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ORIGINAL ARTICLE

Orbital Elements of visual binary stars with very short arcs: With application to double stars from the 1829 southern double star catalog of James Dunlop

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Abstract

Binary double stars are those whose binding energies are less than zero. Obtaining binary star orbits from short arcs has been a long-standing problem in astrophysics. A method is presented and tested here, which addresses the problem by using space-based astrometry, photometry, and astrophysical data, together with historic measures, to generate and constrain a range of possible first-order Grade 5 orbits. After testing the method on an established binary star, we apply the method to eight double stars from the first published catalog of southern double stars, that of Dunlop (1829) and generate orbits for five. The mean orbital period is ~81,000 years, and the mean semi-major axis is ~76″ with a typical uncertainty of the Orbital Elements of ~37%. Their Orbital Elements and associated plots are also presented.

KEYWORDS

astrometry, binaries: visual – celestial mechanics, stars: kinematics

1 | INTRODUCTION

The uniqueness of two stars in abnormal astrometric proximity has been the subject of study for over 200 years and was a major area of astrophysical study from the late 18th to the early 20th centuries. The earliest measure of a double star in the United States Naval Observatory's *The Washington Double Star Catalog* (WDS, Mason et al. 2001) is μ Draconis (STFA 35), which was first measured in 1690. Now over 154,000 double stars are cataloged in the WDS. Of these, computed orbits are available for just over 3300 in the *Sixth Catalog of Orbits of Visual Binary Stars* (Matson et al. 2020). The study of orbits of binary star systems incorporating the physical laws of Newton and Kepler is the fundamental method of determining the mass of stars. The accuracy of the Orbital Elements, and subsequent physical properties of the stars, depends on the precision of the astrometric measures and the computational methodology.

Increasingly precise astrometry is now available from space-based missions such as HIPPARCOS (Perryman et al. 1997) and *Gaia DR2* (Gaia Collaboration et al. 2018), not withstanding that observational constraints in the *Gaia* instrument have resulted in a scarcity of double star measures (position angle [PA] θ, and separation ρ) for close pairs separated by less than ~2″. The Epochs of the HIPPARCOS and *Gaia DR2* missions are similar (1991.25 and 2015.5 respectively) and therefore, for pairs that have

TABLE 1 Statistics of the *Sixth Catalog of Orbits of Visual Binary Stars*

Grade	Number	$P^{\text{years}}_{\text{mean}}$	$P^{\text{years}}_{\text{median}}$	a''_{mean}	a''_{median}
1	89	17.3	11.4	18.72	0.33
2	374	48.0	27.7	8.52	0.25
3	697	106.7	69.4	6.62	0.26
4	988	483.5	218.2	4.57	0.55
5	660	18,244	484.2	813.41	1.57

long periods, it is still imperative that historic measures of lower precision, and which may extend over 200 years, are used to better define the orbit (White et al. 2018). Ultimately this work will advance through future missions with micro-arcsecond precision.

The *Sixth Catalog of Orbits of Visual Binary Stars* is divided into five Grades based on the accuracy of the orbits, where Grade 1 orbits are those with "well-distributed coverage exceeding one revolution" and Grade 5 orbits are those where "the [orbital] elements may not even be approximately correct" and the observed arc is short with little curvature (Hartkopf et al. 2001). Grades 6–9 are reserved for pairs with incomplete elements or lacking measures of relative astrometry. Therefore, they cannot be evaluated with conventional residual analysis.

Table 1 gives a statistical breakdown of the *Sixth Catalog of Orbits of Visual Binary Stars*. A superficial examination of the orbits in Grades 1–5 shows clearly the selection effects resulting from the observational precision and computational techniques. For example, the Grade 1 orbits are for pairs that have short periods (mean and median orbital period of 17.3 and 11.4 years, respectively, where the longest Grade 1 orbit is ∼88.4 years) and the orbit is observed over one or more complete orbits. Such pairs have statistically smaller physical separation (mean and median semi-major axis of 18.72″ and 0.33″, respectively). These close binary star orbits are characteristically the physical size of a planetary system. Even Grade 5 orbits have a median semi-major axis of only ∼1.57″. Hence all physical parameters determined from the Orbital Elements in the *Sixth Catalog of Orbits of Visual Binary Stars* are statistically biased to binaries in close orbits.

Distinguishing between optical and binary doubles has important ramifications for many aspects of astrophysics. This is especially the case for stellar formation models by contributing to multiplicity statistics (Guinan et al. 2007), the potential to distinguish between the mainstream-accepted *Weakly Interacting Massive Particles* based hypothesis of dark matter, and in Modified Newtonian Dynamics (Chanamé & Gould 2004; Longhitano & Binggeli 2010; Németh et al. 2016). The first successful estimation of the Orbital Elements of an assumed binary star was carried out by Félix Savary (Savary 1827a, 1827b). Since then numerous methods have been devised (Aitken 1964; Heintz 1978; See 1896).

The problem of estimating the Orbital Elements of wide binaries (with angular separation $\rho > 2''$) with few measures has resulted in numerous computational methods (see Docobo et al. 2018). For example, there is the so-called "Kovole" method (Ćatović & Olević 1992, 1995; Olević & Cvetković 2004, 2005), which is a modified form of the analytical Kowalski method. The Kowalski method derives five of the seven Orbital Elements from the constants of the Cartesian form of the general equation for the apparent relative ellipse (Belorizky 1949; Glasenapp 1889; Kowalski 1873; Smart 1930), and forms a part of the method presented in this paper (Section 3.2). Another is a modified form of the "Thiele-Innes-Van-den-Bos" method (Docobo 1985, 2012; Docobo et al. 2018; van den Bos 1926). However, to our knowledge, no method to date has successfully addressed the particular case of a very short arc.

This paper proposes a computational method utilizing the convergence of randomized fits (not a once off analytical method), which is now possible because of the availability of computing power for the determination of orbits of binary pairs in longer orbits: those that display short arcs in orbital plots.

Such work has been suggested to be of little value by van den Bos 1962, Worley 1990 and Dommanget 1995, who each published articles with the provocative title "Is this orbit really necessary?" The three authors criticized the publication of orbits that were unreliable because they were based on too few observations over too short an arc, or deemed useless because the quality of such recalculated orbits rarely increases with successive attempts. This paper is to show that improved first-order estimates of orbits can be now obtained using the new method coupled with space-based astrometry and photometry.

The aim of this paper is fourfold. (a) To offer an objective method of detecting binary double stars using space-based astrometric and astrophysical data from *Gaia DR2*; (b) to dispense with the traditional intellectual driver associated with orbits, which is to determine stellar mass; (c) to present a technique of determining first-order orbits of binary stars with short arcs using data from *Gaia DR2* and HIPPARCOS (via *All-sky Compiled Catalog of 2.5 million stars*; ASCC, Kharchenko 2001); and (d) to apply these aims to a subset of wide double stars.

The subset chosen for illustration of the technique is taken from the first published catalog of southern double stars by Dunlop (1829), this is referred to here and elsewhere as the *Dunlop Catalog*. A previous paper by the authors (Letchford et al. 2022) describes and analyses the *Dunlop Catalog* and gives ASCC and *Gaia DR2* source identifiers where available. For our subset used here, all pairs from the *Dunlop Catalog* must have an entry (and therefore historic measures) in the WDS (Mason et al. 2001), and the primary and secondary must both have ASCC and *Gaia DR2* source identifiers. In total, 182 wide doubles from the original *Dunlop Catalog* satisfy this criterion. We define this subset of the original Dunlop Catalog as the *Working Dunlop Catalog*.

Previous work (Letchford et al. 2018, 2019) has presented material on the orbital motion of binary double stars. Here, we refine that work, and in Section 2 we select a sample of eight pairs from the *Working Dunlop Catalog* that satisfy the definition of a binary system. Section 3 gives a proposed new technique for determining the Orbital Elements of binary double stars, and tests the method against a set of Orbital Elements from the *Sixth Catalog of Orbits of Visual Binary Stars*. Section 4 presents and discusses the results of applying the technique to the eight double stars from Section 2.

2 | METHOD: DETECTING VISUAL BINARY DOUBLE STARS

By definition, binary double stars are pairs of stars whose binding energy (E_{binding}) is less than zero (Aarseth et al. 2008; Benacquista 2012; Kouwenhoven et al. 2010; Wiley & Rica 2015). Conversely, any double star with a binding energy greater than zero is an optical double star. The binding energy of two stars is represented by the following equation:

$$E_{\text{binding}} = \frac{1}{2}\frac{m_A m_B}{m_A + m_B} v^2 - \frac{G m_1 m_2}{D} \tag{1}$$

where E_{binding} is in $m_{\odot}\left(\text{km s}^{-1}\right)^2$; m_A and m_B are the masses of the primary and secondary in solar masses ($m_{\odot}$); v is the galactic space speed *difference* between the two stars in km s^{-1}; D is their physical separation, in parsec (pc); and G is the gravitational constant (4.30091×10^{-3} pc$\left(\text{km s}^{-1}\right)^2/M_{\odot}$).

Reasonable estimates of the stellar masses can be obtained via luminosity data from *Gaia DR2* and using the following relationship between individual luminosity (L in solar units) and individual mass (m in solar masses) as given by Duric (2004):

$$m = \begin{cases} \left(\frac{L}{0.23}\right)^{1/2.3}, & \text{if } m < 0.43 \\ L^{1/4}, & \text{if } 0.43 < m < 2 \\ \left(\frac{L}{1.4}\right)^{1/3.5}, & \text{if } 2 < m < 55 \\ \frac{L}{32{,}000}, & \text{if } m > 55 \end{cases} \tag{2}$$

To calculate D, the physical separation of the components in pc, the cosine rule is invoked:

$$D = \sqrt{D_A^2 + D_B^2 - 2D_A D_B \cos(\rho/3{,}600)} \tag{3}$$

where D is the separation distance of the two stars in pc; D_A is the distance from the Sun to the primary in pc, obtained from parallax measures; D_B is the distance from the Sun to the secondary in pc, and ρ is the apparent separation in arcseconds (″).

To calculate the speed of each component (v_1 and v_2), the method of Johnson & Soderblom (1987) will be followed, except that the J2000 transformation matrix (T, Equation (5)) to galactic coordinates is taken from the introduction to the HIPPARCOS catalog (Perryman 1997), and the method tested by first applying it to the pair *α Cen AB,C*, making sure that the results obtained were the same as those in Kervella et al. (2017), in their Table B.1. The galactic space velocity components (U, V, W) are then:

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = (\text{T.A}).\begin{bmatrix} \text{RV} \\ k.\text{pmRA/Plx} \\ k.\text{pmDE/Plx} \end{bmatrix} \tag{4}$$

where:

$$\text{T} = \begin{bmatrix} -0.0548755604 & +0.4941094279 & -0.8676661490 \\ -0.8734370902 & -0.4448296300 & -0.1980763734 \\ -0.4838350155 & +0.7469822445 & +0.4559837762 \end{bmatrix} \tag{5}$$

$$\text{A} = \begin{bmatrix} +\text{cosRA.cosDE} & -\text{sinRA} & -\text{cosRA.sinDE} \\ +\text{sinRA.cosDE} & +\text{cosRA} & -\text{sinRA.sinDE} \\ +\text{sinDE} & 0 & +\text{cosDE} \end{bmatrix} \tag{6}$$

and RV is the radial velocity of the star in km s^{-1}; pmRA is the proper motion in RA in mas year^{-1}; pmDE is the proper motion in DE in mas year^{-1}; Plx is the parallax in mas; and the constant $k = 4.74057$ km s^{-1} is the speed in km s^{-1} required to travel 1 AU in one tropical year.

So the difference in the velocity magnitude (v, speed) between the primary and secondary is:

$$v = \sqrt{\left(U_1^2 - U_2^2\right)^2 + \left(V_1^2 - V_2^2\right)^2 + \left(W_1^2 - W_2^2\right)^2} \tag{7}$$

Of the 182 double stars in the *Working Dunlop Catalog* only 40 had sufficient *Gaia DR2* data to allow the calculation of binding energies (Table 2).

TABLE 2 Catalog numbers from the *Dunlop Catalog* for the 40 double stars for which binding energies E_{binding} could be calculated

Nos. $E_{binding} < 0$ 2 in total	Nos. $E_{binding} > 0$ 38 in total
38, 55	2, 4, 5, 23, 26, 27, 28, 29, 40, 41, 52, 57, 73, 77, 79, 80, 114, 116, 118, 146, 155, 175, 176, 178, 184, 186, 200, 215, 225, 232, 236, 238, 241, 242, 245, 246, 248, 250

The formal uncertainties in E_{binding} will be large due to the compounding uncertainties in the observable parameters, especially from the uncertainties in stellar masses from Equation (2). Here we are assured that numbers 38 and 55 are binary systems as they have $E_{\text{binding}} < 0$. We include in our list of probable binaries numbers 5, 80, 116, 232, 242, and 245, which have binding energies <1 (thus allowing for some uncertainties in E_{binding}), and observables consistent with a binary system (see next paragraph). Number 5 (DUN 5) has a Grade 5 orbit in the *Sixth Catalog of Orbits of Visual Binary Stars* and clearly shows orbital motion (see Appendix).

The approximate binding energies (E_{binding}), physical separations (D, column 4) in pc, and the proper motions of the eight double stars (columns 5 and 6, where pmRA$_A$ and pmDE$_A$ are the proper motions in right ascension and declination for the primary, respectively, and similarly for the secondary, B), are presented in Table 3. Column 1 (No.) is the catalog number of the double star from the *Dunlop Catalog*. Column 2 contains the WDS system identifier, underneath which is the Discoverer code of the particular double star (from the WDS). All physical separations are less than 1 pc (Dommanget 1967 and Sinachopoulos 1991 had a separation limit of 0.01 pc, recently others have demonstrated a limit approaching 1 pc, Longhitano & Binggeli 2010, others to the galactic tidal limit of ∼1.7 pc for solar type stars, Halbwachs et al. 2012; Moeckel & Bate 2010), and each of the eight also exhibit a common proper motion (following Hartkopf et al. 2013), justifying their inclusion as possible binary double stars.

3 | METHOD: ORBITAL MOTION

We present here a computationally-based method of determining orbits that are defined by short arcs. High precision astrometric measures are utilized as historic measures of less accuracy. Stellar mass data are incorporated and physical constraints are applied. Multiple random orbits are generated and compared with modern and historic measures, and the fit to the measures is optimized to define the orbit and its uncertainties.

The problem with dealing with short arcs is that the range of possible ellipses can be considerable with large associated uncertainties. This difficulty is overcome by introducing as many constraints on the solutions as possible.

If the double star is a binary star (the primary and secondary are gravitationally bound), the following corollaries must hold true:

1. The primary position must always be within any apparent orbit of the secondary.
2. The orbit of the secondary must cross the meridian line through the primary twice: only once to the north of the primary, and only once to the south of the primary.
3. Similarly, the orbit of the secondary must cross a line parallel to the equator and passing through the primary twice: only once to the east of the primary, and only once to the west of the primary.

By convention, binary orbit calculations are conducted in Cartesian coordinates, where the primary is fixed at the origin. Our method incorporates the Kowalski method (Section 1), which uses the Cartesian form of the apparent relative ellipse, that is, the ellipse formed on the celestial sphere of a secondary orbiting a fixed primary. The apparent relative orbit of a binary star will fit the following general form of the Cartesian equation of an ellipse:

$$Ax_i^2 + 2Hx_iy_i + By_i^2 + 2Gx_i + 2Fy_i + 1 = 0 \tag{8}$$

where A, H, B, G, and F are constants, and x_i and y_i are the Cartesian coordinates of the secondary at times t_i, given that the primary is fixed at the origin.

At least five positions on the Cartesian plane ($i \geq 5$) are needed to fit an ellipse to Equation (1). The normal procedure, where at least half of the orbit has been measured, is to convert the historic polar measures to Cartesian coordinates and fit a first ellipse to them using either ordinary least squares (OLS), or better, total least squares (TLS). The historic measures can be weighted before fitting (Mason et al. 1999). In the present case, a least squares approach to the historic data will not yield a suitable first ellipse because we consider the cases where much less than half an orbit has been measured. The five or more fitting points must initially be found by another method.

Four of those points can be supplied by positions described in corollaries 2 and 3, above. Two more positions can be supplied by HIPPARCOS (via ASCC) and *Gaia DR2* (Gaia Collaboration et al. 2018) at $t = 1991.25$ and 2015.5 (both at Equinox 2000.0), respectively. In this paper,

TABLE 3 Table of the approximate binding energies (E_{binding}), physical separations (D) in parsec (pc), and the proper motions of the eight double stars, whose binding energies <1, physical separation <1 + 1σ, and whose common proper motions (CPM) are consistent with binarity

				CPM, mas year^{-1}	
				pmRA$_A$	pmRA$_B$
Nos.	**WDS discoverer code**	E_{binding} $M_\odot\,(\text{km s}^{-1})^2$	D, pc	pmDE$_A$	pmDE$_B$
5	01398-5612	~ +0.55	0.005 ± 0.005	+262.378 ± 0.081	+309.102 ± 0.080
	DUN 5			+15.333 ± 0.072	+10.686 ± 0.065
38	07040-4337	~ −2.2	0.002 ± 0.023	−105.060 ± 0.146	−101.764 ± 0.043
	DUN 38AB			+389.550 ± 0.162	+382.276 ± 0.047
55	07442-5027	~ −0.43	0.009 ± 0.035	−114.435 ± 0.047	−111.783 ± 0.060
	DUN 55AB			+143.459 ± 0.042	+142.603 ± 0.053
80	09450-4929	~ +0.14	0.39 ± 0.18	−21.344 ± 0.071	−23.581 ± 0.066
	DUN 80AB			+98.409 ± 0.062	+97.158 ± 0.066
116	11567-3216	~ +0.26	0.016 ± 0.077	−171.610 ± 0.072	−178.910 ± 0.073
	DUN 116AB			−8.250 ± 0.042	−6.668 ± 0.039
232	20417-7521	~ +0.47	0.059 ± 0.071	+156.596 ± 0.039	+163.555 ± 0.048
	DUN 232			−162.079 ± 0.045	−171.231 ± 0.050
242	22397-2820	~ +0.17	0.36 ± 0.84	+96.822 ± 0.109	+96.340 ± 0.106
	H 6 119AB			−40.596 ± 0.083	−36.578 ± 0.079
245	23086-5944	~ +0.12	0.40 ± 0.19	+60.074 ± 0.043	+62.034 ± 0.041
	DUN 245			−63.923 ± 0.046	−67.227 ± 0.047

Note: Column 1 (Nos.) refers to the catalog numbers from the Dunlop Catalog.

we chose to obtain HIPPARCOS data from the ASCC (Kharchenko 2001).

Four random points are generated along the north, south, east, and west axes centering on the primary, and added to them are the two points obtained from the ASCC and *Gaia DR2*. Random iterative searches in increments of ~30″ out to 4′ locate the coordinates of these crossing points. For each increment, we generated ~10^7 sets of points. The search along the primary star axes is limited to a maximum of 4′, determined by the fact that all but two orbits in the *Sixth Catalog of Orbits of Visual Binary Stars* have semi-major axes less than 240″.

The algorithm of Halir & Flusser (1998) is used to find the coefficients A, H, B, G, and F for each generated ellipse, rather than an OLS or TLS fit. The Halir and Flusser algorithm has the advantage of generating an ellipse every time, whereas the random alignment of fitting points may not always generate an ellipse using the OLS or TLS methods.

This random search domain is reduced by noting that because the secondary is slow moving, the positions obtained from the space-based HIPPARCOS (via ASCC) and *Gaia DR2* missions will define a tangent to any orbit at a mean Epoch of ~2003. Therefore the elliptical orbit will only exist on the side of this tangent containing the primary.

3.1 | First constraint

The first constraint is to delete obviously non-viable ellipses by discarding all generated ellipses where the origin was outside the ellipse.

3.2 | Orbital Elements

Next, Orbital Elements associated for each ellipse are calculated. This involves a two-step process.

First, five of the seven Orbital Elements (a, i, Ω, e and ω) are calculated using the Kowalski method (Section 1).

The longitude of the ascending node of the apparent relative orbit, Ω, is:

$$\Omega^{\text{radians}} = \begin{cases} \frac{1}{2}\arctan 2(Q, R) \\ \Omega = \Omega + \pi, \text{if } \Omega < 0 \\ \Omega = \Omega - \pi, \text{if } \Omega > \pi \end{cases} \tag{9}$$

where $Q = -2(FG - H)$, $R = F^2 - G^2 + A - B$;
and,

$$\arctan 2(Q,R) = \begin{cases} \arctan(Q/R), \text{ if } R > Q \\ \frac{\pi}{2} - \arctan(Q/R), \text{ if } Q > 0 \\ -\frac{\pi}{2} - \arctan(Q/R), \text{ if } Q < 0 \\ \arctan(Q/R) \pm \pi, \text{ if } R < 0 \\ \text{undefined, if } R = 0 \text{ and } Q = 0 \end{cases} \tag{10}$$

If the ascending node is unknown, then $\Omega < \pi$.

The inclination of the plane of the true relative orbit to that of the celestial sphere, i, is given by either Equation (11a) or (11b):

$$i^{\text{radians}} = \arctan 2(\sqrt{|2Q|}, \sqrt{|S\sin(2\Omega) - Q|}) \tag{11a}$$

$$i^{\text{radians}} = \arctan 2(\sqrt{|2R|}, \sqrt{|S\cos(2\Omega) - R|}) \tag{11b}$$

where $S = F^2 + G^2 - A - B$ and $i = \pi - i$ if the apparent orbit is retrograde (clockwise).

The argument of periastron, ω, is given by:

$$\omega^{\text{radians}} = \begin{cases} \arctan 2(-U\cos(i), -V) \\ \omega = \omega + 2\pi, \text{ if } \omega < 0 \\ \omega = \omega - 2\pi, \text{ if } \omega > 2\pi \end{cases} \tag{12}$$

where $U = F\cos(\Omega) - G\sin(\Omega)$ and $V = F\sin(\Omega) + G\cos(\Omega)$.

By convention, $0 \le \omega \le 2\pi$.

The eccentricity of the true relative orbit, e, is either Equations (13a) or (13b):

$$e = \left| \frac{-U\cos(i)}{\sin(\omega)} \sqrt{\frac{2}{S - \frac{Q}{\sin(2\Omega)}}} \right| \tag{13a}$$

$$e = \left| \frac{-V}{\sin(\omega)} \sqrt{\frac{2}{S - \frac{R}{\cos(2\Omega)}}} \right| \tag{13b}$$

For ellipses, $0 < e < 1$.

The semi-major axis of the true relative orbit, a, is given by Equations (14a) or (14b):

$$a'' = \frac{1}{1 - e^2} \sqrt{\frac{2}{S - \frac{Q}{\sin(2\Omega)}}} \tag{14a}$$

$$a'' = \frac{1}{1 - e^2} \sqrt{\frac{2}{S - \frac{R}{\cos(2\Omega)}}} \tag{14b}$$

The remaining two Orbital Elements, the orbital period (P) and time of periastron (T) are estimated using the mean anomaly (M_A).

To find the mean anomaly for the HIPPARCOS (via ASCC), M_{HIPP}, and *Gaia DR2* $M_{GaiaDR2}$, positions, the true anomaly (V_A) is followed by the eccentric anomaly (E_A), which is first found for each position:

$$V_A = \arctan 2\left[(\sin(\theta - \Omega)\sec(i), (\cos(\theta - \Omega)\right] - \omega \tag{15}$$

$$E_A = 2\arctan\left(\sqrt{\frac{1-e}{1+e}}\tan(V_A/2)\right) \tag{16}$$

$$M_A = E_A - e\sin(E_A) \tag{17}$$

where $-2\pi < V_A < 2\pi$; $-\pi < E_A < \pi$; and $-\pi < M_A < \pi$. θ is the PA of the secondary with respect to the primary at time t and at Equinox 2000.0.

Continuing:

$$P = \left| 2\pi \frac{2015.5 - 1991.25}{M_{GaiaDR2} - M_{HIP}} \right| \tag{18}$$

$$T = \frac{M_{GaiaDR2}1991.25 - M_{HIP}2015.5}{M_{GaiaDR2} - M_{HIP}} \tag{19}$$

The time of periastron, T, is the year of closest approach normally presented as the one nearest 2000.0, and given as a decimal year. The orbital period, P, is measured in decimal years.

3.3 | Second constraint

The second constraint is to delete all orbits where the calculated total mass of the system (M_{system}) is greater than a preset tolerance of ±10% of the known combined masses of the primary and secondary.

In most double star work, the ultimate aim is to obtain direct measures of the stellar masses using Kepler's third law:

$$m_{\text{system}} = m_A + m_B = \frac{a^3}{\Pi^3 P^2} \tag{20}$$

where m_A and m_B are the masses of the primary and secondary respectively, in solar masses; a is the semi-major axis of the real relative orbit in arcseconds; Π is the parallax of the system (the primary star) in arcseconds; and P is the orbital period of the secondary, in years. In this paper, we reverse the procedure and begin with good estimates of the combined masses obtained from luminosity data from *Gaia DR2* and using the mass-luminosity function, Equation (2). We set the mass constraint to ±10% of the combined masses.

The parallax of the system (Π) is deemed to be the parallax of the primary, obtained from *Gaia DR2*. Thus:

$$\frac{a^3}{P^2}\begin{cases} > \Pi^3\left(m_A + m_B - \frac{m_A+m_B}{10}\right) \\ < \Pi^3\left(m_A + m_B + \frac{m_A+m_B}{10}\right)\end{cases} \tag{21}$$

3.4 | Third and fourth constraints

Finally, we assumed that the best results will be obtained if the calculated positions of the secondary at 1991.25 and 2015.5 fell within the 1σ uncertainty ellipse of the HIPPARCOS (via ASCC) and *Gaia DR2* positions. HIPPARCOS positions (via ASCC) in right ascension have a 1σ uncertainty of ~6 mas and ~3 mas in declination. *Gaia DR2* positions in right ascension and declination have a 1σ uncertainty of ~0.1 mas. Requiring the calculated positions at Epochs 1991.25 and 2015.5 to be within the 1σ uncertainty ellipse of the HIPPARCOS (via ASCC) and *Gaia DR2* positions proved to be the most severe constraints.

The uncertainties of the secondary position in Cartesian coordinates (σ_x, σ_y) for both the HIPPARCOS (via ASCC) and *Gaia DR2* positions is approximated by:

$$\sigma_x = \pm\sqrt{\cos(\mathrm{DE1})^2\left(\sigma^2_{\mathrm{RA1}} + \sigma^2_{\mathrm{RA2}}\right) + (\mathrm{RA2} - \mathrm{RA1})^2 \sin(\mathrm{DE1})^2 \sigma^2_{\mathrm{DE1}}}$$
$$\sigma_y = \pm\sqrt{\sigma^2_{\mathrm{DE1}} + \sigma^2_{\mathrm{DE2}}} \tag{22}$$

where $x = (\mathrm{RA2} - \mathrm{RA1})\cos(\mathrm{DE1})$ and $y = \mathrm{DE2} - \mathrm{DE1}$.

RA1 and DE1 are the right ascension and declination coordinates, respectively, of the primary at Equinox 2000.0; RA2 and DE2 are the right ascension and declination coordinates, respectively, of the secondary at Equinox 2000.0; and σ_{RA1}, σ_{DE1}, σ_{RA2}, and σ_{DE2} are their respective uncertainties (errors), obtained from ASCC and *Gaia DR2*.

To find the position at time t of the secondary implied by the recalculated orbit, the following ephemeris procedure must be undertaken.

First, the true anomaly of the *calculated* position must be found via the mean and eccentric anomalies:

$$M_A = \frac{t - T}{2\pi P} \tag{23}$$

where M_A is the mean anomaly of the calculated position. Next, E must be calculated using Kepler's transcendental equation:

$$M_A = E_A - e\sin(E_A) \tag{24}$$

where e is the eccentricity of the calculated orbit.

Now, the true anomaly of the calculated position can be determined from:

$$V_A = 2\arctan\left[\tan\left(\frac{E_A}{2}\right)\sqrt{\frac{1+e}{1-e}}\right] \tag{25}$$

And the polar coordinates of the ephemeris position can be calculated:

$$\theta^{\mathrm{radians}} = \arctan 2\left[(\sin(V_A) + \omega, \cos(V_A) + \omega\right] + \Omega \tag{26}$$

$$\rho'' = \frac{a\left(1 - e^2\right)\cos(V_A) + \omega}{(1+e)\cos(V_A)\cos(\theta - \Omega)} \tag{27}$$

where θ and ρ are converted to Cartesian coordinates using:

$$x'' = \rho\cos(\theta) \tag{28}$$

$$y'' = \rho\sin(\theta) \tag{29}$$

3.5 | Bringing in the historic data

It is only at this point that historic measures of the double stars, as recorded in the WDS, are introduced into the computation.

The measures of PA contained in the WDS are given at Equinox of Epoch, and are converted to Equinox 2000.0 (θ°). PA°s from HIPPARCOS (via ASCC) have Epochs of 1991.25 and *Gaia DR2* have Epochs of 2015.5. Both PAs are given in the WDS at Equinox 2000.0 and therefore are not converted.

The formula for conversion of PA$_i$ (other than those at 1991.25 and 2015.5) at Equinox of Epoch to Equinox 2000.0 (θ_i) is:

$$\theta_i^\circ = \mathrm{PA}_i^\circ + \left[0.00417 \cdot \mathrm{pmRA1} \cdot \sin(\mathrm{DE1}) + 0.00557\frac{\sin(\mathrm{RA1})}{\cos(\mathrm{DE1})}\right](2000.0 - t_i) \tag{30}$$

where pmRA1 is the proper motion of the primary in arcseconds, and t_i is the Epoch of observation given in Besselian years.

Thus converted, θ_i together with the separation (ρ_i) are then converted to Cartesian coordinates via Equations (28) and (29). Using an ephemeris algorithm (Equations (23)–(29)), the positions of the historic data at times t (t_i, x_i, y_1), based on the Orbital Elements of each surviving orbit are calculated.

For each orbit calculated, the residuals (R_o) are determined by the sum of the squares of the distances between the historic positions and their predicted ephemeris positions. The orbits are then ranked in ascending order of R_o.

TABLE 4 Test double star

Orbital Elements	OLS	TLS	This paper	Sixth orbit
P^{years}	375.1	404.3	444.9	433.8
a''	2.5	2.4	2.4	2.4
$i°$	121.3	128.4	134.85	132.7
$\Omega°$	19.1	27.6	16.4	13.0
T^{year}	1911.7	1912.0	1905.8	1907.2
e	0.70	0.80	0.80	0.81
$\omega°$	208.6	215.6	199.2	193.1

Note: Orbital Elements of WDS J16160+0721 (STF 2026AB) were obtained by four different methods. See Section 3.8. Results have been truncated to one decimal place (except for eccentricity) for clarity of comparison. Columns 2 (OLS) and 3 (TLS) contain initial Orbital Elements where the ellipses were determined by ordinary least squares and total least squares, respectively, and the Orbital Elements then calculated using the Kowalski method. Column 4 contains initial Orbital Elements obtained using the method proposed in this paper. Column 5 (sixth orbit) contains the Orbital Elements currently listed in the *Sixth Catalog of Orbits of Visual Binary Stars*.

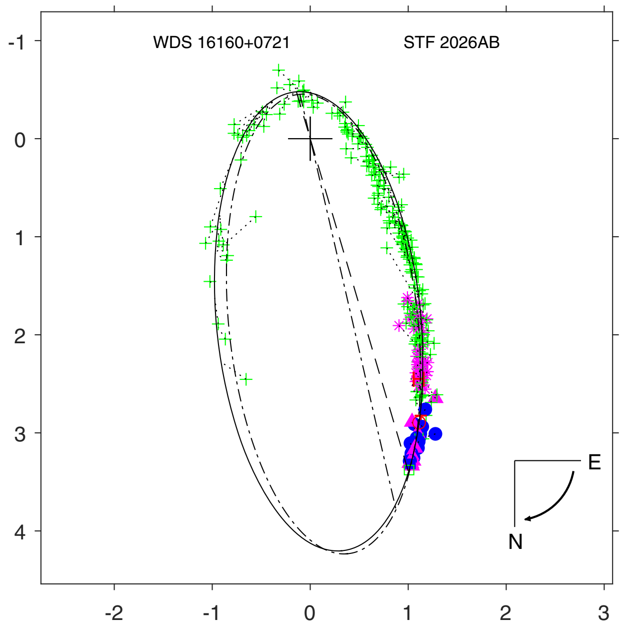

FIGURE 1 Test double star. WDS J16160+0721 (STF 2026AB). The orbit obtained in this paper is marked by the solid black ellipse. The orbit currently in the *Sixth Catalog of Orbits of Visual Binary Stars* is the dash-dot ellipse. The primary is at 0,0 (represented by the large + sign) and the axis scales are in arcseconds. The long straight dashed line is the ascending node calculated in this paper, and the long straight dash-dot line is the ascending node from the *Sixth Catalog of Orbits of Visual Binary Stars*. Individual measures (from the *Washington Double Star Catalog* [WDS]) are color-coded: green, blue, and purple indicate micrometric, interferometric, and photographic/CCD measures, respectively, while red symbols indicate measures from space-based instruments (Hartkopf & Mason 2020). Measures are connected to their predicted locations by dotted lines

3.6 | Final constraint

Finally, we rejected all orbits where $R_o > R_r$, where R_r is the sum of the squares of the distances between the historic positions and the ephemeris positions calculated from a rectilinear line drawn through the HIPPARCOS (via ASCC) and *Gaia DR2* positions. Clearly, if an orbit produces residuals (R_o) larger than those of a rectilinear motion (R_r) then either rectilinear motion is more likely to be present (i.e., it is a possible optical double star) or the orbital arc is still too small to distinguish between rectilinear and orbital motion.

3.7 | Estimating the uncertainties of the Orbital Elements

By the above method, the orbit with the lowest sum of residuals is deemed to be the best estimate of any possible orbit, provided $R_o < R_r$.

In an earlier paper, Letchford et al. (2018), we estimated the uncertainty of each Orbital Element (σ_{OE}) with the following simple equation:

$$\sigma_{\text{OE}} = \mid \text{OE} - \overline{\text{OE}} \mid \tag{31}$$

where OE is the particular Orbital Element and $\overline{\text{OE}}$ is the mean of those found, which satisfy $R_o < R_r$.

The problem with Equation (31) is that it implies that the best estimate of an Orbital Element is its mean ($\overline{\text{OE}}$). Here however the best set of Orbital Elements is *not* the mean but the set with the lowest found R_o, which satisfies $R_o < R_r$.

A better estimate is to use the R_o for each set of Orbital Elements that satisfy $R_o < R_r$ as normalized *weights* (w_i), so that:

$$\sigma_{\text{OE}} = \pm\sqrt{\sum w_i \left(\text{OE}_i - \overline{\text{OE}}\right)^2} \tag{32}$$

where,

$$w_i = \frac{1}{R_{o_i} \sum \frac{1}{R_{o_i}}} \tag{33}$$

and,

$$\overline{\text{OE}} = \sum w_i \text{OE}_i \tag{34}$$

3.8 | Test binary double star

Any new or modified method of computing an orbit must be proven with respect to existing methods. To this end, the binary star WDS J16160+0721 (STF 2026AB) was chosen for testing as it was studied by Lobão (1994) and Sharaf et al. (2014) in their work on the Kowalski method.

TABLE 5 Orbital Elements for five double stars from the *Working Dunlop Catalog*, at Equinox 2000.0, where $E_{\text{binding}} < 1$

No.	WDS	Discoverer code	P^{year} +/−	a'' +/−	i° +/−	Ω° +/−	T^{year} +/−	e +/−	ω° +/−
5	01398-5612	DUN 5	620.0	9.9	120.0	21.0	1800.0	0.35	61.0
			750.0	5.7	4.8	43.0	210.0	0.15	63.0
38	07040-4337	DUN 38AB	55,000.0	110.0	81.0	170.0	−3500.0	0.87	130.0
			—	—	—	—	—	—	—
55	07442-5027	DUN 55AB	230,000.0	160.0	95.0	110.0	91,000.0	0.41	83.0
			97,000.0	37.0	1.9	6.0	71,000.0	0.24	23.0
116	11567-3216	DUN 116AB	24,000.0	30.0	110.0	82.0	3600.0	0.69	100.0
			24,000.0	16.0	2.6	1.9	180.0	0.08	20.0
245	23086-5944	DUN 245	37,000.0	23.0	110.0	110.0	4200.0	0.60	260.0
			25,000.0	9.3	2.7	4.0	500.0	0.13	22.0

Note: All results and uncertainties have been rounded to two significant figures, due to the large uncertainties. Below each Orbital Element (except for No. 38) is an estimate of its uncertainty ("+/−," Section 3.7) is given.

Both component stars of STF 2026AB have HIPPARCOS (via ASCC) and *Gaia DR2* data. It is also a Grade 3 orbit in the *Sixth Catalog of Orbits of Visual Binary Stars*, where Grade 3 orbits are described as "at least half of the orbit defined, but the lesser coverage (in number or distribution), or data consistency, leaves the possibility of larger errors than in Grade 2" (Matson et al. 2020).

Table 4 presents four sets of Orbital Elements where (a) OLS is used to obtain the ellipse coefficients, prior to determining the orbit elements by the Kowalski method (column 2); (b) TLS is used followed by the Kowalski method (column 3); (c) the method in this paper (column 4); and finally; (d) the Orbital Elements from the current *Sixth Catalog of Orbits of Visual Binary Stars* (6th orbit, source paper, Izmailov 2019) (column 5). It should be noted that the results from the method presented in this paper (column 4) and the OLS (column 2) and TLS (column 3) methods were conducted *without* weighting the historic data and *without* any outliers removed. Thus, they do not represent a final solution, but initial estimates that can be improved upon by further processing.

From Table 4 and Figure 1, it can be seen that of the three methods presented, the results obtained by the method presented in this paper are closest to the currently published values in the *Sixth Catalog of Orbits of Visual Binary Stars* in six of the seven Orbital Elements.

4 | RESULTS AND DISCUSSION: ORBITAL ELEMENTS

Of the 40 double stars from the *Working Dunlop Catalog* whose binding energies were calculated, eight are considered probable binaries (Table 3). Of the eight candidate binaries selected, five generated orbits (Nos. 5, 38, 55, 116, and 245). The remaining three (Nos. 80, 232, and 242) failed to generate orbits probably due either to the stringent constraints (namely: a $\pm 10\%$ combined mass tolerance; the requirement that the 1991.25 and 2015.5 (equinox 2000.0) calculated positions fall within the 1σ error ellipse of the HIPPARCOS (via ASCC) and *Gaia DR2* positions; and a search limited to $4'$ from the primary, see Section 3), or to the possibility that the arcs are still too short for the technique to distinguish between rectilinear and orbital motion (Section 3.6).

Sets of Orbital Elements for the five binary double stars are given in Table 5. All Orbital Elements in Table 5 are at Equinox 2000.0. Column 1 of Table 5 is the number of the pair in the *Dunlop Catalog*, column 2 is the designation of the double star system in the WDS, column 3 is the Discoverer code of the particular double star used in the WDS (Disc), and columns 4–10 are the Orbital Elements as defined in the *Sixth Catalog of Orbits of Visual Binary Stars* (Matson et al. 2020). Associated orbital plots are given in the Appendix.

Table 6 presents the mean and median values of the orbital periods (P) and semi-major axes (a) of the five orbits found from the computational technique presented above. The mean orbital period is ~81,000 years and the mean semi-major axis is $\sim 76''$. Comparing our results in Table 6 with those of the *Sixth Catalog of Orbits of Visual Binary Stars* in Table 1, it is clear that these orbits are Grade 5 orbits and extend the scope of possible orbits beyond those of "close" binaries.

As expected, the uncertainties of the Orbital Elements are large. The average percentage uncertainties of the Orbital Elements in Table 5, in P, a, i, Ω, e, and ω are 83, 44, 3, 54, 34, and 40%, respectively. The average value of the uncertainty in T is 3600 years, ~4% of the average period, P, of 81,000 years.

TABLE 6 Statistics on orbits for Nos. 5, 38, 55, 116, and 245

Grade	$P^{\text{years}}_{\text{mean}}$	$P^{\text{years}}_{\text{median}}$	a''_{mean}	a''_{median}
5	81,000	37,000	76	30

Note: Numbers in columns 3–6 have been rounded to two significant figures, because of the large uncertainties inherent in each of the three sets of Orbital Elements.

In the case of No. 38, the proposed method did not produce uncertainties because only one orbit could be found that satisfied the curvature criteria $R_o < R_r$.

One binary double star detected in the *Working Dunlop Catalog*, DUN 5 has a Grade 4 orbit in the current *Sixth Catalog of Orbits of Visual Binary Stars*, taken from Scardia et al. (2018). Our Orbital Elements are close to those in the *Sixth Catalog of Orbits of Visual Binary Stars* (for example, our orbital period and semi-major axis of ∼623 years and ∼9.9″, respectively, are both only ∼25% larger than the current respective measures in the *Sixth Catalog of Orbits of Visual Binary Stars*). We also include uncertainties for DUN 5, as none are given in the current *Sixth Catalog of Orbits of Visual Binary Stars*.

5 | CONCLUSION

Presented here is an unbiased, computationally-based method of determining orbits that are defined by short arcs. High precision astrometric measures were utilized as historic measures of less accuracy. Stellar mass data were incorporated and physical constraints were applied. Multiple random orbits were generated and compared with modern and historic measures, and the fit to the measures was optimized to define the orbit and its uncertainties. Our test binary star STF 2026AB (Section 3.8), and DUN 5, yielded results close to those in the current *Sixth Catalog of Orbits of Visual Binary Stars*.

We proposed Grade 5 Orbital Elements for five double stars from the *Working Dunlop Catalog* (Table 6), Nos. 5, 38, 55, 116, and 245. We recommend that Orbital Elements for Nos. 38 and 55 be included in the *Sixth Catalog of Orbits of Visual Binary Stars*.

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APPENDIX: PLOTS OF BINARY DOUBLE STARS

A total of 40 double stars from the *Working Dunlop Catalog* had sufficient *Gaia DR2* data for their binding energies to the calculated. Of those 40, just two had binding energies < 0 (Nos. 38, and 55), indicating that they are most probably binary double stars.

Plots of the two probable binary double stars and No. 5 are presented here, and their Orbital Elements (rounded to two significant figures) are listed in Table 6. Refer to Figure 1 for an explanation of the contents of each Figure. Plots of two other orbits are displayed; No. 116 and 245. Double stars Nos. 5, 116, and 245 returned $0 < E_{\text{binding}} < 1$. No. 5 is currently in the *Sixth Catalog of Orbits of Visual Binary Stars*.

Note that the orbital plots rely on the non-rounded Orbital Elements to ensure that each orbit passes through the 1σ uncertainty ellipses of the HIPPARCOS (via ASCC) and *Gaia DR2* positions.

The orbit plot of DUN 5 (Section 4), a double star for which there are currently Orbital Elements in the *Sixth Catalog of Orbits of Visual Binary Stars*, includes a comparison plot of the Orbital Elements in the *Sixth Catalog of Orbits of Visual Binary Stars*, marked by a dash-dot line. It is discussed in Section 4 (Figures A1–A5).

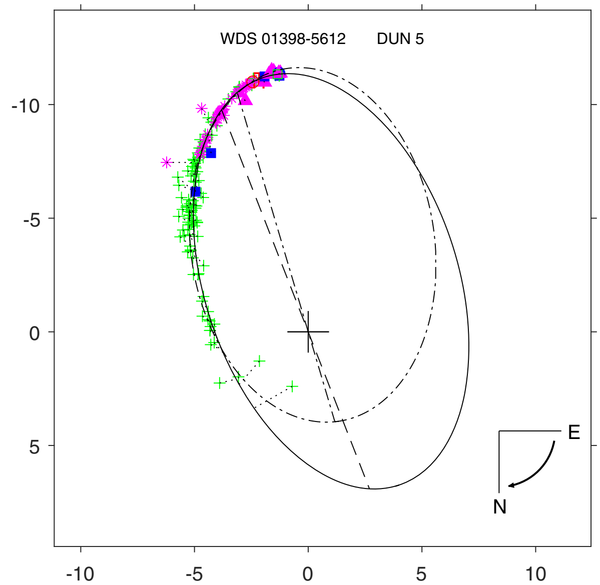

FIGURE A1 No. 5 01398-5612 DUN 5

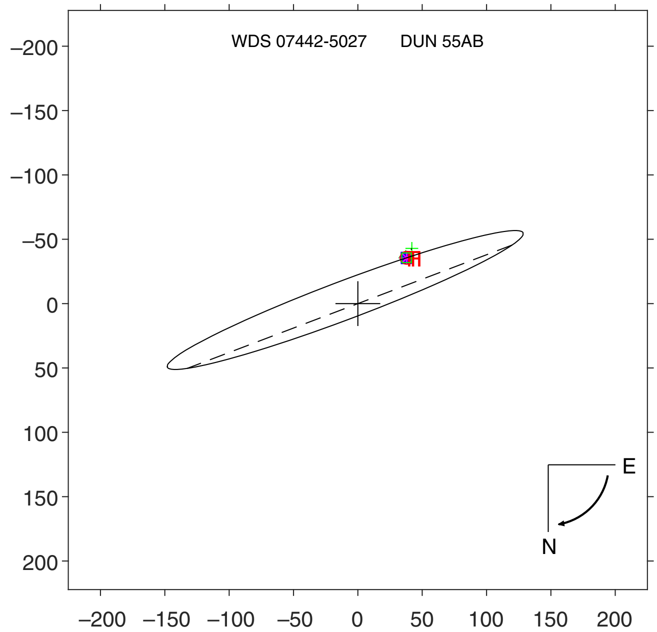

FIGURE A3 No. 55 07442-5027 DUN 55AB

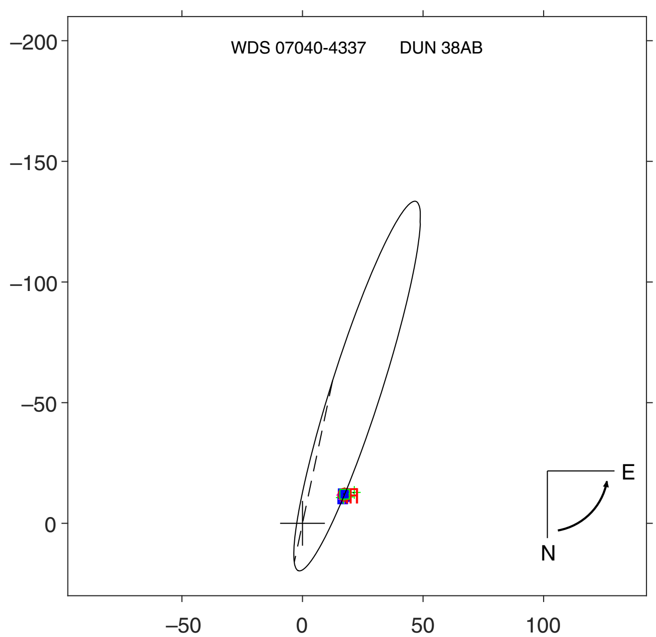

FIGURE A2 No. 38 07040-4337 DUN 38AB

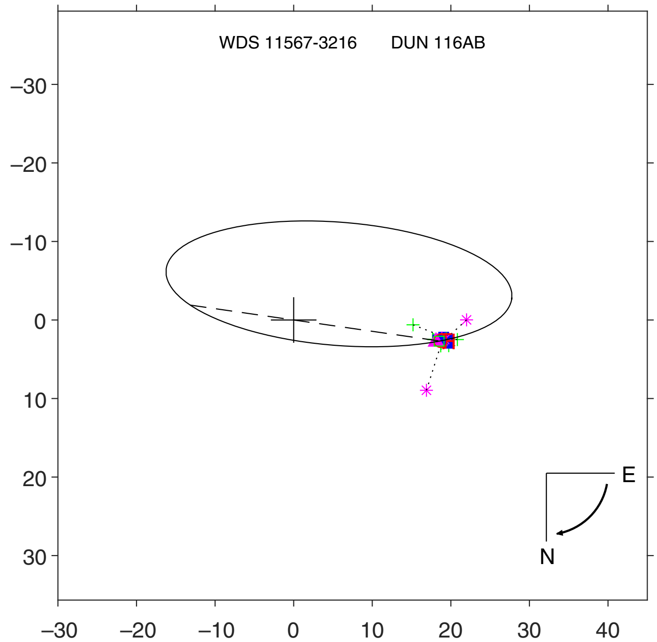

FIGURE A4 No. 116 11567-3216 DUN 116AB

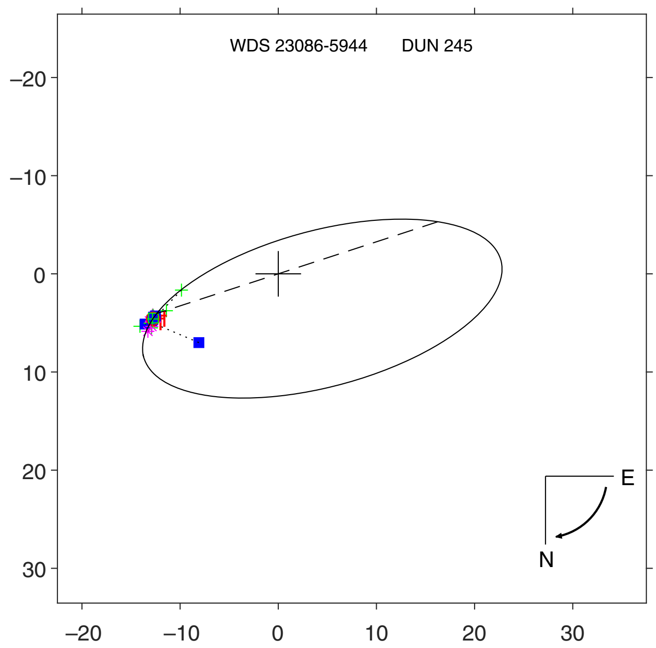

FIGURE A5 No. 245 23086-5944 DUN 245