Technology and the Knowledge Quartet

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Rowland's Knowledge Quartet (KQ) model can provide illuminating insights into teacher practice in mathematics classrooms. This paper explores a framework developed for pedagogical technology knowledge (TPACK) to consider teacher practice in integrated technology classrooms alongside the KQ model. Observations of a Year 11 class in an Australian school provides evidence that technology may impact practice in each of the four component areas, Foundation, Connections, Transformation and Contingency knowledge of the Knowledge Quartet model.

The Powerful Knowledge Project

Many studies in recent years have explored teachers' knowledge for teaching mathematics (e.g., Ball, Thames, & Phelps, 2008; Rowland, Turner, Thwaites, & Huckstep, 2009). The Powerful Knowledge Project, from which this paper reports, aimed to consider changes in the ways that teachers drew on their subject and pedagogical content knowledge as the subject matter they taught became more specialised and advanced. The project was implemented in two curriculum areas—Mathematics and English—across two Australian states where the curriculum is similar, but the context of schooling differs, and in New Zealand, where there is a different curriculum and schooling structure. In volunteer teachers' classrooms, single lessons were videotaped for later analysis and the videotape was supplemented by field notes taken by an experienced observer. Brief interviews were held with the teachers concerned before and after lessons. The lessons were not specially staged for the project but aimed to be a snapshot of regular lesson. Videotapes were viewed, transcribed and analysed by the research team using indicators described in the four quadrants of the *Knowledge Quartet* (Rowland et al., 2009).

This paper reports on observations from a Year 11 mathematics lesson in an Australian classroom where technology was highly integrated.

Theoretical Perspectives: The Knowledge Quartet and Technology

Since the seminal work on *Pedagogical Content Knowledge* (PCK) by Shulman (1987), there has developed a consensus that teachers need to be competent in both pedagogy and content to be effective mathematics teachers (e.g., Ballet al., 2008; Hill, Schilling, & Ball, 2004). Studies have shown that professionally competent, experienced teachers demonstrate substantive knowledge regarding mathematics content and pedagogy with respect to mathematics specifically, and in general teaching practices (e.g., Blömeke, Suhl, & Kaiser,

2011; Beswick, Callingham & Watson, 2012; Schmidt et al., 2008). These different knowledge components should be reflected meaningfully in classroom teaching for effective student learning outcomes. Such understandings underpinned the research by Rowland and his colleagues (e.g., Rowland, 2013; Rowland et al., 2009), which used a grounded theory approach to analyse the behaviours and practices of novice pre-service teachers of mathematics in primary and middle school classrooms from the United Kingdom. Within their framework known as the *Knowledge Quartet* (KQ), they identified four dimensions of knowledge and the processes for using that knowledge for teaching mathematics, as described in Table 1.

Table 1
Summary of the KQ framework (drawn from Rowland, 2013, p. 26)

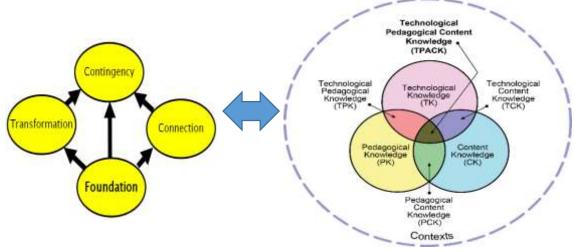| Categories	Brief description
Foundation	Teachers' mathematical content knowledge which includes teachers' beliefs about and understandings of mathematics itself and their ability to make meanings and create descriptions of relevant mathematical concepts, and of relationships between them
Transformation	Teachers' ability to transform their foundation mathematical knowledge into pedagogical actions, for example using powerful analogies, illustrations, explanations, and demonstrations
Connection	The ability of teachers to make connections between different meanings and descriptions of mathematical concepts and make these explicit. It also examines the teacher's skill in suggesting alternative ways of representing ideas and carrying out procedures
Contingency	The teacher's response to classroom events that were not planned or anticipated in the teachers' planning, and the subsequent decisions they make (i.e., their capacity to "think on one's feet")

The KQ framework has proved effective in the analysis of lessons recorded for the Powerful Knowledge Project to date (e.g., Getenet & Callingham, 2019 [submitted]). However, there is some uncertainy of its generalisability outside of the context in which it was developed (novice-primary teachers in the UK). How applicable is it at higher secondary levels and what value does it have in examining the practice of expert teachers? A review of the analysis of lessons (Rowland et al., 2009) indicates that the pre-service teachers were frequently operating within a traditional teacher-led instructional paradigm with teacher talk the main medium of instruction. How might the framework change, for example, in an inquiry-focused or a technology-rich classroom with experienced teachers? In addition, we have explored the value of additional frameworks to further inform our analysis. In one of these, Getenet and Callingham (2019, submitted) used the framework developed by Behr et al. (1997) to examine the teaching of fractions. They used the five interrelated constructs of the Behr et al. (1997) framework (part-whole, ratio, operator, quotient, and measure) to investigate a fraction lesson in a New Zealand Year 7 classroom. Their findings showed students often initiated unexpected uses of fractions as quotient and as operator, with implications for the teacher's *Foundation* knowledge and *Contingency*. More recently, Oates, Callingham and Hay (2019, submitted) trialled the *Pedagogical Technology*

Knowledge (PTK) framework developed by Thomas and Hong (2013) to examine the use of integrated technology in a Year 9 probability lesson.

The integration of technology in teaching has received increasing attention as a factor for the development of students' learning, and changing classroom dynamics (e.g., Getenet, 2017; Mishra & Koehler, 2006; Thomas & Hong, 2013). In this respect, researchers have considered the three intersecting domains of technical knowledge, content knowledge, and pedagogical knowledge for effective teaching (e.g., Chee et al, 2005; Forster, 2006; Mishra & Koehler, 2006). Various attempts have been made to conceptualise and analyse teachers' teaching using integrated technology, for example, in the PTK of Thomas and Hong (2013), which extended Shulman's PCK-model (1987) to incorporate technology:

> A teacher's PTK applied to mathematics includes the principles, conventions and techniques required to teach mathematics through the technology (Thomas & Hong, 2013, p. 69).

In this paper, we adopt the conceptual framework Technological Pedagogical Content Knowledge (TPACK) proposed by Mishra and Koehler (2006) as depicted in the right-hand component in Figure 1. The model shows three intersecting components of teacher knowledge, which in effect adds a third component, *Technological Knowledge* (TK), to the conceptualisations of *Pedagogical Knowledge* (PK) and *Content Knowledge* (CK) described in previous studies (e.g., Ball et al., 2008; Shulman, 1987). To integrate technology in effectively into teaching practice (the intersection of all three components conceptualised as TPACK), the claim is that teachers need to be competent in all three domains (Mishra & Koehler, 2006). However, Mishra and Koehler's (2006) definitions of knowledge types are based on a generic definition of content and hence of pedagogical knowledge and PCK. Only a few studies have related the TPACK framework to specialised subject contexts and mathematics in particular (Getenet, 2017; Guerrero 2010).

Figure 1. Complementing frameworks: The KQ (Rowland et al., 2009)
and TPACK (Mishra & Koehler, 2006)

Figure 1 describes how the TPACK framework might be used to complement the KQ, as is explored in this paper. On the one hand, the KQ, which explicitly defines PCK specific to mathematics teaching, can be used to analyse teacher practice across the four components of the model; on the other hand, we can use TPACK as an additional framework to help explain the influence of technology on teachers' *Foundation, Transformation, Connection,* and *Contingency* knowledge.

The Research Setting

The research for this paper took place in an independent (non-government, non-denominational) boys' school in Australia. It offered education from Year 5 to Year12 with the school identified as being in a high socio-economic band (Index of Community Socio-educational Advantage (Australian Curriculum, Assessment and Reporting Authority, 2013) value 1215) and makes extensive use of digital learning practices. Teachers were asked not to prepare a special lesson but to go about their regular teaching. Researchers took notes from brief interviews with the teachers before and after the lesson, to determine where this lesson fitted in with their overall teaching plan and the nature of their students, so the researchers might better interpret the progression of the lesson and the teachers' actions. Participating teachers were given a wireless lapel microphone linked to a single portable tripod-based video camera set up at the back of the room. One researcher operated the camera, while another observed and took notes. Recordings were transcribed identifying the teacher and generic student contributions.

The class on which this paper reports was a Year 11 class of 18 boys, seated in four long-row tables. The teacher (Neil, not his real name) was an experienced registered teacher of secondary school mathematics with a degree in mathematics and science and a postgraduate university qualification in Education. He had taught mathematics at the school for more than 10 years and the lesson occurred towards the end of the school year, so the teacher knew his students well. The observed lesson was a 60-minute part of a calculus unit that the teacher had been developing over several lessons. The main theme of the lesson was visually connecting the graphs of the gradient function with its initial function before differentiation and followed a previous lesson in which they had looked at differentiation from first principles. After a brief review of the learning, the students worked individually and with their immediate neighbours through the prepared activities and problems. Each student used a tablet-laptop computer (i.e., a laptop with the facility to write on the screen), connected to the internet and the school's e-portal, which provided access to the lesson and enabled students to revise and review past content. The teacher used a tablet and a large-screen television monitor at the front of the room as his instructional focus point, connected through the e-portal to students' computers. At different times he brought the class group together and used the screen to explain something to the whole group or to have the students review the problem under investigation and its "solution". Although the teacher could monitor all laptops from the front, he constantly moved around the room talking to students in groups and individually.

Linking TPACK and the Knowledge Quartet: Evidence from the Lesson

Transcripts of the lesson were first analysed separately by three of the researchers to identify examples of practice within the four KQ components (Rowland et al., 2009) with close agreement in the resultant coding. We then looked for ways in which the seven categorisations of the TPACK framework (Mishra & Koehler, 2006) might distinguish between, or afford additional insights into, the examples identified within the KQ model. We also looked for examples from the lesson that might reflect elements of the TPACK model not previously identified in our KQ analysis. Figure 1 shows the relationship between the seven components of the TPACK Model as described earlier (PK, CK, TK, TPK, TCK, PCK, & TPACK), and we summarise some examples of these in Table 2.

With respect to the KQ framework, the lesson clearly demonstrated the teacher's strong *Foundation* knowledge across all parts, but more so at the start and end, in setting up and

drawing the lesson together. At the start of the lesson for example, Neil reviewed the previous lesson and positioned the new lesson, stating:

> …yesterday, we were looking at differentiation by first principles and I kind of said, look, you could probably be very successful Year 11 and 12 calculus at the university if you didn't really, if you got that right once and then you forgot about it, you could go on and be very good in terms of your calculus knowledge. What we're doing today is pretty central, okay? So, you're going to learn a whole bunch of things over the next couple of years, chain rule, product rule, quotient rule, integration, a whole lot of algebraic techniques, but this conceptual understanding and the graphical representation of what we mean by differentiation's really important.

This example illustrates Neil's *Foundation* content knowledge (CK) in terms of the purpose and value in teaching differentiation from first principles; what Ball et al. (2008) would further categorise as *Horizon Knowledge*. It also identifies Neil's knowledge of the importance of graphical representations for students to understand the concepts that they will meet in coming years and the need to make *Connections* between the various concepts (PK).

Table 2
Technology influencing Foundation, Transformation, Connection, and Contingency knowledge

Categories	Observed evidence ("N" is the teacher Neil; "S" is used generically for student comments)	TPACK Code(s)
Example 1 *Foundation*	Instead of using the formula for differentiation, the teacher made a deliberate decision for the students to use a graphics calculator to estimate the gradients of a function on the graph of $f(x)$ and connect this to the graphical representation of the gradient function.	CK TPK PCK
Example 2 *Transformation*	N: So, you're going to get your graphics calculator out and for $y = x^2$, you're going to find the gradient when x equals negative three. Or, the gradient when x equals negative two, etcetera. And then you're going to plot those points on this set of axes, the one underneath… So for $y = x^2$, you're going to plot the gradient function using those points here [*Neil demonstrates on the screen*]… so this gets the gradient function and state the equation of it. Then you're going do the same thing for the second function… So guys, if you want to open up your tablet so you can fill that in, it's fine but I'll leave it up on the board.	TPK TCK TK
Example 3 *Connection*	N: So, guys, we've come across our first important point here. If you put negative 3, you said negative 3 squared is 9 and so you're filling in the table and say oh (-3, 9). That's great, you just found a point on the curve [*Neil indicates on the screen*]. But that's not what the question asks you to do. It asks you to find the gradient at that point. Now clearly the gradient in there is negative. So, whatever you get has to be a negative value.	CK TPK
Contingency	N: Okay. So, you reckon the gradient, when x is negative 3 is 9? You reckon it's got a positive gradient? S: Yeah, not really	CK

N: You need to use the trace function. You found the y values. You've got to find the gradient.	TPACK

Other components became evident as the lesson progressed. *Connections* were seen for example in the procedures and the sequencing of concepts in the lesson. Neil began with concrete examples of calculating gradients on the curve of a function $f(x)$ using estimates from drawing tangents on the curve (estimating $\frac{\Delta y}{\Delta x}$), and moved towards connecting this with the graph of the gradient function $f'(x)$. Soon after explaining the importance of what they would be doing, Neil demonstrated a powerful example of *Transformation* in describing the way he had set up the graphs, with the graphs of two functions $f(x)$ drawn at the top, and another set of axes aligned immediately below each function for sketching the points calculated for the gradient of $f(x)$. While one function was easily recognisable by the students as $f(x) = x^2$, the second function was more complex and there were no scales on the axes to help them determine its equation. Students had to make connections between the procedures and the concept of $\frac{dy}{dx}$ for $f(x) = x^2$ to help them with the second more complex example, using the aligned axes to help them and transform this knowledge to a new context.

> Well, it just so happens that you went through a process to find the gradient function and it gave you the line $\frac{dy}{dx} = 2x$. Alright? So, even before you start this one, you should be able to work out what you're expecting in advance…this is deliberately a graph with no scale. That's the origin, but you've got no numbers to work with. I want you to sketch… again, the Y axes or the vertical axis is lined up. I want you to sketch a possible gradient function of that on the axes below. No numbers. You can't use your calculator. You've got to look at it and interpret, right, "What have I just learnt and how can I apply that to a function that has no scale when I don't have any numbers?"

Contingency is shown across the lesson as the teacher exchanges dialogue and stops and starts many times as the students come to an issue needing assistance. At one point in the lesson, Neil could see many students were confused about what turning points for a function itself told them about the gradient of that function, as distinct from the turning points of the gradient function: Neil asked one student to demonstrate on the screen how he used the turning points to help sketch the graph of the gradient function:

> S: So, firstly just these two points, the turning points, you know that they're going to be… the intercept's on the next graph because the gradient here is zero and so, you can plot them easily. And the next thing is… the turning point and the turning point's going to be the highest gradient on the graph, which is here.
>
> N: Hang on, so you mean the turning point of your gradient function?
>
> S: [student is thinking and gesturing to the screen]
>
> N: Sorry, I'm interrupting here, but you're telling me that… you're saying that this point has the highest gradient in this section?

This extract shows Neil dealing with *Contingency* and connecting this back to *Foundation* knowledge using the definitions of turning points and gradients. This example also illustrates frequent observations from the lesson with respect to the sophisticated language used by the students in their conversations; further support for Neil's attention to content knowledge (CK) and making connections (PCK). Neil revisited the concept of maximum and minimum turning points and their relationship to the gradient of the function later in the lesson to reinforce these connections. His choice of difficult examples was causing the students to struggle, requiring him to go back over terms and procedures. The suggestion here is that Neil's strong *Foundation* knowledge and confidence in dealing with *Contingency* allowed for more effective *Transformation* and scaffolded the students in their

struggle. This in turn helped students to make connections between the higher concepts.

With respect to the use of technology, it is useful here to distinguish between "demonstration" technologies used by the teacher (e.g., OneNote to prepare the class materials provided to students and project on the screen) and tools for actually "doing" mathematics such as the graphics calculators used by the students and the teacher's use of his tablet to show calculations and sketch directly onto graphs shown on the digital screen. Example 1 in Table 2 demonstrates how Neil used his awareness of the affordances of the graphics calculator (TPK) to design a lesson with an innovative approach to differentiation (CK) rather than what may be the more common approach using the formula (PCK). Example 2 shows how Neil used the technology to transform students' knowledge, making connections between the gradients of a function calculated using technology (TK) and the gradient function itself, again lotted using technology (TPK). Examples 3 and 4 show how Neil responded to the *Contingency* by using the technology to distinguish between the gradient of a function, and the value of the function itself at a certain point (CK). In doing this, he demonstrated his technical knowledge (TK) in the value of the calculator's trace function and his technological pedagogical knowledge (TPK) in how this may help students understand this potentially confusing but critical conceptual difference.

Summary

The evidence and discussion presented here reinforce the findings from Oates et al. (2019, submitted) that there is indeed value in using the KQ framework (Rowland, 2013) to examine an experienced teacher's practice at the upper secondary level, and thus supports extending the use of the KQ framework across curriculum levels (i.e., from primary to secondary). There are multiple examples that demonstrate how the teacher's practice may be examined within all four components of the KQ model. The preliminary analysis by Oates et al. further suggested a greater exploration of the role of technology within the KQ framework when they observed that:

> There is an implication that the contributing codes within each of the four components may need some modification or extension to accommodate the impact of technology on teacher practice and the mathematics classroom (Oates et al., 2019 submitted, p. 7).

The findings here add weight to this proposition, with clear examples of the impact of technology on the lesson within each of the four KQ components, and the additional insights into these provided by the TPACK framework. The teacher had a strong understanding of the purpose of the lesson and was working to build *Foundation* and *Connection*. He allowed the students to have control of the software but continually monitored their outputs to address any misconceptions (*Contingency*). Technology (OneNote) played a significant role by consolidating students' work within one place while allowing for individual exploration. In this respect, *Transformation* is seen at the macro-control of the software and technology while the choice of activities and the time on task indicated the teacher had high levels of TPK (awareness of the technology's affordances) and PCK in making *Connections* between procedures and concepts.

Casting a TPACK lens over the *Transformation, Connection,* and *Contingency* dimensions of the KQ framework suggests the technology helped to facilitate students' active engagement in learning and helped them connect concepts and consolidate their learning (TPK). There is no "dumbing down" of the software (TK) or the mathematics (CK); instead, we see the technology used to scaffold their learning (TPK) as the students worked at the high end of their knowledge (PK). Contingency was typically linked back to the

foundation knowledge (PK) and the connection between concepts (CK). For the teacher, the technology was seen as a tool to advance the students' mathematics knowledge (TCK) in a seamless, integrated manner. The teacher may therefore be regarded as working within the intersection of the three components of TK, PK, and CK, conceptualised as TPACK by Mishra and Koehler (2006, see Figure 1).

We thus conclude that using complementary models such as depicted in Figure 1 provides valuable insights into teachers' classroom practice. The next steps might be to compare different lessons and examine to what extent such an approach allows us to identify the implications for effective teacher classroom practice.

Acknowledgments

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