Stress-strain model for high strength concrete confined by FRP

W. P. Lokuge
Faculty of Engineering and Surveying, University of Southern Queensland, Toowoomba, Australia

S. Setunge
School of Civil, Environmental and Chemical Engineering, RMIT University, Melbourne, Australia

J. G. Sanjayan
Faculty of Engineering & Industrial Sciences, Swinburne University of Technology, Hawthorn, Australia

ABSTRACT: Ductility of High Strength Concrete (HSC) columns can be increased by lateral confinement. The conventional confinement with steel reinforcement may not always be adequate to provide the ductility levels desired by the engineer. The lateral confinement by FRP can provide significantly higher confinement stresses than the conventional steel reinforcement, and convenient for repair applications. Confining pressure applied by FRP is a function of the lateral strain of concrete. Therefore information of axial stress, axial strain and lateral strain relationships of concrete is fundamental in the design of confinement. Many existing models are based on test results with low confining pressures which are not suitable for modelling FRP confined concrete. The authors present a model in this paper which is suitable for use in designing FRP confinement of concrete. Model results are compared with experimental results of FRP confined columns. The results demonstrate the model is suitable for this use.

1 INTRODUCTION

The use of high strength concrete (HSC) in building systems is increasing around the world due to many advantages it offers. However its use is constrained by the well-known fact that HSC is more brittle than normal strength concrete (NSC). Using steel reinforcement is the conventional method of improving ductility. Fibre-Reinforced Polymer (FRP) as the method of confinement in concrete columns has gained increased popularity because of increased strength and ductility (Teng et al. 2002, Lorenzis & Tepfers 2003).

Design of such FRP jackets needs a proper understanding of the stress-strain behaviour of the concrete they confine. As a result, extensive research work has been carried out in the past and a number of stress-strain models for FRP confined concrete have been proposed (Jiang & Teng 2007, Teng et al. 2009). Most of these models are based on an actively confined concrete (Teng & Lam 2004, Spoelstra & Monti 1999, Binic 2005, Teng et al. 2009).

In this paper, behaviour of plain concrete is studied under multi-axial stress conditions with the aim of establishing a constitutive material model for short-term loadings. Passive confinement (using lateral reinforcement such as spirals, ties, steel tubes or other form of material like carbon fibre) depends on lateral dilation of concrete under axial load and the stress-strain relationship of confining material which necessitates establishing of the axial stress lateral strain behaviour of confined concrete.

This paper describes a novel approach in predicting the stress-strain relationship of HSC subjected to active lateral confinement. It is further developed for concrete with passive confinement. The model results are then compared with experimental results of concrete confined by FRP.

1.1 *Existing models*

Various models for the stress-strain relationship of FRP confined concrete have been developed in the past. Teng et al. (2009) categorized them as design oriented models and analysis oriented models. Design oriented models are normally in simple closed form and they can be used directly in practical design (Xiao & Wu 2000, Lam & Teng 2003). Analysis oriented models predict the stress-strain behaviour using iterative process (Spoelstra & Monti 1999, Binici 2005, Jiang & Teng 2007). Many of these models are based on an active confinement base model.

Basic concept of using active confinement base model and predict the stress-strain behaviour of steel confined concrete was first developed by Ahmad & Shah (1982). Further development of this concept to predict the stress-strain behaviour of FRP confined concrete was first reported by Mirmiran & Shahawy (1996). However Jiang & Teng (2007) conducted an extensive review and comparison of the available analysis oriented models. Key elements considered were stress-strain equation, peak axial stress point of the active confinement model and axial and lateral strain relationship. Jiang & Teng (2007) concluded that the model proposed by Teng et al. (2007) with a modification can better predict the test results. Teng et al. (2009) reported a latest version for the same with a better descending branch of the stress-strain curves. However these models do not perform well either with poor confinement or with higher concrete strengths. As a result the authors have approached the problem using a stress-strain model developed for HSC with active confinement as the base model.

1.2 *The Stress-Strain Model*

The model is based on 24 triaxial tests on four grades of concrete (40, 60, 75 and 100 MPa) and three confining pressures (4, 8 and 12 MPa) conducted in duplicate at Monash Univerity (Candappa et al. 2001). Model formulation is described in detail in Lokuge et al. (2005). However, for the convenience of the reader, model is briefly presented here. The proposed stress-strain relationship consists of different exponential functions for ascending branch and descending branch for unconfined and confined concrete as illustrated in Figure 1.

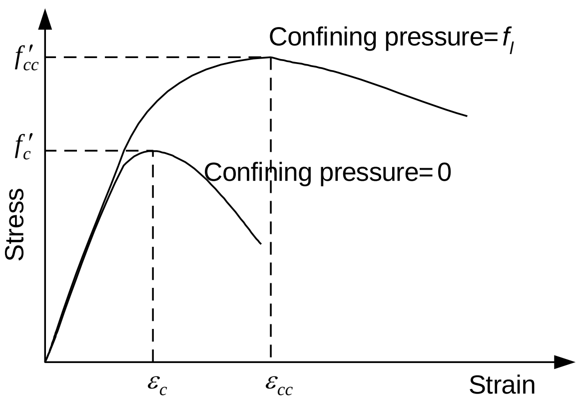

Figure 1. Typical stress-strain relationship for confined and unconfined concrete.

1.2.1 *Predicting a relationship between axial strain and lateral strain*

A relationship between axial strain ε_1 and lateral strain ε_2 was developed based on experimental data reported by Candappa (2000).

$$\frac{\varepsilon_2}{\varepsilon'_{cc}} = \begin{cases} \nu_i^a \left(\dfrac{\varepsilon_1}{\varepsilon_{cc}} \right) & \text{if} \quad \varepsilon_1 \le \varepsilon' \\ \left(\dfrac{\varepsilon_1}{\varepsilon_{cc}} \right)^{a} & \text{if} \quad \varepsilon_1 > \varepsilon' \end{cases} \tag{1}$$

where ε_{cc} = axial strain corresponding to peak axial stress; ε'_{cc} = corresponding lateral strain; and *a* = a material parameter which depends on the uniaxial concrete strength. *a* can be approximated by:

$$\mathbf{a} = 0.0177 f_c + 1.2818 \tag{2}$$

ε' can be obtained by equating the right hand side of Equation 1. Initial Poisson's ratio (ν_i^a) is defined in Candappa (2000).

$$\nu_i^a = 8 \times 10^{-6} (f_c)^2 + 0.0002 f_c + 0.138 \tag{3}$$

Therefore if axial strain ε_{cc} and lateral strain (ε'_{cc}) corresponding to peak axial stress can be expressed, the relationship between axial strain and lateral strain at any time is fully defined by Equation 1.

Attard & Setunge (1996) suggested equations for axial strain corresponding to peak axial stress ε_{cc}:

$$\frac{\varepsilon_{cc}}{\varepsilon_{co}} = 1 + (17 - 0.06 f_c)\left(\frac{f_l}{f_c} \right) \tag{4}$$

where f_l = confining pressure; and ε_{co} = axial strain corresponding to the peak uniaxial compressive strength (generally assumed to be 0.002). Peak axial stress for confined concrete f_{cc} is defined as:

$$\frac{f_{cc}}{f_c} = \left(\frac{f_l}{f_t} + 1 \right)^k \tag{5}$$

where k = a constant given by:

$$k = 1.25 \left(1 + 0.062 \frac{f_l}{f_c} \right) (f_c)^{-0.21} \tag{6}$$

f_t = tensile strength. As silica fume was not used in this project tensile strength is given by:

$$f_t = 0.9 \times 0.32 (f_c)^{0.67} \tag{7}$$

In this study Equation 4 is used to predict the axial strain corresponding to peak axial stress, which is determined by Equation 5.

1.2.2 *Predicting lateral strain at peak axial stress*

In order to find a relationship between axial and lateral strain at peak axial stress, normalized volumetric strain factor ($\bar{\varepsilon}_v$) versus normalized axial strain factor ($\bar{\varepsilon}_1$) graphs were plotted for all concrete batches and all confining pressures. It is interesting to note that there is a point where volumetric strain changes its sign. That is, at this point of zero volume, the concrete comes back to its original volume. When the experimental data are carefully analyzed for the peak axial stress it is observed that volumetric strain becomes zero when peak axial stress is reached. Therefore at peak stress:

$$\bar{\varepsilon}_v = \frac{\varepsilon_1 + 2\varepsilon_2}{\varepsilon_{v,\max}} = 0 \tag{8}$$

$$\varepsilon_{cc} = 2\varepsilon'_{cc} \tag{9}$$

By introducing Poisson's ratio, Equation 9 can be written as:

$$\nu_f^a = 0.5 \tag{10}$$

where ν_f^a = secant value of Poisson's ratio at peak stress. This has been addressed in the literature by few researchers as reported by Lokuge (2004). Based on the experimental results by Candappa (2000) and observing the volumetric behaviour for both normal and high strength concrete, it was confirmed that secant value of Poisson's ratio at peak stress is 0.5.

1.2.3 *Relationship between axial stress, axial strain and lateral strain*

Based on shear stress factors and shear strain factors, axial stress (σ_1), axial strain (ε_1) and lateral strain (ε_2) behaviour of concrete can be expressed as:

$$\sigma_1 = \begin{cases} 2\tau_{mp}\left(1 - e^{-c\left(\frac{\varepsilon_1+\varepsilon_2}{2\gamma_{mp}}\right)}\right) + f_l & before\ \ peak \\ 2\tau_{mp}\left(1 - e^{d\left(\frac{\varepsilon_1+\varepsilon_2}{2\gamma_{mp}}\right)^2} - d\right) + f_l & after\ \ peak \end{cases} \tag{11}$$

where c and d = material parameters. They depend on the uniaxial concrete strength. Their values for each concrete strength were found using the best fit curves and given in Figure 3. τ_{mp} is the maximum shear stress at peak and γ_{mp} is the maximum shear strain at peak. They are defined as:

$$\tau_{mp} = \frac{f_{cc} - f_l}{2} \quad \text{and} \quad \gamma_{mp} = \frac{\varepsilon_{cc} + \varepsilon'_{cc}}{2} \tag{12}$$

Therefore complete deformational behaviour of concrete can now be generated.

2 FRP CONFINED CONCRETE

In the last two decades, the use of fibre reinforced polymer (FRP) composites as the method of confinement has been gaining increasing popularity. The FRP reinforcement can provide significantly higher confinement stresses than the conventional steel reinforcement and therefore provide good level of ductility to high strength concrete.

2.1 *Confining pressure*

The confining pressure exerted by the fibre reinforced plastic sheets on the concrete core is of passive type. As the axial stress increases, the corresponding lateral strain in concrete increases and a tensile hoop stress is developed in the confining sheets which is balanced by the uniform radial pressure due to the lateral expansion of concrete (Fig. 2).

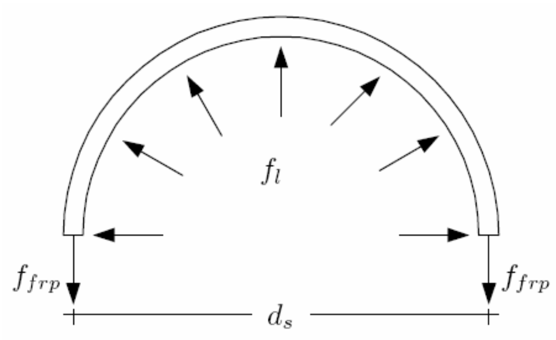

Figure 2. Confinement action in FRP composites.

Confining pressure can then be found by the equilibrium of forces.

$$f_l = \frac{f_{frp} n t_{frp}}{d_s} \tag{13}$$

where, f_l = confining pressure; d_s = diameter of the cylinder; f_{frp} = hoop stress of the carbon fibre sheet; n = number of sheets; and t_{frp} = thickness of a sheet.

Many researchers have noted that the strain measured in the confining FRP at rupture is in many cases lower than the ultimate strain of FRP tested for tensile strength (Lorenzis 2001). The recorded hoop strains corresponding to rupture had a range of 50 to 80% of the failure strain obtained in the tensile tests (Xiao & Wu 2000). This phenomenon considerably affects the accuracy of the model.

2.2 *Stress-strain model for FRP confined concrete*

There are several steps involved in developing a stress-strain relationship for FRP confined concrete using an active confinement base model.

For a given axial strain, find the lateral strain using Equation 1.

Based on the lateral strain, find the confining pressure exerted by the FRP using Equation 13.

Using Equation 11 find the axial stress corresponding to the given axial strain.

Repeat the above steps for incremental axial strains.

This procedure is described in Figure 3.

3 EXPERIMENTAL RESULTS

The experimental results reported by Candappa (2000) using carbon fibre sheets as the method of confinement are used for comparative purposes. The average tensile strength of carbon fibre composites was 741.3 MPa and the average Young's modulus was 101,920 MPa. The hoop strength of the carbon wrap was assumed to be the tensile strength of the carbon fibre composite (741.3 MPa).

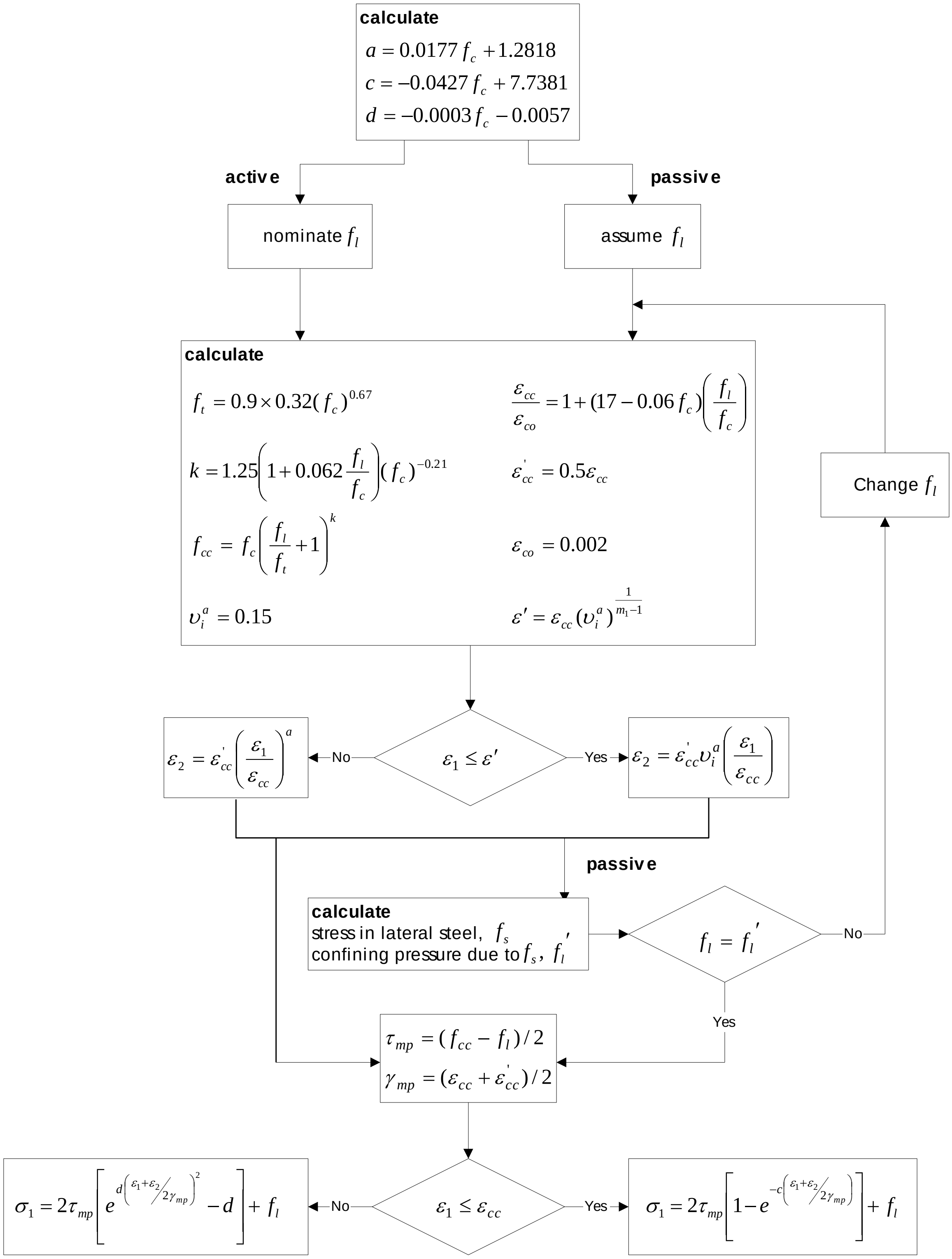

Figure 3. Procedure in drawing stress-strain curves for confined concrete.

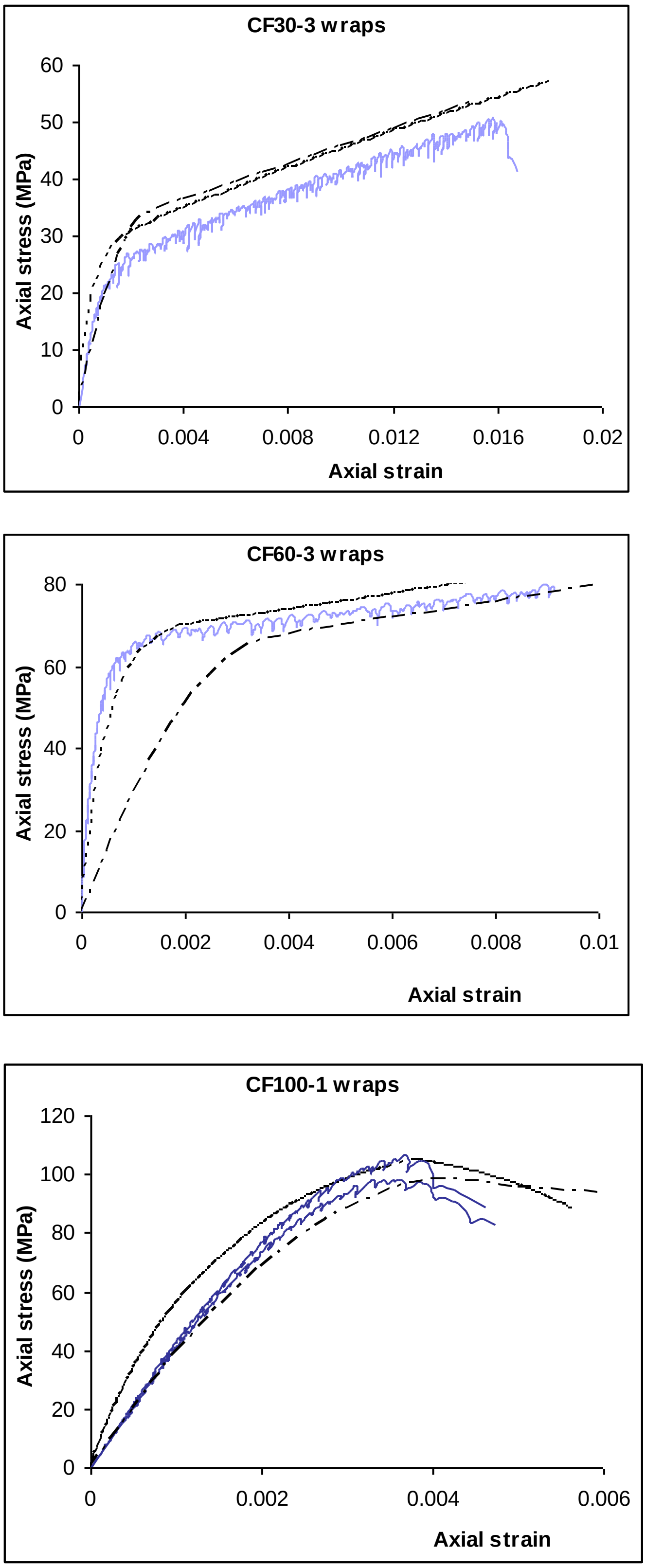

Figure 4. Model comparison using Candappa (2000).

The diameter of the specimen was 150 mm and the thickness of one layer of carbon fibre was 0.24 mm.

Another two test results are used in this analysis from Teng et al. (2009). Elastic modulus of FRP in hoop direction was 80100MPa, hoop rupture strain was 0.0174, thickness of one layer of glass fibre was 0.17mm and diameter of the specimen was 152mm.

Some test results shown by Teng et al. (2009) for GFRP confined concrete have also been used during this analysis. Test results are compared with the proposed model in this paper and the latest version of Teng et al. (2009) model.

Model comparisons using Candappa (2000) test results are shown in Figure 4 and those for Teng et al. (2009) are shown in Figure 5 respectively. For both figures,

——— Test

—·—·— Teng et al. (2009)

- - - - - - Proposed model

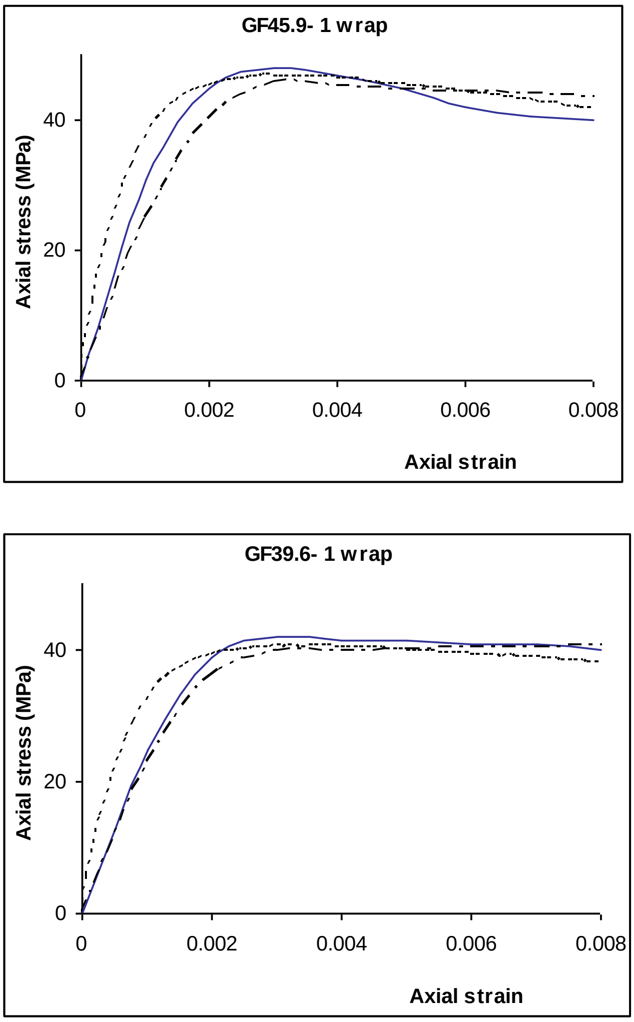

Figure 5. Model comparison using Teng et al. (2009).

4 COMPARISON

The constitutive model for HSC presented in this paper, is applied to obtain the behaviour of carbon fibre wrapped concrete columns and glass fibre wrapped concrete columns. The analytical findings are compared with the experimental results outlined above.

Both the models give perfect agreement when used for NSC with strong confinement. However when 60MPa concrete is used Teng et al. (2009) model overestimates the axial strain at peak axial stress. Though as not as significant as for 60MPa concrete, Teng et al. (2009) overestimates the axial strain for 100MPa concrete as well. Both the models predict the post peak behaviour accurately for NSC. However the proposed model predicts the descending curve more accurately for HSC with poor confinement.

5 CONCLUSIONS

A new stress-strain model for passively or actively confined concrete is presented.

The model prediction compares well with the experimental results of high strength concrete confined by carbon fibre and glass fibre wraps.

The proposed model has good prediction capability for low levels of confinements.

6 REFERENCES

Ahmad, S. H. & Shah, S. P. 1982. Stress-strain curves of concrete confined by spiral reinforcement. *ACI Journal* 79(6): 484-490.

Attard, M. M. & Setunge, S. 1996. Stress-strain relationship of confined and unconfined concrete. *ACI Materials Journal* 93(5): 432-442.

Binici, B. 2005. An analytical model for stress strain behavior of confined concrete. *Engineering Structures* 27(7): 1040-1051.

Candappa, D. 2000. The constitutive behaviour of high strength concrete under lateral confinement. PhD thesis. Monash University.

Candappa, D. P., Sanjayan, J. G., & Setunge, S. 2001. Complete triaxial stress-strain curves of high-strength concrete. *Journal of Materials in Civil Engineering* 13(3): 209-215.

Jiang, T. & Teng, J.G. 2007. Analysis-oriented stress-strain models for FRP-confined concrete *Engineering Structures* 29(11): 2968-2986.

Lam, L. & Teng, J.G. 2003. Design-oriented stress-strain model for FRP-confined concrete. *Construction and Building Materials* 17(6-7): 471-489.

Lokuge, W. P. 2004. Stress-strain behaviour of confined high strength concrete under monotonically increasing and cyclic loadings. PhD thesis. Monash University.

Lokuge, Weena P., Sanjayan, J. G., & Setunge, S. 2005. Stress strain model for laterally confined concrete. *Journal of Materials in Civil Engineering, ASCE* 17(6): 607-616.

Lorenzis, L. D. 2001. A comparative study of models on confinement of concrete cylinders with FRP composites. Research report, Chalmers University of Technology, Goteborg, Sweden.

Lorenzis, D.L. & Tepfers, R. 2003. Comparative study of models on confinement of concrete cylinders with fiber-reinforced polymer composites. *Journal of Composites for Construction, ASCE* 7(3): 219-237.

Mirmiran, A. & Shahawy M. 1996. A new concrete filled hollow FRP composite column. *Composites Part B-Engineering* 27(3-4): 263-268.

Spoelstra, M.R. & Monti, G. 1999. FRP-confined concrete model. *Journal of Composites for Construction, ASCE* (3): 143-150.

Teng, J.G. & Lam L. 2004. Behaviour and modelling of fiber reinforced polymer confined concrete. *Journal of Structural Engineering, ASCE* 130(11): 1713-23.

Teng, J.G, Chen J.F, Smith S.T & Lan L. 2002. FRP-strengthened RC structures. UK: John Wiley and Sons Ltd.

Teng, J.G, Huang, Y.L, Lam, L & Ye, L.P. 2007. Theoretical model for fiber reinforced polymer confined concrete. *Journal of Composites for Construction, ASCE*: 11(2).

Teng, J.G, Jiang, T, Lam, L. & Luo, Y.Z. 2009. Refinement of a design-oriented stress-strain model for FRP-confined concrete. *Journal of Composites for Construction, ASCE* 13(4): 269-278.

Xiao, Y. & Wu, H. 2000. Compressive behaviour of concrete confined by carbon fibre composite jackets. *Journal of Materials in Civil* Engineering 12(2): 139-146.