Modeling Performance of CSMA/CA with Retransmissions in Wireless Personal Area Networks

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Abstract

Despite the widespread uses in military, smart building, habitat monitoring, Wireless sensor networks (WSNs) just begun showing their superiority in the medical and healthcare domain. Facing same challenges as other applications, deployment of WPANs in medical domain exert many more stringent requirements, due to the nature of high accessibility and the privacy sensitive requirement of medical environment, the application of WPANs must be carefully examined before practical use. This paper considers the performance of WPANs deployed in the specific domains, particularly on the behavior of sensor nodes while accessing the shared channels during communicating each other, by theory of Markov chain.

1. Introduction

The latest advances in wireless communication technology and device sensing technology have given rise to a new communication paradigm. Wireless sensor networks (WSNs) are one of such autonomous distributed communication systems. Sensor nodes in wireless sensor networks are severely constrained by the limited power supply, and small memory size and low bandwidth. Notwithstanding, there has been a huge potential of applications in many demanding areas, application of wireless sensor networks in healthcare and medicine domain [1, 2] has been long awaited. One prototyping application of WSNs in healthcare is Wireless Personal Area Networks(WPANs), where WPANs are expected to monitor chronicle diseases, collect physiological data in order to detect early the deterioration of patients in hospital and to improve the life quality of elderly in nursing homes.

In case that WSNs are deployed in medical and healthcare domain, extra and stronger constraints verily emerge and surpass [3] the common ones all WSNs have been facing all the time. It is well known that the use of WPANs is potentially very beneficial to people who has taken advantages of it, but also ethically controversial to them. For instance, the deployment of WSN in medical domains such as telemedicine in hospitals or healthcare in nursing homes, the constraints caused by limited communication range become very prominently stringent. The deployment of WPAN in the medical and healthcare domains usually relies on many medical devices or medical sensor nodes. For example, CodeBlue and MEDiSN [4] are two exemplary systems. Medical sensor nodes are small devices with limited capacity of processing, and short-range communication, and scary power supply. Relatively lightweight than IEEE 802.11, IEEE 802.15.4 has been adopted by most of sensor nodes in the markets.

To avert from the constraints of medical sensor nodes, the channel access schemes in the medium access control are designed with much energy conscious. Normally the networks have two modes: beacon-enable and non-beacon. Beacon-enabled networks use a slotted carrier sense multiple access mechanism with collision avoidance (CSMA/CA), and the slot boundaries of each device are aligned with the slot boundaries of the short-range networks coordinator [5].

The rest of paper is organized as follows. In Section 2, a brief discussion on IEEE 802.15.4 Media Access Control (MAC) protocol is given. Following it, Section 3 highlights the modeling process of the IEEE 802.15.4 MAC based on Markov Chain theory. In Section 4, we consider the parameters which affect the performance of the protocol. At the same time, there are some important assumptions that simplify the complicated situations in order to derive the performance indices formula in Section 5. A comparison of performance is given in Section 6, which show that the theoretical results are very close to that of simulation. Finally, Section 7 concludes the paper.

2. WPAN vs. IEEE 802.15.4 MAC Protocol

Wireless Personal Area Network (WPANs) is a popular choice for many applications, particularly in medical environment such as hospitals or nursing homes; whereas IEEE 802.15.4 is the protocol chosen in the MAC layer of WPANs.

2.1. *Low rate wireless network*

In a low-rate wireless personal area network (LR-WPAN), a coordinator or central controller builds the network in its personal operating space. Communications from nodes to coordinator (called *uplink*), from coordinator to nodes (called *downlink*), or from node to node (ad hoc) are allowable.

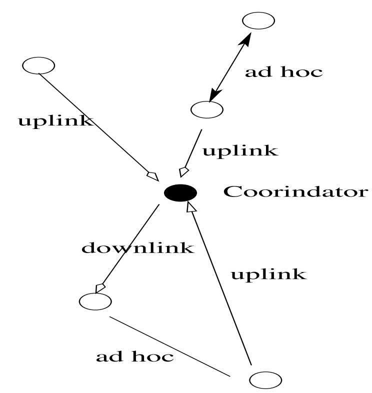

Fig. 1. WPAN with a PAN coordinator and four nodes

Fig 1 depicts a simplified Wireless personal network with one PAN coordinator node and six nodes.

2.2. *Slotted CSMA/CA mechanism*

IEEE 802.15.4 standard has been adopted for WSNs with a little modification. When considering beacon-enable mode, the network uses a slotted carrier sense multiple access (CSMA) mechanism with collision avoidance (CA), and the slot boundaries of each device are aligned with the slot boundaries of the PAN coordinator. The slotted CSMA/CA mechanism is mainly reliant on special designed data structure, namely superframe. The superframes are bounded by network beacons and divided into 16 equally sized slots. The beacon frame is sent in the first slot of each superframe shown in Figure 2.

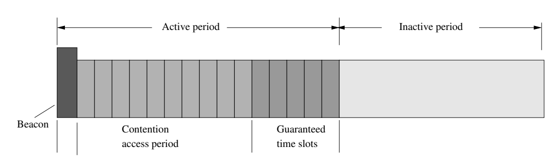

Fig. 2. Superframe

Each sensor node in WPANs has three variables: NB, CW and BE. NB is the number of times the CSMA/CA algorithm was required to delay while attempting the current transmission. `CW` is the contention window length, which defines the number of slot periods that need to be clear of activity before the transmission can start. `BE` is the backoff exponent, which is related to how many slot periods a node must wait before attempting to assess the channel.

2.3. *Slotted CSMA/CA with retransmission*

Fig. 3 depicts the slotted CSMA/CA with retransmission. The slotted CSMA/CA mechanism works as follows. `NB`, `CW` and `BE` are initialized and the boundary of the next slot period is located. The MAC layer delays for a random number of complete slot periods in the range 0 and $2^{\mathtt{BE}} - 1$ and then requests PHY to perform a CCA (clear channel assessment). If the channel is sensed to be busy, the MAC sublayer increments both `NB` and `BE` by 1, ensuring that `BE` is not more than `aMaxBE`, and `CW` is reset to 2. If the value of `NB` is less than or equal to `macMaxCSMABackoffs`, the CSMA/CA must return to step 2 (i.e. delay for a random number ranging in 0 and $2^{\mathtt{BE}}$). Otherwise (i.e. `NB` reaches `macMaxCSMABackoffs`), the CSMA/CA must terminate.

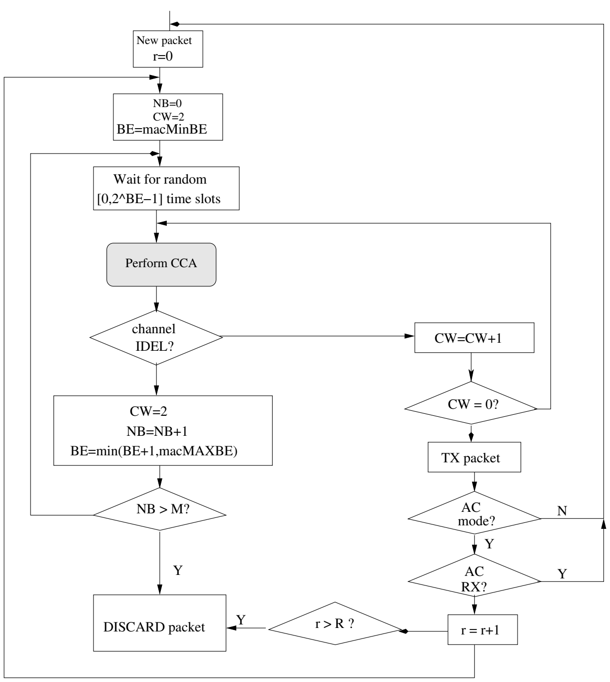

Fig. 3. Procedure of channel access with retransmission

If the channel is assessed to be idle, the MAC sublayer must ensure that the contention window is expired before starting transmission. What needs more attention is on the right side of the flowchart. The left side of the flowchart is a typical CSMA/CA procedure.

The packet has to be retransmitted only under one situation. That is when the packet ACK mode although the channel access was successful, the retransmission is necessary subject to the number of retransmission has not reached the preset limit(R).

3. Markov Chain Model

The behavior of a single node per attempt is modeled using a 2D Markov chain with states represented by $\{s(t), c(t)\}$ at a given backoff period t. $s(t)$ is a stochastic process representing the delay line stages; and $c(t)$ is also a stochastic process representing the delay line.

3.1. *States*

The behavior of the network can be analyzed by looking only at the per-attempt Markov chain model of a node, the behavior of a single node per attempt are represented with $\{s(t), c(t)\}$ at a given backoff period t. A slot is referred to a backoff period. All events happen at the beginning of a slot.

At a given slot t, the stochastic process $s(t)$ represents

1. the transmission stage when $s(t) = -1$. $c(t) \in \{0, \ldots, L-1\}$ represents the state of packet transmission. L is the packet size.

2. the ACK stage when $s(t) = -2$. When $c(t) = 0$ represents the state used for switching the transceiver from TX to RX.

3. the backoff stage when $s(t) = \{0, 1, \ldots, M\}$. $c(t) \in \{0, \ldots, W_i - 1\}$ represents the value of the backoff counter, where $W_i = 2^{\min(\mathtt{macMinBE}+i, \mathtt{macMaxBE})}$ is the size of the backoff window.

4. when the node is performing one of CCA1 or CCA2, $c(t) = 0$ represents the node is performing CCA1 and $c(t) = -1$ represents the node is performing CCA2. Note that the state

$\{s(t), c(t)\} = \{i, 0\}$ has to be seen as a CCA1 and not as a backoff state.

All states of Markov chain are summarized in Table 1. For example, when $s(t) \in [0, M]$, and $c(t) = -1$, the state of the node is in CCA2.

Table 1: States

		$s(t)$		
		$0, \ldots, M$	-1	-2
	-2	-	-	ACK No RX
	-1	CCA2	-	ACK No RX
	0	CCA1	P	ACK Idel Wait
	1	B	K	ACK &
$c(t)$	2	A	T	RX
	3	C		-
	$\cdot$	K	T	-
	L		R	-
	$L+1$	O	A	-
	$\cdot$	F	N	-
	W_{M-1}	F	S	-

Fig. 4 shows 2D Markov model of a single node per attempt with all the states.

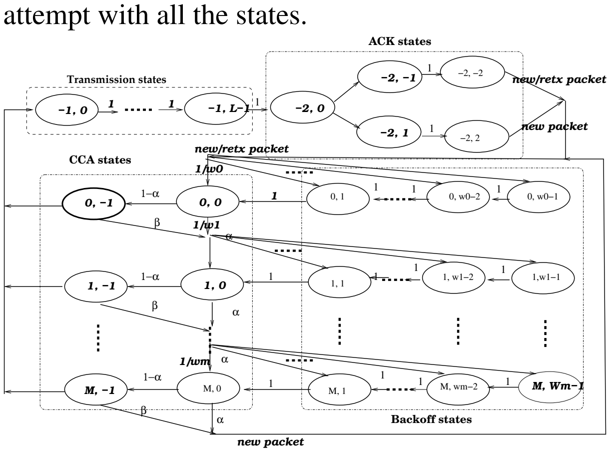

Fig. 4. Markov model for slotted CSMA/CA per node

There are four categories of states: (1) Transmission states, (2) ACK states, (3) Backoff states, and (4) CCA states.

3.2. *Calculating Markov chain parameters*

Let $b_{i,k}$ be the steady-state probability of being in state $\{i, k\}$. That is $b_{i,k} = \lim_{t\to\infty} P\{s(t) = i, c(t) = k\}$.

From Fig. 4, these steady-state probabilities can be seen to related through the following equations

$$b_{i,k} = \frac{W_i - k}{W_i} b_{i,0}, \quad 0 \leqslant i \leqslant M, \ 0 \leqslant k \leqslant W_i - 1 \tag{1}$$

$$b_{i,0} = (1-y)^i b_{0,0}, \ 0 \leqslant i \leqslant M \tag{2}$$

$$b_{i,-1} = (1-\alpha) b_{i,0}, \ 0 \leqslant i \leqslant M \tag{3}$$

$$b_{-1,k} = y \sum_{j=0}^{M} b_{j,0} = y\phi, \ 0 \leqslant k \leqslant L-1 \tag{4}$$

$$b_{-2,k} = \begin{cases} p_{c_o} y\phi & -2 \leqslant k \leqslant -1 \\ y\phi & k = 0 \\ (1-p_{c_o}) y\phi & 1 \leqslant k \leqslant 2 \end{cases} \tag{5}$$

The access probability can be defined as

$$y = (1-\alpha)(1-\beta) \tag{6}$$

Hence, ϕ is the probability that a given node spends on a randomly picked slot performing CCA1,

$$\phi = \sum_{j=0}^{M} b_{j,0} = \frac{1-(1-y)^{M+1}}{y} b_{0,0} \tag{7}$$

and p_{c_o} is the probability that the transmission of a given node ends up in a collision.

Within a network with N nodes, the probability α of finding the channel busy during CCA1 is

$$\alpha = [L + L_{ack}(1-p_{c*})][1-(1-\phi)^{N-1}]y \tag{8}$$

where p_{c*} is the probability that at a random slot a collision takes place in the network. L and L_{ack} are the length of the packet and ACK packets in slots.

The probability that the channel is busy when the considered node does its its second sensing, is represented by β.

$$\beta = \left[1 - \frac{2-p_{c*}}{2-p_{c*} + \frac{1}{1-(1-\phi)^N}}\right](1-(1-\phi)^{N-1}) + \frac{1-p_{c*}}{2-p_{c*} + \frac{1}{1-(1-\phi)^N}} \tag{9}$$

The p_{c*} and p_{c_o} will be derived in Section 5.

3.3. *Events*

Table 2 lists all the events that can take place in the network. In the table, C_X stands for CCA1 and X

defines the number of nodes or the set of nodes involved.

Table 2: Events Notation

Notation	Event Description
CF_1	Channel Free in 1 slot
CF_2	Channel Free in 2 connective slots
$C_{\in\{j\}}$	At least node i node in CCA1
$C_{\geqslant i}$	At least i nodes in CCA1
$C_{\{i\}}$	Only node i in CCA1
C_i	Exactly i nodes in CCA1
$\hat{C}_i$	Exactly i nodes including node 1 in CCA1

CF_1 and CF_2 represent Channel Free in slot 1 or 2 connective slots. Moreover, we use TX_X to represent the corresponding nodes being in a transmission state. For instance TX_i represents that exactly i nodes are in transmission. Note that in the $\hat{C}_i$, node 1 will be used as the reference node. Because the event is symmetric with respect to all nodes, there is no loss of generality.

4. Determining Markov Model

The Markov model has been represented by the values of ϕ, α, and β. In this section, we will discuss how to numerically solve these parameters.

4.1. *Model parameters*

The constant $b_{0,0}$ in Equation (7) deserves a bit more explanation. Its value can be determined by imposing the normalizing condition, which means all probabilities must sum to 1, i.e.

$$\sum_i \sum_k b_{i,k} = 1 \tag{10}$$

From Equation (5), we can get

$$b_{-2,-2} = b_{-2,-1} = p_{c_o} y\phi \tag{11}$$

$$b_{-2,0} = y\phi \tag{12}$$

$$b_{-1,-2} = b_{-1,-1} = 0 \tag{13}$$

From Equations (2) and (3) when $0 \leqslant i \leqslant M$, we can obtain

$$b_{i,-2} = 0 \tag{14}$$

$$b_{i,0} = (1-y)^i b_{0,0} \tag{15}$$

$$\begin{aligned} b_{i,-1} &= (1-\alpha) b_{i,0} \\ &= (1-\alpha)(1-y)^i b_{0,0} \end{aligned} \tag{16}$$

When the node is in backoff, the backoff windows is $W_i = 2^{\min(\texttt{aMinBE}+i,\texttt{aMaxBE})}$, which has been determined sort of exponential, i.e. $W_0 = 2^{\texttt{aMinBE}}$, $W_1 = 2^{(\texttt{aMinBE}+1)}$, …, $W_M = 2^{\texttt{aMaxBE}}$. For instance, suppose $R = 4$, $\texttt{aMinBE} = 3$, $\texttt{aMaxBE} = 5$, then $W_0 = 8$, $W_1 = 16$, $W_2 = W_3 = W_4 = 32$. Equation (10) becomes

$$\begin{aligned} 1 &= \sum_i \sum_k b_{i,k} \\ &= \sum_{k=-2}^{W_M-1} b_{-2,k} + \sum_{k=-2}^{W_M-1} b_{-1,k} + \sum_{i=0}^{M} \sum_{k=-2}^{W_M-1} b_{i,k} \\ &= 2p_{c_o} y\phi + y\phi + 2(1-p_{c_o})y\phi + Ly\phi \\ &\qquad + \sum_{i=0}^{M} \sum_{k=0}^{W_i-1} b_{i,k} \\ &= 2y\phi + Ly\phi + \sum_{i=0}^{M} \sum_{k=0}^{W_i-1} b_{i,k} \\ &= (2+L)y\phi + \sum_{i=0}^{M} \sum_{k=0}^{W_i-1} \frac{W_i-k}{W_i} b_{i,0} \\ &= (2+L)y\phi \\ &+ b_{0,0} + \sum_{i=1}^{M} \sum_{k=0}^{W_i-1} \frac{W_i-k}{W_i} (1-y)^i b_{0,0} \\ &= (2+L)y\phi \\ &+ b_{0,0} \left(1 + \sum_{i=1}^{M} \sum_{k=0}^{W_i-1} \frac{W_i-k}{W_i} (1-y)^i \right) \end{aligned} \tag{17}$$

We substitute $b_{0,0}$ in Equation (7) to get a non-linear equation. Combining this non-linear equation with Equations (8) and (9), we get a system of non-linear equations with ϕ, α, and β. To get α, β and ϕ, we have to solve numerically this non-linear equations, by use of computer software such as MATLAB, etc.

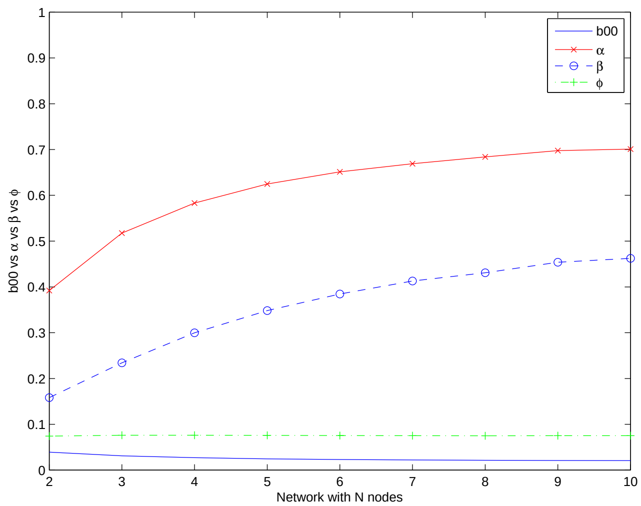

Fig. 5. b_{00}, α, β and ϕ vs. N

4.2. *Simplifying assumptions*

IEEE 802.15.4 CSMA/CA scheme has made no discrimination to each node when it comes to each of operation such as channel access, backoff count, and clear channel assessment (CCA). Based on these characteristics, we can make the following simple assumptions while modeling the networks using Markov chain theory.

- A1 The probability to sense the channel busy during CCA1 and CCA2 does not depend on the backoff stage where the corresponding CCA is performed.
- A2 The probability to sense the channel busy during CCA1 and CCA2 does not depend on r, ie. the number of attempts the current packet has gone through.
- A3 The probability of sensing the channel busy during a CCA does not depend on the random backoff value drawn in the backoff stage preceding the CCA.
- A4 The probability ϕ that at a given time a given node starts sensing the channel is independent of what other nodes in the network are doing at the same time.
- A5 The probability of sensing the channel free in two consecutive slots is equal to $y = (1 - \alpha)(1 - \beta)$ from the per-node Markov chain.

5. Calculating Network Metrics

By the use of values of ϕ, α, and β, we are ready to derive the different network metrics. In many cases, the derivation of these metrics can be simplified based on the assumptions made in previous section.

5.1. *Throughput*

Definition 5.1 (Throughput S_0 and S_*)**.** *We define the per-node throughput S_o as the fraction of time that a node spends in the successful transmission. The network throughput S_* is*

$$\begin{aligned} S_* &= N \cdot S_o \\ &= Pr\{at\, t : TX_1\} \\ &= N \cdot Pr\{at\, t : TX_{\{1\}}\} \\ &= N \cdot L \cdot Pr\{at\, t' \in [t-1-L, t-2] : \\ &\qquad C_{\{1\}} \cap CF2\} \end{aligned} \tag{18}$$

Using assumption A5), we can get

$$S_* = NL\phi(1-\phi)^{N-1}y \tag{19}$$

5.2. *Transmission probability*

Definition 5.2. *Let p_{tx_o} denote the probability that, at a random slot, a particular node is transmitting. This probability is the same for all nodes due to symmetry and can be written as*

$$\begin{aligned} p_{tx_o} &= Pr\{TX_{\supseteq\{1\}}\} \\ &= LPr\{C_{\supseteq\{1\}} \cap CF2\} \end{aligned} \tag{20}$$

Using assumption A5, we get

$$p_{tx_o} = L\phi y \tag{21}$$

5.3. *Collision probability*

Definition 5.3. *The per-node collision probability p_{c_o} is defined as the probability that at a time in which a given node (i.e. node 1) is transmitting, one or more other nodes are also transmitting.*

$$\begin{aligned} p_{c_o} &= Pr\{TX_{\geqslant 2} | TX_{\supseteq\{1\}}\} \\ &= Pr\{C_{\geqslant 2} | C_{\supseteq} \cap CF2\} \end{aligned} \tag{22}$$

By taking advantage of assumption A4, it is easily to see

$$p_{c_o} = 1 - (1 - \phi)^{N-1} \tag{23}$$

While the network collision probability p_{c_*} is the probability that at a time in which at least one nodes is transmitting, one or more other nodes are also transmitting.

$$\begin{aligned} p_{c_x} &= Pr\{TX_{\geqslant 2} \mid TX_{\geqslant 1}\} \\ &= 1 - Pr\{TX_1 \mid TX_{\geqslant 1}\} \\ &= 1 - \frac{S_*}{p_{tx_*}} \end{aligned} \tag{24}$$

Replacing S_*, we get

$$p_{c_*} = 1 - \frac{N\phi(1-\phi)^{N-1}}{1-(1-\phi)^N} \tag{25}$$

5.4. *Discard probability*

Definition 5.4. The packet-discard probability p_d is the fraction of generated packets that are not transmitted with success. If p_{dc} and p_{df} are the probabilities that a packet is discarded due to reaching the retransmission limit R or due to FAIL, then

$$p_d = p_{dc} + p_{df} \tag{26}$$

The p_{df} and p_{dc} are

$$\begin{aligned} p_{dc} &= Pr\{A_1, \ldots, A_{R+1} : \text{COL}\} \\ &= p_{\text{COL}}^{R+1} \end{aligned} \tag{27}$$

$$\begin{aligned} p_{df} &= \sum_{i=0}^{R} Pr\{A_1, \ldots, A_i : \text{COL} \cap A_{i+1} : \text{FAIL}\} \\ &= \sum_{i=0}^{R} p_{\text{FAIL}} p_{\text{COL}}^i \\ &= p_{\text{FAIL}} \frac{1 - p_{\text{COL}}^{R+1}}{1 - p_{\text{COL}}} \end{aligned} \tag{28}$$

Taking advantage of assumption A2), then

$$\begin{aligned} p_d &= p_{dc} + p_{df} \\ &= p_{\text{COL}}^{R+1} + p_{\text{FAIL}} \frac{1 - p_{\text{COL}}^{R+1}}{1 - p_{\text{COL}}} \end{aligned} \tag{29}$$

where R is the number of retransmission, and M represents

$$p_{\text{FAIL}} = (1 - y)^{M+1} \tag{30}$$

$$p_{\text{COL}} = p_{c_o}(1 - p_{\text{FAIL}}) \tag{31}$$

$$p_{SUC} = (1 - p_{c_o})(1 - p_{\text{FAIL}}) \tag{32}$$

5.5. *Power consumption*

Definition 5.5 (Average Power Consumption)**.** Let us represent the average power consumption per node, by $\bar{P}$. $\bar{P}$ depends on whether a node is idle, receiving, or transmitting, the node uses different power levels denoted by P_{id}, P_{rx}, and P_{tx}, respectively.

$$\begin{aligned} \bar{P} = & \frac{\bar{n}_B P_{id} + \bar{n}_C P_{tx}}{\bar{n}_B + \bar{n}_C + (3 + L)(1 - p_{\text{FAIL}})} \\ & + \frac{(1 - p_{\text{FAIL}})[(P_{id} + 2P_{rx}) + LP_{tx}]}{\bar{n}_B + \bar{n}_C + (3 + L)(1 - p_{\text{FAIL}})} \end{aligned} \tag{33}$$

where $\bar{n}_B$ and $\bar{n}_C$ are the average number of slots per attempt in Backoff and CCA, respectively.

When using the simplifying assumptions, the mean number of Backoff, $\bar{n}_{B_{tx}}$

$$\bar{n}_{B_{tx}} = \sum_{i=0}^{M} \left(\sum_{k=0}^{i} \frac{W_k - 1}{2} \right) \frac{p_{S_i}}{1 - p_{\text{FAIL}}} \tag{34}$$

where p_{S_i} is the probability that the first time two connective CCAs are successful in backoff stage i,

$$p_{S_i} = y(1 - y)^i, \quad 0 \leqslant i \leqslant M \tag{35}$$

The mean number of slots that a node spends per attempt in backoff is

$$\bar{n}_B = \bar{n}_{B_{tx}}(1 - p_{\text{FAIL}}) + \bar{n}_{B_f} p_{\text{FAIL}} \tag{36}$$

Similar to the mean number of backoffs, the mean number of CCAs, $\bar{n}_C$

$$\bar{n}_C = \bar{n}_{C_{tx}}(1 - p_{\text{FAIL}}) + \bar{n}_{C_f} p_{\text{FAIL}} \tag{37}$$

where $\bar{n}_{C_{tx}}$ and $\bar{n}_{C_f}$ are the mean number of CCAs before a successful access procedure and the mean number of CCAs due to an access failure, respectively.

The $\bar{n}_{C_{tx}}$ is given by

$$\bar{n}_{C_{tx}} = y \sum_{i=0}^{M} \sum_{k=0}^{i} (i+k+2) \frac{p_{f_{i,k}}}{1-p_{\text{FAIL}}} = 2 + [2(1-y)-\alpha] \left[\frac{1}{y} - (M+1) \frac{(1-y)^M}{1-p_{\text{FAIL}}} \right] \quad (38)$$

where $p_{f_{i,k}}$ is the probability of a node having k successful CCA1 and failing CCA2 for some $k \leqslant i$.

$$p_{f_{i,k}} = \binom{i}{k} [(1-\alpha)\beta]^k \cdot \alpha^{i-k} \quad (39)$$

Similarly, the $\bar{n}_{C_f}$ is

$$\bar{n}_{C_f} = \sum_{k=0}^{M+1} (M+1+k) \frac{p_{f_{M+1,k}}}{p_{\text{FAIL}}} = (M+1) \left(2 - \frac{\alpha}{1-y} \right) \quad (40)$$

5.6. *Delay for transmitting packets*

Definition 5.6 (Average Delay for Successful Transmission)**.** The average delay for a successfully transmitted packet is the number of slots a packet takes from the moment it reaches the head of the line to the moment it arrives at its destination.

$$\bar{D} = (\bar{n}_{B_{tx}} + \bar{n}_{C_{tx}} + L + 3)(\bar{r}_{suc} + 1) - 3 \quad (41)$$

where $\bar{n}_{B_{tx}}$ and $\bar{n}_{C_{tx}}$ are the mean number of slots spend performing Backoff and CCA, respectively. $\bar{r}_{suc}$ is the average number of times a packet has to be retransmitted until it is transmitted with success.

Each packet has $R+1$ possible RXs, $\bar{r}_{suc}$ is defined as

$$\bar{r}_{suc} = \sum_{i=0}^{R} i Pr\{A_1 \cdots A_i : \text{COL} \cap A_{i+1} : \text{SUC} \mid \text{pkt SUC}\} = \sum_{i=0}^{R} i Pr\{\text{pkt SUC} \mid A_1 \cdots A_i : \text{COL} \cap A_{i+1} : \text{SUC}\} \times \frac{Pr\{A_1 \cdots A_i : \text{COL} \cap A_{i+1} : \text{SUC}\}}{Pr\{\text{pkt SUC}\}} \quad (42)$$

It is obvious that

$$Pr\{\text{pkt SUC}\} = 1 - p_d = p_{\text{SUC}} \frac{1-p_{\text{SUC}}^{R+1}}{1-p_{\text{COL}}} \quad (43)$$

By using assumption A2) and Equation (43), then Equation (42) turns into,

$$\bar{r}_{\text{SUC}} = \sum_{i=0}^{R} i \frac{p_{\text{SUC}} p_{\text{COL}}^i}{1-p_d} = p_{\text{SUC}} \cdot \frac{1-(R+1)p_{\text{COL}}^R + R p_{\text{COL}}^{R+1}}{(1-p_{\text{COL}}^{R+1})(1-p_{\text{COL}})} \quad (44)$$

6. Performance Analysis

To validate the performance of the Markov Chain model, we adopt a computer simulator that mainly focuses on IEEE 802.15.4 MAC layer in WSNs. The performance indicators of interest are the throughput, the collision probability, the transmission probability and the average number of CCA per channel access. The average power consumption and average packet delay are also considered.

6.1. *Algorithm*

The channel accessing mechanism in WPAN MAC layer is achieved based on IEEE 802.15.4 MAC layer protocol. It is a modified slotted CSMA/CA with retransmission, shown in Algorithm 6.

We consider the network having N nodes, each performs CCA on its own. The simulation is run for T seconds. The maximal number of retransmission is R. All these parameters plus the parameters for the WPAN are given in Table 3. In all experiments, two constants `aMinBE` and `aMaxBE` are chosen as 3 and 5, and it should satisfy `aMinBE` $\leqslant$ `BE` $\leqslant$ `aMaxBE`.

Table 3. The parameters for the simulation and the WPAN

Parameter	Values
Nodes: N	2, 4, 6, 8, 10,
BackOff: M	4 (stages)
RX: R	3 (round)
Slots: T	10^6 (slots)
Length: L	7 (slots)
L_{ack}	2
aMinBE	3
aMaxBE	5
$W_i = 2^{\min(\texttt{aMinBE}+i,\texttt{aMaxBE})}$	[8 16 32 32 32]
P_{idel}	0.5e-5
P_{tx}	26.9e-3
P_{rx}	26.7e-3

```
Set RX round and State vectors
r = 0  BE(1:.N) = macMinBE
NB(1:,N) = 0  CW(1:.N) = 2
for T slots do
    check node i read to for CCA (delay == 0)
    update nbCCA(i)   if channel idle then
        decrement CW for nodes i  check nodes j
        from i with CW == 0  if more than one node
        j then
            increment nbCollision(j)
        else
            increment nbTransmission(j)  set
            busyFor(j) to the transaction length in
            slots
        end
    else
        Channel busy  check nodes j transmit-
        ting  for node transmitting j do
            i.e.  s(t)=-1  decrement BusyFor(j)
            if BusyFor(j)==0 then
                reset NB(j)=0, CW(j)=2 and
                delay(j)
            else
                BACKoff or ACK , i.e. s(t) ⩾ 0
                or s(t) = −2  decrease CW by 1  if
                CW == 0 then
                    transmit packet  increase r
                    by 1
                else
                    CCA1 or CCA2 or ACK  go
                    back to performing CCA1
                end
            end
        end
        for node i doing CCA do
            update CW(i),NS(i),BE(i) ac-
            cording to CSMA/CA algorithm
            check nodes k from i with
            NB(k)==maxMaxCSMABackoffs
        end
    end
end
```

Fig. 6. Slotted CSMA/CA with Retransmission

Given a network with N nodes, the channel will be competed at a random time. The Chipcon CC2430 are used when computing the power consumption, where CC2430 Chipcon is IEEE 802.15.4 compliant Radio Frequency transceiver.

6.2. *Performance indices comparison*

In this section, we present the network performance indices under the simulation and the theoretic model results.

First of all, we take the network success probability as the network throughput. Fig. 7 depicts the network throughput in the experiments

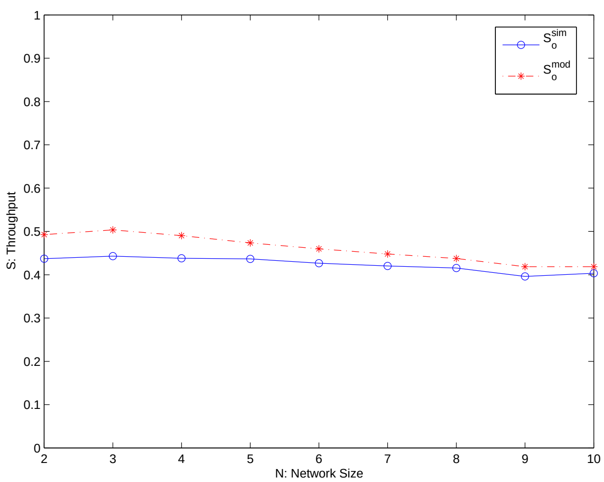

Fig. 7. Network throughput vs Size

The network throughput is represented with the overall probability of packets successfully transmitted irrespective of the number of retransmission. From Fig. 7, we can clearly see that the simulation result converge to the modeled results, although the difference between two when there are a small number of nodes are relatively large around 10%.

The network transmission probabilities are given in Fig. 8. From Fig. 8, we can see that the transmission probability of per-node is declining, but the overall probabilities of per network is climbing up. Despite of this divergence in the transmission probabilities, the average transmission probabilities per node and the per network are approaching close to each other.

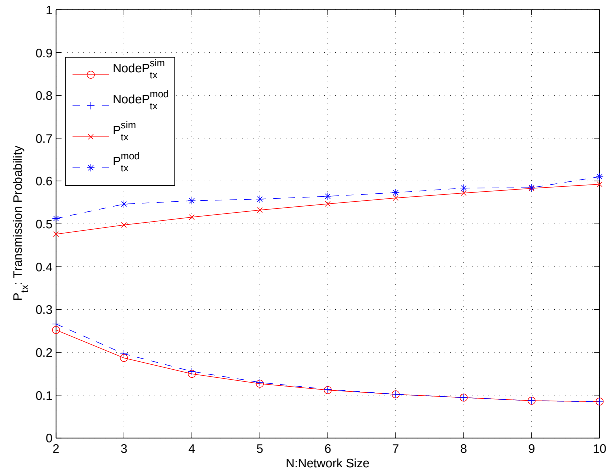

Fig. 8. Transmission probability

The relation between the per-node collision probability and the the probability that one or more nodes are transmitting is not simple, ie. they are not in a linear relationship. Fig. 9 has given the results. We have contrasted the average collision probabilities of the node and the network. The simulation always have a higher collision probability than the network despite looking at per-node or per-network.

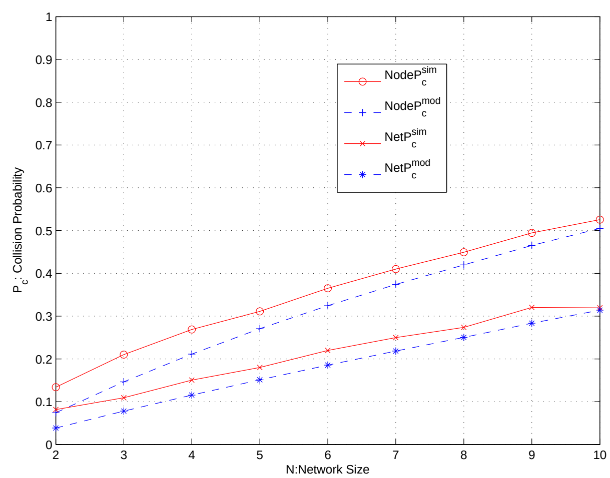

Fig. 9. Collision probability

The power consumption is shown in Fig. 10. The simulation results well match the theoretic results. The interesting thing is that the average power consumption of the network is decreasing along with the number of nodes in the networks. There are noticeable differences in the average power consumption when the size of networks are small, say 2 or 3.

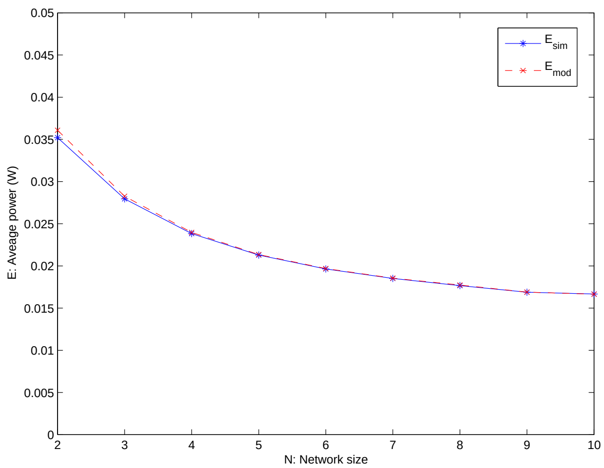

Fig. 10. Power consumption

The average delay shown in Fig. 11 show a trend of increasing along with the size of networks. The difference between the simulation result and the theoretic model result is relatively large and noticeable when the size of the network are small, however, the differences are fast disappearing when coming to the networks with large number of nodes.

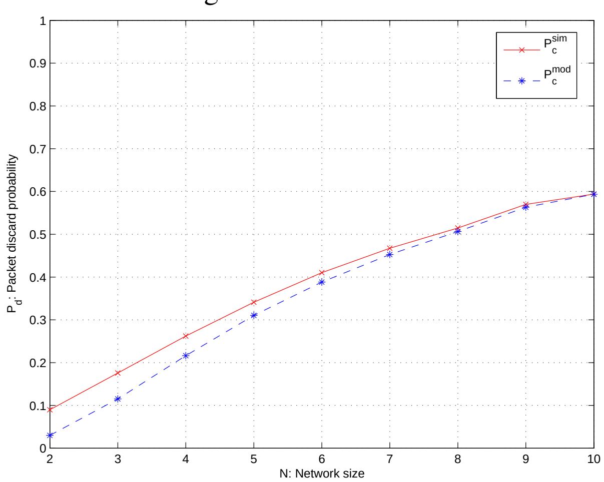

Fig. 11. Average delay

7. Conclusion

We envisaged that WSNs technology will play a significant role in many areas. The deployment of WSN in medical and healthcare domains such as WPANs surely takes increasingly important position as they have done in other fields. We have looked into the performance of IEEE 802.15.4 MAC layer protocol by modeling WSNs using the Markov chain in this paper. From the comparison of the performance in metrics, we come to conclude that the derived mathematical model gives excellent results very close to the CSMA/CA mechanism with retransmissions that most of simulated WPANs in the terms of the network throughput, the transmission and collision probability, the power consumption and the average packet delay. This indicates that it is possible and appropriate to study the performance of WPANs deployed in the medical and healthcare environment by examining the derived mathematical model.

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