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Performance of Timber Girders with End-Notch: Experimental and Numerical Investigation

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Abstract

Many bridges built during the colonial times in Australia have timber girders as load transferring elements and they are still in service with increased traffic loads and consistent deterioration. Most of the timber girders in those bridges are notched at the ends for better seating arrangement. Therefore, it is necessary to quantify the strength characteristics of notched girders in order to ensure structural safety and make necessary intervention to extend their service lives. Hence experimental tests were conducted on the notched rectangular timber girder samples with three different notch depths (i.e.10%, 15% and 30% of the depth of beam) having the notched angle of 1:4. Consequently, detailed finite element models were developed for notched timber girders, and the models were validated with experimental results. The validated model was used to predict the shear and flexural strengths and stiffnesses of typical circular and rectangular timber girders with two different spans (i.e. 6 m and 9 m), three different notch angles (i.e. 1: 0, 1:2 and 1:4) and three depths (i.e.15%, 30% and 45% o of the depth of beam). Strength data developed for notched timber were used to compare the applicability of the design provisions in various timber design standards. Experimental and finite element model test results show that when the notch depth increased from 15% to 45%, the load carrying capacity of rectangular timber girder was reduced by 50%. Whilst, the reduction of the load carrying capacity of the corresponding circular timber girder was slightly low (i.e. 37%). Further, when the notched angle changes from 1:0 to 1:4, the load carrying capacity of circular and rectangular girders increased about 50%, 69% and 110% for the notch depths 15%, 30% and 45%, respectively.

Keywords: *Timber bridges; Timber girder; Notch depth; Notch angle; Finite element model; Design standards*

1 Introduction

Most of the road network bridges constructed during the colonial times in Australia consist timber girders as the load transferring members under the deck [1]. Many of these bridges and their timber girders still remain in service despite been exposed to increased traffic loads and continuous weathering. All of these timber bridges were constructed as per the design standards at that time which are now obsolete and the current traffic loadings are more than the original designed situation. Hence there is a stern need to repair or replace the timber girders to meet the present traffic loadings and design methodologies [2, 3]. Further these structural timber members suffer from a range of degradation mechanisms which means, systematic inspection and repair/strengthening regimes are needed to guarantee continued operation.

Moreover, most of the timber girders in the bridges are sniped/notched at both ends. The notching at the ends enables the girders to have adequate seating and create levelness on the top of the piles/corbels, which will ease to build flat decks. Conversely notching at the supports reduces the strength of the girders around the notched region, where the concentration of relatively higher shear stress and cross-grain tensile stresses at the notched angle corner can create cracks to propagate along the grain leading to brittle failure [4, 5]. Subsequently, several experimental studies have been dedicated to investigate the failure behaviour of notched timber girders/beams in the past and detailed design guidelines are provided in various national design standards [6-18]. Jockwer et al., [19] provides a comprehensive review on the past experimental programmes and current design approaches available for notched beam/girders. It was highlighted that most of the past studies were focused on testing beams with relatively small depth and design approaches were primarily developed based on those data. Therefore, the need of comprehensive research studies on the influence of different beam/girder depths, notch lengths and angles were emphasised.

Nonetheless most of the existing notch configurations in the old timber bridge girders in the bridges are out of the provisions given in the current design standards. The old timber girders are often notched to depths of up to 50 percent of the total depth [16] and hence these excessive notching reduces the cross-sectional area available to resist bending and shear forces. In practice 1:4 is recommended as the preferred notch angle [20, 21] and excessive notching of corbels is considered to be less severe as the notch is in compression rather than tension. Therefore, there is a concern that the excessive notch depths may cause the corbel to lose its strength and then fail in bending. For an example, the guidelines provided in Queensland Department of Transport and Main Roads (QTMR) manual [2] is given in Table 1. In actual practices in QTMR, it is recommended to replace the girders with greater than 30% notched and any girders with notched depth between 15% to 30% are to be strengthened with anti-splitter bolts. Therefore, the actual resistance of the timber girders with different notched configurations should be accurately verified to assess the overall performance of the bridges [22-24].

Table 1: Condition states based on notched depth [2].

Notched depth	Condition state	Remedial action
0%	1	Non
<10%	2	Non
10-15%	3	Non
16-30%	4	Anti-splitting bolts
> 30%	-	Replace

In summary, it can be said that even though the failure mechanism of notched timber beam/girders are well understood, the parameters that influence the failure pattern and the corresponding resistance are not comprehensively understood in the literature, particularly for the notched configurations that are not specified in the current design standards similar to those found in old timber girder bridges. Therefore, in order to better understand the behaviour of notched timber girders, an experimental testing programme followed by numerical analyses were carried out in this research. Initially the outcome of the experimental programme undertaken to test twenty timber girder samples are presented in terms of failure modes observed and corresponding the load resistance behaviour reported. Thereafter a finite element based numerical modelling method developed to verify the experimental results is outlined in the paper. Later the verified numerical modelling technique was used to investigate critical parameters such as span, section type (rectangular and circular) notched depth and notched angle that influence the behaviour of notched timber girders. Finally, the formulations given in various timber design standards were verified using the numerical data generated and suitable formulation is recommended.

2 Experimental Programme

The experimental programme comprised of testing twenty timber girder samples under three point bending as outlined in AS4063.1 [25] to determine the shear and flexural resistance when the load is applied perpendicular to the grain orientation. The schematic diagram of the testing arrangement is shown in Fig 1. The span to depth (un-notched) ratio of the timber specimens were kept as 6:1 and the notched angle was maintained as 1:4, while the percentage (i.e. ratio between the notch depth and depth of beam) of the notched depth was varied (0, 10%, 15%, 30%) among the samples. The selected three different notch profiles were cut using the bandsaw. Table 2 presents the details of the timber girder specimens tested. For the notched specimen combinations, both ends were notched, and simply supported at both ends. Based on the AS4063.1 [25], the displacement controlled loading rate of 2.5 mm/min was applied to the samples and mid span deflections were measured using the displacement transducers. In total, five samples were tested for each notch depth configuration to verify the average response.

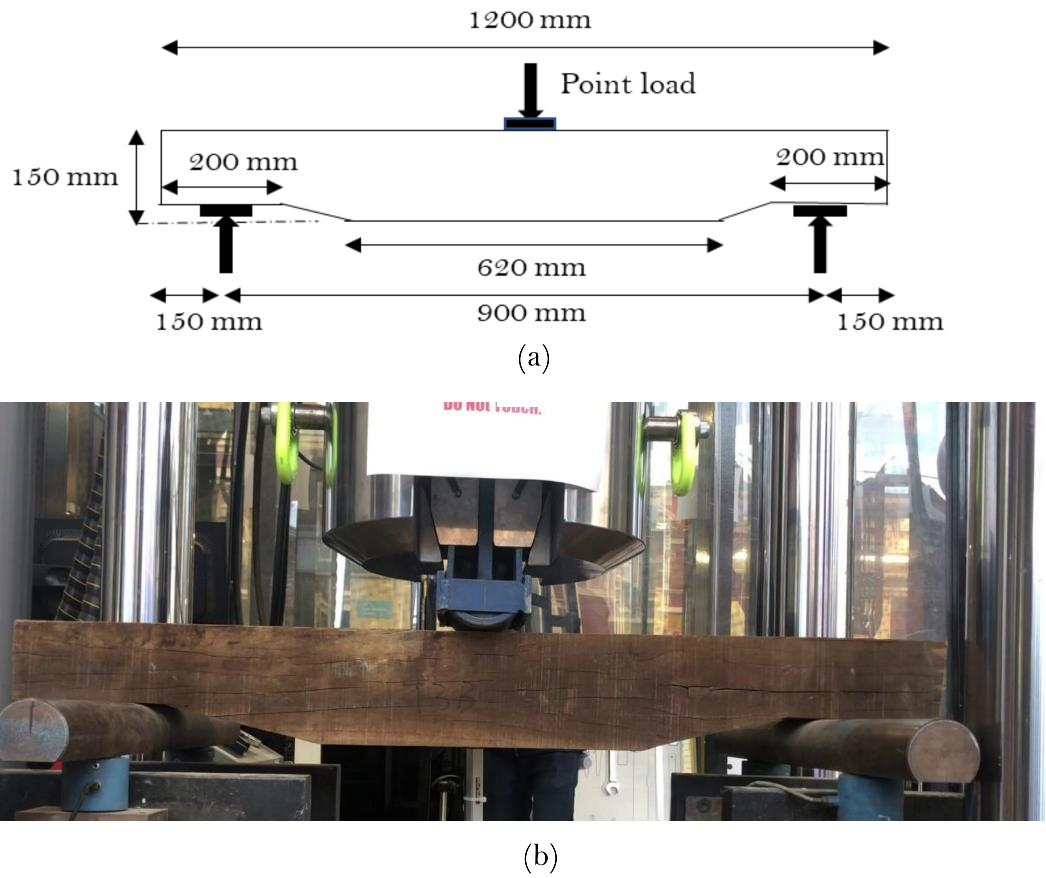


Fig 1: Three-point bending test setup: (a) schematic diagram and (b) sample ready to test

All the timber girder samples tested were F27 Spotted Gum timber /Corymbia maculate species, which is a commonly unitised hard timber in Australia and widely used as girders in the bridges. The timber girder samples were selected such a way that they do not possess any defects such as knots, splits and sloping grain, especially near the notched region. Prior to the experiments, density and moisture content of each girder samples were measured. The density of timber was 1058 kg/m^3 with a coefficient of variation (COV) of 10%. The moisture content of each timber was found to be 13.8% on average with a COV of 12%. This timber class is designated as S2 strength group as per AS/NZS 2878 [26]. The characteristics shear, tensile, flexural and compressive strengths parallel to the grain as per AS/NZS 2878 [26] are 5.1 MPa, 42 MPa, 67 MPa and 51 MPa, respectively.

Table 2: Details of tested samples.

Type	Number of specimens	Width (mm)	Depth (mm)	Notch depth (mm)	Notch angle	Span (mm)
A	5	150	150	0	0	900
B	5	150	150	15	1:4	900
C	5	150	150	22.5	1:4	900
D	5	150	150	45	1:4	900

2.1 Experimental results

2.1.1 Failure mode

The failure patterns of the different notched configured timber girder samples under three-point bending load are shown in Fig 2. Type A girder samples have depicted conventional flexural failure as shown in Fig 2(a). However, the notched timber girder samples have portrayed three distinct failure modes: (1) Mode 1 failure is caused by tensile stresses perpendicular to the timber grain at the notch; (2) Mode 2 failure occurs by sudden brittle failure when the shear stress of the parallel to the grain exceeds when shear strength of the timber; (3) Mode 3 is the out of plane shear failure of the beam due to the cracking and instability under in-plane splitting of section along the parallel grain [27]. Subsequently the failure patterns of all the notched samples were quite similar in nature as shown in Fig 2(b) to (c), however when the notched depth percentage increased, the failure became more rapid and brittle. Furthermore, when the notched percentage increases from 10% to 30%, significant splitting cracking was noted and the crack propagated towards the neutral axis as shown in Fig 2(d).

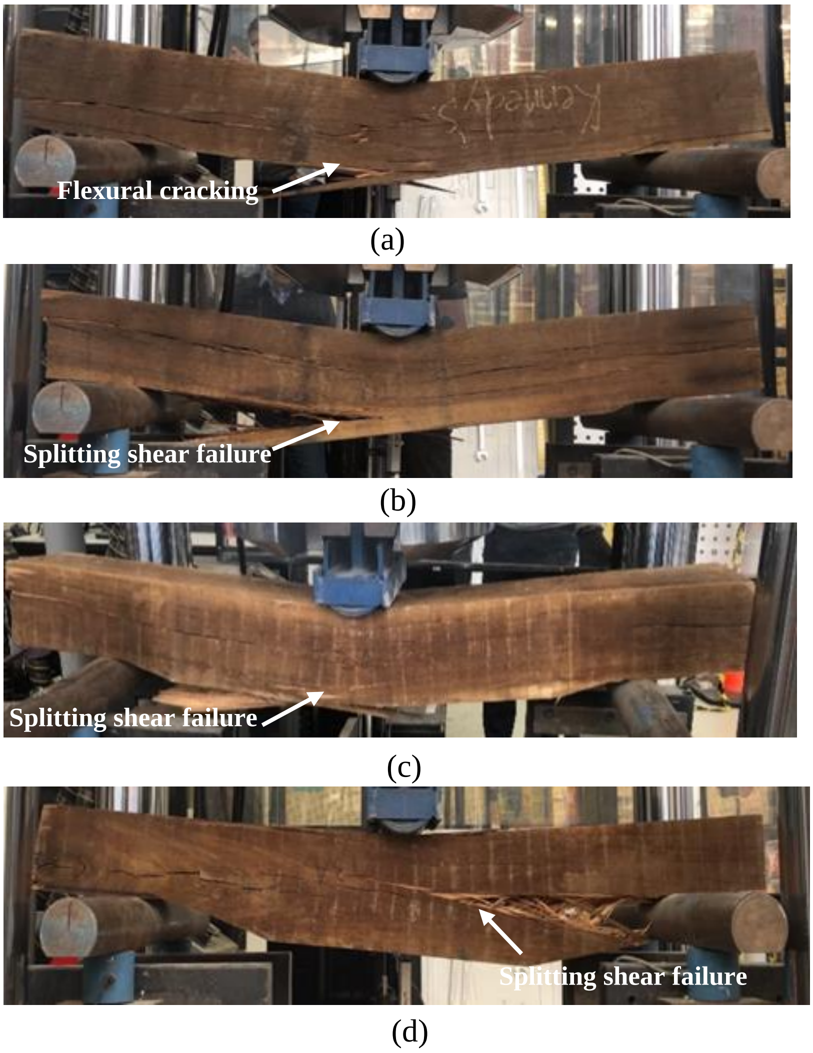


Fig 2: Failure modes of the timber specimens under three-point bending: (a) Type A (b); Type B (c); Type C; and (d) Type D.

2.1.2 Load-deflection response

The load-deflection responses recorded during the tests are presented in Fig 3. The load-deflection behaviour of un-notched specimens is mainly characterised by three distinct regions (1) linear (2) non-linear hardening and (3) post-peak softening. The non-linear behaviour was associated with the initiation of flexural cracks at the middle bottom fibre of the samples and the post peak softening was related to the widening of those cracks. Further, the load-deflection behaviour of notched timber girder samples are quite similar as un-notched specimens, however a clear distinction between the linear and non-linear regions could be observed in the curves, where they are mainly linked to mode 1 and mode 2 failure commencement during the testing. Moreover, from the load-deflection responses, the mode 1 and mode 2 failure loads and critical stresses were identified as given in Dewey et al., [12] and presented in Table 3. The shear stress (τ), flexural stress (σ) and flexural stiffness (EI) were computed and presented in Table 3. The coefficients of variations of the critical stresses are also given in the parentheses. The shear stress and flexural stress are calculated using the following equations:

$$\tau = \frac{VQ}{Ib} \tag{1}$$

$$\sigma = \frac{My}{I} \tag{2}$$

Where: V is shear force; Q is the first moment of area above horizontal plane; I is the moment of inertia; b width of timber girder; M is bending moment; and y is the distance from neutral axis to extreme fiber. EI of timber girder was derived from the Eq. 3.

$$EI = \frac{L^3(P_2 - P_1)}{48(\Delta_2 - \Delta_1)} \tag{3}$$

Where, L is the span of the timber girder; P_1 and P_2 are the loads corresponding to 10% and 40% of the ultimate load from the load displacement curve; Δ_1 and Δ_2 are the displacements corresponding to loads P_1 and P_2, respectively.

Apparently, the un-notched sample has shown the highest load carrying capacity among the specimens tested. However, in terms of mode 1 failure load, when the 10% notching was introduced the load carrying capacity of the specimen has dropped by 33% compared to un-notched specimen. No significant difference in the load carrying capacity between the 10% and 15% notching was noticed. Nevertheless, when the notching percentage was increased to 30%, the load carrying capacity was dropped to about 55% compared to Type A samples. Quite similar trend could be observed among the other critical stresses computed from the load-deflection responses. Also, the flexural stiffness of timber girder was reduced by about 20% when notch depth increased from 10% (i.e. Type B) to 30% (i.e. Type D).

Table 3: measured critical failure loads loads/stresses.

Type	Failure Load (kN)		Shear stress (MPa)		Flexural stress (MPa)		Flexural stiffness (EI) ×10⁸ kNmm²
	Mode1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	
A	-	243 0.06)	-	33	-	97	4.24
B	154 (0.07)	221 0.05)	24	33	76	109	4.14
C	150 (0.05)	204 (0.04)	24	32	83	113	3.65
D	110	131	21	25	90	107	3.85

	(0.08)	(0.04)

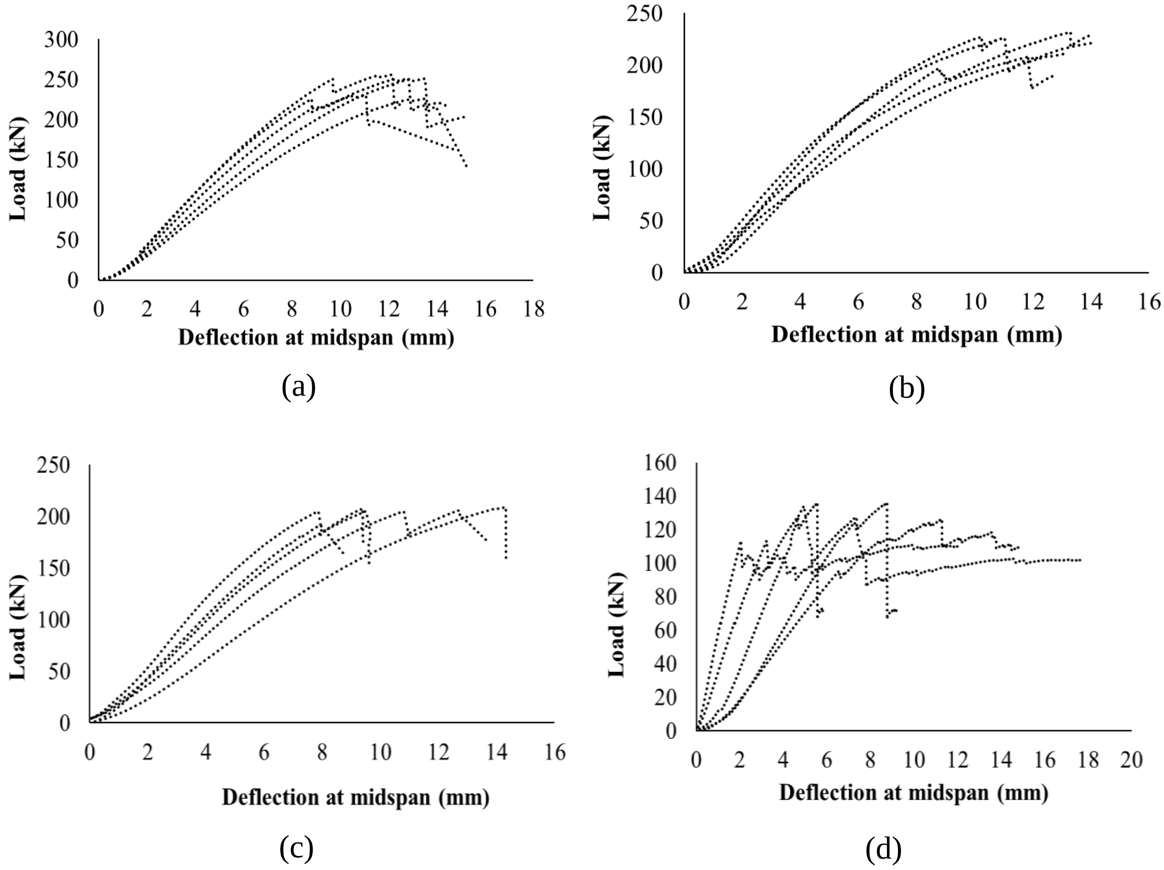


Fig 3: Load-Deflection responses of the tested specimens: (a) Type A; (b) Type B; (c) Type C; and (d) Type D.

3 Finite Element Modelling

In order to further understand the behaviour of notched girder specimens, a numerical modelling technique based on the finite element (FE) method was developed in this research. The ABAQUS [28] FE package was used to develop the models. The developed FE model of the notched girder sample is shown in Fig 4. In the FE model, the timber profile was assumed to be free of defects, idealised grain and fibres are expected to be in the longitudinal direction. The elasto-plastic constitutive law was used to develop this model [29] and the plastic yield stress of timber at radial direction was used to define the failure. Elsener [30] have studied the material characteristics of F27 Spotted Gum timber with different density. Based on the measured density and moisture content measure in this experimental tests (Section 2), this study obtained the material properties of timber from Elsener [30] and given in Table 4. The orthotropic behaviour was considered for the timber section and the parameters were derived from the experimental data from Elsener [30]. Experimental stress-strain curve obtained by Elsener [30] under loading in the radial (90°) and longitudinal (0°) directions was used to derive the plastic material properties (i.e. F, G, H, N, M and L) of the stated timber. Where, F, G, H, N, M and L are Hill's anisotropic constants and were derived by using yield stress under compression in radial ($\sigma_{C,90}$) and longitudinal ($\sigma_{C,0}$) directions (Eqs. 4 and 5). Based on the isotropic Von Mises criterion, Hill's constants L, M and N assumed 1.5, then it was adjusted via comparing results from simulation and experimental, similar method was used by Oudjene and Khelifa [29], Navaratnam et al., [31, 32] and Tran et al., [33]. The timber material properties used in the FE modelling are given in Table 4. The mesh and element convergence studies were conducted to select the appropriate element size in the model (Table A in Appendix). This mesh sensitivity analysis highlighted that an eight-node hexahedral solid element (C3D8R) with mesh size of 5 × 5 × 5 mm gives more accurate results than the other elements (Figure A in Appendix). Therefore, the timber and steel bearing sections were created with eight-node hexahedral solid element (C3D8R) with mesh size of 5 × 5 × 5 mm. A surface to surface friction contact was employed between steel bearing and timber and the boundary conditions of FEM were based on the experimental arrangement (i.e. one end roller and other end was pin support).

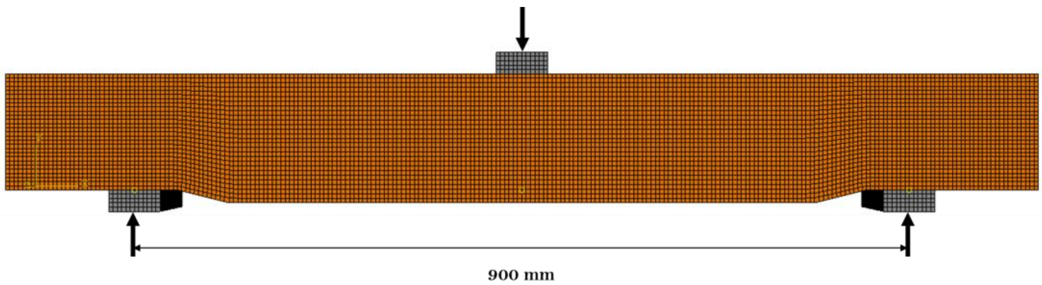


Fig 4. FE model of the timber girder.

Table 4: Material properties of timber used in the FE model.

Details	Values
Density (kg/m³)	1060
Youngs modulus X direction (MPa)	1093
Youngs modulus Y direction (MPa)	1655
Youngs modulus Z direction (MPa)	16107
Poisson's ratio XY	0.480

Poisson's ratio YZ	0.047
Poisson's ratio XZ	0.045
Shear modulus XY (MPa)	630
Shear modulus YZ (MPa)	1148
Shear modulus XZ (MPa)	609
$\sigma_{C,90}$ (MPa)	10.9
F	0.75
G	0.15
H	0.25
L=M	1.19
N	1.24
Friction coefficient	0.6

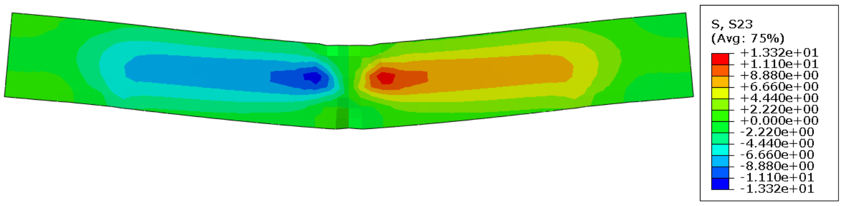
$$\sqrt{F+H} = 1 \quad (4)$$

$$\sigma_{C,0} = \frac{\sigma_{C,90}}{\sqrt{H+G}} = 68.13 \text{ MPa} \quad (5)$$

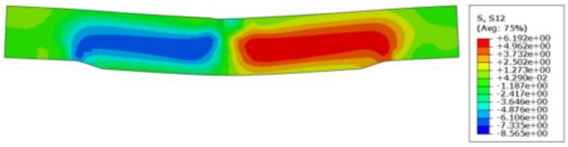
3.1 FE model validation

The developed FE modelling technique of the timber girders were validated with the experimental results obtained. The validation was carried out in terms of failure modes and load-deflection responses acquired and compared with the experimental observation. Subsequently, FE models of all four types of girder specimens (A, B, C and D) tested were created and simulated for verification. Fig 5 shows the failure modes obtained in the FE models. It can be noted that the FE model of Type A (un-notched) specimens portrayed similar failure mode as observed in the experimental testing. Further, the FE results of notched specimens (B, C and D) have depicted clearly the initiation of mode 1 and the mode 2 failures as observed in testing. Thus, it can be concluded that the developed FE modeling technique of the timber girder is able to capture the typical failure patterns of the notched and un-notched girders/beams.

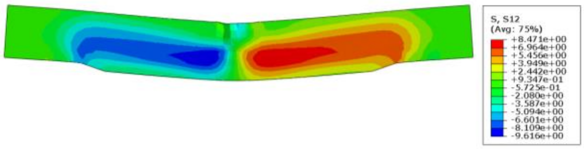
Moreover, the load-deflection responses obtained in the FE models and the experimental testing are compared in Fig 6. For the comparison purposes, only the average load-deflection curves from experimental results are presented in Fig 6. It can be noted that fairly good agreement between the experimental and FE model were obtained in the load-deflection responses. Table 5 shows the critical loads measured in the load-deflection response of FE model and experimental results. One can note that the FE models accurately predict the critical loads in the load-deflection response and the maximum variation noted is only about 15%. Thus, it can be said that the developed FE modelling technique is capable of predicting the failure patterns as well as load-deflection response of notched girders.



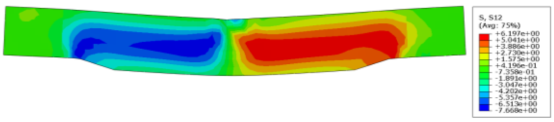
(a) Flexural failure (Type A)



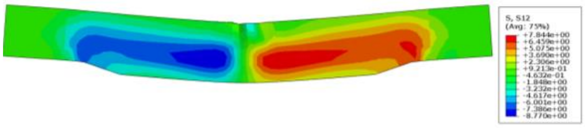
(b) Mode 1 (Type B)



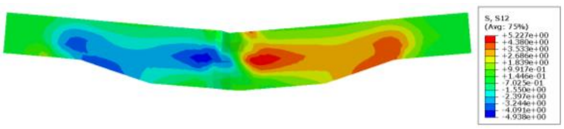
(c) Mode 2 (Type B)



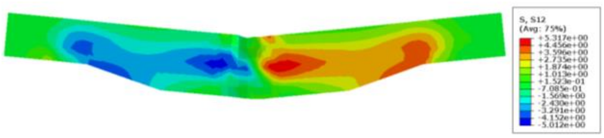
(d) Mode 1 (Type C)



(e) Mode 2 (Type C)

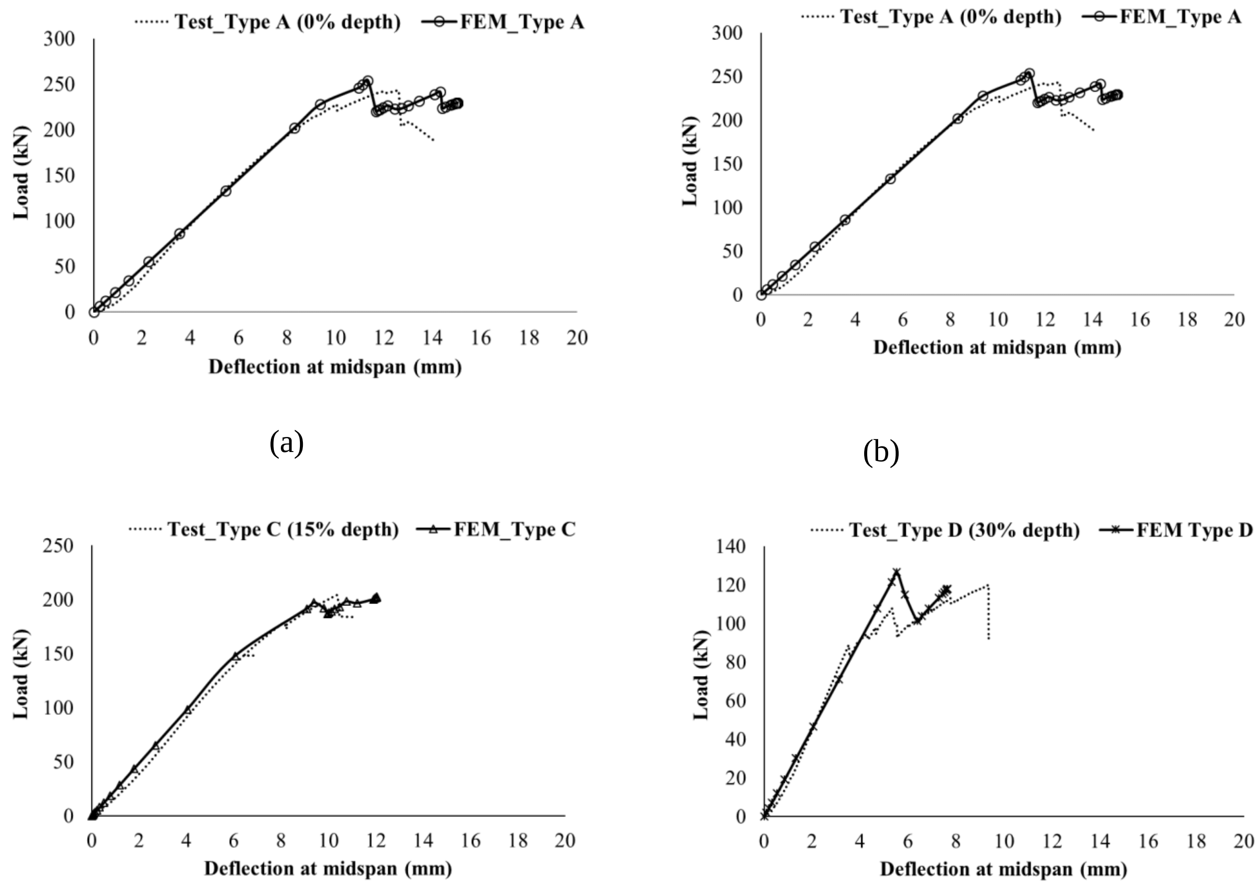


(f) Mode 1 (Type D)



(g) Mode 2 (Type C)

Fig 5: Failure patterns observed in the FE models.



(a) (b) (c) (d)

Fig 6: Comparison of experimental and numerical average load-deflection response: (a) Type A; (b) Type B; (c) Type C; and (d) Type D.

Table 5: Comparison of experimental and numerically predicted critical failure loads/stresses.

Type		Failure Load (kN)		Shear stress (MPa)		Flexural stress (MPa)		Flexural stiffness (EI) $\times 10^8$ kNmm^2
		Mode1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	
A	Exp	-	243	-	33	-	97	4.24
	FE		254		34		91	3.70
	Variation (%)		4		3		-7	-15
B	Exp	154	221	24	33	76	109	4.14
	FE	152	226	23	34	75	112	3.80
	Variation	-1	2	-4	3	-1	3	-9

	(%)							
C	Exp	150	204	24	32	83	113	3.65
	FE	148	198	23	31	82	110	3.73
	Variation (%)	-1	-3	-4	-3	-1	-3	2
D	Exp	110	131	21	25	90	107	3.85
	FE	108	127	20.5	24.1	88	104	3.50
	Variation (%)	-2	-3	-2	-4	-2	-3	-10

4 Parametric Study

In order to further extend the understanding of the behaviour of notched girders, parametric study was carried out using the validated numerical modelling technique. Subsequently following variables were considered for the parametric study (1) cross section of the girder (rectangular and circular); (2) span (6 m and 9 m); (3) notched depth percentage (0%, 15%, 30% and 45%) and notched angle (0, 1:0, 1:2, and 1:4). The girders were modeled similar to the sections and span used for Class A and B timber bridges on Queensland state-controlled roads [2, 34]. These A and B class bridges are now considered superseded and they are in need of maintenance and rehabilitation [2]. Further, the timber girders were modeled with rectangular (depth = 406 mm × width = 406 mm) and circular (406 mm diameter) sections with 6 m and 9 m spans. Also selected notched depth and angle were based on an expert advice from a timber bridge asset manager.

Moreover, in the parametric study, the notched depth and angle were varied as mentioned above to examine the influence of these two parameters to the flexural behaviour of timber girders. The rectangular and circular girder sections with notched depth percentage of 15% and notched angle 1:0 used for the parametric analysis are shown in Fig 7(a) and (b) respectively. The results of the parametric analysis are presented in Table 7. The sample notation is given each girder as the first letter denotes the section of girder (i.e. R-rectangular or C-circular), the second letter indicates the length of specimen in meter (i.e. 6 m or 9 m) and third letter indicates the notch depth percentage (i.e. 15%, 30% and 45%) and the final letter specifies the notch angles (i.e. 1:0, 1:2 and 1:4).

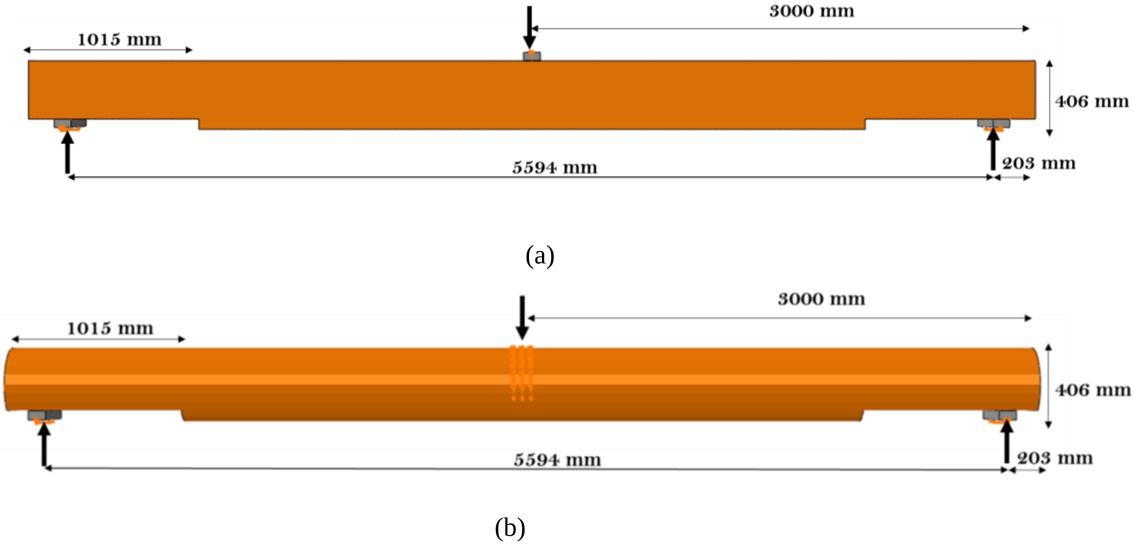


Fig 7: Schematic diagram FE models used for the parametric analyses: (a) rectangular girder; and (b) circular girder

Table 6: Results of the parametric study.

Type	Notch depth (%)	Notch angle	Failure Load (kN)		Shear stress (MPa)		Flexural stress (MPa)		Flexural stiffness (EI) ×10⁸ kNmm²
			Mode1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1
5594 mm span girder									
R-6-0-0	0	0	-	685	-	12	-	86	318
R-6-15-	15	1:0	252	399	5	9	44	69	306
R-6-15-	15	1:2	424	522	9	11	74	91	308
R-6-15-	15	1:4	438	616	9	13	76	107	307
R-6-30-	30	1:0	248	307	6	8	63	79	259
R-6-30-	30	1:2	256	395	7	10	66	101	282
R-6-30-	30	1:4	412	518	11	13	105	133	280
R-6-45-	45	1:0	179	194	6	6	74	80	250
R-6-45-	45	1:2	271	301	9	10	112	125	242
R-6-45-	45	1:4	374	407	12	13	155	169	229
C-6-0-0	0	0	-	725	-	17	-	134	228
C-6-15-	15	1:0	358	465	14	18	132	171	190
C-6-15-	15	1:2	490	537	19	21	181	198	192
C-6-15-	15	1:4	519	594	20	23	191	219	191
C-6-30-	30	1:0	329	394	5	6	65	77	183
C-6-30-	30	1:2	391	462	6	8	77	91	179
C-6-30-	30	1:4	506	516	8	9	100	102	172
C-6-45-	45	1:0	226	297	1	1	13	17	161
C-6-45-	45	1:2	290	361	1	1	16	20	154
C-6-45-	45	1:4	394	446	1	2	22	25	148
8594 mm span girder									
R-9-0-0	0	0	-	433	-	8	-	83	386

R-9-15-	15	1:0	152	257	3	6	41	69	337
R-9-15-	15	1:2	263	338	6	7	70	90	336
R-9-15-	15	1:4	267	397	6	9	71	106	336
R-9-30-	30	1:0	147	196	4	5	58	77	322
R-9-30-	30	1:2	252	262	7	7	99	103	326
R-9-30-	30	1:4	262	331	7	9	103	130	324
R-9-45-	45	1:0	95	127	3	4	60	81	312
R-9-45-	45	1:2	143	198	5	7	91	126	306
R-9-45-	45	1:4	210	262	7	9	134	167	300
C-9-0-0	0	0	-	456	-	10	0	129	222
C-9-15-	15	1:0	186	293	7	11	105	166	203
C-9-15-	15	1:2	277	373	11	14	157	211	203
C-9-15-	15	1:4	279	422	11	16	158	239	202
C-9-30-	30	1:0	203	250	3	4	61	76	197
C-9-30-	30	1:2	248	307	4	5	75	93	198
C-9-30-	30	1:4	256	367	4	6	77	111	197
C-9-45-	45	1:0	148	201	1	1	13	17	189
C-9-45-	45	1:2	195	237	1	1	17	21	188
C-9-45-	45	1:4	265	291	1	1	23	25	185

It can be noted that the failure load reduces with the increasing level of notched depth percentage. For an example, in the 6 m span rectangular girder, when the notched depth percentage varies from 15% to 45%, the failure load reduces by 42% to 72% compared with the control sample (i.e. Unnotched sample). From the cases analysed, it is observed that if the notched depth percentage of 45% is applied, the failure load will be reduced between 60% to 72%. Additionally, notched angle of 1:0 has shown to significantly reduce the failure load reduction of the timber girders irrespective of the cross-section type, subsequently when the notched angle is increased from 1:0 to 1:4 the failure load of the girders have shown to improve the capacity considerably. For an example, for the 9 m span circular girder with 45 % notched depth, when the notched angle changes from 1:0 to 1:4, the load carrying capacity increased from 201 kN to 297 kN, thus it was about 48% increment.

Moreover, comparison of the performances of circular and rectangular timber girders reveals that the circular timber girders perform slightly better than the rectangular girders. One could compare the failure load reductions in both sections between the un-notched section and the 45% notched depth section with 1:0 notched angle, where maximum failure load reduction is in the range of 71-72 % for the rectangular section, whereas in the circular section the failure load reduction was about 56-60%. The comparisons of other critical stress parameters in the load deflection responses of the timber girder follow similar trend as the failure load, where an increased notched angle reduced the rate of shear and flexural stresses development (mode 1 and mode 2) in the girders.

5 Design verification

The design methodologies applied to verify the shear resistance of the notched beam/girder section in different national standards differ from each other in the aspects of treating the shear resistance of the material and the effect of notched profile through capacity reduction factors. More detailed review of the design approaches of notched timber sections can be found in Dewey et al., [5] and Jockwer [19]. Thus, in this section, the design approaches outlined in different standards have been used to verify against the data generated using the numerical analyses in Section 4. Primarily, the design approaches outlined in EN 1995-1-1 [13], AS1720.1 [14], American Institute of timber construction (AITC) [16] and CSA 086-09 [15] have been considered to verify the prediction of shear resistance of the notched timber beam/girder sections. Table 7 outlines the equations given in those different standards to determine the shear resistance of the notched timber sections.

The EN 1995-1-1 [13] provisions to predict the resistance of notched section is based on the fracture mechanics principles proposed by Gustafasson [34]. A reduction in shear resistance in notched section is incorporated using k_v factor, where h is the depth of un-notched section, α is the notched depth ratio, i is the notched angle, x is the distance of the notched to support and k_n is the material constant that account different timber types. Further AS 1720.1 [14] design provisions are based on the linear elastic fracture mechanics taking into the effects of both tension parallel to the grain and shear stress perpendicular to the grain faced at the change of the section profile. A modification factor g_{40} is incorporated to account the effect of notched depth percentage and the constants k_1, k_4, k_6 and k_{12} are given to load duration class, in-service moisture variation, temperature and humidity effect and stability factor, respectively.

The AITC [16] provisions are based on the resistance reduction at the notch section using mainly the notched depth ratio. The A_n, d and d_n refer to the net cross sectional area, total depth and remaining depth above the notch respectively. The CSA 086-09 [15] outlines K_n factor to account the influence of notching in the section. It has to be mentioned that in CSA 086-09 [15], the notch angle is not explicitly considered, whereas distance from the notch corner to the support (n) and the notched depth ratio (α) are incorporated in the method.

The shear strength of timber F_f was found using the shear strength of timber along the grain (f_f) and other factors such as load duration (K_D), system (K_H), service (K_{sf}) and treatment factor (K_T).

Table 7: Equations provided in different standards to predict the shear resistance of notched section.

Standards	Equation	Constant parameters
EN 1995-1-1 [13]	$k_v f_{v,d}$, where $k_v = min\left[1 \colon \frac{k_n\left(1+\frac{1.1 i^{1.5}}{\sqrt{h}}\right)}{\sqrt{h}\left(\sqrt{\alpha(1-\alpha)+0.8\frac{x}{h}\sqrt{\frac{1}{\alpha}-\alpha^2}}\right)}\right]$	$k_n = 5$
AS1720.1 [14]	$\emptyset g_{40} k_1 k_4 k_6 k_{12} f_{sj}$	$k_1 = 1$; $k_4 = 0.7$, $k_6 = 1$; $k_{12} = 1$
AITC [16]	$\frac{2}{3} f_v A_n \left(\frac{d_e}{d}\right)^2$	
CSA 086-09 [15]	$\emptyset F_f A K_n$, where $K_n = \sqrt{\left\{0.006d\left(1.6\left(\frac{1}{\alpha}-1\right)+n^2\left(\frac{1}{\alpha^3}-1\right)\right)\right\}}$; $F_f = f_f K_D K_H K_{sf} K_{DT}$	$K_D = 0.65$; $K_H = 1$; $K_{SF} = 1$, $K_{DT} = 1$

The calculated design shear resistances of the notched girders/beams using standards are presented in the Table B of the Appendix. Subsequently one can note among the predicted shear resistances from the standards considerably vary from each other. The variations are mainly due to the different approaches followed to incorporate the notching effect in the shear resistance. CSA 086-09 [15] formulations consider the influence of notch depth and distance from the support, thus no variations are noted with the difference in the notch angle. AITC [15] formulation only considers the influence of notched depth in calculation, hence the effect of notch angle and distance from support did not show any changes in the values calculated. AS1720.1 [14] only allows the notched depth of 10% into the capacity reduction using g_{40} factor, high values of notched depths are not recommended. Only EN 1995-1-1 [13] formulations incorporate all the parameters into the calculation of the shear resistance of notched section, therefore they follow similar trend as found in the numerical results. In order to compare the predictability of these different design formulations, the model error (ME) of each combination verified were computed and given in Table B. The ME is defined as the ratio of FE model prediction divided by the standard design prediction.

The basic statistical values of the ME values computed for each design standard are given in Table 8 for comparison. It is evident that the EN 1995-1-1 [13] provisions are the most conservative as the mean ME value is relatively higher than that for other standards. Further, the EN 1995-1-1 [13] are more reliable as the coefficient of variation is also comparatively less than the MEs of other design approaches. The predictions of other design formulations show that they sometimes (e.g. higher notched depths 45%) predict the resistance relatively higher (i.e. ME <1) than the numerical value, thus they are un-conservative in certain situations. Subsequently these higher notched depth timber girders need appropriate strengthening to sustain increased loading demands.

Table 8: Statistical parameters of ME derived from the design verification.

Parameters	EN 1995-1-1 [13]	AS1720.1 [14]	AITC [16]	CSA 086-09 [15]
Mean	2.49	1.11	1.51	2.16
COV	0.38	0.46	0.58	0.6
Minimum	1.11	0.43	0.38	0.48
Maximum	4.63	2.52	4.14	6.02

6 Conclusions

Three-point bending tests were conducted on the rectangular notched timber girders with notch angle of 1:4 and three different notch depths. Further FE modelling technique was developed analyse the behaviour of notched girders. Experimental results were used to validate the FE model. This validated FE model was extended to quantify the load carrying capacity, shear and flexural stress of typical circular and rectangular notched girder with two different spans (i.e. 6 m and 9 m), three different notch depths (i.e. 15%, 30% and 45%) and angles (i.e. 1:0, 1:2 and 1:4). The parametric analysis results were compared with different timber design standards. Based on the experimental tests, FE model analyses and design verifications following conclusions are highlighted:

- Increase of notch depths from 15% to 30%, the load carrying capacity of rectangular timber girder was reduced by 42%. This reduction further increased to 51% when notch depth reached 45% from the girder original depth. However, the load carrying capacity reduction in circular girders are comparatively less than the reduction found in rectangular section, thus the cross-sectional shape influence the behaviour of notched timber girders.

- Changing the notch angle of rectangular girder from 1:0 to 1:4, the load carrying capacity of timber girder increased by 50%, 69% and 110% for the girder with notch depth of 15%, 30% and 45%, respectively. Similar behaviour was observed in the circular timber girder. Thus, the notched angle greatly affects the stress characteristics of the timber girders.
- Out of four design standards considered for the verification, the EN 1995-1-1 [13] conservatively predict the shear capacity of the timber girder section, whereas the prediction of other standards vary considerably as they have certain limitations in incorporating the different parameters influencing the behaviour of notched timber section.

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Appendix

Table A: The detail of model used for the mesh sensitivity analysis

Model type	Number of elements (length × width × height)
M1	50 x 50 x 50 mm
M2	40 x 40 x 40 mm
M3	30 x 30 x 30 mm
M4	20 x 20 x 20 mm
M5	10 x 10 x 10 mm
M6	5 x 5 x 5 mm

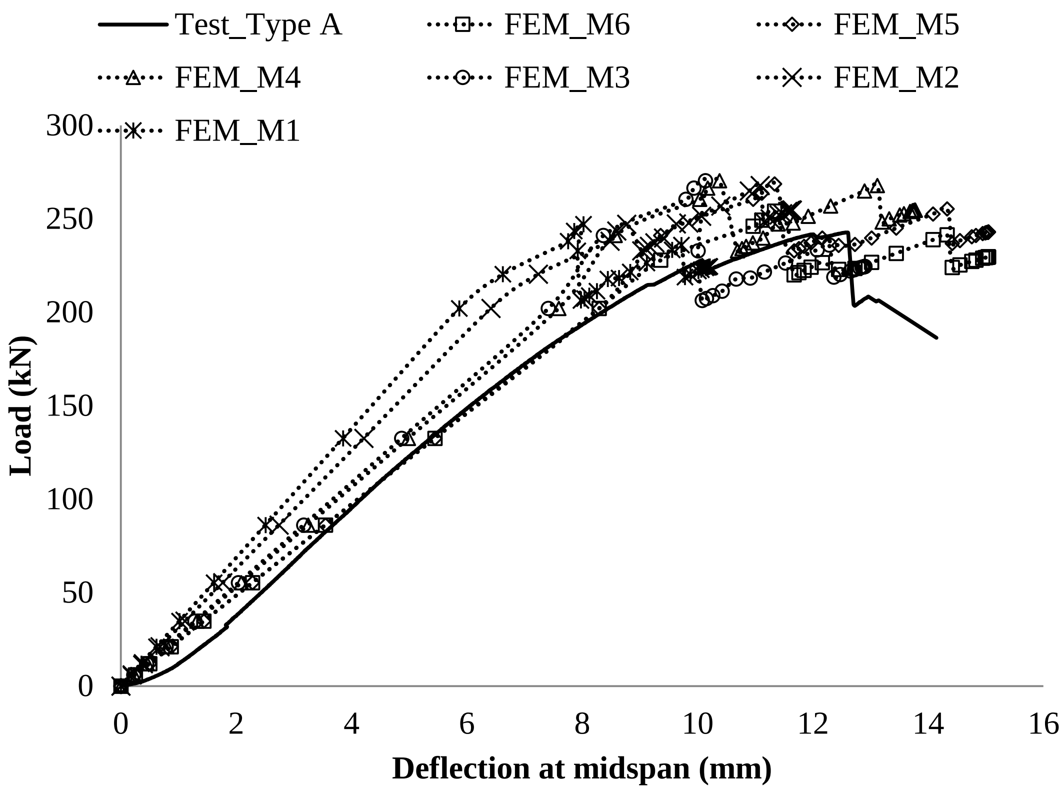

Figure A: Load–deflection curves of different mesh sizes

Table B: Design verification with different standards.

Type	EN 1995-1-1 [13]		AS1720.1 [14]		AITC [15]		CSA 086-09 [16]	
	Predicted	ME	Predicted	ME	Predicted	ME	Predicted	ME
R-6-15-	164	1.53	175	1.44	400	0.63	212	1.19
R-6-15-	189	2.24	360	1.18	400	1.06	318	1.33
R-6-15-	236	1.85	618	0.71	400	1.10	318	1.38
R-6-30-	100	2.47	144	1.72	223	1.11	152	1.64
R-6-30-	115	2.22	297	0.86	223	1.15	152	1.69
R-6-30-	144	2.86	509	0.81	223	1.85	152	2.72
R-6-45-	67	2.66	113	1.58	108	1.65	74	2.40
R-6-45-	77	3.50	233	1.16	108	2.50	74	3.64
R-6-45-	97	3.86	400	0.93	108	3.45	74	5.02
C-6-15-	152	2.35	162	2.20	371	0.97	295	1.21
C-6-15-	175	2.80	334	1.47	371	1.32	295	1.66

Type	EN 1995-1-1 [13]		AS1720.1 [14]		AITC [15]		CSA 086-09 [16]	
	Predicted	ME	Predicted	ME	Predicted	ME	Predicted	ME
C-6-15-	219	2.37	573	0.91	371	1.40	295	1.76
C-6-30-	91	3.62	131	2.52	203	1.63	138	2.39
C-6-30-	105	3.73	269	1.45	203	1.93	138	2.84
C-6-30-	131	3.86	462	1.09	203	2.50	138	3.68
C-6-45-	59	3.82	100	2.27	95	2.37	65	3.45
C-6-45-	68	4.27	205	1.42	95	3.05	65	4.44
C-6-45-	85	4.63	351	1.12	95	4.14	65	6.02
R-9-15-	88	1.73	175	0.87	400	0.38	318	0.48
R-9-15-	236	1.11	360	0.73	400	0.66	318	0.83
R-9-15-	164	1.62	618	0.43	400	0.67	318	0.84
R-9-30-	115	1.28	144	1.02	223	0.66	152	0.97
R-9-30-	144	1.75	297	0.85	223	1.13	152	1.66
R-9-30-	100	2.61	509	0.51	223	1.17	152	1.73
R-9-45-	77	1.23	113	0.84	108	0.88	74	1.28
R-9-45-	97	1.48	233	0.61	108	1.32	74	1.92
R-9-45-	67	3.12	400	0.52	108	1.94	74	2.82
C-9-15-	152	1.22	162	1.14	371	0.50	295	0.63
C-9-15-	175	1.58	334	0.83	371	0.75	295	0.94
C-9-15-	219	1.27	573	0.49	371	0.75	295	0.95
C-9-30-	91	2.23	131	1.55	203	1.00	138	1.47
C-9-30-	105	2.37	269	0.92	203	1.22	138	1.80
C-9-30-	131	1.96	462	0.55	203	1.26	138	1.86
C-9-45-	59	2.50	100	1.48	95	1.55	65	2.26
C-9-45-	68	2.87	205	0.95	95	2.05	65	2.98
C-9-45-	85	3.12	351	0.75	95	2.78	65	4.05