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HOW TO PEG A STRAIGHT LINE
PART 1
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This paper is designed to introduce several geodetic concepts in a practical surveying context – the chosen context is investigation of long survey lines. This was done largely in response to many years of retrieving enquiries on these matters from practicing surveyors and other practitioners. It is hoped that this paper will prove useful to practitioners dealing with long lines, and that this paper might generate thought on how to handle long lines in the field and on survey plans.

This paper will also find its way into the study materials for a course in which I lecture at the University of Southern Queensland. It is not meant to be definitive, nor is it designed to provide full discussions on all concepts. Rather, it sets out some practical problems, shows how to solve them, and provides ample data sets for interested readers to follow through with their own calculations. It is expected that the references and the standard geodesy texts (or information freely available on the Internet) will provide sufficient information to supplement what is presented here if necessary to aid understanding.

The Problem

Suppose a client asked you to set out a straight line between two points 20 metres apart, and place a mark at the centre point as well as any other intermediate marks deemed necessary along the line. Most surveyors would be able to set up a total station (or even a string line for that matter) and position marks along the line. While doing this it would not be difficult to segment the line into two equal parts and place a mark midway between the two points. But what if the points were a long way apart – so far apart in fact that they were not intervisible?

Let's look at an example where the two points are about 150km apart. In this case a client has had a conveyor belt designed between two points as shown below:

Terminal Point GDA94 Coordinates

Point 1	Point 2
φ -27° 00′ 00″	λ -26° 00′ 00″
φ E155° 00′ 00″	λ E156° 00′ 00″

The client wants you to set out the conveyor belt, including a midpoint, and further tells you that it is critical that this is exactly a straight line. This is because 'that is how it was designed on the CAD package', and if it is not exactly straight the conveyor belt will wear too much on one side and therefore need replacing prematurely (at significant cost to the client). At this point I will deviate from the practical nature of the scenario so as not to confuse the general concepts. We are going to assume the conveyor belt and the two terminal points are exactly on the ellipsoid (h=0). The same principles apply when the terminal and intermediate points have varying heights, but this is ignored for this paper in the interests of keeping the number of variables to a manageable level.

Given that you may be uncomfortable dealing with geographic coordinates (perhaps of the belief that latitudes and longitudes would be easier to understand if only they were in metres), the first task you carry out is to convert the coordinates of Points 1 and 2 into grid coordinates (Eastings and Northings).You can do this simply by entering the geographic coordinates into a good quality CAD package (with an appropriate coordinate system configured), by using various calculator programs, or by using spreadsheets that contain Refrearn's Formula. One such spreadsheet is available from the Intergovernmental Committee on Surveying and Mapping (ICSM) (2013). You will find several useful spreadsheets here and for the remainder of this paper I will refer to these as the ICSM spreadsheets. Below we see the coordinates of the two points converted to MGA94 grid coordinates.

Terminal Point MGA94 Coordinates

Point 1	Point 2
φ -27° 00′ 00″	λ -26° 00′ 00″
λ E155° 00′ 00″	λ E156° 00′ 00″
E 698454.234 Zone 56	E 800318.075 Zone 56
N 7011991.862 Zone 56	N 7120867.466 Zone 56

Now you have a plan on how to approach this job. You can put these points into your CAD package, draw a line between them (thus replicating what the client told you the designers had done) and segment the line as appropriate - starting with the centre point to see how it works out. These points can then be set out with a total station or perhaps RTK GNSS.

Note that in this paper I am only concerned with the general geodetic principles and not the field set out process itself. So you do just that, and as an additional check you simply take an arithmetic mean of the Easting and Northing of the two points since this should also give you the centre point … right? These calculations provides you with a bearing and distance from Point 1 to Point 2 of 43° 05′ 39.6″ and 149097.751 as well as coordi-

Figure 1 (left):

Gnd azimuth:	43°05'40"
Gnd distance:	149097.751 m
Δ Elevation:	-2.185 m
Geodetic azimuth Forward:	42°12'14"
Backward:	221°45'28"
Ellipsoid distance:	149041.324 m
Ground distance:	149041.324 m
Δ Height:	0.000 m

Figure 1: Inverse Report from Point 1 to Point 2

The bearing agrees with your manual calculations (although the terminology of 'Grid azimuth' is a little confusing) and the distance is perfect as well. So at this stage you are fairly confident you can just go into the field and start pegging the conveyor belt line. There is a bit of concern over the 'Geodetic azimuth' shown in the inverse report though, and you're not too sure what those numbers are. And the ellipsoidal distance is significantly different from the grid distance, but you remember from your university studies that this is what is expected ... you just can't quite remember why though. (Note the ground distance is the same as the ellipsoidal distance because I have assumed ellipsoidal height of zero for the points.) Although you are not too concerned at this stage, you figure it is probably worth one last check before you head out into the field to start pegging. You decide to put your coordinates into the ICSM spreadsheet GRIDCALC.XLS and use this as a final check. Results are provided in Figure 2.

Now this is bit of a worry and you are concerned with a number of issues (good thing you decided to do this last check). The ellipsoidal distance from 1 to 2 (on the top calculation) agrees with the CAD inverse report and the plane distance agrees with the grid distance from your earlier calculations. But now you have two grid bearings (forward from 1 to 2 and the reverse from 2 to 1), and neither of them agree with your calculated bearing – your bearing is close to the mean of the two bearings shown, but not exactly. You realise that even a small bearing discrepancy over 150km will be important so

Grid Bearing and Ellipsoidal Distance from Grid Coordinates   MGA

	Name	East (E)	North (N)	Zone
From (1)	1	698,454.234	7,011,991.862	56
To (2)	2	800,318.075	7,120,867.466	56
Ellipsoidal Distance (s)	149041.325			
Plane Distance (L)	149097.751			KEY
Grid Bearing (β_1)	43°	06'	44.08"	User input
Grid Bearing (β_2)	223°	04'	25.81"	Result

Arc to Chord correction (δ_1)	-64.43"
Arc to Chord correction (δ_2)	73.84"
Line scale factor (K)	1.000 378 59

Grid Bearing and Ellipsoidal Distance from Grid Coordinates   MGA

	Name	East (E)	North (N)	Zone
From (1)	1	698,454.234	7,011,991.862	56
To (2)	3	749,386.155	7,066,429.664	56
Ellipsoidal Distance (s)	74532.351			
Plane Distance (L)	74548.876			KEY
Grid Bearing (β_1)	43°	06'	09.51"	User input
Grid Bearing (β_2)	223°	05'	07.44"	Result

Arc to Chord correction (δ_1)	-29.86"
Arc to Chord correction (δ_2)	32.21"
Line scale factor (K)	1.000 221 72

Grid Bearing and Ellipsoidal Distance from Grid Coordinates   MGA

	Name	East (E)	North (N)	Zone
From (1)	2	800,318.075	7,120,867.466	56
To (2)	3	749,386.155	7,066,429.664	56
Ellipsoidal Distance (s)	74508.976			
Plane Distance (L)	74548.875			KEY
Grid Bearing (β_1)	223°	05'	00.38"	User input
Grid Bearing (β_2)	43°	06'	16.57"	Result

Arc to Chord correction (δ_1)	39.27"
Arc to Chord correction (δ_2)	-36.92"
Line scale factor (K)	1.000 535 49

Figure 2 – Grid bearing and distances between Points 1, 2 and 3

determine that you will have to sort that discrepancy out. Even more of a worry though is that the plane distance from 1 to 3 agrees with 2 to 3, but the ellipsoidal distances are about 23 metres different. You figure that if the two points are on the ellipsoid, and you go half way between them, then the ellipsoidal distance from 1 to 3 and from 2 to 3 should be the same ... but they are not ... so what is the problem here?

You note at the bottom of the calculations in Figure 2 that there are some scale factors quoted. It then dawns on you that simply meaning the coordinates of Points 1 and 2 will not put you in the middle, even though this seems intuitive from bisecting the line between 1 and 2 on your CAD system. This is because the map projection (UTM/MGA) scale factor is different for each line. You figure you could balance the ellipsoidal distances by trial and error, but this is a bit longwinded, not terribly satisfying, and does not explain the problem with the bearings. It's about now you start to realise it is not as easy as you thought to peg this straight line, and that maybe you should be charging your client a little more than first thought. To understand this problem and ultimately solve the dilemma, we first need to agree on some background knowledge.

Background

One of the key problems here is to define what we mean by a straight line between the two points: does this mean a straight line on the ground, or a straight line on the design CAD package? Unfortunately, information from your client is ambiguous. Initial instructions indicated that the conveyor belt had to be in a straight line to minimise wear, and this would suggest it had to be a straight line on the ground (or on the ellipsoid in this example since we ignored the heights). It was also stated that this is how the conveyor belt was designed on the CAD package. Unfortunately it can't be both. In general, and this manifests itself particularly over long lines, a straight line on the ground will not plot as a straight line on the UTM, nor on a CAD package using almost any other map projection. And the converse is true as well – a straight line designed on CAD will not in general be a straight line on the ground. We tend to ignore this effect because we are used to working with shorter lines, and this is a sensible thing to do ... until the lines get a little longer. Another point to remember is that most CAD packages will simply use plane bearings and

plane distances and not forward and reverse grid bearings or azimuths.

So how might certain 'straight lines' be plotted on a CAD package? We will begin by thinking about the earth as a sphere. We can visualise a straight line (and shortest distance) between two points on the sphere. In this case the normal (line at right angles to the surface) at each point will go through the centre of the sphere (refer to Figure 3). We can form a plane through the two normals at each point and intersect this with the sphere and get a line of intersection as a great circle.

If we now consider the earth as an ellipsoid, you can visualise a great ellipse between two points on the ellipsoid in a similar manner to the great circle on the sphere. As before a plane can be formed between each point that will also go through the centre of the ellipsoid (refer to Figure 3). The great ellipse is the intersection of this plane with the ellipsoid.
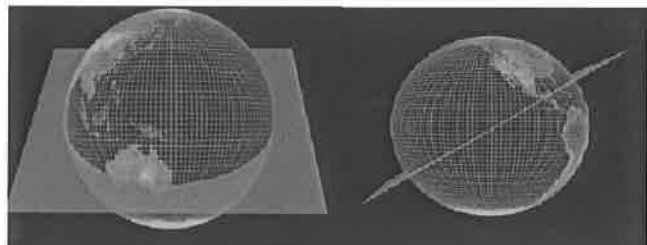

Figure 3 – Great Circle/Great Ellipse

There are an infinite number of planes that will contain two points on an ellipsoid: the one we are looking at now is the one that also contains the centre of the ellipsoid. Therefore the great ellipse would be related to the dashed line from the centre of the ellipsoid to point A in Figure 4.
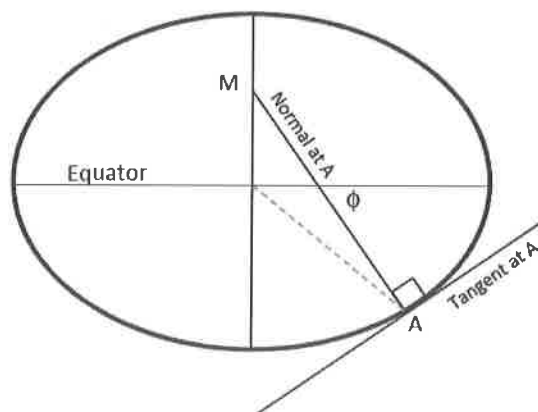

Figure 4 – Meridian Plane Showing Normal at Point A

Unfortunately on an ellipsoid the normal at a point (in general) does not go through the centre of the ellipsoid.

Therefore the great ellipse, or great elliptic arc as it is commonly known, does not represent a straight line on the ellipsoid, and nor is it the shortest distance between two points on an ellipsoid. For this reason, and the fact that they don't have a practical connection with field
observations such as total stations (R. E. Deakin, 2010a), great elliptic arcs are not used often in practical surveying. Their main use is that they would be the lines traced out on the earth by a satellite orbiting the earth in an orbital plane containing the earth's centre of mass (ignoring earth rotations and of course making the assumption the geometric centre of the ellipsoid is coincident with the centre of mass). Readers are directed to Tseng and Lee (2010) for further information on the great ellipse and formula for navigation calculations.

By definition the normal to an ellipsoid at any point (A in this case) must be at right angles to the plane tangential to the surface of the ellipsoid at that point. The line A-M in Figure 4 fulfils this condition. Note that in Figure 4 the normal at point A will not pass through the centre of the ellipsoid. For interest the normal at A intersects the equatorial plane at an angle known as the geodetic latitude shown as ϕ.
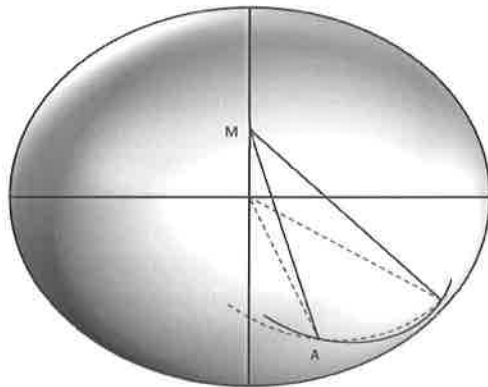

Figure 5 – 3D View of Great Ellipse and Normal Section at Point A

We now consider another of those infinite number of planes that will contain two points on an ellipsoid: the one we are looking at this time is the one that also contains Point M. Where this plane intersects the ellipsoid is known as a normal section and this line is obviously different from the great elliptic arc (refer to Figure 5). Therefore the great ellipse will always plot closer to the equator than the normal section.

Now we complicate this just a little. Since the normal at a point will change as the latitude changes, unless two points are at the same latitude, their normals will not be in the same plane. We therefore have two new

planes to consider, and since these are related to normals at the two points they are called normal planes. Each normal plane intersects the ellipsoid to form a different normal section. Therefore, in general, we have two normal sections between two points on the ellipsoid at different latitudes.

I won't go into a long discussion on this since it is not important from a practical perspective, and this is covered in great detail in reference texts. In our example above, the maximum distance between the two normal sections is only 27mm (formula are available from Krakiwsky & Thompson, 1974 and similar texts) and we can safely ignore this in all but the most accurate work, particularly over a line of 150km.

Note that the shortest distance between two points on the ellipsoid is not a normal section: strictly it is the geodesic (R.E. Deakin & Hunter, 2007). In most geodesy texts the geodesic is drawn between the two normal sections, and this is where it lies in our example, but this is not always the case and sometimes the geodesic will fall outside the normal sections. The good news, and we sure can use some about now, is that for most practical purposes we can assume the geodesic length, and the length along the normal sections (and the great elliptic arc for that matter) are the same. There is less than 1mm difference in length between a great elliptic arc and geodesic over 1,200km (R. E. Deakin, 2010a), and less than 1mm difference in length between a normal section and a curve of alignment over 1,600km (R. E. Deakin, 2010b), so we are not concerned about being able to calculate the length of the line in our example. From a practical perspective then in our example, we don't really mind whether we follow a normal section
or a geodesic (or a curve of alignment for that matter – refer to geodetic texts for a definition) since they are so close together.

Looking now at our scenario, what we peg on the ground will all depend on that we mean by a straight line and whether or not this is on the ground or on the CAD package.

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How to Peg a Straight Line Part 2 **will appear in the March edition on Spatial Science Queensland, when we will discover The Solution, Expanded Cadastral Context, Lease Context and Lessons learned.**