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Stability of planetary, single M dwarf, and binary star companions to *Kepler* detached eclipsing binaries and a possible five-body system

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ABSTRACT

In this study, we identify 11 *Kepler* systems (KIC 5255552, 5653126, 5731312, 7670617, 7821010, 8023317, 10268809, 10296163, 11519226, 11558882, and 12356914) with a flip-flop effect in the eclipse timing variations O − C diagrams of the systems, report on what these systems have in common and whether these systems are dynamically stable. These systems have previously reported high eccentric binary stars with highly eccentric third bodies/outer companions. We find that all of the additional bodies in the system are dynamically stable for the configurations previously reported and are therefore likely to exist as described. We also provide additional evidence of KIC 5255552 being a quadruple star system composed of an eclipsing binary pair and non-eclipsing binary pair with the possibility of a fifth body in the system. With the advent of the NASA's *Transiting Exoplanet Survey Satellite* (*TESS*) exoplanet survey, its precision photometric monitoring offers an opportunity to help confirm more local eclipsing binary star companions, including planets.

Key words: binaries: eclipsing.

1 INTRODUCTION

The *Kepler* Eclipsing Binary Catalog contains more than 2000 eclipsing binary stars that have been observed during the *Kepler* mission (Prša et al. 2011; Slawson et al. 2011). The high precision observations from *Kepler* enable eclipse time studies to be performed where variations in the eclipse times of binary stars can be used to detect third bodies (e.g. Borkovits et al. 2016; Getley et al. 2017). Binary stars that have orbits aligned with the Earth will eclipse each other, and detached and isolated binary stars should have eclipses that occur at predictable intervals. Plots of observed eclipse times (O) minus the calculated eclipse times (C), or O − C plots, may show variations from these predicted intervals. If these variations are also periodic, it may be the result of a third body orbiting the binary stars (Beuermann et al. 2010).

When performing an eclipse timing study on the eclipsing binaries contained in the *Kepler* catalogue, several O − C diagrams were found where the values begin to decrease, or increase, and then suddenly and rapidly reverse direction and change sign, i.e. eclipses that occur earlier than expected change to later than expected, or vice versa. The O − C curves for these systems then rapidly reverse sign again, or flip-flop (see Fig. 1 for a visual example). The secondary eclipse O − C curve is out of phase with the primary eclipse O − C curve by a half orbital period. Examples of these flip-flop systems can be seen in Borkovits et al. (2016). Most of these systems also appear to have eclipse depth variations with differing magnitudes for each system. These systems all have similar reported eccentricities of the eclipsing binary and the highly eccentric orbit of the reported third body/outer companion. For the purposes of this paper, eclipsing binary is defined as the primary and secondary stars that eclipse each other, i.e. producing the eclipses seen in the system O − C diagrams while third body/outer companion refers to one (or more) additional bodies orbiting the eclipsing binary.

The flip-flop features of the O − C diagrams and the high eccentricities raise the question of the dynamical stability of the systems and whether the systems with the reported configurations are stable. The dynamical stability of systems is important as outer companions in unstable orbits may result in the outer companion being ejected from the system within a short time period. However, stable orbits suggest the outer companion will remain within the system and are, therefore, more likely to exist as described and be observed (Horner et al. 2012a,b). If an outer companion is stable for a range of configurations, then the outer companion is more likely to exist as any detection errors will not have a dramatic effect on the determination of the stability of the system.

The aims of this study are to perform a dynamical stability analysis on the systems found with highly eccentric binary star orbits and extremely high eccentric outer companion orbits; report on the source of the flip-flop effect and the stability of the systems KIC 5255552, KIC 5653126, KIC 5731312, KIC 7670617, KIC 7821010, KIC 8023317, KIC 10268809, KIC 10296163, KIC 11519226, KIC 11558882, and KIC 12356914 with the proposed third bodies; comment on the likelihood of these proposed third bodies existing; and comment on the likelihood of more of these flip-flop systems existing that continue to go undetected.

2 METHOD

The *Kepler* data were used to produce O − C diagrams for detached eclipsing binaries to study eclipse timing variations. We created a C++ program, called Binary Eclipse Timings (BET), to determine the

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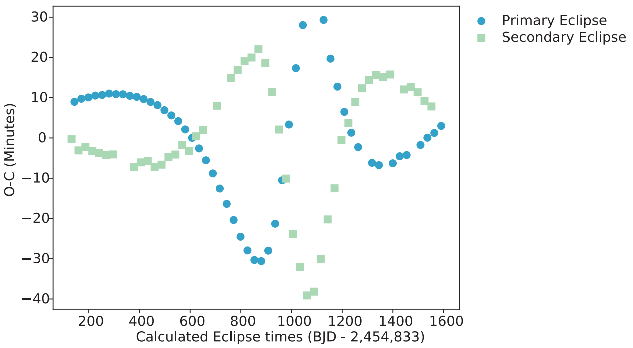

Figure 1. Observed minus calculated (O − C) diagram of KIC 12356914 showing the sudden and rapid period flip in the primary (blue circles) and secondary (green squares) eclipses. For example, at ∼900 d the primary eclipses go from occurring ∼30 min earlier than calculated to ∼30 min later than calculated in the space of ∼200 d.

mid-eclipse times of as many primary and secondary eclipses in the *Kepler* detached binary systems as possible (see Getley et al. 2017). BET is based on the software Transit Analysis Package (TAP; Gazak et al. 2012) that uses the analytic formulae from Mandel & Agol (2002). The analytic formulae describe a system of two objects, using parameters including orbital period, radius ratio of the two objects, mid-eclipse time, orbital inclination, and eccentricity, during various points throughout an orbit. The O − C diagrams of the *Kepler* systems shown in this paper were created using BET and found to contain rapid variations with the primary and secondary eclipse O − C curves out of phase.

REBOUND is an *N*-body integrator with PYTHON and C implementations (Rein & Spiegel 2015). Systems of bodies are able to be set up and integrated over time to estimate the orbital characteristics, such as semimajor axis and eccentricity, at various intervals. By simulating the positions and the evolution of the estimated orbital characteristics of a system over a long time period, we can determine if the proposed system is in a stable orbit (allowing it to have been observed) or if it is in an unstable orbit and likely to eject one or more of the bodies. Eclipse times were obtained from the simulation and an O − C diagram produced to make sure that the distinctive characteristics of the actual O − C diagrams were present. The systems with these orbital characteristics were also integrated for 10^6 yr. These same systems were then integrated again 40 times for 10^4 yr with random values for the mean longitude, argument of pericentre and longitude of ascending node of the orbit of the outer companion, and the eclipsing binary. The purpose of the random values was to see if the third bodies were stable in this very specific configuration or if third bodies were stable for a range of configurations.

For the REBOUND models used, the value for the longitude of the ascending node for the eclipsing binary (i.e. Ω_{binary}) was fixed to 90°. We found when it was fixed to 0°, although the flip-flop features of the O − C diagrams still occurred, the primary and secondary eclipse O − C curves were in phase rather than out of phase as seen within the real O − C diagrams. The value for the longitude of the ascending node for the outer companion (i.e. $\Omega_{\mathrm{companion}}$) was set such that $\Omega_{\mathrm{companion}} = \Omega_{\mathrm{binary}} + \Delta\Omega$.

The individual masses for the primary and secondary star were calculated using the sum of the masses in Borkovits et al. (2016) and the temperature of the systems. Making the assumption that the primary star significantly dominates the temperature of the system, we can search for the corresponding mass of a star at that temperature from Pecaut & Mamajek (2013).[1] This becomes the estimate for the mass of the primary star. Using either a calculated mass ratio or a sum of masses of the primary and secondary star with the estimated primary star mass, calculating the estimated mass of the secondary star becomes trivial. Finally, we compare the $J - H$ colour/magnitude difference of the system with the estimate for the primary star in order to perform a check on the assumption that the primary star significantly dominates the system. We tested this process for estimating masses against *Kepler* systems with known masses for the primary and secondary stars, Kepler-16 (Doyle et al. 2011), Kepler-34 and Kepler-35 (Welsh et al. 2012), Kepler-38 (Orosz et al. 2012b), and Kepler-47 (Orosz et al. 2012a), all with at least one confirmed planet. Our estimates for the primary and secondary masses agree with the reported masses within ∼10 per cent or less. We also tested against systems with no confirmed outer companions, KIC 9851142 (Çakırlı 2015) and KIC 1571511 (Ofir et al. 2012), and found our mass estimates agreed with the reported masses within ∼20 per cent or less.

The first systems to be selected for the dynamical stability study were KIC 5255552, KIC 5731312, KIC 7670617, KIC 10268809, and KIC 12356914 as these systems were identified as part of our own eclipse time study of the *Kepler* eclipsing binaries that had matching entries in Borkovits et al. (2016). These systems all contained a unique flip-flop feature or sudden period change in their O − C diagrams as seen in Fig. 1. The inferred properties of these systems were compared to see what all the systems had in common. The systems were found to have binary eccentricities ranging between ∼0.25 and ∼0.42 and third bodies with eccentricities of at least 0.385. The rest of the systems in Borkovits et al. (2016) were checked to see if there were any other systems that matched these criteria. Finally, the O − C diagrams of the systems were visually compared to find other possible candidates. The complete list of systems and their orbital properties can be found in Tables 1 and 2. Two systems, KIC 4055092 and KIC 9715925, were found to match the selection criteria, however these systems were not a part of the dynamical stability study as both of these systems have mass estimates for the primary star that exceed the mass estimates for the total system. Another two systems, KIC 6794131 and KIC 7177553, were also possible candidates for the dynamical stability study, however accurate values for $m_{\mathrm{a+b}}$ were not obtainable from Borkovits et al. (2016). As such, reliable models in REBOUND were unable to be made for these four systems and they were not included in the study.

The systems in Table 2 are listed separately due to the long-period nature of the outer companions. These third bodies all have periods longer than the window of *Kepler*'s observations, and so, while models and fits can give us an indication of the properties and type of third bodies located within the systems, the margin of error in the values is likely too great to make firm conclusions. We can expect any estimate of the orbital period to be a lower limit due to the uncertainty involved in observing a system for less than one complete orbital period. The outer companion mass is likely to be an upper limit as lower masses are more detectable at longer periods (Watson & Marsh 2010). For those systems with orbital periods less than the period of *Kepler*'s observations, the values listed in Table 1 are likely to be accurate with a smaller margin of error.

The primary and secondary masses for the systems listed in Tables 1 and 2 were calculated as described and are listed in Table 3.

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[1] With additional details from http://www.pas.rochester.edu/~emamajek/EEM_dwarf_UBVIJHK_colors_Teff.txt

Table 1. A list of orbital properties for the systems used in the dynamical stability studies. Values from Borkovits et al. (2016). With orbital periods for the third bodies less than, or approximately equal to, the *Kepler* viewing window the orbital periods of the third bodies and their properties will be likely to reflect the true nature of the systems.

KIC no.	P_1 (d)	P_2 (d)	m_{a+b} ($M_\odot$)	m_c	e_1	e_2	i_1 (°)	i_2 (°)	ω_1 (°)	ω_2 (°)	$\Delta\Omega$ (°)
5255552	32.465339	862.1	1.7	0.7 $M_\odot$	0.30668	0.4342	83.8	89.5	105.27	37.3	−2.8
5653126	38.49233	968	1.8	1.1 $M_\odot$	0.247	0.189	87	78	313	326	−5
5731312	7.9464246	911	1.1	0.13 $M_\odot$	0.4196	0.584	88.5	77.3	183.9	25.9	36.4
7821010	24.2382191	991	2.3	2.6 M_{Jup}	0.6791	0.372	88	105	239.234	126	−19
8023317	16.57907	610.6	1.3	0.15 $M_\odot$	0.2511	0.249	88	93	177.7	164	−49.3
11519226	22.161767	1437	1.44	1.25 $M_\odot$	0.18718	0.332	88	89	358.4	321.7	17.0

Table 2. A list of orbital properties for the systems used in the dynamical stability studies. With orbital periods for the third bodies larger than the *Kepler* viewing window the ability to accurately resolve these properties is difficult, however they still give an indication of the possible configuration of these systems. Values from Borkovits et al. (2016).

KIC no.	P_1 (d)	P_2 (d)	m_{a+b} ($M_\odot$)	m_c ($M_\odot$)	e_1	e_2	i_1 (°)	i_2 (°)	ω_1 (°)	ω_2 (°)	$\Delta\Omega$ (°)
7670617	27.70317	3304	0.9	0.55	0.249	0.707	86	89	135	86.4	−147.8
10268809	24.70843	7000	1.5	1.4	0.314	0.737	84	94	143.1	292.6	21.6
10296163	9.296847	15271	1.4	0.5	0.354	0.73	86	127	45.7	355	−40
11558882	73.9135	4050	1.9	0.4	0.365	0.30	88	84	169	105	−43
12356914	27.3083183	1804	1.8	0.41	0.325	0.385	88	60	113.2	36.5	−30.4

Table 3. Additional information about the systems found in Tables 1 and 2. The temperature of the system comes from the *Kepler* Eclipsing Binary Catalog. Pecaut & Mamajek (2013) and the temperature are used to estimate the primary star mass and, with the values of m_{a+b} from Tables 1 and 2, the secondary star mass.

KIC no.	Temperature (K)	m_a ($M_\odot$)	m_b ($M_\odot$)
5255552	4775	0.96^a	0.74
5653126	5766	1.02	0.78
5731312	4658	0.73	0.37
7821010	6298	1.23	1.07
8023317	5625	0.98	0.32
11519226	5646	0.98	0.46
7670617	4876	0.75	0.15
10268809	5787	1.07	0.43
10296163	6229	1.21	0.19
11558882	6066	1.14	0.76
12356914	5368	0.90	0.90

[a] Mass of the primary star is larger than would be expected from the temperature of the system, though the total mass of the binary star system matches and is expected to be useful to determine the stability of the outer companion.

A number of the systems in Tables 1 and 2 have outer companion masses that are almost as large, or even larger, than one or both of the stars in the binary system. If these third bodies significantly contribute to the flux of the system, then the individual mass for the primary star would be larger than estimated and the mass for the secondary star would be lower (although the total mass of the binary system would be unaffected).

With the systems set-up in REBOUND and integrated, plots are produced showing eccentricity versus time and semimajor axis versus time. By considering these plots, we are able to view the evolution of the system over the defined period and determine whether any object is likely to be ejected from the system. For example, by considering the change in semimajor axis we can tell if an outer companion stays within the system or is moving further away from the binary stars and being ejected out of the system.

The light curves for the systems with inclinations of close to 90° were also visually inspected to look for any additional eclipsing events. Additional eclipsing events are a direct way of confirming the existence of additional bodies and may provide additional information about the characteristics and orbital properties of any additional bodies.

3 RESULTS

The *Kepler* flip-flop systems appear visually unique upon the first consideration of their $O-C$ diagram (Fig. 1). The primary and secondary eclipses $O-C$ variations are out of phase with each other, and there are sharp and rapid flip-flops indicating eclipses rapidly transitioning from earlier than expected to later than expected (or vice versa). An example of a simulated model's $O-C$ diagram can be seen in Fig. 2. The simulated $O-C$ diagram shows the same out of phase and rapid variations that can be seen in the actual $O-C$ diagrams from observed data.

The models from REBOUND allowed us to produce visual representations of the bodies and their orbits within the systems found in Tables 1 and 2. By producing visual representations of the binary star orbits (Fig. 3), animating the binary star and outer companion orbits and the inclination evolution of the systems (Fig. 4), we were able to determine that all systems with the flip-flop $O-C$ variations exhibit similar behaviour/orbital configurations as described in Section 2. The binary stars are locally bound together and both orbit and exhibit apsidal precession around the centre of mass of the entire system. The period of the eclipsing binary apsidal precession around the centre of mass appears to be the same as the orbital period of the outer companion, likely due to the dynamical interactions between the outer body and primary and secondary stars. The third bodies orbit the centre of mass opposite the binary stars. The orbits of the binary stars and the system as a whole are provided as animations available as additional supplementary material online. The models

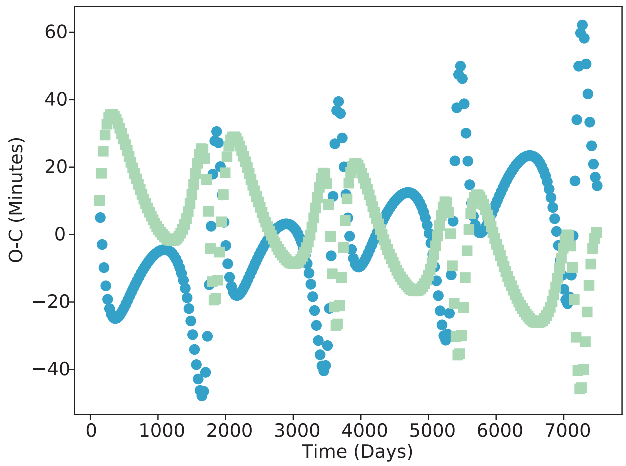


Figure 2. Simulated observed minus calculated (O − C) diagram of KIC 12356914 showing the sudden and rapid period flip in the primary (blue circles) and secondary (green squares) eclipses like the sudden flip-flops seen in Fig. 1.

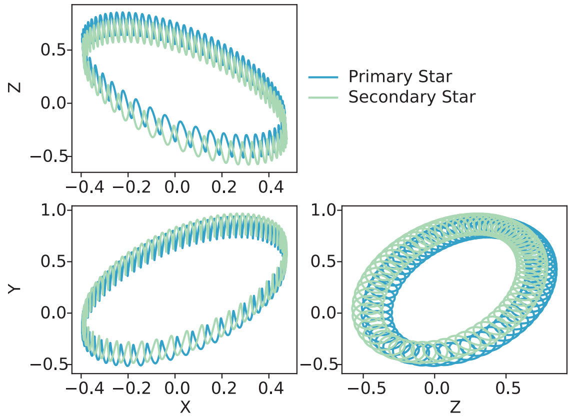


Figure 3. Plot of the *XYZ* coordinates of the two stars in the eclipsing binary of KIC 12356914 showing a wobble around the centre of mass of the systems and the apsidal precession (particularly noticeable in the *YZ* plot) throughout a single orbit of the outer companion. Note: animations of the binary star and outer body orbits will be available online as supplementary material. The observer is in the positive *X* direction with the *Y*-axis running horizontal and the *Z*-axis vertical.

provide clarity on the orbits of the bodies within the system and explain the features seen in the O − C diagrams.

The results of integrating the systems for 10^6 yr can be seen in Figs 5(a)–(f). All of these systems were found to be stable over 10^6 yr. The eccentricities of the eclipsing binary combined with the high eccentricities of the outer companion do not appear to compromise the long-term stability of the systems. While the eccentricities of the objects in the systems varied over differing time-scales and by differing amounts, the semimajor axis remained relatively constant and, therefore, the outer companions remained within each system. As illustrated by Figs 5(d) and (f), while the eccentricity of the binary stars can vary significantly, this did not necessarily translate to a major change in eccentricity of the outer companion or the semimajor axis of the system. The systems were also found to be stable for 10^4 yr when random values were used for the mean longitude, argument of pericentre and longitude of pericentre of the outer companion, and the eclipsing binary. This increases the likelihood of the outer companions existing as slight changes or deviations from the proposed orbital properties still produced stable orbits.

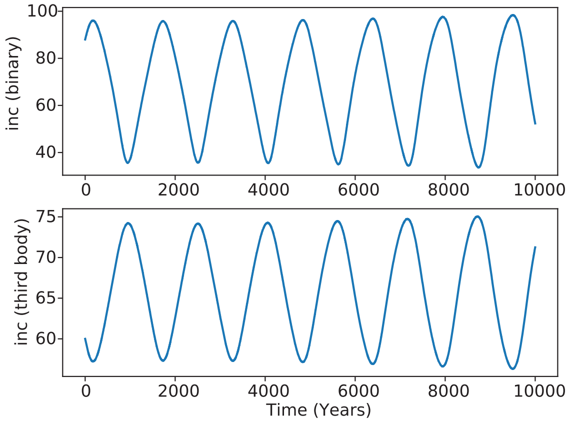


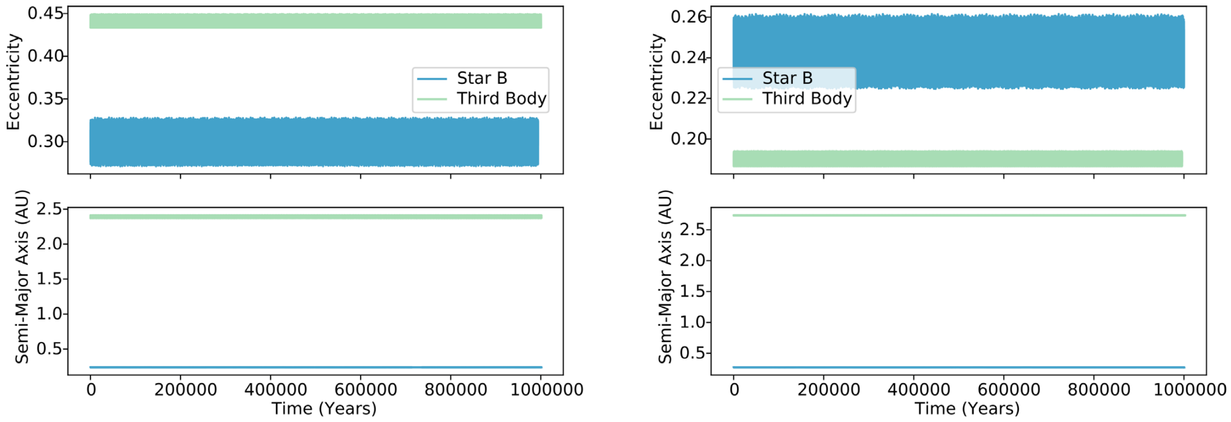
Figure 4. The change in inclination of the eclipsing binary (top) and third body (bottom) for the system KIC 12356914 over 10^4 yr. The eclipsing binary inclination changes between ∼40° and ∼100°. As a result, there are likely to be extended intervals of time when no eclipses of the eclipsing binary will be seen from the Earth.

4 DISCUSSION

The out of phase variations in the O − C diagrams for primary and secondary eclipses are likely the result of apsidal motion (Zasche et al. 2015). The apsidal motion and rapid eclipse time transitions are features that appear in all of the O − C diagrams of the models when an outer companion as described in Tables 1 or 2 is present. The light curves of some of these systems also show significant eclipse depth variations. The eclipse depth variations are likely due to the dynamics of the system at play due to apsidal and nodal precession (Kane, Horner & von Braun 2012), and the evolution of the inclination in the system over time. Apsidal motion and nodal precession are illustrated in the simulated orbits in Fig. 3, while inclination evolution over time for a system can be seen in Fig. 4. Inclination evolution does not necessarily only change the depth of the eclipses seen but also whether we see the eclipses at all. For example, secondary eclipses for KIC 11558882 are not initially seen in the light curve but begin to appear around 800 d (BJD – 245 4833) and remain for the rest of the observing window (Fig. 6).

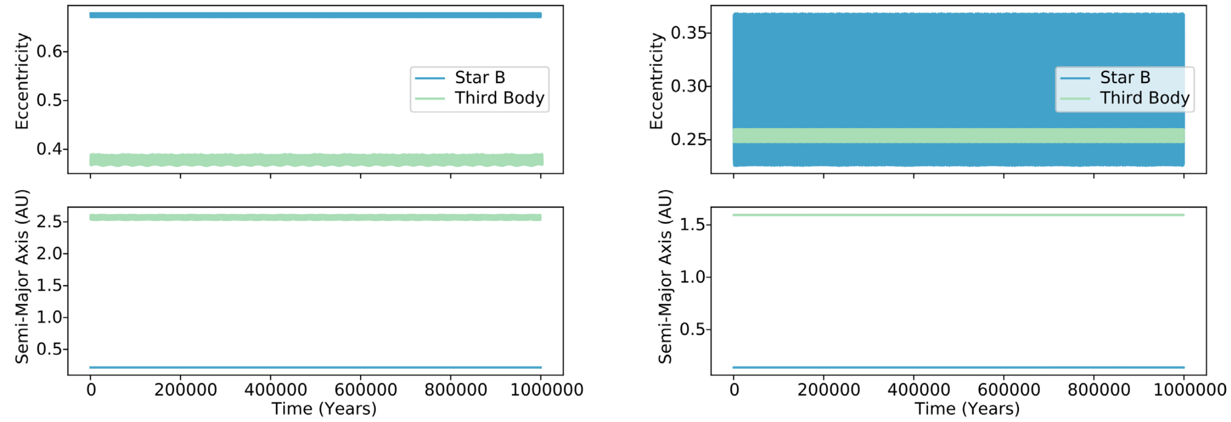
The *Kepler* mission viewed these systems for approximately 1400 d (Conroy et al. 2014), and it is fortunate that the observation period of *Kepler* coincided with the point in the outer companion's orbit that results in the sudden flip-flop nature of the period changes. For third bodies that have orbital periods greater than 1400 d, part of the orbit will be unobserved and the flip-flop effect potentially missed. The greater the orbital period of the outer companion, the greater the chance of missing this dynamical effect in the observations. The sudden period changes are so rapid, some occurring over approximately 100 d, that even an orbital period of ∼1700 d could result in this system characteristic going undetected in the *Kepler* data.

The set of orbital properties within a system jointly influences the potential for transits or eclipses to be seen in the light curve. The probability of a transit occurring decreases as the orbital period increases (Kane & von Braun 2009) so while KIC 10268809, for example, has inclinations that may indicate the possibility of transits (84° and 94° for the binary stars and outer companion, respectively), the very long orbital period of the outer companion results in transits being unlikely to occur. Extra events can be seen in the light curve



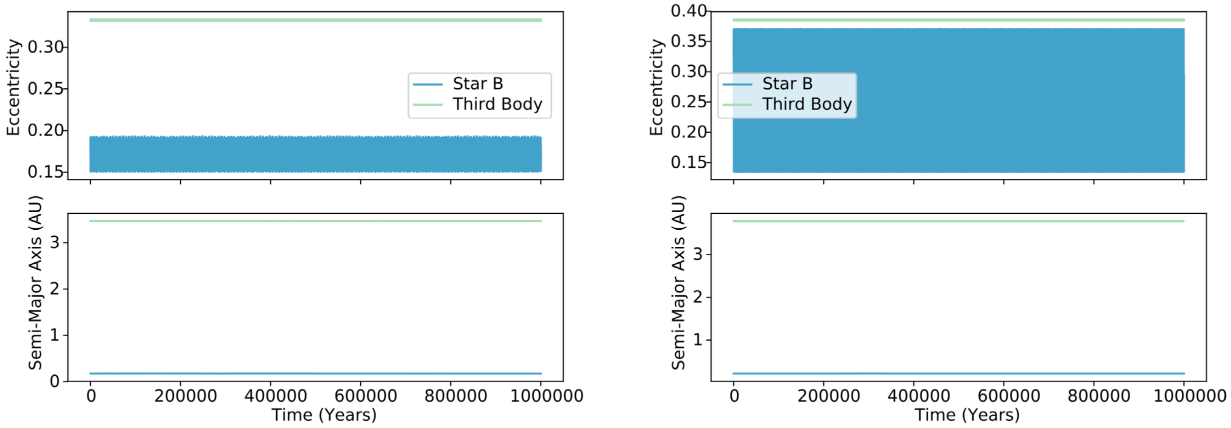
(a) KIC5255552

(b) KIC5653126



(c) KIC7821010

(d) KIC8023317



(e) KIC11519226

(f) KIC12356914

Figure 5. Eccentricity and semimajor axis of the secondary star and third body/outer companion after integration in REBOUND for a period of 10^6 yr for the systems listed in Tables 1 and 2. Note: figures for additional systems will be available online as supplementary material.

of KIC 5255552, indicating that transits occur, and there are also additional eclipses that a third body may not account for, thus indicating the possibility of a quadruple system (Zhang et al. 2018). None of the other systems considered in this study have definite or clear additional events occurring within the light curve, however it is possible KIC 11519226 contains an additional eclipse (described in Section 4.4). The equation for the probability of a third body transit/eclipse being seen from the Earth is

$$P_{\rm tr} = 0.0045 \left(\frac{1\,\mathrm{au}}{a}\right)\left(\frac{R_\star + R}{\mathrm{R}_\odot}\right)\left[\frac{1 + e\cos(\frac{\pi}{2} - \omega)}{1 - e^2}\right], \qquad (1)$$

where a is the semimajor axis, e is the eccentricity, and ω is the longitude of periastron of the third body and the orientation of the

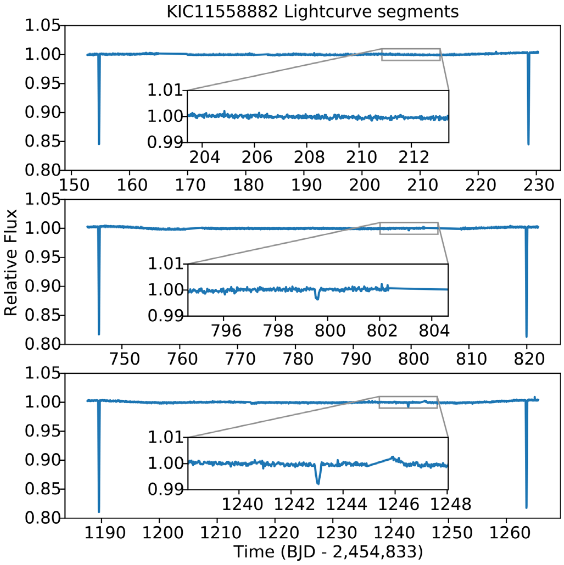


Figure 6. Secondary eclipses for KIC 11558882 are not initially viewable. However, as time progresses and the inclination/binary star orientation changes secondary eclipses come in to view.

orbit of the third body is assumed to be random (Charbonneau et al. 2006). Using equation (1) and the mean radius of stars from Pecaut & Mamajek (2013) with the masses and other orbital characteristics in Table 1, we can calculate the probabilities of seeing transits from the systems with third bodies. We find that the probability of extra events occurring in KIC 5255552, KIC 8023317, and KIC 11519226 to be less than 1 per cent and that the extra events seen in KIC 5255552 must be due to an extremely fortuitous occurrence.

In some systems, the sudden period flip in the O − C diagram may be the only indication of the presence of an outer companion. It is likely, given the large number of eclipsing binary stars observed with *Kepler*, that there are a number of systems that have been observed and classified as not containing an outer companion when in actuality the observations of *Kepler* have not been long enough to observe the effects of an outer companion. With only 11 systems displaying the flip-flop behaviour out of the more than 2000 *Kepler* eclipsing binary systems and almost half of the systems having an outer companion reported with greater than a ∼1400 d orbital period, it is likely that there are many more systems that have outer companions that remain undetected due to orbital configurations that did not result in notable O − C diagrams within the *Kepler* viewing window. The approximately 1400 d viewing window of *Kepler* will necessarily bias the detection results to systems that have outer companions with orbital periods of less than 1400 d. As Tables 1 and 2 contain a similar number of systems, it is possible, if not likely, that the flip-flop characteristic seen in the O − C diagrams will exist in a wide range of systems that have already been observed but not during this flip-flop window.

All of the systems in Tables 1 and 2 were integrated 40 times each for 10^4 yr with random initial values for the mean longitude, argument of pericentre and longitude of pericentre of the third body orbit, and the eclipsing binary. While the random values can produce systems with O − C diagrams that vary significantly from the previously calculated values, the systems are still found to be stable. This exercise shows that even for a wide range of (though not necessarily all) orbital configurations systems with these mass and eccentricity values are likely to be stable.

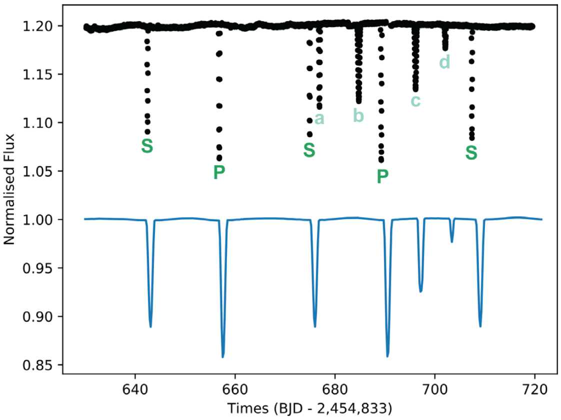


Figure 7. Top (black): a vertically shifted segment of the KIC 5255552 light curve showing the regular primary (P) and secondary (S) eclipses and the additional eclipsing events (a, b, c, and d). Bottom (blue): a segment of the modelled light curve of KIC 5255552 eclipsing binary system with a single companion as described. While there are extra eclipsing events (corresponding to events c and d in the actual light curve), a single companion does not account for the a and b eclipsing events.

4.1 KIC 5255552

The KIC 5255552 reported outer companion mass of 0.7 $M_\odot$ (Borkovits et al. 2016) closely matches the estimated mass of the secondary star at 0.74 $M_\odot$. If this system were to contain a similar tertiary star to the secondary star, we would expect this to have an effect on the system, for example, in the reported colours of the system and therefore affect mass estimates. KIC 5255552 has a Two Micron All Sky Survey (2MASS) $J - H$ magnitude difference of 0.507 that approximately matches a K3V star (Pecaut & Mamajek 2013). Larger mass dwarf stars will have a smaller $J - H$ magnitude difference, while smaller mass stars have a larger $J - H$ value. If the mass of the outer companion was as large as or larger than that of the secondary star, we would expect a smaller $J - H$ magnitude difference, and therefore earlier spectral type. The estimated primary star mass was higher than expected from the temperature of the system and it is possible the $J - H$ magnitude difference indicating a K3V star with a mass of 0.75 $M_\odot$ more accurately reflects the primary star mass.

KIC 5255552 is unique amongst all of the systems considered in this study as it showed clear eclipsing events that cannot be attributed to the binary star alone. The light curve of KIC 5255552 has a number of groups of extra eclipsing events, one group is shown in the top plot of Fig. 7. Four extra observed eclipses (a, b, c, and d) can be seen in this group. This system was then modelled using the PhysicsofEclipsingBinaries (PHOEBE; Horvat et al. 2018) with the binary stars and a third body as described in Table 1. However, only two additional eclipsing events can be seen in the modelled light curve in the bottom section of Fig. 7, corresponding to eclipses c and d seen in the actual light curve. The number of observed eclipsing events indicates that KIC 5255552 contains a fourth body, while the grouping of eclipsing events suggests that the third and fourth body are themselves in a binary star configuration. As no eclipses from the companion binary star are seen in the light curve, we interpret this system as a non-eclipsing binary that itself eclipses an eclipsing binary.

There are clear additional groups of eclipsing events located around approximately 690 and 1542 d, representing the eclipsing binary passing in front of the companion binary, and 948 d, representing the eclipsing binary passing behind the companion binary (Zhang et al. 2018). A particularly large eclipsing event occurs at approximately 1548 d and is expected to be the primary star of the eclipsing binary blocking the light from both stars of the companion binary. Zhang et al. (2018) note a possible additional eclipsing event occurs at approximately 1278 d; however, it is a very shallow and isolated event. It is possible other events occurred slightly earlier than this event. However, they correspond to a time when no observations were taken. Given the probable binary nature of the companion if this is an independent, physical, eclipsing event, it may indicate the presence of a fifth body in the system rather than a fourth body suggested by Zhang et al. (2018).

4.2 KIC 5653126

The mass of the outer companion around KIC 5653126 is reported to be 1.1 $M_\odot$ (Borkovits et al. 2016). Using the method described in Section 2, we estimate the masses of the eclipsing binary primary and secondary stars to be 1.02 and 0.78 $M_\odot$, respectively.

The 2MASS $J-H$ magnitude difference of KIC 5653126 is 0.247 that approximately matches an F9.5V star that is consistent with the mass of the reported outer companion. This may be because the outer companion is a single star that dominates the $J-H$ colour of the system. The presence of significant third light can result in unreliable mass ratio determinations (Hambálek & Pribulla 2013). As a result the mass estimates for the primary and secondary stars of the eclipsing binary would not be accurately determined. Alternatively, the primary star of the eclipsing binary may dominate the temperature of the system with the outer companion contributing only slightly to the $J-H$ colour of the system. However, assuming relatively accurate combined mass estimates, in either case the outcome of the stability check performed would remain the same.

In the second case, if the outer companion contributes slightly to the $J-H$ colour of the system, it is possible that the outer companion is itself an additional binary rather than a single star companion. As there are no additional eclipsing events seen in the light curve of KIC 5653126, this potential additional binary is unlikely to be eclipsing, nor is it likely that a star in either the eclipsing binary or this potential companion binary eclipses a star in the other binary. This is the expected result with the inclination of the outer companion being 78°.

4.3 KIC 7821010

Another system of note is KIC 7821010 that has a third body mass of just ~2.6 Jupiter masses (Borkovits et al. 2016). The evidence for this third body mass (i.e. the eclipse timing fit, the models reproducing the O − C effects, and the stability of the system) all strongly point to the existence and viability of this as a planetary candidate. The third body in this system is in an orbit with an inclination of 105° and with a configuration similar to that of the planetary mass third body found orbiting KIC 5095269 (Getley et al. 2017). It is also further evidence that low-mass objects can have a significant effect on the orbital properties of the host stars and also that, for at least some orbital configurations, eclipse timing variations are a valid way of detecting planetary mass bodies. Eclipse timing variations are particularly useful for detecting planetary mass bodies in orbital configurations that would go undetected with other methods such as searching for transits that require specific orbital characteristics (such as a compatible inclination) to be viewed from the Earth. The $J-H$ magnitude difference of KIC 7821010 from 2MASS is 0.195 and approximately matches the $J-H$ magnitude difference of a 1.25 $M_\odot$ F6V star that is consistent with the mass estimated for the primary star of the system. A planetary mass third body would contribute essentially nothing to the colours of the system and therefore allows for more accurate estimates of the masses of the primary and secondary stars.

4.4 KIC 11519226

KIC 11519226 comprises an outer companion with a mass of 1.25 $M_\odot$ (Borkovits et al. 2016), and eclipsing binary primary and secondary star mass of 0.98 and 0.46 $M_\odot$, respectively. Like KIC 5653126 in Section 4.2, a third body with such a large mass relative to the binary stars would dominate the light from the system.

The inclination of 89° for an additional body around KIC 11519226 indicates the possibility of additional eclipse events taking place within the light curve; however, there is a lot of variability within the light curve of KIC 11519226 that could hide such events. The long-period nature of the additional bodies would also limit the number of eclipses that could be observed. Period04 (Lenz & Breger 2005) was used to attempt to clean the periodicity from the light curve of KIC 11519226 in an attempt to locate additional eclipsing events without success. Despite this, there is a possible additional eclipse event located within the light curve as seen in Fig. 8, however, more observations would be required to confirm if this is an additional eclipse or some other kind of variability.

The 2MASS $J-H$ magnitude difference of KIC 11519226 is 0.321 that is approximately equivalent to a G6V star and closely matches the estimate for the primary star. This $J-H$ colour, coupled with the possibility of an additional shallow eclipsing event despite the 89° inclination, suggests that similar to KIC 5653126 the reported third body may contribute nothing to the colours of the system. An outer companion with a larger mass than the primary and secondary star that does not contribute to the colour of the system suggests the outer companion may be an additional binary, comprised of two smaller stars, or a white dwarf.

A periodogram of the variability was produced using the Lomb–Scargle approach in GATSPY (VanderPlas & Ivezić 2015) and is shown in Fig. 9. Two large peaks can be seen, the first at 5.3023 d and the second at 13.3276 d, while a smaller peak can be seen at 2.7084 d. The variability periods of 2.7084, 5.3023, and 13.3276 d are in an approximately 1:2:5 ratio.

δ Scuti variable stars exhibit pulsations in the orders of hours (Rodríguez & Breger 2001), while γ Doradus variable stars are typically early F- to late A-type stars (Van Reeth Tkachenko & Aerts 2016) as opposed to the G6 primary star estimated in this system. The vast majority of γ Doradus candidates listed in Handler (1999) have variability periods of less than 2 d. One system, HD 109838, stands out as an exception to the typical periods of a γ Doradus star with possible periods of 14 and 2.9 d, however the periods are listed as uncertain. The variability periods for HD 109838 are comparable to the variability periods seen in KIC 11519226.

5 SUMMARY AND CONCLUSIONS

In this study, we used custom software BET based on TAP to perform an eclipse timing study on *Kepler* eclipsing binary stars. During the eclipse timing study we found systems that had O − C diagrams that displayed flip-flop or out of phase variations between the primary and secondary eclipse O − C curves and rapid period change variations. REBOUND was used to simulate these systems. The systems in Tables 1 and 2 were chosen as they all exhibited a unique flip-flop effect within their O − C diagrams. Outer companions with

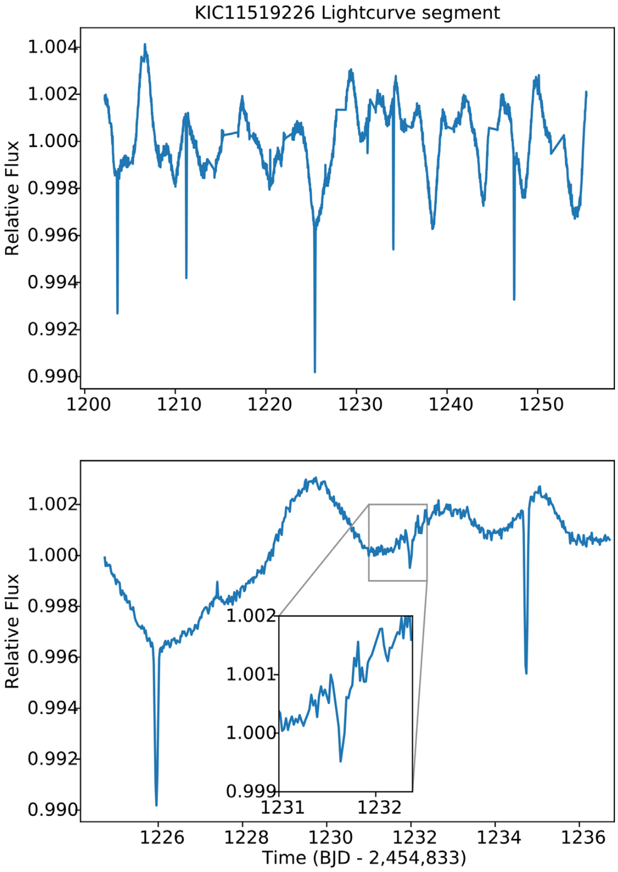


Figure 8. Top panel: a segment of the light curve of KIC 11519226 showing a number of primary and secondary eclipses, the variability in the light curve, and a possible extra eclipsing event. Bottom panel: a possible extra eclipsing event in the light curve of KIC 11519226. The primary eclipse can be seen on the left, the secondary eclipse on the right, and the possible extra eclipsing event is shown in the rectangle. Given the long period of the outer companion, no additional eclipses would be seen and secondary eclipses are likely lost in the variability of the light curve itself.

the characteristics described all account for the features seen in the O − C diagrams such as the out of phase eclipse time variations and the flip-flop effect. With the systems simulated in REBOUND we then integrated the systems as described for 10^6 yr. We found that all systems were dynamically stable for at least 10^6 yr and, therefore, bodies in these orbital configurations are likely to be stable and observable. We also integrated these systems with random values for the mean longitude, argument of pericentre and longitude of pericentre of the third body, and the binary star for 10^4 yr and found that the systems were stable for a wide range of orbital configurations. The evidence suggests the outer companions for the systems listed in Table 1 are an additional pair of stars in a binary configuration (KIC 5255552, KIC 5653126, and KIC 11519226), a single M dwarf star (KIC 5731312 and KIC 8023317), and a planet (KIC 7821010).

We also suspect that a larger number of systems that have been observed would also show similar flip-flop characteristics if observed over longer or much longer time spans. However, due to the limits of the *Kepler* viewing window and large orbital periods estimated for the third bodies/outer companions the flip-flop effect continues to go undetected. As more and more systems are found with multiple bodies, the dynamical stability of the system as a whole is an important consideration when determining the likelihood of their existence. Of particular note is KIC 7821010 that has a third body mass of ∼2.6 Jupiter masses. At ∼2.6 Jupiter masses it is well within planetary mass range and shows that even a relatively small mass can have large effects on the motion of its parent stars.

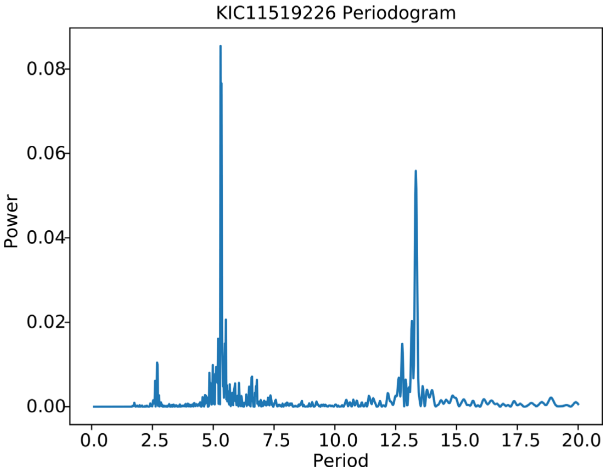


Figure 9. A periodogram of the variability in the out of eclipse light curve of KIC 11519226. Two large peaks can be seen, first at 5.3023 d and second at 13.3276 d, and one smaller peak can be seen at 2.7084 d.

Other stand-out systems from this study include KIC 5255552, where there are additional eclipses in the light curve (Zhang et al. 2018) that may indicate the presence of a fourth star bound in a binary with the third star. A fifth body in the KIC 5255552 system is a possibility and further observations of the system are crucial in determining the true nature of this system. While a triple star explanation cannot be ruled out for the systems KIC 5653126 and KIC 11519226, the photometric and dynamical analysis performed for this study suggests these systems are detached eclipsing binary stars with binary star companions.

Some of the systems presented, for example KIC 11558882, cannot be reliably studied with ground-based observations. The orbital period of the binary stars can be so great that observing eclipses to get meaningful data were only made possible with *Kepler*. Without space-based observations these systems, and their O − C variations, may have continued to go undetected.

Transiting Exoplanet Survey Satellite (*TESS*; Ricker et al. 2015) is an all-sky survey of bright local stars with the ability of detecting planets with orbital periods of a few hours to a year or more. The launch of *TESS* provides more opportunities to locate comparable systems that are more local to the Solar system and capable of follow-up studies. With the launch of *TESS* and future projects, we expect the number of systems that have similar characteristics to increase significantly.

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DATA AVAILABILITY

The data underlying this paper will be shared on reasonable request to the corresponding author.

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SUPPORTING INFORMATION

Supplementary data are available at *MNRAS* online.

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