Ponder this

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This edition marks the start of a regular problems section in the ASMJ. I would like to express my sincere gratitude to the editors of the journal, Robyn Pierce and Sue Garner, without whose energetic work and inspiration, this section would not have come to be.

This section aims to give readers an opportunity to exchange interesting mathematical problems and solutions. The set in each issue will consist of up to five problems; some may be easy, others more difficult, but each problem will involve an interesting idea, result or solution method that will contribute to a further development of certain problem solving skills and knowledge.

Solutions will be made available on the AAMT website. Readers, especially teachers and their students, are also encouraged to submit solutions to the editor; solutions of note will be published in the next issue.

Leonard Euler, a truly great mathematician, was interested in many aspects of mathematics — in analysis, applied mathematics, theory of numbers and geometry. His contribution to each of them was enormous. However, it is not widely known that Euler also created much of the mathematical notation that we use today. For example, he introduced the letter e to represent the base of the system of natural logarithms; the use of the Greek letter π for the ratio of circumference to diameter in a circle also is largely due to Euler; the symbol i for $\sqrt{-1}$; the use of the small letters a, b and c for the sides of a triangle and of the corresponding capitals A, B and C for the opposite angles stems from Euler, as does the application of the letters r, R and s for the inradius, circumradius and semiperimeter of the triangle respectively; the designation lx for logarithm of x; the use of Σ to indicate a summation; and, perhaps most important of all, the notation $f(x)$ for a function of x are all due to Euler.

The first problem is a tribute to Leonard Euler and his great achievements in mathematics. Thanks also to Gregory Galperin, Bohdan Rublev and Michel Bataille who provided problems. Please send your interesting problem (with a solution — if one exists) to the editor.

Problem set 1

In memoriam of Leonard Euler

1. Prove that for each positive integer $n \geq 3$, a number 2^n can be represented as $2^n = 7x^2 + y^2$ where x and y are both odd numbers.

Proposed by Gregory Galperin

2. There are two non-congruent triangles, T_1 and T_2. For any pair of sides of T_1, there is a pair of sides of T_2 with the same sum of their lengths. Likewise, for any pair of sides of T_2, there is a pair of sides of T_1 with the same sum of their lengths. The triangle T_1 has two sides of lengths 25 and 35 units, respectively. Prove that the triangles have different perimeters. Find the triangle with the smaller perimeter and evaluate that perimeter.

Proposed by Gregory Galperin

3. A square grid on the Euclidean plane consists of all points (m, n), where m and n are integers. Is it possible to cover all grid points by an infinite family of discs with non-overlapping interiors:
   (a) if each disc in the family has a radius of 1.3 units?
   (b) if each disc in the family has a radius of at least 5 units?

Proposed by Bohdan Rublev

4. Janet and Jack play a game on an $n \times n$ grid. Each player in turn draws a polygon (not necessary a convex one) with vertices at the grid points (i.e., points (i, j), where i, j are integers) and area 1. Janet goes first and then the players alternate. Each new polygon cannot share any common point with polygons drawn before. Janet wins if Jack cannot draw a polygon and vice versa. Find, with proof, a winning strategy for one of the players.

Proposed by Michel Bataille

5. Let points B and C of a semicircle with diameter DE satisfy $BD + CE = DE$. If lines BD and CE intersect at A, prove that

$$\frac{2}{BC} \leq \frac{1}{AC} + \frac{1}{AB}$$

Solutions to this set or problems for publication should be submitted to:
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or by email to yevdokim@usq.edu.au.
Solutions to this set will be made available on the AAMT website
(www.aamt.edu.au) after 1 August 2008.