI	2. Plotting and modelling the daily UVI

The presented *Mathematics Enrichment* activities were planned with a specific focus on the mathematical modelling process that has proved successful in secondary school settings (Stillman et al. 2007). Another focus was to show students how mathematics is part of their everyday life, by creating well-planned teaching sessions that involved input from mathematicians, mathematics educators and pre-service teachers (Galligan, 2015). The sessions involved guided group discussions to work through the planned activities. In summary, this involved group formulation of the problem, discussion of assumptions and variables, development of a mathematical solution, modelling of possible solutions, and interpretation of the real world meaning and further model refinement.

Activity 1 (Concept Discussion): Solar Ultraviolet Radiation

During the middle schooling years it is not uncommon for students to engage with problems involving the calculation of simple physical laws and their units of measurement. A common unit of measurement relevant to the use and conservation of the world's energy resources is the watt. A watt (W) is the derived SI unit that defines Power, or energy use per unit time in joules per second (J/s) (Thompson & Taylor, 2008). As an example, a household light bulb may be rated at 100 W, effectively using 100 joules of electrical energy per second. Compare this type of energy usage with a Light Emitting Diode (LED). Such diodes are typically found in sets, arrayed for use as traffic lights, bicycle lamps, or flashlights. This is because a typical LED uses energy at a much lower rate (and is therefore a much more efficient use of battery life) than a common light bulb. A LED may produce a very bright light, comparable to a light bulb and be rated at only 25 mW, effectively using only 0.025 joules of energy per second. The Sun (Figure 1) uses energy at a much higher rate.

In comparison, the Sun outputs energy across the electromagnetic spectrum with a rated power of approximately $3.86\text{x}10^{26}$ W. An astronomical number! However, the inverse square law of light (Equation 1) greatly reduces the total solar irradiance received by the Earth. At an approximate distance of 150 million km, the Sun's output as measured above the Earth's Atmosphere is reduced to a constant of 1360 to 1361 W/m^2 (NASA, 2015a), with the variation being accounted for by the Earth's elliptical orbit and the corresponding distance to the Sun (d).

$$E(l) = \frac{1}{d^2} \qquad (1)$$

The energy received per second per m^2 is defined as the total solar irradiance (E). This irradiance is the summation of the radiant energy received per second for all wavelengths (λ) in the electromagnetic spectrum, ranging from the very short wavelength gamma-rays to the very long wavelength radio-waves. The *solar constant*, being between 1360 and 1361 W/m^2 is the integral (or area under the curve) of the received spectral irradiance measured over the full range of wavelengths in the solar spectrum. The solar spectrum from the ultraviolet to near infrared is shown in Figure 2 including the ultraviolet (UV) wavelengths from 290 to 400 nm that reach the Earth's surface.

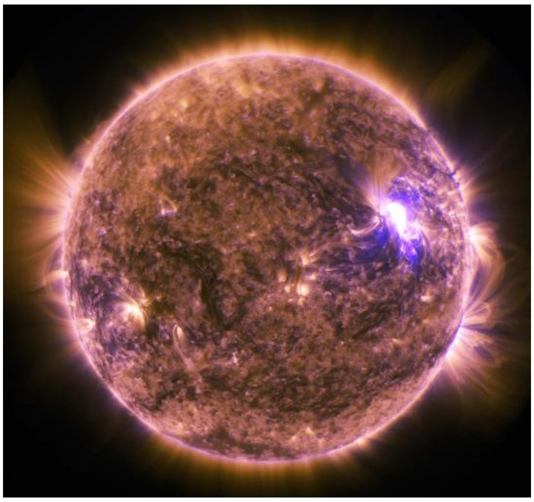

Figure 1. NASA Solar Dynamics Observatory image (NASA 2015b). Although the Sun's surface appears to vary regularly on a day to day basis the radiation leaving the surface and reaching Earth varies very little and remains close to a constant 1360 W/m^2.

It can be seen from Figure 2 that the peak irradiance received from the Sun lies in the visible region of the electromagnetic spectrum. The peak irradiance of the Sun may be explained effectively using a *black-body* model. The peak emission of a *black-body* is described by Wien's law which states that the peak wavelength of the radiant energy (Q_{peak}) emitted by any body above absolute zero is inversely proportional to its absolute temperature (T). Students may note an inversely proportional relationship is observed by tracing a line through the peak irradiance of the *black-bodies* depicted in the Figure where the irradiance is the equivalent of radiant energy received per m^2 per second.

$$Q_{peak}\ \mu\ \frac{1}{T} \qquad (2)$$

The surface of our Sun is approximately 6000 K and as such its spectral irradiance peaks in the visible region of the electromagnetic spectrum at approximately 500 nm with cooler bodies peaking at a lower irradiance and longer wavelength. Like other bodies above absolute zero, the Sun emits radiation across the electro-magnetic spectrum although the distribution of this radiation is not as smooth as predicted by a theoretical *black-body* curve for an emitter at 6000 K. In reality, there are dips in the solar spectrum caused by the discrete absorption of radiation in the Sun's atmosphere.

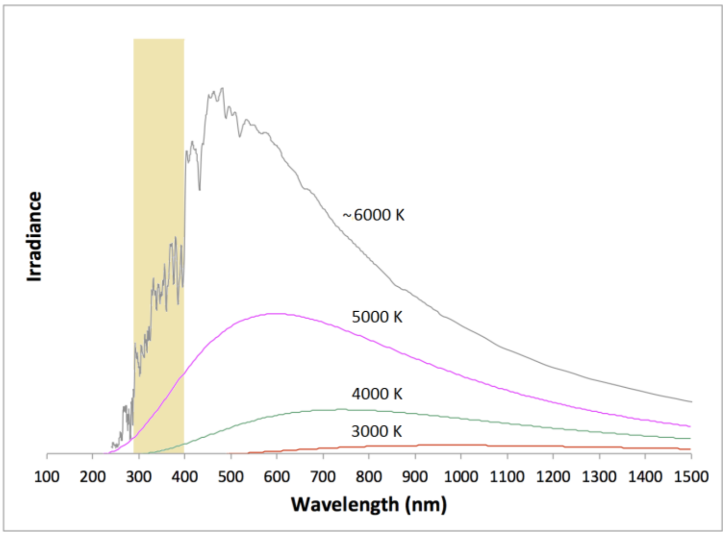

Figure 2. Theoretical black-body emission curves at 3000 K, 4000 K and 5000 K and the solar spectral irradiance above the Earth's atmosphere measured from 240 nm to 1500 nm by SIM (Spectral Irradiance Monitor) onboard the SORCE (Solar Radiation and Climate Experiment) satellite (Woods et al. 2015). The highlighted ultraviolet wavelengths that reach the Earth's surface range from 290 to 400 nm.

Solar Spectrum at the Earth's Surface

Absorption of solar radiation across the electromagnetic spectrum is wavelength dependent and occurs in the Earth's atmosphere due to the presence of atmospheric gases. Backscatter by larger particulates such as dust and water droplets in the atmosphere account for a further reduction of the incoming solar radiation. Atmospheric absorption is particularly strong in the shorter wavelengths of the solar spectrum, and accounts for the complete removal of radiation below 280 nm (Madronich & Flocke, 1997).

Absorption of solar radiation in the UV waveband by stratospheric ozone (O_3) is particularly important for life on Earth and plays a critical role in human health. Solar radiation that reaches the surface in the UVB wavelengths (290 to 315 nm) contributes significantly to sunburn (erythema) and snow blindness (photokeratitis) as well as long term health ailments, including cataract and immunosuppression (Longstreth et al. 1995). Radiation in the longer UVA wavelengths (315 to 400 nm) penetrates deeper into the skin tissue than UVB (Agar et al. 2004), and is strongly associated with skin photo-ageing (loss of elasticity and wrinkling). Both UVA and UVB wavelengths are significant environmental carcinogens linked with the development of skin cancers in fair skinned population groups (Ramos et al. 2004, Armstrong & Kricker 2001).

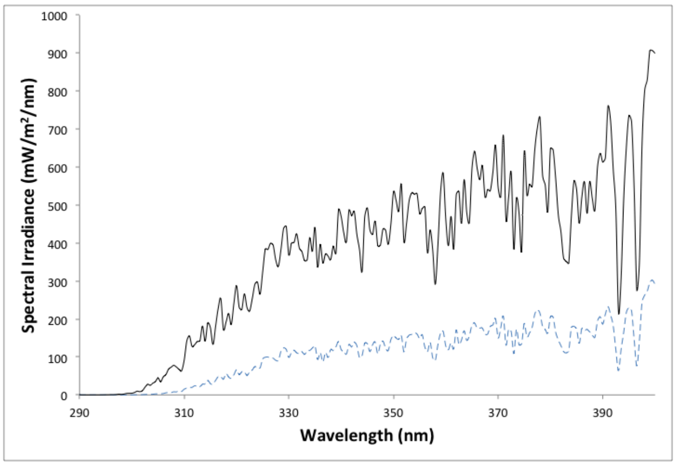

Figure 3. The measured ultraviolet spectral irradiance on a cloud free, mid-autumn day at solar noon, 12:00 pm (solid) and in the early morning, 8:00 am (dashed) for spectra downloaded from the south east Queensland ultraviolet data set.

The measured surface irradiance from 290 to 400 nm is shown in Figure 3 during the early morning (8:00 am) and near solar noon (12:00 pm) at a subtropical latitude in southern Queensland, Australia. Unlike the extra-terrestrial solar irradiance, the spectral irradiance reaching the Earth's surface is not constant but depends on several variables. Although the spectra in Figure 3 resemble the UV waveband of Figure 2 the spectra in Figure 3 show the progressively stronger influence of absorption by O_3 at shorter wavelengths with very little solar radiation being recorded at wavelengths below 300 nm. The influence of a longer optical path in reducing the measured spectral irradiance is also evident at 8:00 am as a result of greater absorption and atmospheric scattering than for the spectrum measured at 12:00 pm. This is an important concept for the mathematical modeler who wishes to take into account daily variations in the received surface irradiance. Figure 3 therefore highlights the significant influence of the Solar Zenith Angle (SZA), that is, the angle of the Sun measured with respect to the zenith.

Activity 1 (Student Practice): Measuring the Solar Spectrum

Following discussion of the preceding solar radiation concepts, students were tasked with calculating the UVI from spectral solar radiation data for a specific date and time of day. This involved students downloading unweighted solar spectra from the South East Queensland ultraviolet monitoring dataset (SEQUV) at seven time stamped intervals from the previous day's measurements, namely 6:00 am, 8:00 am, 10:00 am, 12:00 pm, 2:00 pm, 4:00 pm and 6:00 pm (Australian Eastern Standard Time, AEST).

The SEQUV data set is a collective of solar radiation and atmospheric monitoring data maintained by the University of Southern Queensland's *Solar Radiation Research Group*. This online data set has been developed and maintained from measurements recorded since 2003 and currently exceeds 6 Terra bytes of historical and real-time atmospheric and solar radiation research data. Divided between three broadband UV radiation monitors, two broadband solar radiation sensors, a sky camera imaging system and scanning UV spectroradiometer, information collected by these instruments from between 1 and 5 minute intervals daily is stored, processed and uploaded to the group's *Solar Radiation* public website available at (http://www.usq.edu.au/faculty/science/depts/biophysci/weather/Biometer/biometer1.htm) and the Queensland Cyber Infrastructure Fund (QCIF) password protected network for long term storage and public access (https://Q0076.qcloud.qcif.edu.au). Data recorded by the USQ scanning spectroradiometer (Bentham Instruments, model DTM300, Reading UK) and stored on the QCIF network was made available online by providing password access to the participating student groups in the *Mathematics Enrichment* program. UV spectra recorded by this instrument is made in 0.5 nm steps between 290 and 400 nm.

Spectral UV irradiance SEQUV data are stored in csv format and may be opened as a delimited spreadsheet listing wavelength in the first column and the calibrated spectral UV irradiance in the second column. Data for 21 April 2015 at 8:00 am and 12:00 pm are plotted from these two spreadsheet columns in Figure 3. Students working in small groups of between 3 to 5 students were asked to estimate the Total UV irradiance for their designated measurement time (6:00 am, 8:00 am, 10:00 am, 12:00 pm, 2:00 pm, 4:00 pm or 6:00 pm) from pre-prepared printouts of the spectral UV irradiance. Printouts resembled the UV irradiance plots shown in Figure 3.

Students were not guided on how to calculate the area under the spectral irradiance curve. A common solution however involved drawing a visual approximation to the line of best fit through the data and calculating the area of a simple triangle to estimate the integral. An example is shown in Figure 4.

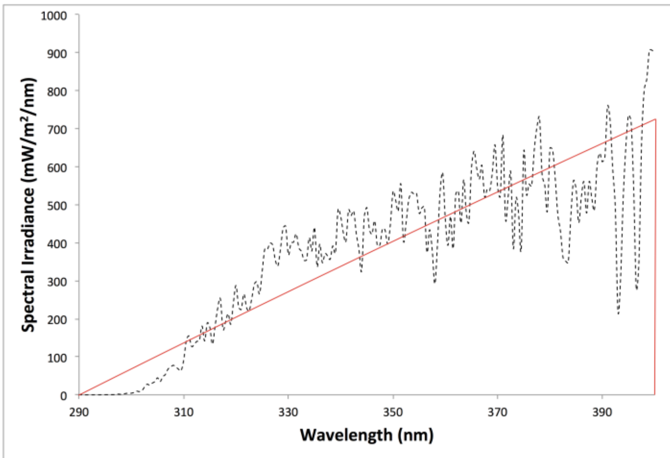

Figure 4. Estimating the UV irradiance on 21 April 2015 at 12:00 pm by the calculation of the area bounded by the solid line (a triangle). In this case the area under the curve is approximated to 35 W/m^2.

Other students divided irradiance plots into rectangles and estimated the integral by the summation of rectangular areas. This concept was then extended to show how the total irradiance could be estimated as a Riemann sum where the UV irradiance, E_{total} (W/m^2) is determined by integrating the spectral irradiance with respect to wavelength (in nm),

$$E_{total} = \int_{\lambda=290}^{400} E(\lambda)d\lambda \qquad (3)$$

or more specifically, the area of each rectangle in a numerical summation is found by the product of the Spectral Irradiance, $E(\lambda)$ and wavelength, $\Delta\lambda$ for discrete step sizes in the range 290 to 400 nm.

$$E_{total} \approx \sum_{\lambda=290}^{400} E(\lambda)\Delta\lambda \qquad (4)$$

This numerical approximation accurately measures the integrated spectral UV surface irradiance provided $\Delta\lambda$ is small. It may be noted for the spectra shown in Figure 3, the incident solar radiation of 1360 W/m^2 available at the top of the atmosphere (Figure 2) has been reduced to 41 W/m^2 in the UV waveband at 12:00 pm. This is also a more precise solution than the estimate of 35 W/m^2 found using the technique illustrated in Figure 4.

Activity 2 (Concept Discussion): Introducing the UV Index

Erythemally Effective UV

The World Health Organization (WHO) promotes a standardized public health measure for reporting the UV irradiance, the UVI (WHO, 2002). The internationally adopted UVI standard (WMO, 1997) includes several categories or bands on an open ended linear scale (Table 2). The UVI weights the spectral surface irradiance to those wavelengths that cause a sunburning (or erythema) reaction in human skin. The relative effectiveness of wavelengths responsible for any biological reaction including an erythema reaction are collectively known as *Action Spectra.*

Table 2. The UV index categories.

Category	Color code	UVI	E_{ery} (mW/m^2)
Low	Green	0 to 2	0 to 75
Moderate	Yellow	3 to 5	75 to 150
High	Orange	6 to 7	150 to 200
Very High	Red	8 to 10	200 to 275
Extreme	Purple	11+	275+

Action spectra are commonly applied to the spectral irradiance to measure many recognized biological responses in humans, animals and plants. Figure 5 shows three common biological UV action spectra for erythema (CIE, 1998), photokeratitis (CIE, 1986) and the production of vitamin D (CIE, 2006).

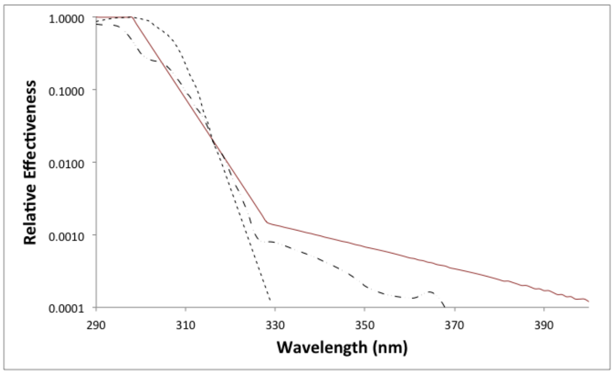

Figure 5. Action spectra for the human erythema biological response (solid), vitamin D production (dashed) and photokeratitis (dot dashed).

Weighting the UV irradiance to the biologically significant erythemally effective Action spectrum greatly reduces the magnitude of the integrated irradiance by effectively removing the influence of the stronger UVA wavelengths. When the erythemal action spectrum is applied to the spectra shown in Figure 3, the erythemally effective UV irradiance, E_{ery} at 12:00 pm becomes 0.155 W/m^2 (and only 0.022 W/m^2 at 8:00 am). This is measured by weighting the incident spectral UV irradiance with the erythemally effective action spectrum, $A(\lambda)$ and determining the integral numerically.

$$E_{ery} = \sum_{\lambda=290}^{400} A(\lambda)E(\lambda)\Delta\lambda \qquad (5)$$

It can be seen that the solar radiation received by the Sun that has a measureable biological impact on human health is very small when compared to the original radiation source. The UVI is typically employed to clearly represent the meaning of these low magnitude integrals of erythemally weighted solar radiation at the Earth's surface. As an index, the UVI has no units and is formally defined by the ratio,

$$UVI = \frac{E_{ery}}{25} \qquad (6)$$

where E_{ery} is expressed in mW. Note that a UVI of 1 corresponds to 25 mW/m^2 of erythemally effective solar radiation, the same amount of energy used per second by a typical LED. Similarly a UVI of 10 represents 250 mW/m^2 of erythemally effective radiation. The spectra from Figure 3, when weighted to the erythemal action spectrum may now be expressed as a UVI of 5 at 12:00 pm and 1 at 8:00 am respectively.

Predicting the UVI

The UVI can reach extreme ranges exceeding 18 in the high altitude locations of tropical South America (Luccini et al. 2006) and often exceeds 10 in tropical to subtropical latitudes during summer. Personal protection and solar radiation avoidance is recommended by the Cancer Council (2014) when the UV index is 3 or higher. In Australia, UV alert times are published in newspapers and broadcast by national media services when the UVI exceeds 3. Figure 6 is a typical UVI chart available from the Australian Bureau of Meteorology (ABOM) showing the predicted UVI and UV alert times for the Tropical North Queensland township of Cooktown (15.5^{o}S, 145.3^{o}E). The figure, published for 11 September 2015 includes UV alert times between 8:50 am and 3:40 pm and is color-coded according to the UVI standard categories.

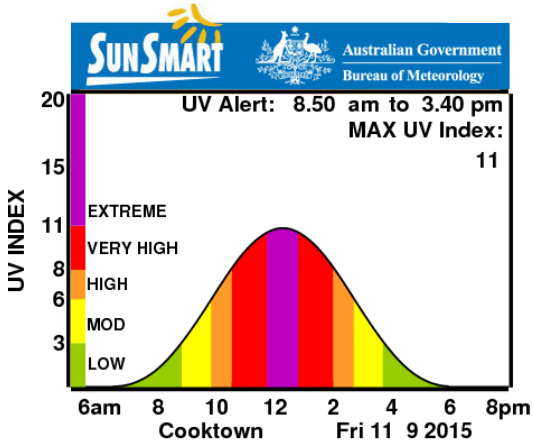

Figure 6. Predicted UVI chart for Cooktown, tropical North Queensland on 11 September 2015 (ABOM, 2015).

The UVI prediction chart shows several key features that may be utilized by middle school aged children developing an understanding of the mathematical modelling process. In fact, a UVI model can be developed relatively easily by application of a periodic sine or cosine function. It can be seen that the most significant factor affecting such a model is the SZA. As shown in Figure 6, the maximum solar irradiance will occur at noon when the Sun is overhead and the optical path through the atmosphere is at a minimum. The minimum solar irradiance will occur when the sun is on the horizon as it is at this time that the optical path through the atmosphere will be the longest. A noteworthy observation of the simple trigonometric sine ratio shows that if the sine ratio were used to model the effective solar irradiance, the ratio will tend toward 0 if the Sun were on the horizon (0^o and 180^o) and will reach a maximum value of 1 if the Sun were directly overhead (90^o).

Activity 2 (Student Practice): Measuring and Modelling the UV Index

Plotting the Measured UVI

Spectral UV data, made available through the QCIF website was weighted to the erythema action spectrum (CIE, 1998). This analysis was done in groups, with each student group determining the erythemal UV irradiance for either 8:00 am, 10:00 am, 12:00 pm, 2:00 pm, 4:00 pm or 6:00 pm. To calculate the erythemal UV irradiance students multiplied each unweighted wavelength in their downloaded spectral irradiance data file with the corresponding relative effectiveness of the CIE (1998) action spectrum. Here, the CIE (1998) action spectrum was provided as a single column of an excel spreadsheet for the wavelengths between 290 to 400 nm. Students were then able to determine E_{ery} as a Riemann sum by multiplying the weighted spectral irradiance by 0.5 nm and calculating the sum of areas for each step size from 290 to 400 nm. The integral was then divided by 25 mW/m^2 (Equation 6) to calculate the measured UVI.

A sample of student group results for the activity run on 22 April 2015 is given in Table 3. Group results were plotted with respect to the local time, showing the measured variability in the UVI for the day before, 21 April 2015 (Figure 7). The group results highlighted several important concepts. Namely, that the Sun does not reach 90^o in elevation (0^o SZA) at the site where the data was collected, the UVI is 0 when the Sun is below the horizon (negative elevation angles), and the actual daily UVI does not necessarily follow the predicted smooth curve typical of daily UVI forecast charts.

Table 3. Erythemal spectral irradiance data and easterly solar elevation angle (ESE) for 21 April 2015.

SEQUV record (yyyymmddhhmm)	Local Time (AEST)	SZA	ESE	*Sin(ESE°)*	UVI
201504210600	6:00 am (0600)	93°	-3°	0.0	0.00
201504210800	8:00 am (0800)	68°	22°	0.4	0.88
201504211000	10:00 am (1000)	47°	43°	0.68	3.97
201504211200	12:00 pm (1200)	39°	51°	0.78	4.79
201504211400	2:00 pm (1400)	50°	140°	0.64	3.48
201504211600	4:00 pm (1600)	72°	162°	0.31	1.83
201504211800	6:00 pm (1800)	97°	187°	-0.12	0.00

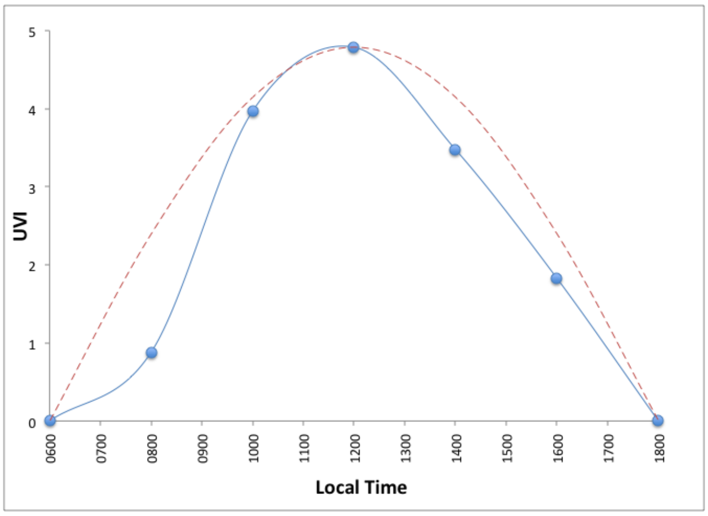

Figure 7. The measured UVI for 21 April 2015 plotted with respect to local time (solid). The UVI modelled as a function of local time closely approximates the measured daily variation (dashed).

Developing a Simple Trigonometric Model

The Australian national curriculum for mathematics introduces trigonometric ratios as part of the *Measurement and Geometry* content strand in years 9 and 10 (ACARA, 2015). Depending on the progression of individual schools and the age of student participants, trigonometric ratios were a relatively new concept for approximately half of the *Mathematics Enrichment* students. Therefore, following the successful group calculation of the UVI, students were introduced to the meaning of the sine ratio as a ratio of side lengths calculated from similar right-angled triangles.

The significance of the sine ratio was illustrated by discussion of the potential effect of vertical and horizontal components of the atmospheric path length in any incident beam of sunlight. Here, the physical reason for significant sine ratios at steep elevations were discussed relative to the proportional increase in solar UV radiation with reference to the more significant vertical component as the component that absorbs the least amount of radiation. Students were shown how to find basic sine ratios using their calculators and were presented with the known solar elevation angle at the time of each UVI measurement. They were then asked to plot their collective time stamped UVI measurements as a function of easterly solar elevation with angles at dawn starting from 0°, approaching solar noon at or near 90° and thereafter extending toward 180° at sunset.

Student models considered the importance of the amplitude of the periodic sine function. Students participating in the *Mathematics Enrichment* activity calculated the sine ratio of each easterly solar elevation angle and plotted the result for each of the spectral UV irradiance measurement times of 6:00 am, 8:00 am, 10:00 am, 12:00 pm, 2:00 pm, 4:00 pm and 6:00 pm. The model was improved by increasing the function amplitude by multiplication of an appropriate scale factor, *k* where,

$$k = \frac{UVI_{\max}}{\sin(ESE)_{\max}} \qquad (6)$$

Here, UVI_{max} is the peak UVI measured in the daily period and $sin(ESE)_{max}$ is the maximum value of the sine function for the given easterly solar elevation angle of each spectral irradiance measurement time. For the year 9 and 10 *Mathematics Enrichment* participants, the inclusion of the scale factor was enough to develop close approximations to the measured UVI.

Extending the Model

For other groups of students with a developing knowledge of trigonometric functions, it may be noted that the daily UV irradiance varies over a period of 12 hours (from 6:00 am to 6:00 pm). A sine function set to repeat 15 times in the interval from 0 to 180^o may therefore be plotted over a new interval of between 0 and 12^o, (or 0 and 12 hours) by inclusion of a trigonometric period factor of 15. To start a trigonometric function from 6 instead of 0, a phase shift of 6^o can also be included. Note also that any model tuned to peak in the middle of a 12 hour daily exposure period should be scaled such that the maximum amplitude matches the expected peak UVI. This is accounted for by the inclusion of a trigonometric amplitude factor, *A.* An appropriate UVI model may then be developed according to the function,

$$UVI = A\sin(15(t - 6)) \qquad (7)$$

where the sine ratio is evaluated in degrees, *t* is local time in hours, and the UVI is plotted over the interval 6 hrs $< t <$ 18 hrs. The UVI model for 21 April 2015 is plotted as a function of local time in Figure 7. Importantly, the developed model shows that the most important factor affecting the UVI is the SZA or in this instance, the easterly solar elevation angle.

Comparison between the measured UVI and developed model shows there are other physical factors that influence the UV index. Students considered these differences in relation to their developed functions. After SZA, cloud cover was discussed as the next most important factor. The measured UVI at 8:00 am on 21 April 2015 clearly showed that any developed trigonometric model (Figure 7) requires modification to take this into account. Thinking about the trigonometric sine model, students discover the importance of amplitude, and note importantly which property in Equation 7 requires modification to take the influence of cloud into account. Students learn that to develop better models requires a clear understanding of function properties.

At the end of the sessions, students were asked to evaluate their level of interest in the topic and generally write about their experience of the session. On average, students rated the session 7.3/10. Comments included:

- *I learned something new, found the topic interesting, enjoyable or fun (10 students specifically mentioned one of these words)*
- *It was a good lesson as we learnt about a real important and exciting topic*
- *It was challenging but not too hard*

Conclusion

School students participating in the University of Southern Queensland's *Mathematics Enrichment* program investigated the concept of numerical integration to measure the UVI from the South East Queensland ultraviolet monitoring data set. Similar pertinent solar radiation and public health information is commonly available worldwide though often not in an unprocessed format for use by the general public. The current study has made such information available through the utility of the internet and provision of password protected public access. The activities presented in the current study promoted the thoughtful application of mathematical problem solving and practical application of computer spreadsheets as a tool to help derive meaningful measurements of the UVI. The development of these skills were practiced with the specific aim of consolidating student understanding of the UVI and the physical variables which play a role in affecting this very important Public Health measure. Students learned that the most significant factor affecting the UVI is the SZA and used this concept to develop their own trigonometric models to explain the diurnal variation of the index as a function of time. This is a significant outcome for the participating students as public perceptions of UV exposure are often incorrect.

In reality, environmental and physical variables influence personal daily exposures to solar UV radiation. A separate follow on activity was therefore developed to test the influence of personal perceptions whereby students who had already completed Activities 1 and 2 were asked to make 'educated' guesses on the UV index at various outdoor locations and compare these to actual index measurements. Thus, students participating in the presented solar radiation activities were introduced to the interpretation of scale variables, categorical UV indices, and the importance of empirical data when needing to draw conclusions that involve several variables at a time. An index, such as the UV index is a simplification of a much more complex reality. The UV index, whether modelled or measured enables data driven decision making rather than intuition and promotes critical objective, rather than subjective decision making.

Public misunderstanding of the international UVI standard is commonly reported in populations worldwide, including populations at high risk of skin cancer (Carter & Donovan 2007, Geller et al. 1997). Developing a better understanding of the UVI is critically important in changing behavioral attitudes of populations at risk of disease caused by poor lifetime sun exposure habits (Carter & Donovan 2007). By developing their own mathematical models, students learn not only what the UVI represents, but also consider the important variables that affect any prediction of the index. Students participating in the developed solar radiation activity series were able to make informed decisions regarding their developed models. They used mathematical knowledge to adjust and redevelop their models. This reinforced the importance of mathematical modelling as a cyclic process involving the development of assumptions, formulation of a solution and continued refinement (Galbraith, 2006). Here, we present a methodology for investigating the UVI using mathematical concepts familiar to middle school aged children. These concepts reinforce mathematical learning and also build early understanding of an important Public Health issue not typically investigated in the school curriculum.

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