A mathematical model for complete stress-strain curve prediction of permeable concrete

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ABSTRACT: An empirical equation to represent the complete stress-strain behaviour for unconfined permeable concrete with compressive strength ranging between 10-35MPa and porosity ranging between 25-15%, made with different combinations of aggregate size and sand ratios is proposed in this paper. A series of compression tests were conducted on 100 × 200 mm cylindrical samples using a modified testing method to determine the complete stress-strain behaviour of permeable concrete. Various existing models for low strength concrete and normal strength concrete were used and compared with the experimental data. Various parameters were studied and their relationships were experimentally determined. The only parameters need to run the model is the ultimate compressive strength and the density. The proposed empirical stress-strain equations were compared with actual cylinder tests results under axial compression, and demonstrated that the present model gives a good representation of the mean behaviour of the actual stress-strain response.

1 INTRODUCTION

There has been an increasing use of permeable concrete in the civil engineering and building construction industries in recent years (Offenberg 2008). However its use is currently limited to low trafficked areas such as pavements in car parks and footpaths, largely due to its low strength and stiffness. It is timely to investigate the stress-strain behaviour of permeable concrete to help enable its wider use in more structural applications. An understanding of the complete stress-strain curve of permeable concrete is essential for rational design, as structural designers are unable to take full advantage of the material with insufficient information about this behaviour.

A number of researchers (Attard & Setunge 1996,Carreira & Chu 1985, Kent & Park. 1971, Kumar 2004, Lokuge et al. 2004, Lokuge et al. 2005, Popovics 1973, Sargin et al. 1971, Tasnimi 2004, etc) have studied the stress-strain behaviours of unconfined and confined conventional concretes under uni-axial compressive loading. Complete stress-strain relationships were developed in some of these studies on the basis of easily measured conventional concrete material parameters such as peak stress f'_c, the corresponding strain ε_o and the initial modulus of elasticity E_o.

The information about the stress-strain behaviour of the permeable concrete is still limited (Hussin et al. 2012) and the only research paper that reported this behaviour was based on experiments conducted by Deo & Neithalath (2010). These authors found that the stress-strain behaviour of the permeable concrete is approximately similar to that of the conventional concrete. It is reported that there were many material parameters such as aggregate size, porosity ratio, and the pore structural features, as well as the test method that influenced stress-strain behaviour. Deo & Neithalath (2010) proposed a mathematical stress-strain model for the permeable concrete that was related to the pore structure. Deo & Neithalath (2010) employed the model proposed by Carreira & Chu (1985) for the conventional concrete with some modifications. The proposed empirical model was found to agree satisfactorily with their own test results but it required extensive computations to determine the essential parameters and cannot be used to represent the stress-strain behaviour of a different strength of permeable concrete that is prepared under different conditions. However, the ascending branch of the stress-strain curve is well–represented by the model; and most discrepancies between the actual and the predicted curve were observed in the descending branch.

2 PROPOSED STRESS-STRAIN RELASHIONSHIP

In an effort to construct a simple mathematical model that can represent the stress-strain curve of permeable concrete with different compressive strengths and porosity ratios, it was found that the model developed by Carreira & Chu (1985) and later modified by different authors such as Tasnimi (2004) is suitable for adopting in the stress-strain relationship of permeable concrete. The advantages of this model are:

- It performs well for the low strength and normal strength concrete, which are both in the same range as the permeable concrete's compressive strength.
- It is simple and has the same general equation for use in both the ascending and the descending branch.
- Its parameters are easy to find from the experimental data.

Equation 1 illustrates the general equation for the stress-strain behaviour as proposed by Carreira & Chu (1985).

$$f_c = \frac{f'_c \beta(\varepsilon / \varepsilon_o)}{[\beta - 1(\varepsilon / \varepsilon_o)^{\beta}]} \tag{1}$$

$$\beta = \frac{1}{\left[1 - (\frac{f'_c}{E_o \varepsilon_o})\right]} \tag{2}$$

Where: f_c = concrete stress; f'_c = maximum stress; β = material parameter; ε =concrete strain; ε_o =corresponding strain at maximum stress; and E_o = initial tangent modulus of elasticity.

3 EXPERIMENTAL PROGRAM

Twenty six specimens with different compressive strength and porosity ratios were tested. Strain gauging and the platen-to-platen methods were used to find the stress-strain relationship. Before testing, the cylinders were capped with a sulphur compound on both ends to produce a smooth surface to ensure uniform transfer of load. Two diametrically opposite 60 mm long strain gauges, were attached to each specimen in its middle third position. A prepared specimen ready to be tested is shown in Figure 1. The signals from the two strain gauges were averaged for a more accurate result. The results from the strain gauges were used to find the ascending branch of the stress-strain curve until the peak stress; the strain gauge would not give reliable results after that due to the development of vertical cracks in the surface of the specimens. The residual parts of the curve were determined using the platen-to-platen method. An Avery 500kN testing machine was employed for this purpose and the experimental set-up is shown in Figure 2.

The strain rates of the test specimens were kept constant to 10µs per second in order to obtain the stress-strain curves. During the experiment, the axial compression load and the vertical deformation of the test specimens were automatically collected by the attached computer. Preliminary low level loading was carried out at least three times primarily for the seating of the gauges and for investigating and correcting any unusual behaviour of the strain gauge according to AS 1012.17(1997).

Figure 1. Permeable concrete sample with strain gauges.

Figure2. Experimental set-up

4 PREDICTION OF THE MODEL PARAMETERS

4.1 *Estimating strain at peak stress* (ε_o)

One of the most important parameters affecting the ascending and descending portions of the concrete stress-strain curve is the corresponding strain at peak stress. Table 1 provides the values of the corresponding strain at peak stress, the initial modulus of elasticity and the density ρ from our testing results. As similar tests were conducted by Goede (2009), his results were also included.

The maximum and minimum values of ε_o for the permeable concrete specimens in this research varied from 0.0009 to 0.00189, the corresponding compressive strength varied from 9MPa to 38MPa and the porosity varied from 25% to 15% respectively. The tests results indicated that, on average, the peak strain increased as compressive strength increased and the porosity ratio decreased.

Table 1 Experimental results

Sample No	f'_c MPa	ε_o %	ρ kg/m^3	E_{it} GPa
1	9.89	0.001146	1976.7	7.8
2	10.62	0.001002	1991.8	15
3	12	0.0013	2050	13
4	12.3	0.0011	2095	9
5	12.77	0.00091	1984.95	7.6
6	12.86	0.001059	1965.9	9.7
7	14.28	0.001	2068.2	12
8	15.69	0.001	2046	12
9	16.15	0.001333	1981.3	11.7
10	17	0.0014	2070	15
11	17.1	0.0009	2068	19.55
12	17.7	0.00127	2220	19
13	20	0.0012	2080	17.25
14	20	0.00116	2211	18.6
15	23	0.0011	2255	26.6
16	24	0.001575	2220	15.6
17	25.6	0.0014	2235	22
18	25.7	0.0016	2208	18.4
19	26.5	0.00115	2298	19.5
20	27.5	0.0015	2210	17.88
21	28	0.0014	2267	13.6
22	30	0.0012	2285	26.6
23	31	0.0014	2300	27
24	36.6	0.00189	2321	25.4
25	38	0.0015	2315	28
26	38	0.00152	2323	30
27*	11.35		1861.4	12.99
28*	9.25		1834.1	12.86
29*	11.45		1880.6	15.19
30*	11.52		1854.9	13.44
31*	9.67		1837.3	12.06
32*	11.34		1875.7	12.57
33*	9.81		1838.9	9.81
34*	11.14		1856.5	14.99
35*	10.69		1893.4	12.99
36*	9.95		1843.1	12.1

* Tests results adopted from Goede (2009)

Several authors reported a linear relationship between the peak compressive stress and the corresponding axial strain (Almusallam & Alsayed 1995, Carreira & Chu 1985), while Tasnimi (2004) reported a polynomial function. These relationships are shown in Equation 3, 4 and 5 respectively.

$$\varepsilon_o = (0.398 f'_c + 18.147)10^{-4} \tag{3}$$

$$\varepsilon_o = (7.1 f'_c + 1680)10^{-6} \tag{4}$$

$$\varepsilon_o = (65.57 f'^{\,0.44}_c - 6.748)10^{-5} \tag{5}$$

The experimental and estimated ε_o that were found according to these equations were plotted against the compressive strength for comparison and are shown in Figure 3.

From Figure 3, a linear relationship was found between the compressive strength and the corresponding strain based on the permeable concrete experimental data. It is also clear that Eq. 4 developed by Carreira & Chu (1985) gives a more accurate estimation for ε_o when compared with other equations. In the current research, a linear relationship similar to Eq. 4 has been proposed as shown in Equation 6. To confirm the reliability of the proposed relationship, the ratio of the estimated strain ($\varepsilon_{o\ cal}$) divided by the experimental strain ($\varepsilon_{o\ exp}$) at peak stress was plotted against the compressive strength (f'_c) as shown in Figure 4. It can be observed from Figure 4 that the proposed linear relationship gives a better estimation for ε_o when compared with other equations. Thus it is proposed that Eq. 6 could be used to find the corresponding strain at peak stress for permeable concrete.

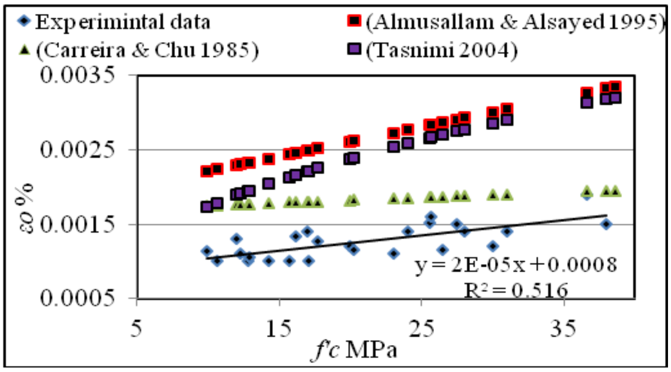


Figure 3 Experimental and estimated ε_o versus f'_c.

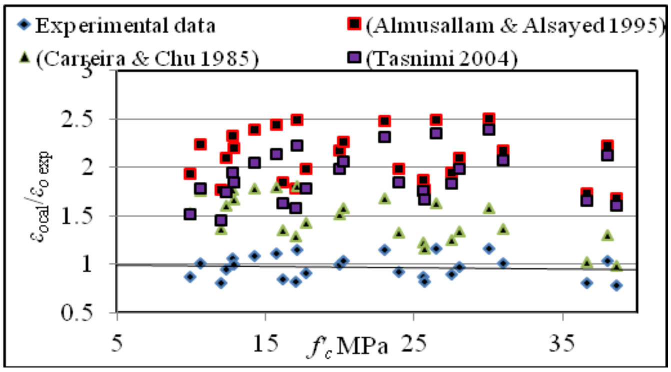


Figure 4.The ratio of the ε_{ocal} to ε_{oexp} versus. $f'c$.

$$\varepsilon_o = (2 f'_c)10^{-5} + 0.0008 \tag{6}$$

4.2 *Estimated the modulus of elasticity (E_o)*

Twenty six cylinders with different compressive strengths (f'_c) and porosity ratios (P) were tested using the strain gauge method in this research program. Results from Goede (2009) using the compressometer method were also adopted to determine the E_o. The compressometer has been used for evaluating deformation and strain characteristics of concrete cylinders while undergoing compression testing. Either a dial gauge or a digital indicator could be attached to collect the deformation/strain inforamtion of the testing sample. The testing results are shown in Table 1.

Various theoretical equations that can be used to determine the modulus of elastrcity of any type of concrete including permeable concrete were presented as follows.

The Australia Standard AS-3600(2009) specified Equation 7 for estimating the E_o of the plain concrete as a function of its unit weight (ρ) and the compressive strength. Equation 7 for normal concrete (when $f'c \leq 40$MPa) is based on the extensive work of Pauw (1960) (Attard & Setunge 1996).

$$E_o = 0.043(\rho)^{1.5}(f'_c)^{0.5} \tag{7}$$

Similarly, the Architectural Institute of Japan AIJ (1985) has specified the following Euation 8.

$$E_o = 21000\left(\frac{\rho}{2300}\right)^{1.5}\left(\frac{f'_c}{20}\right)^{0.5} \tag{8}$$

Tasnimi (2004) specified Equations 9 and 10 for determining the E_o for low strength concrete and normal strength concrete, respectively.

$$E_o = 2.1684(f'_c)^{0.535} \tag{9}$$

$$E_o = 2.25\ln\left[\frac{(f'_c)^{2.8}}{(\rho)^{0.2}}\right] + 0.05f'_c \tag{10}$$

Ghafoori & Dutta (1995) derived Equation 11 to find the E_o of the permeable concrete as a function of the unit weight and the compressive strength.

$$E_o = 4.258*10^{-5}(\rho)^{1.5}(f'_c)^{0.5} \tag{11}$$

In all these equations, E_o = initial tangent modulus of elasticity; ρ = the concrete density; f'_c = maximum compressive stress.

In order to find the most suitable equation for predicting E_o of permeable concrete, the values of the initial tangent modulus of elasticity, which were obtained experimentally using the strain gauges, were combined with the results obtained by Goede (2009) using the compressometer, are plotted in Figure 5. A comparison was then made between the experimental and the calculated E_o according to the equations proposed by Ghafoori & Dutta (1995) (Eq. 11), AS-3600 (Eq. 7) and AIJ (Eq. 8), against the ($\rho^{1.5} \times f'_c{}^{0.5}$) to highlight the excellent agreement of the these equations with the experimintial data.

It can be observed from Figure 5 that all of the equations proposed by various researchers, gave a close approximation of the experimental data. The conclusion can thus be drawn from this that the modulus of elasticity of the permeable concrete is similar to that of the low strength concrete and normal strength concrete. The equation proposed by Ghafoori & Dutta (1995) will thus be used to determine the E_o of the permeable concrete in the proposed model.

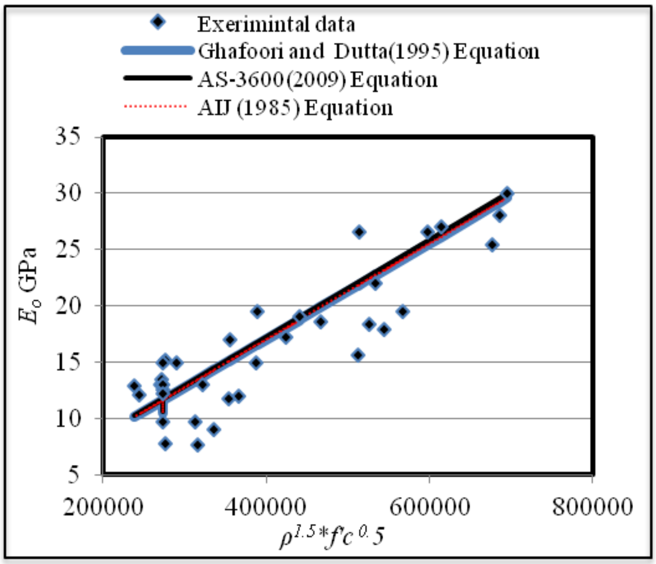


Figure 5. The relationship of experimental and calculated E_o versus $\rho^{1.5} \times f'_c{}^{0.5}$

4.3 *The effect of the porosity (P) on the stress-strain curve*

As expected the strain-stress curves for the permeable concrete are similar in shape to those of the conventional concrete as shown in Figure 6. It can be observed that the stress-strain behaviours of permeable concrete for different aggregate sizes and sand ratios follow similar trends. The porosity ratio showed a direct effect on the shape and descending branch of the stress-strain curve of the permeable concrete. With the reduction of the porosity ratio, the compressive strength, corresponding strain and modulus of elasticity increased. Moreover, the descending branch showed more ductile behaviour and gradual cracks with increasing porosity ratio. As a result, the original equation proposed by Carreira & Chu (1985) (Eq. 1 and 2) needs to be modified to be used for permeable concrete with different porosity ratios. Therefore the descending branch of the model will be multiplied by a correction factor which is directly related to the porosity ratio. The proposed new equation thus suggested for the stress-strain curve of the permeable concrete is shown in Equation 12.

Figure 7 shows the relationship between n and the porosity ratio (P) and presented in Equation 13. Furthermore, the experimental results from this research and from Goede (2009) showed that there is a direct relationship between the porosity and the compressive strength as shown in Figure 8 and presented in Equation 14.

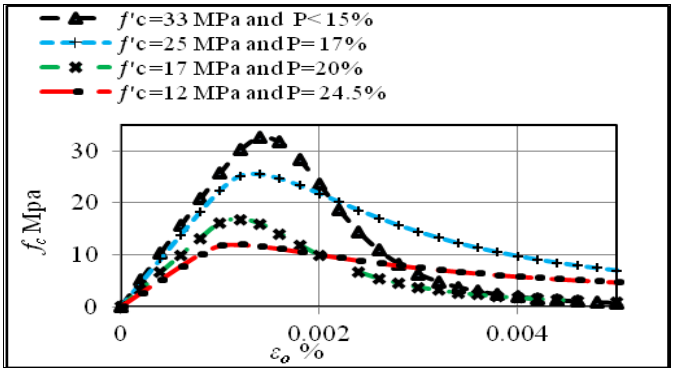


Figure 6 Effect of porosity on the stress-strain curve of the permeable concrete specimen.

$$f_c = \begin{cases} \dfrac{f'_c\,\beta(\frac{\varepsilon}{\varepsilon_o})}{[\beta - 1(\frac{\varepsilon}{\varepsilon_o})^{\beta}]} & \text{if} \quad \varepsilon \le \varepsilon_o \\ \dfrac{f'_c\,n\beta(\frac{\varepsilon}{\varepsilon_o})}{[n\beta - 1(\frac{\varepsilon}{\varepsilon_o})^{n\beta}]} & \text{if} \quad \varepsilon > \varepsilon_o \end{cases} \tag{12}$$

Where n= material parameter related to the porosity ratio. β = material parameter as shown in Eq. 2.

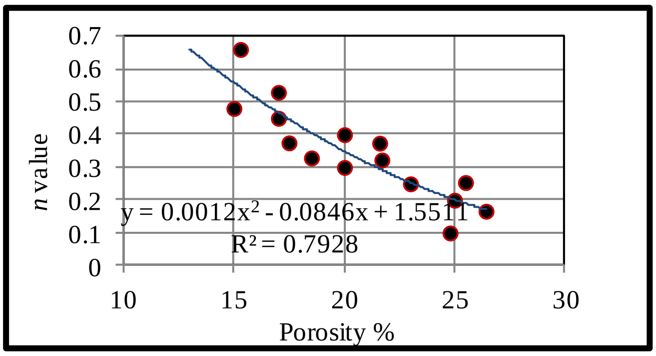


Figure 7 Relationship between the porosity and the material parameter (n).

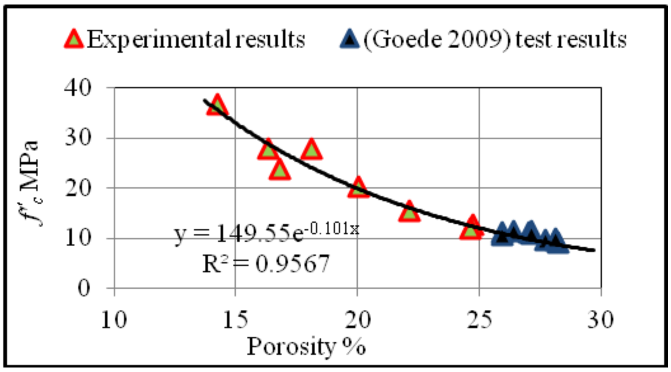


Figure 8 Relationship between the porosity and the compressive strength.

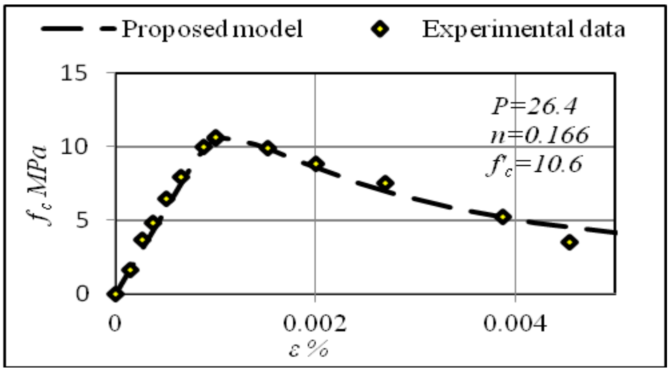
$$n = 0.0012P^2 - 0.0846P + 1.551 \tag{13}$$

$$f'_c = 149.55e^{-0.1P} \tag{14}$$

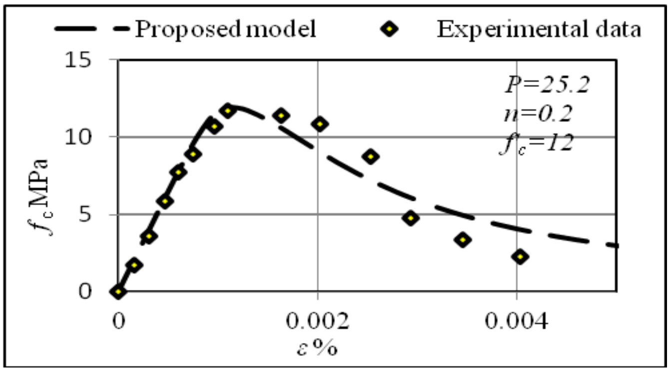
It will thus be a simple matter to calculate the porosity even if it is not tested in the laboratory.

4.4 *Examination of the proposed mode*

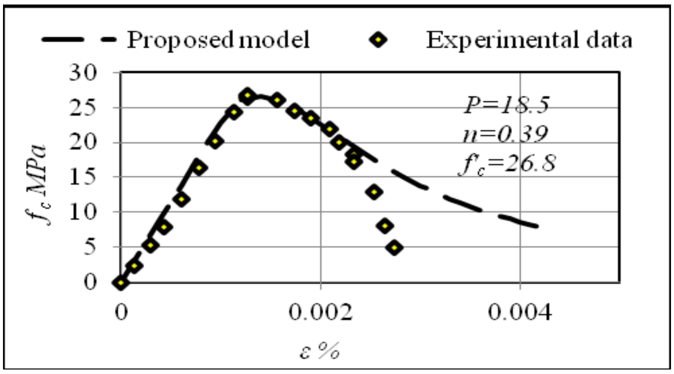
In order to verify the new empirical stress-strain model for unconfined permeable concrete under uni-axial compression strength, the stress-strain curve that was generated using the empirical model (Eq. 12) was compared with the experimental data over a wide range of strengths and porosities. Figure 9 (A-D) shows the proposed model in relation to the experimental data.



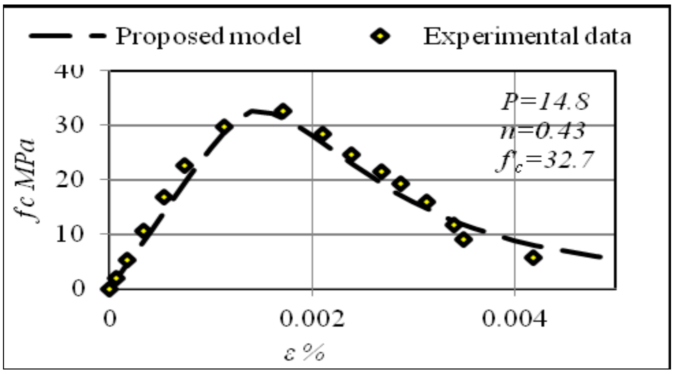
(A)



(B)



(C)



(D)
Figure 9 Comparison of proposed model against the experimental data.

It can be observed from Figure 9 that the proposed model is in good agreement with the experimental results for permeable concrete with varying compressive strength and porosities.

5 CONCLUSIONS

The major conclusions that can be drawn from this research are outlined in point form below:
The stress-strain curve for permeable concrete follows similar trends for that of conventional concrete curve.

- The strain corresponding to the peak stress of permeable concrete increases with an increase in peak compressive strength. This follows the same trend for conventional concrete.
- Existing equations used to predict the initial modulus of elasticity of low strength concrete and normal strength concrete can also be used for permeable concrete.
- The proposed numerical model is able to generate complete-stress-strain curve for unconfined permeable concrete under uni-axial compression. This is applicable to permeable concretes having different porosity ratios and compressive strengths.
- The stress-strain relationships are controlled by a few controlling parameters and the empirical expressions for these parameters based on f'_c and ρ are derived so that these relationships can be used in the absence of accurate experimental results.
- The model was validated against the experimental data and gave good predictions for both the ascending and the descending branches; it also demonstrates that it is capable of generating the complete stress-strain curve for permeable concrete with different porosity ratios.

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