The Stability of Inclined Plate Anchors in Purely Cohesive Soil

Richard S Merifield
Scott W Sloan
Andrei V Lyamin

richard.merifield@usq.edu.au

http://www.usq.edu.au/users/merifield/

October, 2003

Faculty of Engineering & Surveying Technical Reports
ISSN 1446-1846

Report TR-2003-03
ISBN 1 877078 05 0

Faculty of Engineering & Surveying
University of Southern Queensland
Toowoomba Qld 4350 Australia
http:/www.usq.edu.au/

Abstract

Soil anchors are commonly used as foundation systems for structures requiring uplift resistance such as transmission towers, or for structures requiring lateral resistance, such as sheet pile walls. To date the design of these anchors has been largely based on empiricism. This paper applies numerical limit analysis and displacement finite element analysis to evaluate the stability of inclined strip anchors in undrained clay. Results are presented in the familiar form of break-out factors based on various anchor geometries. By obtaining both upper and lower bound limit analysis estimates of the pullout capacity, the true pullout resistance can be bracketed from above and below. In addition, the displacement finite element solutions provide an opportunity to validate these findings thus providing a rigorous evaluation of anchor capacity.

Contents

Nomenclature

A	anchor area
B	anchor width
D	anchor diameter
E_u	Undrained Young's Modulus
L	anchor length
H	anchor embedment depth
H', H_a	depth to top and middle of anchor
i	anchor inclination factor
β	anchor inclination angle
γ	the soil unit weight
c_u	the soil cohesion
ϕ_u	soil friction angle
N_c	the anchor break-out factor
N_{co}	anchor break-out factor for weightless soil
$N_{co\beta}$	break-out factor for inclined anchor in weightless soil
N_{co90}	anchor break-out factor for a vertical anchor
H/B	anchor embedment ratio
H/D	anchor embedment ratio
L/B	anchor aspect ratio
q_u	the ultimate anchor pullout capacity
S_F	the dimensionless anchor shape factor

1. Introduction and Background

During the last thirty years various researchers have proposed approximate techniques to estimate the uplift capacity of soil anchors. The majority of past research has been experimentally based and, as a result, current design practices are largely based on empiricism. In contrast, very few rigorous numerical analyses have been performed to determine the ultimate pull-out load of anchors.

Most of the results from studies of anchors in purely cohesive soil either consist of simple approximate solutions or are derived empirically from laboratory model tests. These results can be found in the works of Adams and Hayes (1967), Meyerhof and Adams (1968), Kupferman (1971), Vesic (1971, 1972), Meyerhof (1973), Das (1978,1980), Ranjan and Arora (1980), and Das *et al.* (1985a,1985b,1989). The uplift capacity of anchors is typically expressed in terms of a break-out factor, which is a function of the anchor shape, embedment depth, overburden pressure and soil properties.

In contrast to the variety of experimental results mentioned above, very few numerical analyses have been performed to determine the pullout capacity of anchors in clay, with the most rigorous studies being by Rowe and Davis (1982) and more recently Merifield *et al* (2001). In these papers, results were presented for both horizontal and vertical strip anchors embedded in homogeneous saturated clay. The results of Rowe and Davis were obtained using an elasto-plastic finite element analysis whereas the results of Merifield *et al* were obtained using the same numerical limit analysis used herein. Other displacement finite element studies on the behaviour of anchors in clay have been made by Ashbee (1969),Davie and Sutherland (1977), and Dewaikar (1988), although very limited results were reported.

To date, most anchor studies have been concerned with either the vertical or horizontal uplift problem. In many instances, anchors are placed at inclined orientations depending on the type of application and load orientation (e.g. transmission tower foundations). However, the important effect of anchor inclination has received very little attention by researchers. A limited number of results for the capacity of inclined square and strip anchors can be found in the works of Meyerhof (1973). The study of Das and Puri (1989) appears to be the most significant attempt to quantify the capacity of inclined anchors. In their tests, the capacity of shallow square anchors embedded in compacted clay with an average undrained shear strength of 42.1kPa was investigated. Pullout tests were conducted on anchors at inclinations ranging between 0°(horizontal) and 90°(vertical) for embedment ratios (H/B) of up to four. A simple empirical relationship was suggested for predicting the capacity of square anchors at any orientation which compared reasonably well with the laboratory observations. Das and Puri (1989) also concluded that anchors with aspect ratios (L/B) of 5 or greater would, for all practical purposes, behave as a strip anchor.

The purpose of this paper is to take full advantage of the ability of recent numerical formulations of the limit theorems to bracket the actual collapse load of inclined anchors accurately from above and below. The lower and upper bounds are computed, respectively, using the numerical techniques developed by Lyamin and Sloan (2002) and Sloan and Kleeman (1995). In addition, the finite element formulation presented by Abbo (1997) and Abbo and Sloan (2000) has also been used for comparison purposes. This research software, named SNAC (Solid Non-linear Analysis Code), was developed with the aim of reducing the complexity of elasto-plastic analysis by using advanced solution algorithms with automatic error control. The resulting formulation greatly enhances the ability of the finite element technique to predict collapse loads accurately, and avoids many of the locking problems discussed by Toh and Sloan (1980) and Sloan and Randolph (1982).

2. Problem of Inclined Anchor Capacity

2.1. Problem Definition

The problem geometry to be considered is shown in Figure 1. An inclined anchor will be defined as an anchor placed at an angle β to the vertical (Figure 1(b)). A horizontal anchor is one where $\beta = 0^\circ$ (Figure 1(a)) while a vertical anchor is one where $\beta = 90^\circ$(Figure 1(c)). The direction of pullout is perpendicular to the anchor face and the depths H', H_a and H are, respectively, the depths to the top, middle and bottom of the anchor from the soil surface. The capacity of anchors inclined at $\beta = 22.5^\circ$, 45°, and 67.5° will be investigated.

After Rowe *et al.* (1982), the analysis of anchor behaviour can be divided into two distinct categories, namely those of “immediate breakaway” and “no breakaway”. In the immediate breakaway case it is as-

sumed that the soil/anchor interface cannot sustain tension so that, upon loading, the vertical stress immediately below the anchor reduces to zero and the anchor is no longer in contact with the underlying soil. This represents the case where there is no adhesion or suction between the soil and anchor. In the no breakaway case the opposite is assumed, with the soil/anchor interface sustaining adequate tension to ensure the anchor remains in contact with the soil at all times. This models the case where an adhesion or suction exists between the anchor and the soil. In reality it is likely that the true breakaway state will fall somewhere between the extremities of the "immediate breakaway" and "no breakaway" cases.

The suction force developed between the anchor and soil is likely to be a function of several variables including the embedment depth, soil permeability, undrained shear strength and loading rate. As such, the actual magnitude of any adhesion or suction force is highly uncertain and therefore should not be relied upon in the routine design of anchors. For this reason, the anchor analyses presented in this paper are performed for the immediate breakaway case only. This will result in conservative estimates of the actual pullout resistance.

2.2. Anchors in purely cohesive soil

The ultimate anchor pull-out capacity of horizontal and vertical anchors in purely cohesive soil is usually expressed as a function of the undrained shear strength in the following form (Merifield *et al* (2001))

$$q_u = \frac{Q_u}{A} = c_u N_c \tag{1}$$

where for a homogeneous soil profile

$$N_c = \left(\frac{q_u}{c_u}\right)_{\gamma \neq 0} = N_{co} + \frac{\gamma H_a}{c_u} \tag{2}$$

and the term N_{co} is defined as

$$N_{co} = \left(\frac{q_u}{c_u}\right)_{\gamma = 0} \tag{3}$$

where, c_u is the undrained soil strength and N_c is known as the anchor break-out factor. $H_a = H$ for horizontal anchors (Figure 1(a)) and $H_a = H - B/2$ for vertical anchors (Figure 1(c)).

Implicit in (1) is the assumption that the effects of soil unit weight and cohesion are independent of each other and may be superimposed. It was shown by Merifield *et al* (2001) that this assumption generally provides a good approximation to the behaviour of anchors in purely cohesive undrained clay.

For an inclined anchor in purely cohesive soil the ultimate capacity will be given by (1) where

$$N_c = N_{co\beta} + \frac{\gamma H_a}{c_u} \tag{4}$$

A new break-out factor $N_{co\beta}$ is introduced which will have a value somewhere between the break-out factors N_{co} given in (3) for vertical and horizontal anchors. Only the homogeneous case is considered and immediate breakaway is assumed.

It should be noted that the break-out factor N_c given in (4) does not continue to increase indefinitely, but reaches a limiting value which marks the transition between shallow and deep anchor behaviour. This process is explained in greater depth by Merifield *et al* (2001) and Rowe (1978). The limiting value of the break-out factor is defined as N_{c^*} for a homogeneous soil profile (Merifield *et al* (2001)).

3. Results and Discussion

The computed upper and lower bound estimates of the anchor break-out factor $N_{co\beta}$ (equation (4)) for homogeneous soils with no soil weight are shown graphically in Figure 2 and Figure 3. The results are for the case where no suction forces exist between the anchor and soil, which constitutes what is known as the "immediate breakaway" condition (Rowe 1978). Sufficiently small error bounds were achieved with the true value of the anchor break-out factor typically being bracketed to within $\pm 7\%$. The greatest variation between the bounds solutions occurs at small embedment ratios ($H_a/B \leq 2$) where the error bounds grow to a maximum of $\pm 10\%$.

Also shown in Figure 2 and Figure 3 are the SNAC results. These results plot close to the upper bound solution and are typically within $\pm 5\%$.

The variation of break-out factor with angle of inclination is clearly presented in Figure 4. In this Figure, the break-out factor is presented as a ratio of the break-out factor for an inclined anchor to that of a vertical anchor. This ratio is defined as the inclination factor i according to

$$i = \frac{N_{co\beta}}{N_{co\,90}} \tag{5}$$

where i is the inclination factor, $N_{co\beta}$ is the break-out factor for an inclined anchor at an embedment ratio of H_a/B (Figure 2 or Figure 3), and $N_{co\,90}$ is the break-out factor for a vertical anchor at the same embedment ratio H_a/B given by

$$N_{co\,90} = N_{co(\beta=90,\, H/B=H_a/B+0.5)}$$

The value of the break-out factor N_{co90} can, with sufficient accuracy, be approximated by the following expression (Merifield *et al.* (2001))

$$N_{co90} = N_{co} = 2.46\ln\left(2\frac{H}{B}\right) + 0.89 \quad \text{Lower Bound} \tag{6}$$

The inclination factor can be seen to increase in a non-linear manner with increasing inclination from $\beta = 0^\circ$ to $\beta = 90^\circ$. This observation is consistent with the laboratory study of Das and Puri (1989). Figure 4 also suggests that there is very little difference between the capacity of a horizontal anchor ($\beta = 0^\circ$) and anchors inclined at $\beta \leq 22.5^\circ$. The greatest rate of increase in anchor capacity appears to occur once $\beta \geq 30^\circ$.

The failure mechanisms observed for inclined anchors are illustrated by the upper bound velocity diagrams and SNAC displacement plots in Figure 5 to Figure 7. As expected, the vector and displacement fields obtained from both types of analyses are very similar. A direct comparison is shown for anchors at $H'/B = 1$ in Figure 5.

The lateral extent of surface deformation increases with increasing embedment depth and inclination angle. This is consistent with the findings for both the horizontal and vertical anchor cases. As expected, the actual magnitude of the surface deformations decreases with the embedment ratio and, at $H'/B = 10$, these are predicted to be negligible (see the results for $H'/B = 6$ in Figure 7).

Localised elastic zones were observed near the soil surface at most embedment ratios and inclination angles. Several of these zones are shown in Figure 8 for anchors at $H'/B = 4$. In addition, very little plastic shearing was observed below the bottom edge of anchors inclined at $\beta < 45^\circ$. This is highlighted in Figure 7.

The only laboratory investigation to determine the effect of anchor inclination was by Das and Puri (1989). Unfortunately, these tests were limited to square anchors and their results cannot be compared directly to those presented here. Das and Puri (1989) proposed a simple empirical relationship, based on their laboratory findings, for estimating the capacity of inclined anchors. This relationship is of the form

$$N_{co\beta} = N_{co(\beta=0^\circ)} + \left[N_{co(\beta=90^\circ)} - N_{co(\beta=0^\circ)}\right]\left(\frac{\beta^\circ}{90}\right)^2 \tag{7}$$

where N_{co} is obtained at the same value of H_a for each inclination angle β. The value of $N_{co(\beta=0)}$ is the break-out factor for a horizontal anchor and can, with sufficient accuracy, be approximated by the following expression (Merifield *et al.* (2001))

$$N_{co(\beta=0)} = N_{co} = 2.56\ln\left(2\frac{H}{B}\right) \quad \text{Lower Bound} \tag{8}$$

Out of curiosity, equation (7) has been used to estimate the break-out factors for strip anchors and a comparison between these estimates and the results from the current study are shown in Figure 9. The limit analysis and SNAC results (90 points) for inclination angles of 22.5°, 45°, 67.5° and embedment depths of H_a/B of 1 to 10 are shown.

Figure 9 indicates that although the empirical equation of Das and Puri (1989) was specifically proposed for inclined square anchors, it also provides a reasonable estimate for the capacity of inclined strip anchors. Equation (7) plots almost central to the data and, on average, the estimated values are within $\pm$ 5% of the actual values. This is considered an adequate level of accuracy for design purposes. The discrepancy between the predicted and actual break-out factors tends to be marginally larger for smaller embedment ratios ($H/B \leq 2$) where the predicted value is expected to be slightly conservative. It is therefore concluded that the empirical relation given by equation (7) may be used to estimate the capacity inclined strip anchors.

3.1. Effect of overburden pressure

The numerical results discussed above are limited to soil with no unit weight, and therefore the effect of soil weight (overburden) needs to be investigated. If our assumption of superposition is valid then it would be expected that the ultimate anchor capacity, as given by equations (1) and (2), would increase linearly with the dimensionless overburden pressure $\gamma H_a/c_u$. The results from further lower bound analyses that include cohesion and soil weight, shown in Figure 10(a), confirm that this is indeed the case. This conclusion is in agreement with the observations of Merifield *et al* (2001) and Rowe (1978).

The error due to superposition can be expressed in the following form

$$F_s = \frac{q_{actual}}{q_{predicted}} \tag{9}$$

and is shown in Figure 10(b). This figure indicates superposition error are likely to be insignificant.

Figure 10(a) indicates that the ultimate anchor capacity increases linearly with overburden pressure up to a limiting value. This limiting value reflects the transition of the failure mode from being a non-local one to a local one. An example of a deep anchor failure is shown by the velocity diagram in Figure 11 for an anchor where $\beta = 45^\circ$. At a given embedment depth the anchor failure mode may be non-localised or localised, depending on the dimensionless overburden ratio $\gamma H_a/c_u$. For shallow anchors exhibiting non-localised failure, the mode of failure is independent of the overburden pressure.

For deep anchors, the limiting values of the break-out factor N_{c^*} were found to be 10.8(lower bound) and 11.96 (upper bound). These values compare well with the analytical solutions of Rowe, who found lower and upper bounds of 10.28 and 11.42 for the horizontal anchor case. For deep anchors, the form of the velocity field at collapse is essentially independent of the overburden pressure.

3.2. SUGGESTED PROCEDURE FOR ESTIMATION OF UPLIFT CAPACITY

1. Determine representative values of the material parameters c_u and γ.
2. Knowing the anchor size B and embedment depth H_a calculate the embedment ratio H_a/B and overburden ratio $\gamma H_a/c_u$.
3. Calculate N_{co90} using equation (6) with $H/B = H_a/B + 0.5$.
4. (i) For an anchor at $\beta = 22.5^\circ$, 45° or 67.5°, estimate the the break-out factor N_{co} using Figure 2 or Figure 3 depending on the anchor orientation.
(ii) For anchors at other orientations, estimate the the anchor inclination factor i using Figure 4(a) and the value of H_a/B obtained in (3). Then calculate N_{co} as per equation (5). A value of N_{co} could also be estimated from equation (7) using equations (6) and (8).
5. Adopt $N_{c^*} = 10.9$.
6. (i) Calculate the break-out factor N_c using equation (2).
(ii) If $N_c \geq N_{c^*}$ then the anchor is a deep anchor. The ultimate pull-out capacity is given by equation (1) where $N_c = N_{c^*} = 10.9$.
(iii) If $N_c \leq N_{c^*}$ then the anchor is a shallow anchor. The ultimate pull-out capacity is given by equation (1) where N_c is the value obtained in 6(i).

Example of Application

We now illustrate how to use the results presented to determine the ultimate pullout capacity of an inclined anchor in clay.

Problem: A plate anchor of width 0.2 m is to be embedded at $H_a = 1.5$m at an orientation of 45°. Determine the ultimate pullout capacity given the clay has a shear strength $c_u = 50$ kPa and unit weight $\gamma = 15$ kN/m^3.

The systematic procedures given above will now be used to determine the ultimate anchor capacity.

1. Given $c_u = 50$ kPa and $\gamma = 15$ kN/m^3.
2. The embedment ratio can be calculated as $H_a/B = 1.5/0.2 = 7.5$
   The dimensionless parameter $\gamma H_a/c_u = (15 \times 1.5)/50 = 0.45$
3. $N_{co90} = 2.46 \ln(2H/B) + 0.89 = 2.46 \ln(2(7.5 + 0.5)) + 0.89 = 7.71$
4. (i) From Figure 2, $N_{co} = 7$ (lower bound)
5. Adopt $N_{c^*} = 10.9$.
6.(i) From equation (2) , $N_c = 7 + 0.45 = 7.45$
   (ii) $N_c < N_{c^*}$ and therefore the anchor is "shallow" and using equation (1)

$q_u = c_u N_c = 50 \times 7.45 =$ **372.5** kPa

4. Conclusions

A rigorous numerical study into the ultimate capacity of inclined strip anchors has been presented. Consideration has been given to the effect of embedment depth and anchor inclination. The results have been presented as break-out factors in chart form to facilitate their use in solving practical design problems.

The following conclusions can be drawn from the results presented in this paper:

(1) Using the lower and upper bound limit theorems, small error bounds of less than $\pm 7\%$ were achieved on the true value of the break-out factor for anchors inclined at 22.5°, 45°, and 67.5° to the vertical in a weightless soil.

(2) The displacement finite element (SNAC) results compare favourably with the numerical bounds solutions, and plot close to the upper bound solution and are typically within $\pm 5\%$.

(3) The effect of anchor inclination on the pullout capacity of anchors has been investigated. A simple empirical equation has been proposed which, on average, provides collapse load estimates within $\pm$ 5% of the actual values.

(4) The ultimate anchor capacity increases linearly with overburden pressure up to a limiting value that reflects the transition from a non-localised to localised (or "deep") failure mechanism.

References

Abbo, A. J. (1997). "Finite element algorithms for elastoplasticity and consolidation." *PhD Thesis*, Department of Civil, Surveying and Environmental Engineering, University of Newcastle, NSW.

Abbo, A. J. and Sloan, S. W. (2000). SNAC users manual Version 2.0.

Adams J. I., and Hayes D. C. (1967). "The uplift capacity of shallow foundations." Ontario Hydro-Research Quarterly, 1-13.

Ashbee R. A. (1969). "A uniaxial analysis for use in uplift foundation calculations." Report RD/L/R 1608. Central Electricity Research Laboratory.

Das, B. M. (1978). "Model tests for uplift capacity of foundations in clay." *Soils and Found., Japan ,* 18(2):17-24.

Das, B. M. (1980). "A procedure for estimation of ultimate capacity of foundations in clay." *Soils and Found., Japan ,* 20(1):77-82.

Lyamin A. V. & Sloan S. W. (2002), "Lower bound limit analysis using nonlinear programming." *International Journal for Numerical Methods in Engineering,* 55(5), 573-611.

Das, B. M., Tarquin A. J., and Moreno R. (1985a). "Model tests for pullout resistance of vertical anchors in clay." *Civil Eng. for Practicing and Design Engineers., Pergamon Press, New York,* 4(2):191-209.

Das, B. M., Moreno R., and Dallo K. F. (1985b). "Ultimate pullout capacity of shallow vertical anchors in clay.", *Soils and Found., Japan ,* 25(2):148-152.

Das, B. M., and Puri, V. K. (1989). "Holding capacity of inclined square plate anchors in clay." *Soils and Found., Japan ,* 29(3):138-144.

Davie J. R., and Sutherland H. B. (1977). "Uplift resistance of cohesive soils." *Journal of the Soil Mechanics and Foundations Division, ASCE,* 103(9), 935-952.

Dewaikar, D. M. (1988). "Finite element analysis of certain aspects of deep anchors in cohesive soil medium." *Numerical methods in Geomechanics (Innsbruck 1988),* Swoboda(ed.), Balkema, Rotterdam, 1437-1441.

Kupferman M. (1971). *The vertical holding capacity of marine anchors in clay subjected to static and cyclic loading.* M.Sc Thesis, University of Massachusetts, Amherst, USA.

Meyerhof G. G., and Adams J. I. (1968). "The ultimate uplift capacity of foundations." *Canadian Geotechnical Journal.,* 5(4), 225-244.

Meyerhof G. G., (1973). "Uplift resistance of inclined anchors and piles." *Proceedings, 8th International Conference on Soil Mechanics and Foundation Engineering, Moscow, Vol 2:*167-172.

Merifield R. S. , Sloan S. W. and Yu H. S. (2001). "Stability of plate anchors in undrained clay." *Geotechnique*, 51(2), 141-153.

Ranjan G., and Arora V. B. (1980). "Model studies on anchors under horizontal pull in clay." *Proc. 3rd Aust, N.Z Conf. Geomech., Wellington, N.Z.,* 1:65-70.

Rowe R. K. (1978). *Soil structure interaction analysis and its application to the prediction of anchor behaviour.* PhD thesis, University of Sydney, Australia.

Rowe R. K., and Davis E. H. (1982). "The behaviour of anchor plates in clay." *Geotechnique*, 32(1), 9-23.

Sloan S. W. and Randolph M. F. (1982). Numerical prediction of collapse loads using finite element methods. *International Journal for Numerical and Analytical Methods in Geomechanics*, 6(1), 47-76.

Sloan, S. W., and Kleeman P. W. (1995). "Upper bound limit analysis using discontinuous velocity fields." *Computer Methods in Applied Mechanics and Engineering*, 127:293-314.

Toh, C. T., and Sloan, S. W. (1980). "Finite element analysis of isotropic and anisotropic cohesive soils with a view to correctly predicting impending collapse." *International Journal for Numerical and Analytical Methods in Geomechanics*, 4, 1-23.

Vesic A. S. (1971). "Break-out resistance of objects embedded in ocean bottom." *Journal of the Soil Mechanics and Foundations Division, ASCE,* 97(9), 1183-1205.

Vesic A. S. (1972). “Expansion of cavities in infinite soil mass.” *Journal of the Soil Mechanics and Foundations Division, ASCE,* 98(3),265-290.

Figures

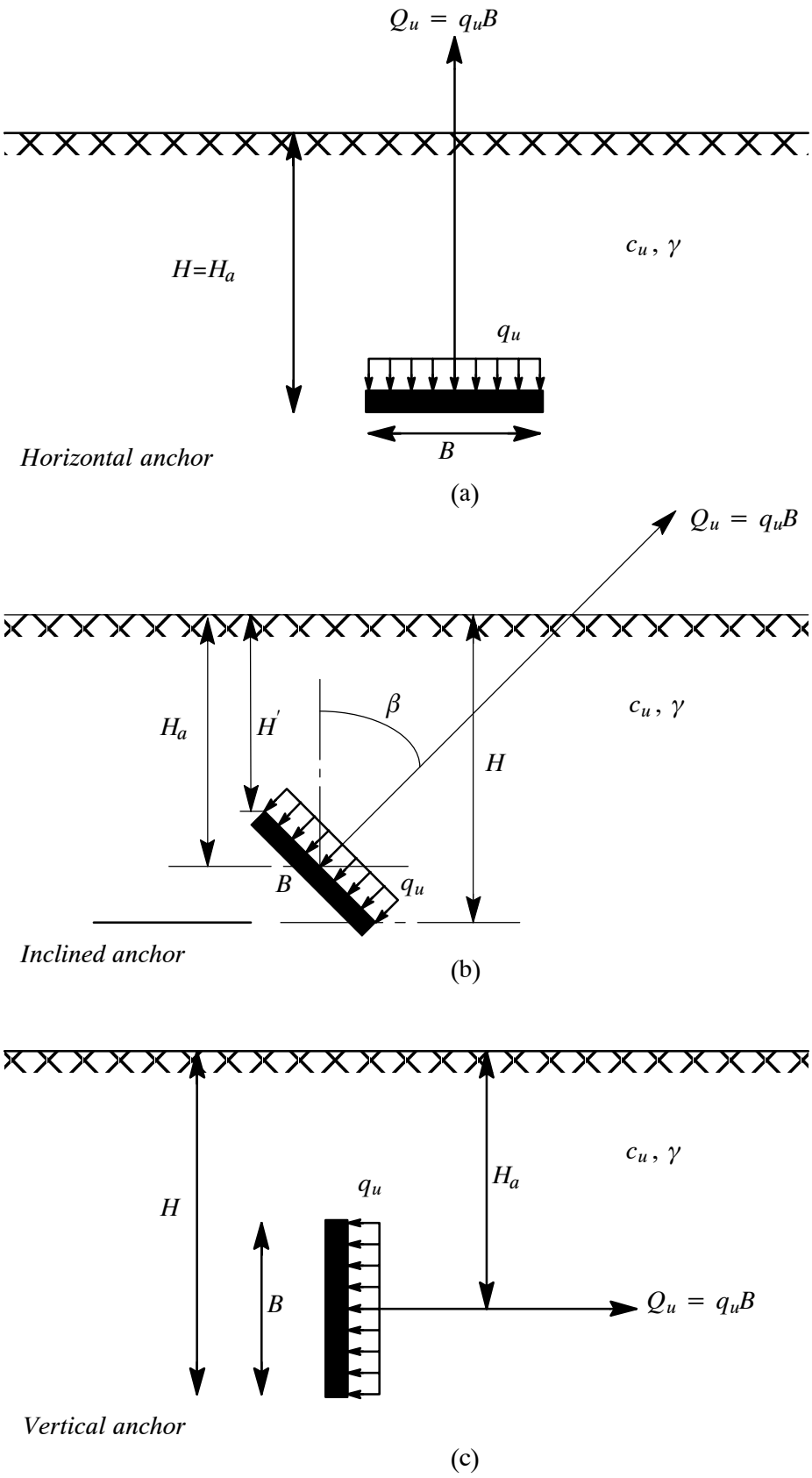
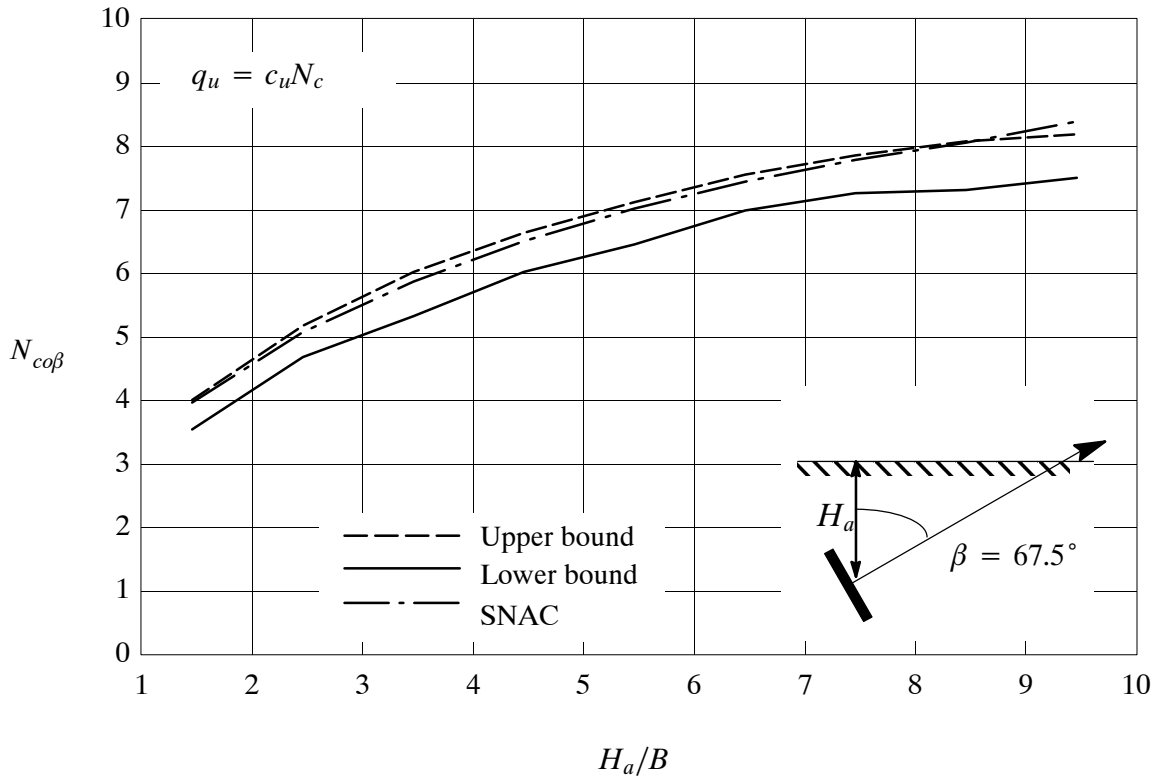


Figure 1 Problem notation for inclined plate anchors.



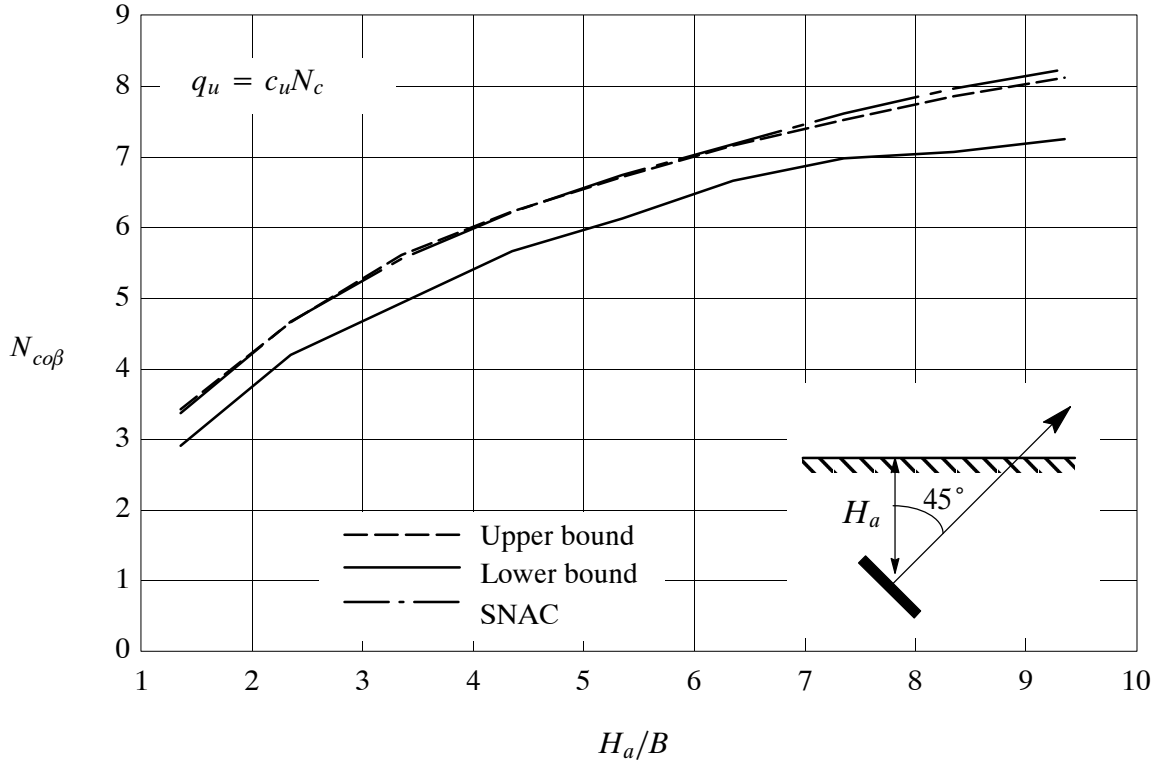


Figure 2 Break-out factors for inclined anchors in purely cohesive weightless soil.

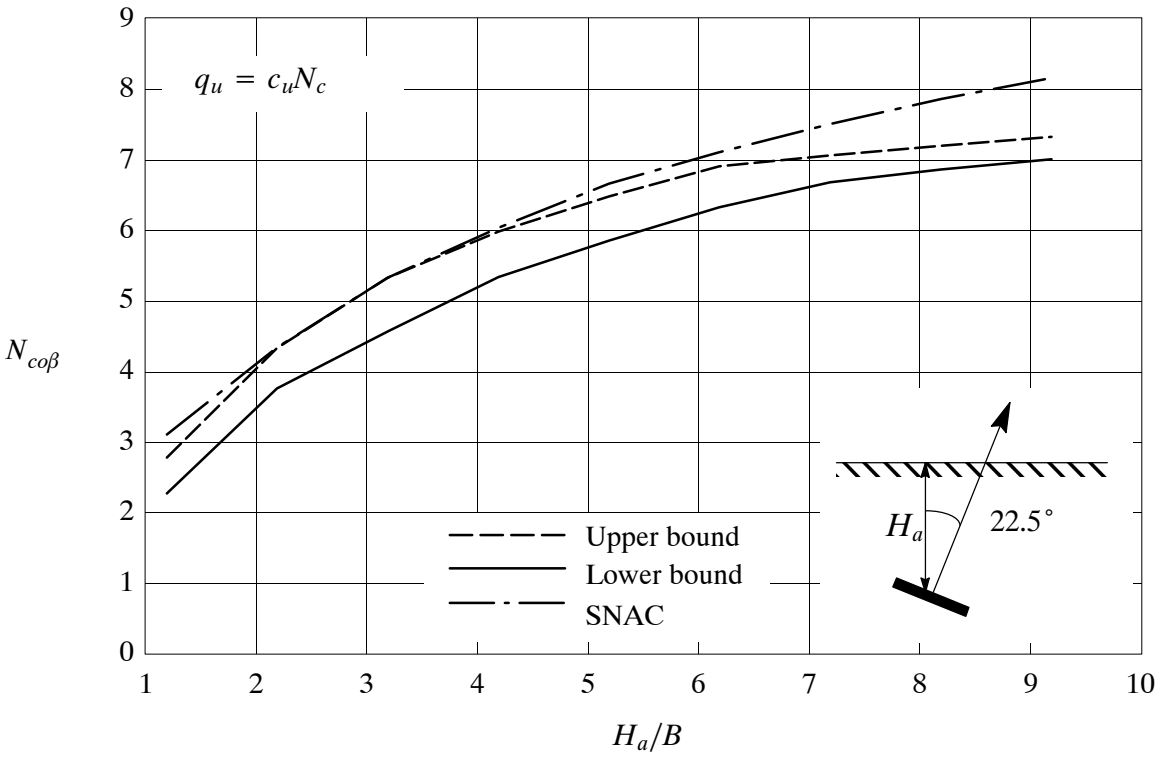
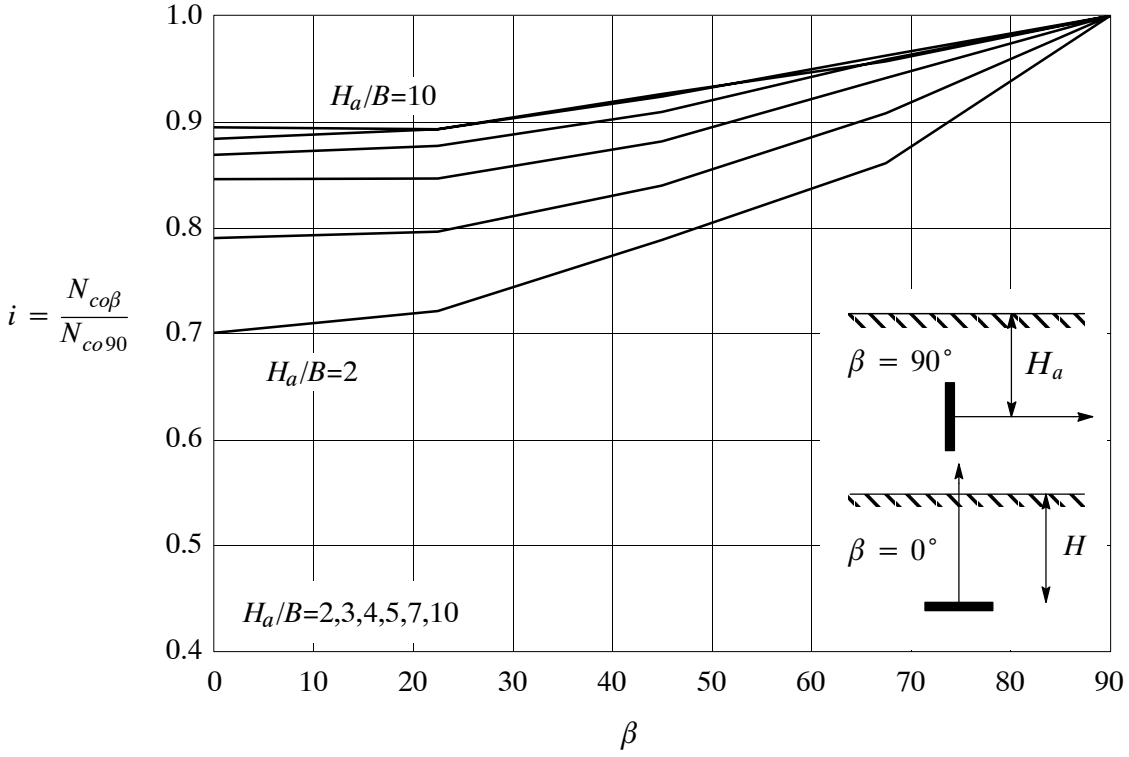
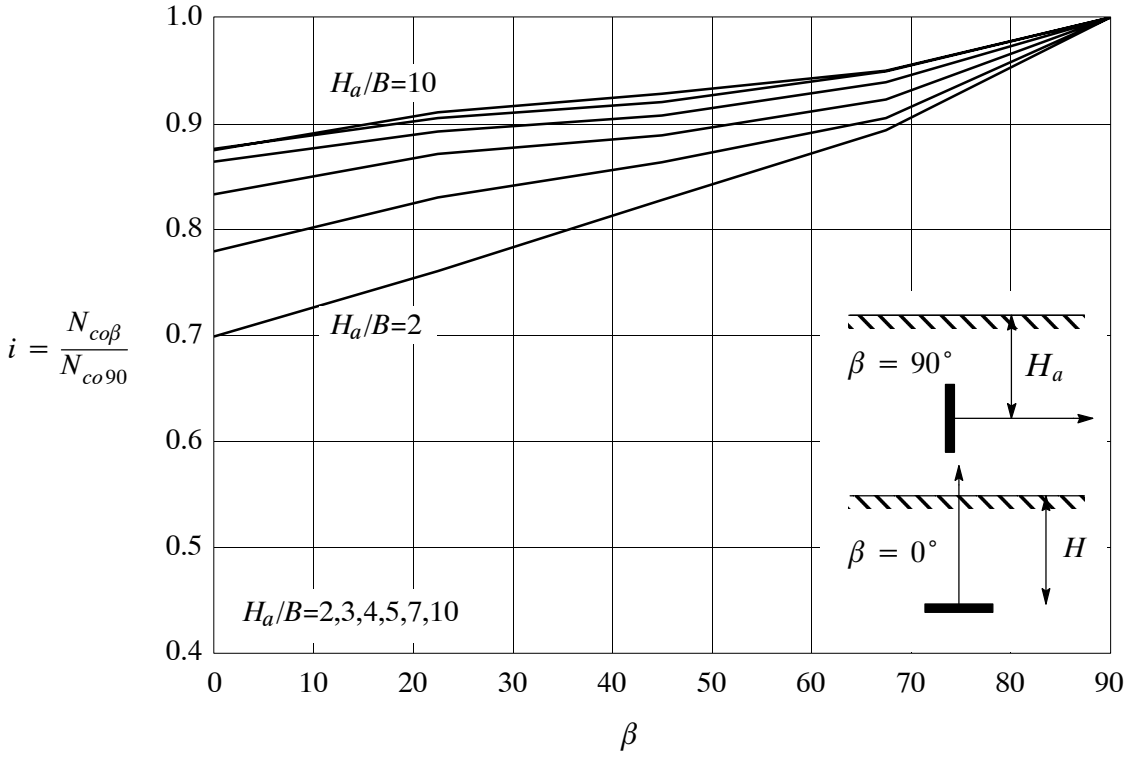


Figure 3 Break-out factors for inclined anchors in purely cohesive weightless soil.



(a)



(b)

Figure 4 Inclination factors for anchors in purely cohesive weightless soil (a) Lower bound, (b) SNAC.

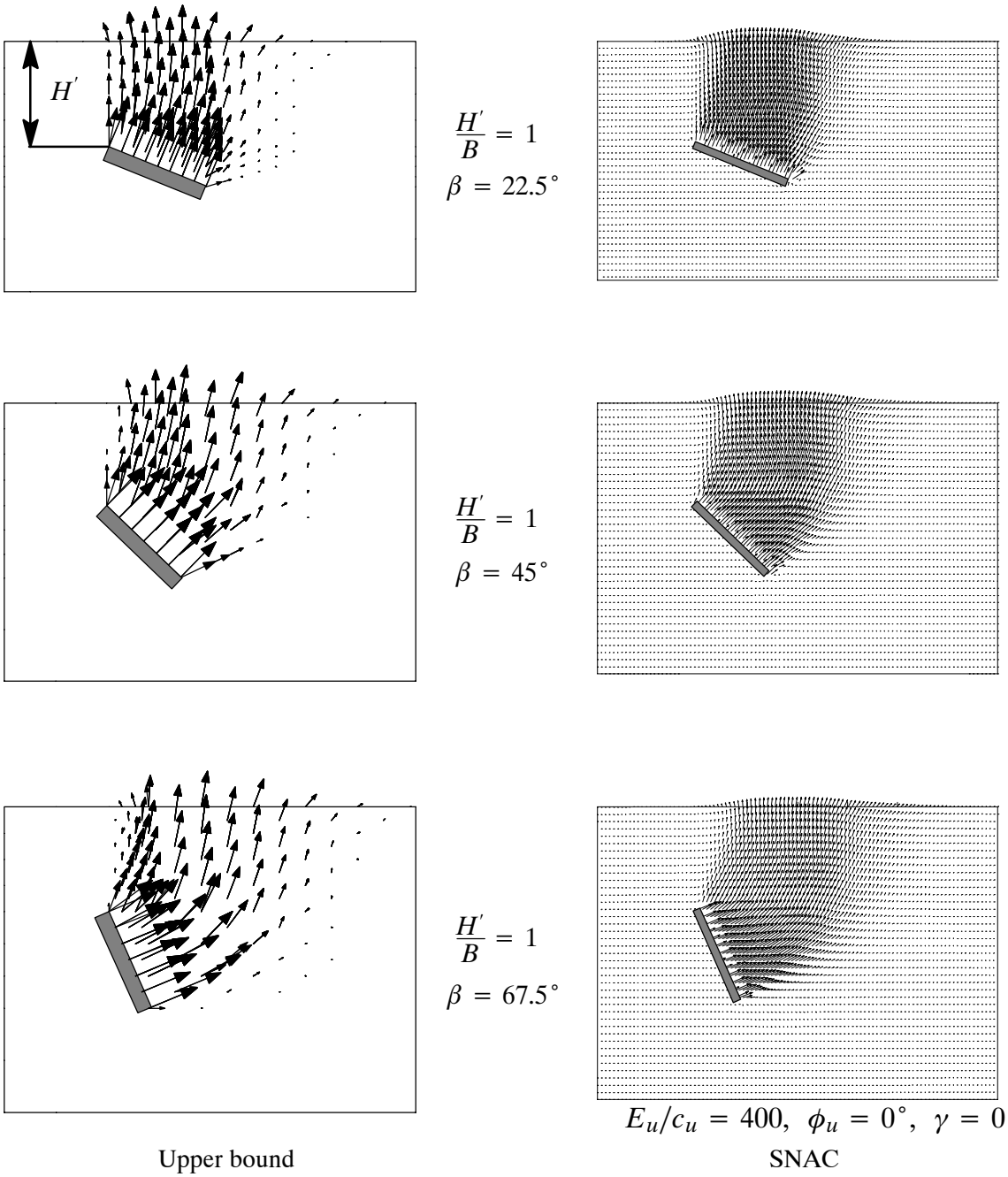
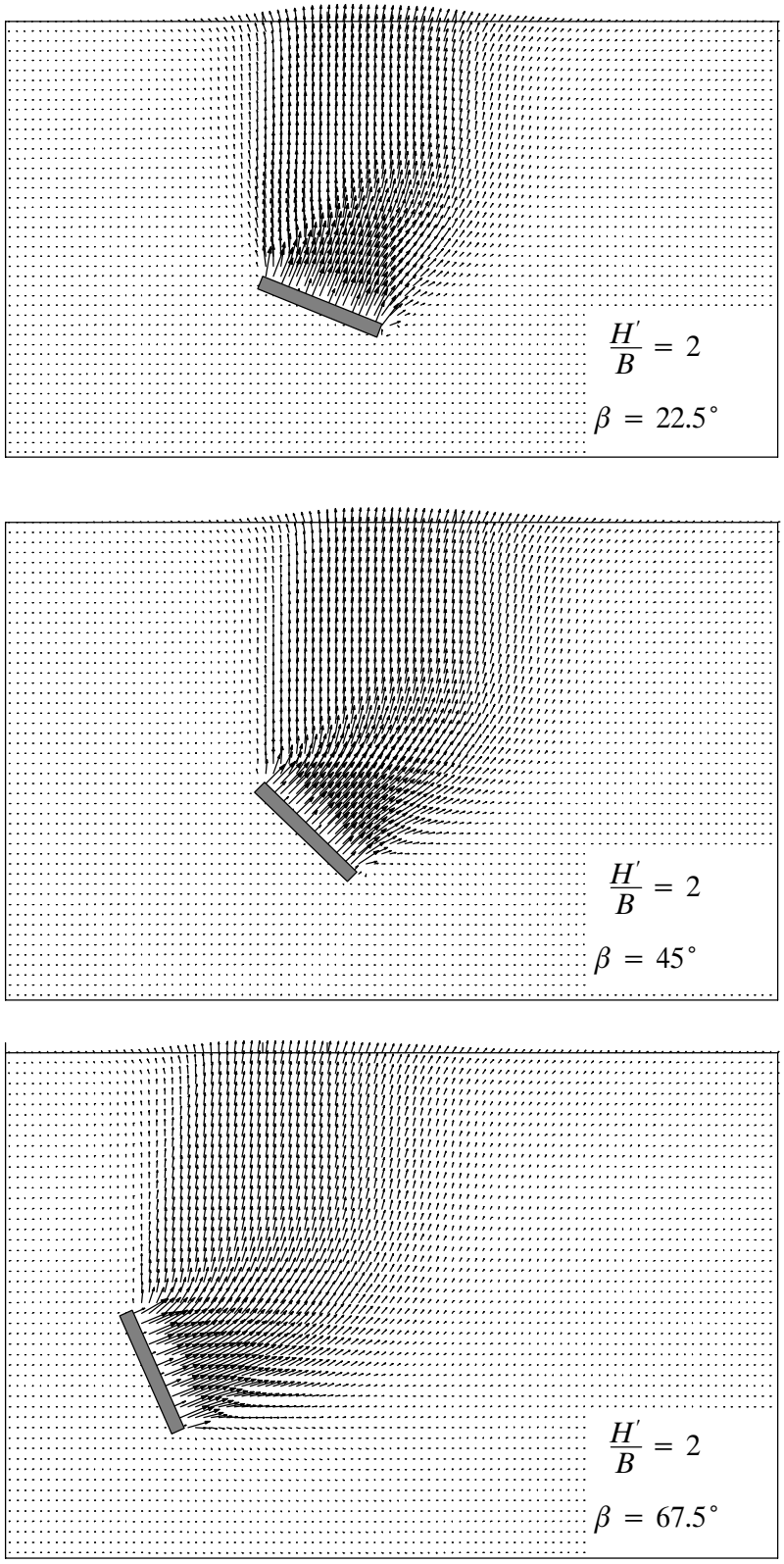


Figure 5 Failure modes for inclined anchors in purely cohesive weightless soil.



$E_u/c_u = 400,\ \phi_u = 0^\circ,\ \gamma = 0$

Figure 6 Failure modes for inclined anchors in purely cohesive weightless soil (SNAC).

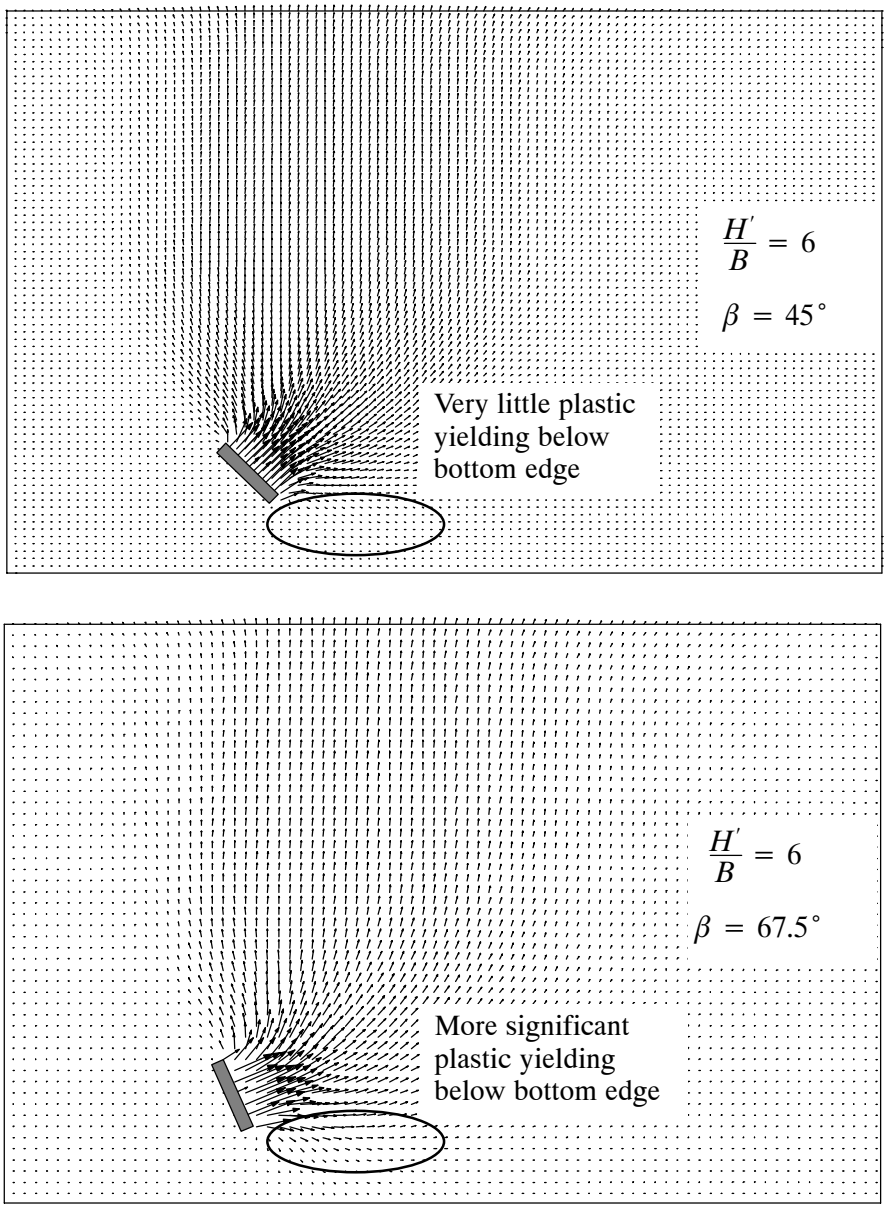
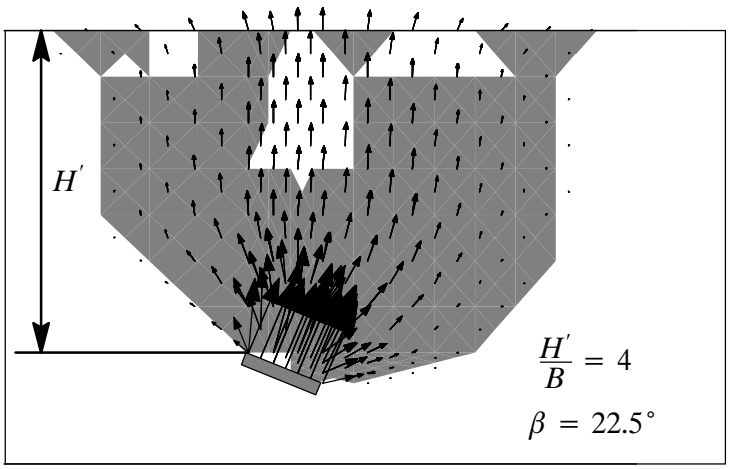
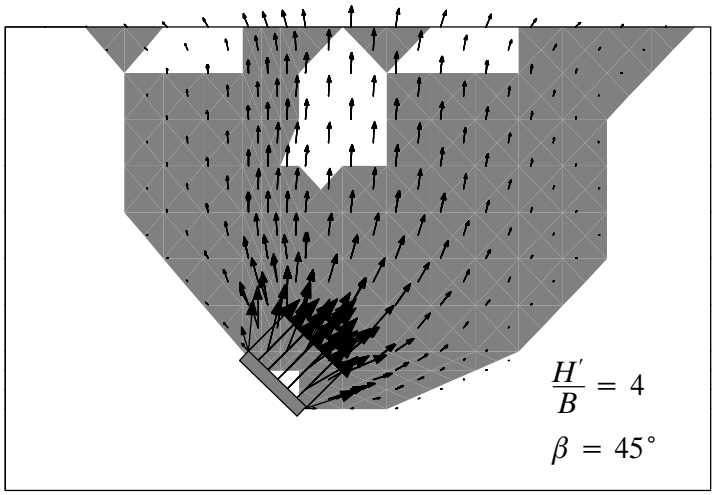


Figure 7 Failure modes for inclined anchors in purely cohesive weightless soil (SNAC).





Plastic zones

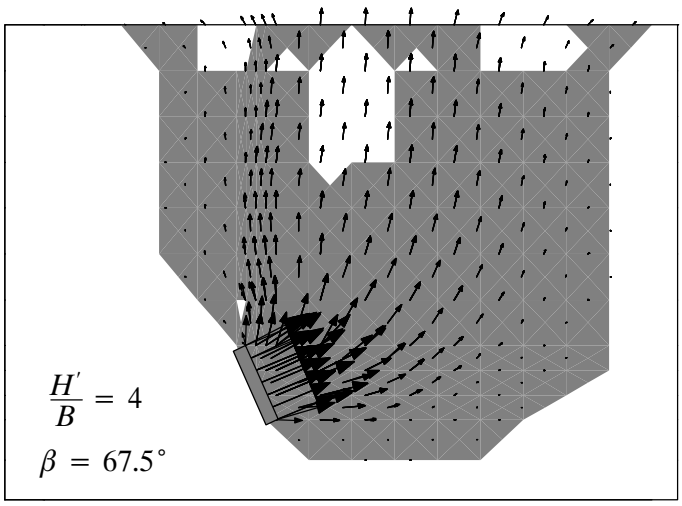


Figure 8 Failure modes and zones of plastic yielding for inclined anchors in purely cohesive weightless soil (upper bound).

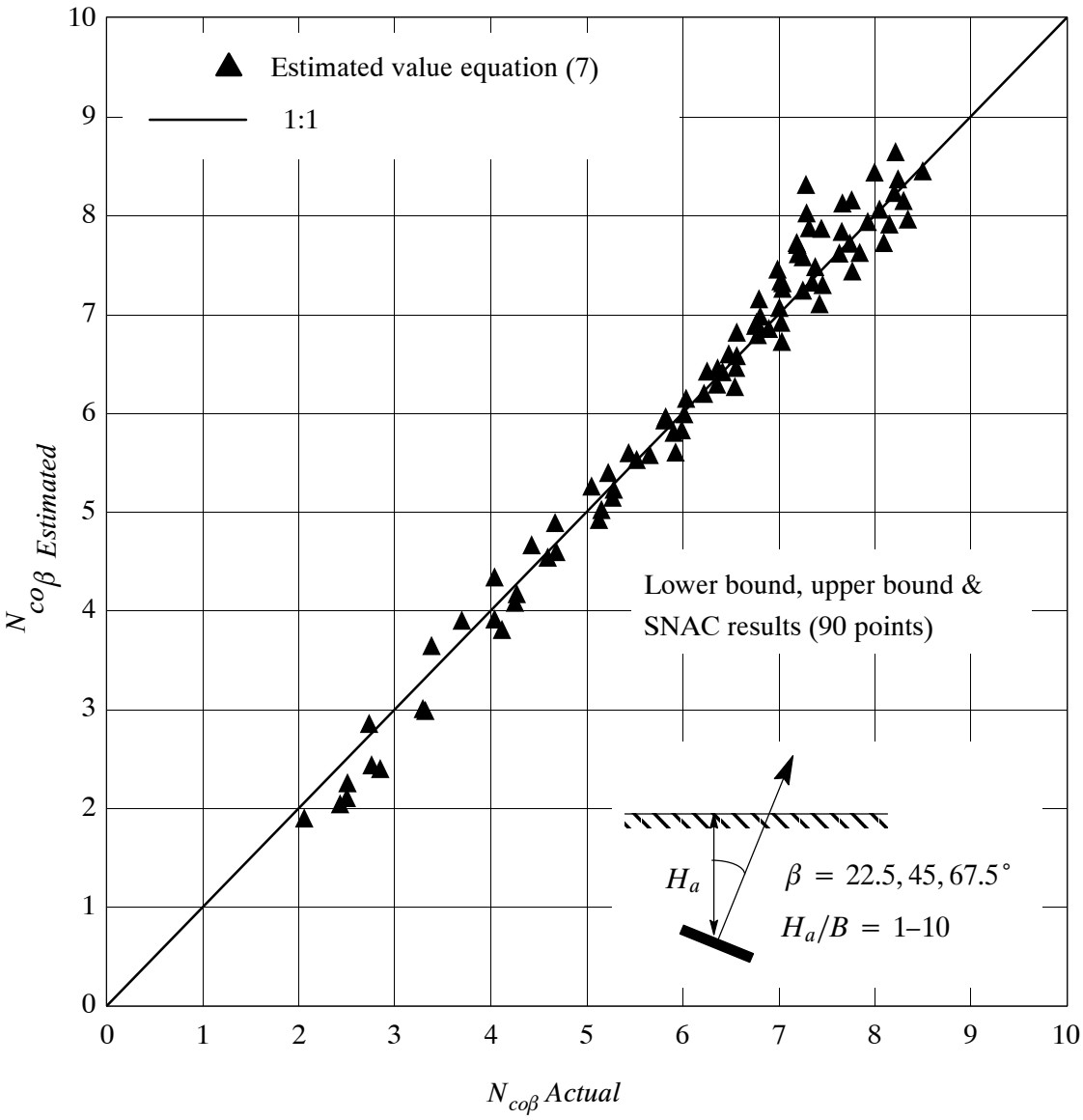
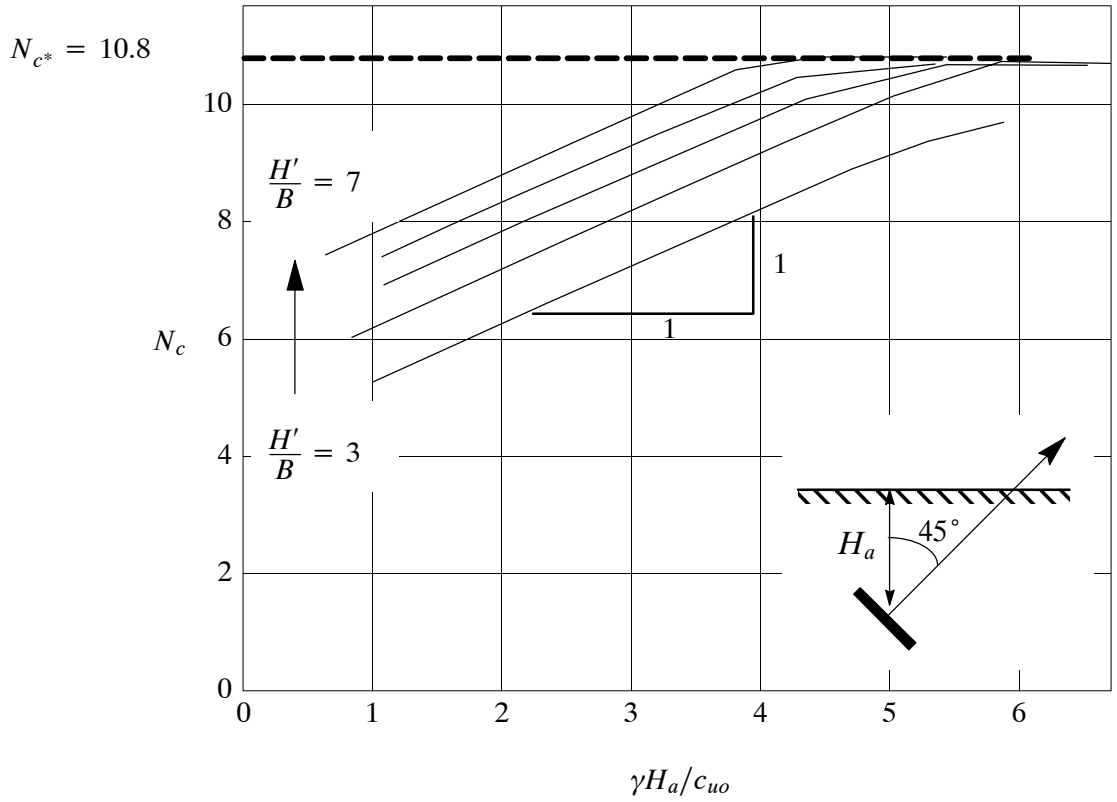
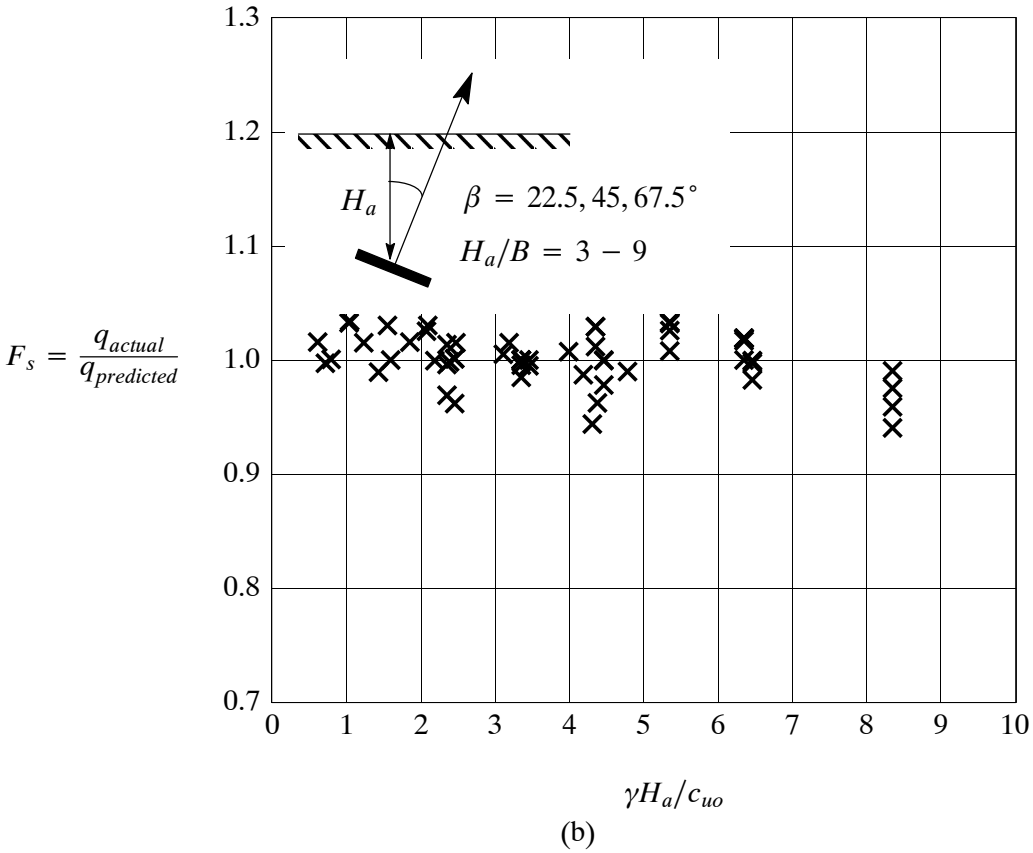


Figure 9 Comparison of break-out factors for inclined strip anchors in purely cohesive weightless soil.



(a)



(b)

Figure 10 Effect of overburden pressure for horizontal anchors in purely cohesive weightless soil - Lower bound.

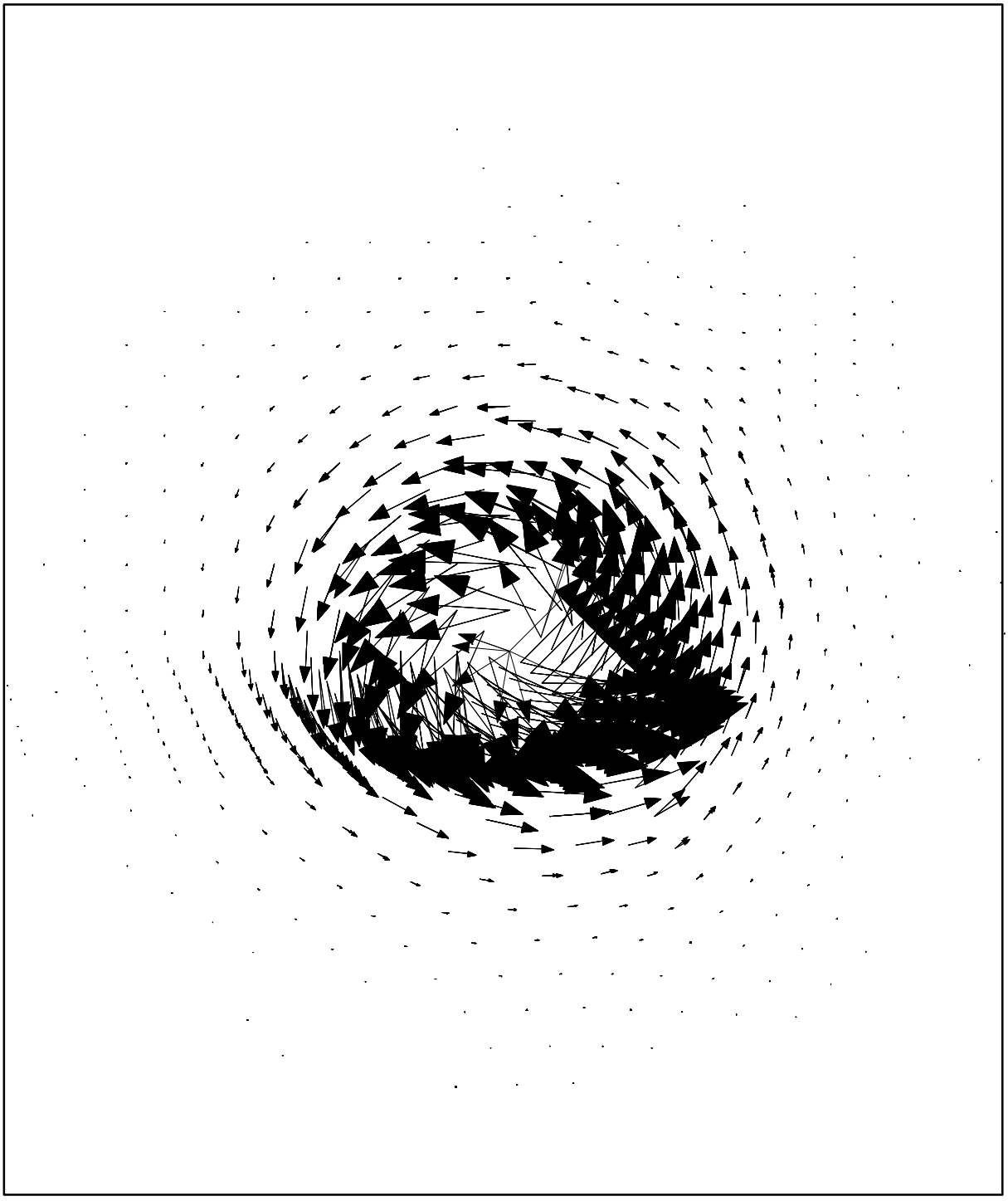


Figure 11 Upper Bound failure mechanism for a deep inclined anchor, $\beta = 45^\circ$