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RESEARCH ARTICLE

Price optimization in supply chain agreements: a comparative analysis of buyback and put option contracts for inventory risk management

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Abstract
This paper aims to provide a model of a supply chain in the integrated system and obtain its optimal decision variables. The paper introduces buyback and put option contracts to reduce inventory risk. These contracts were compared in three different cases via a numerical analysis approach. In the first case, the holding cost (h) of a retailer for surplus orders in the buyback contract is equal to the option price (o) in the put option model. The relationship between exercise price (e) in the put option model and buyback price (b) in the buyback contract was obtained by comparing the optimal values in the models. This study found that the exercise price in the put option contract will be greater than the buyback price. Furthermore, it is more likely that the retailer gave more benefits under the buyback agreement than the time the retailer chooses the put option contract. Therefore, it can be concluded that if the retailer chooses the buyback agreement in this situation, can gain more benefits. The study provides essential managerial insights to compare agreements and presents recommendations to choose a suitable contract.

Keywords Buyback contract · Put option contract · Sales effort · Mean value theorem · Price · Supply chain

Introduction

Research in the supply chain (Awudu et al. 2023; de Bastos 2023) and the effects of price have received strong attention in recent years (Taleizadeh et al. 2023; Xia et al. 2022). Calculating put option values will help producers who invest in the put option markets to get rid of price risk (Contreras and Rodríguez 2014). The contract that allows the retailer to return the surplus goods to the wholesaler is the put option contract. This event occurs when the retailer pays the option price at the beginning of the sales season per option order. The wholesaler pays the exercise price to the retailer per order when goods are returned by the retailer at the end of the sales season (Wang and Chen 2018). Hence, the put option contract is an appropriate contract to reduce the retailer's inventory risk, and it has the highest initial order quantity of all kinds of option contacts (Yang et al. 2017). Put option contract usually does not have benefits for the supplier, especially in a case that has high service constraints. The contract can strongly improve the performance of the decentralized system when the service restriction is low (Chen et al. 2019).

The retailer can return the excess goods to the wholesaler by the usage of a buyback contract under the predetermined condition at the end of the sales season (Heydari et al. 2022; Wu et al. 2021). A buyback agreement can coordinate the supply chain (Heydari and Momeni 2021). Therefore, if we use it to provide a coordination mechanism between members of the supply chain, they will satisfy that agreement equally (Shi et al. 2008). Retailers sometimes have competition with each other and the competition may happen when there is a buyback contract in a chain. Besides, biding the buyback contract to retailers which are competitors can have benefits for all the members (Xue et al. 2019). The buyback contract has higher

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advantages than the non-buyback contract in both channel strategies for two rival supply chains (Wu 2013). Yet, limited studies have been done on the put option and buyback contracts within the supply chain context. Therefore, the current study aims to address this particular research gap.

The supply chain which includes a retailer and a wholesaler is coordinated by a buyback contract when the retailer's demand function is related to the sales effort variable and reaches to win–win result between members of a chain (Taylor 2002). Moreover, the two parameters that are loss aversion and sales effort have less impact on the supply chain. If optimal sales effort decreases, loss aversion will increase as well. The total benefits of a supply chain in situations that are centralized and decentralized, increase as the sales effort increases. As well as the wholesaler's benefit and the retailer's benefits increase similar to the last condition (Ke and Liu 2017). Under circumstances where the problem is loss-averse newsvendor, all kinds of contracts cannot manage the supply chain when the demand function is affected by sales effort variables. Hence, this research could lead to better supply chain management by understanding the implications of the put option and the buyback contracts.

In this study, the main question is *"What is the better contract in terms of reducing surplus inventory risk on the retail side of the supply chain?"* To address this question, the put option contract and the buyback contract have been compared with each other. Therefore, the more appropriate contract will opt for these two contracts. The reason for this comparison is the very similar nature of the two contracts as both of them pursue the goal of reducing the risk of surplus inventory in the supply chain. This comparison will be accomplished in three cases ($o = h$, $e = b$, $o = h$, $e \neq b$, and $o \neq h$, $e = b$). According to the existing historical data, the probability of their occurrence in the supply chain considered in the case study is high. Therefore, considering these three various cases and also considering the sales effort factor as a decision variable leads to complicating mathematical relations in the problem. Hence, the mean value theorem is employed to solve this complexity and find the final answers.

The next section presents the issues in the current literature. The paper then explains the issues, describes the notation, and defines the assumption. Next, the paper formulated a centralized supply chain model and represent the model of buyback and put option contracts. The study then compares contracts which provide before in two different cases. Next, we provide a numerical example and show the effect of parameters in the model. Lastly, the study presents a conclusion and managerial insight which get from the comparison of contracts.

Literature review

Put option contract

The demand for fast-moving consumer goods (FMCG) products is often random and affected by the price (Buckinx et al. 2004; Pourhejazy et al. 2019). Besides, price is often a key factor that plays a role in the supply chain network and demand (Aliahmadi et al. 2023; Sajadieh and Jokar 2009). In the case of a put option contract, it is considered that the buyer can buy the products from the factory and if extra goods remain, the buyer can return them to the factory by receiving the selling price after the end of the sales season (Wang and Chen 2018). To manage the uncertainty in the demand function and the supply of products, a put option contract has been provided to reduce the risk of the supply chain (SC) (Luo et al. 2018). In fact, Hu et al. (2019) have compared the buyback, the put option, and the wholesale price contract under the same conditions and environments. Their results show that the put option contract could reduce the risk of surplus inventory on the retail side (Yang et al. 2017).

The put option contract has been used to create a balance between the demand and the supplier's ability to produce the products required by retailers (Kume and Fujiwara 2016). The supply chain's coordination has been examined with the option contract. In this case, the retail price, the option price parameter of the option agreement, the exercise price parameter, and the optimal order quantity could be optimized (Hu et al. 2019).

On the other hand, the issue of fairness has been studied in an SC that involves a retailer and wholesaler retailers procuring goods by the option contract from the wholesaler (Sharma et al. 2019). An option agreement has been considered for the supply chain offered by the retailer, while both the retailer and the wholesaler are at risk and have examined the effect of the option agreement and the exercise price parameter of an option agreement (Fan et al. 2020). The SC was coordinated by the option agreement considered by Tang et al. (2019) and has been proven that retailers with limited capital make more profit when they place their order under an option contract. Also, if the producer does not have enough information about the market demand, the producer will get more profit by offering the option contract to the retailer (Tang et al. 2019). Besides, the use of the option contract has been examined in a single period chain considering the asymmetry of cost information. It has been shown that the asymmetry of cost information has great efficacy on the optimal decision variable of the option agreement and supply chain benefits (Chen et al. 2019).

Sales effort

Under a fixed demand if the supply chain is decentralized the sales effort variable of offline stores, the benefit of the chain will be less than when the supply chain is centralized (Pu et al. 2017). The combination of sales effort and an attempt to decrease carbon emissions in a home appliance supply chain system has been explored by Lou and Ma (2018). A problem of coordination for the supply chain which includes a wholesaler and a retailer is considered and demand depends on the sales effort variable (Wang et al. 2019). In this situation, the cost-sharing contract and buyback contract cannot coordinate the chain. Combining the two contracts proves that the production cost has an inverse effect on the benefit of the retailer, wholesaler, and the whole chain (Wang et al. 2019). The effect of the timing of the retailer's obligation of sales efforts on investiture in a decentralized SC has also been considered by Liu et al. (2018).

On the one hand, an optimal decision is assessed based on price and demand depends on the sales efforts (Dalalah et al. 2022; Phumchusri et al. 2023) until customers decide how to evaluate wholesale prices, collection rates, retail prices, and sales efforts under various decision-making approaches (Rabbani et al. 2020). A supply chain has been investigated under the effect of retail prices and sales efforts. It has been considered demand, cost of production, and sales efforts as uncertain variables. The centralized model and the decentralized model have relied on data about retail parameters and the optimal decision is found by games theory Shen and Zhu 2018). For instance, a producer sells an interchangeable green product by direct channel (sells by producers) and sells a non-green product by the retailer who is a supply chain member. The results show that the demand function is affected by sales effort level, green quality level, and online and offline prices (Ranjan and Jha 2019).

On the other hand, a supply chain that involves a wholesaler and a retailer has been considered. The demand is affected by the sales effort variable and this study examines the effect of cost-sharing revenue-sharing agreements on the decisions of supply chain members. The analysis of rival producers and rival retailers is considered by Li et al. (2018). The results show the impact of price rivalry and sales effort rivalry in the channel strategies. A supply chain that includes the manufacturers and several retailers was analyzed by Du and Lei (2018). They found that retailers would make a sales effort to find more customers by using an attractive price in the final market. In centralized and decentralized cases, the balance solution is developed. Then according to the level of the sales effort, the effect of the supply chain's profit is analyzed (Du and Lei 2018).

Buyback contract

A generic mathematical structure can be an effective way to analyze the buyback contract in seller inventory control models and retailer inventory control (Sainathan and Groenevelt 2019). Under decentralized conditions, the optimal wholesale price was determined and it has been proven that the members of the chain in the buyback contract gain more than the wholesale price contract (Gu et al. 2018). A supply chain is presented that has a tier of retailers who offer a return policy with partial refunds to the customer. The demand for this chain is random and price dependent (Duc et al. 2018). To maximize profits for supply chain members, a buyback contract is considered, which focuses on a buyer in the supply chain to satisfy quality and product demand (Gao et al. 2019).

In addition, producers and retailers use a buyback contract to reduce the risk of surplus inventory and improve their efficiency (Huang and Ip 2019). A buyback contract has been studied in a supply chain that involves a wholesaler and retailer, in which the retailer is exposed to uncertainty with a downward slope demand curve that is dependent on price (Torun and Canbulut 2018). This is also formulated by the Stucklerberg model (Zhao et al. 2019). The problem of pricing and ordering in a dyadic supply chain has been investigated by Chen et al. (2017). They reported that a retailer who is limited budget and is facing random demand could use the buyback contract to receive financial support buyback.

Besides, the value of the buyback agreement has been investigated by analyzing the SC which contains a producer and two retailers competing with each other. For instance, three modes of decision making are considered, which are coordinated and examined by a buyback agreement, and optimal values are obtained (Qin et al. 2017). It has been found that buyback contracts can coordinate the chain, as well as, buyback contracts can gain more profit to the supply chain, chiefly when retailers are at high risk (Luo et al. 2018). The optimal decision variables in the decentralized and centralized chain have been acknowledged that a buyback agreement can coordinate a supply chain that is dual channel. More related research can be found in the works of Taleizadeh et al. (2016, 2018), Lashgari et al. (2016), Taleizadeh (2017), and Taleizadeh and Noori-Daryan (2016). Table 1 is a summary of the relevant literature review.

Table 1 Brief literature review

Author	Year	Contract		Demand			Compare put option and buyback contract		
		Put option	Buyback	Stochastic	Affected by the retail price	Affected by sales effort	$e = b$ $o = h$	$e = b$ $o \neq h$	$e \neq b$ $o = h$
Xue et al.	2018		*	*					
Lou and Ma	2018			*	*				
Basiri and Heydari	2017			*	*				
Wang and Song	2020			*	*				
Shen and Zhou	2017			*	*				
Wang and Liu	2018			*	*				
Hu et al.	2019	*	*	*			*		
Yang et al.	2017	*		*	*	*			
Zhou et al.	2017	*		*					
Zhao et al.	2019		*	*	*				
Chen et al.	2017		*	*					
Duc et al.	2018		*	*	*				
Chen et al.	2020	*		*	*				
Chen et al.	2017		*	*					
Liu et al.	2018		*	*	*				
Gu et al.	2018		*	*					
This paper	2022	*	*	*	*	*	*	*	*

Problem definition, notation, and assumption

This current study has considered a cosmetic SC which includes one retailer and a wholesaler. In this SC on the retailer side, there is a high risk of surplus inventory. For this purpose, it needed to provide a contract that reduces inventory risk for surplus goods in-retailer side. The reason for insisting on reducing inventory risk is to reduce the cost of surplus inventory for the retailer. When the retailer's costs increase, it causes the chain's costs to increase too. Hence, the supply chain will get out of the competitive market. One of the important parameters in the competitive market is retail price "P." As a result, one of the factors affecting the amount of profit is the price of goods which is given to the customer. The amount of this price could increase or decrease the market demand. Other factors that can affect supply chain demand are advertising, which has a direct impact on market demand. This variable has shown by "S" in the models and the cost of this variable can be formulated as follows: $\frac{\alpha S^2}{2}$.

α in this function is the coefficient of the sales effort. Therefore, in this research, regular demand function (D) has been affected by sales effort variables (S) and price variables (p) so it can be formulated as follows: $D(S, p) = D\frac{S}{p}$. This means that the demand function will increase while the sales effort variable increases and it will decrease while the price variable increases. Linear demand function without price variables has been employed in the supply chain contract (Yang et al. 2017). In the cosmetic supply chain and the supply chains with extensive advertising and promoters, the price variable is a significant factor in the demand function in addition to the sales effort variable. Therefore, in this research, price adds to the demand function in addition to the sale effort.

After providing the model of a centralizing supply chain, we provided buyback and put option contracts that could reduce the inventory risk of surplus goods on the retailer side. This study aims to conclude in which circumstances, which contract is more suitable and can reduce the risk of surplus goods more and also, as a result, gain more profit for the chain.

The models' notations are as follows:

C	Production cost per order
W	Wholesaler price
P	Retailer price
S	Sales effort level
α	Sales effort coefficient, $0 < \alpha < 1$
D (S, p)	Demand for goods
F	PDF of demand
F	CDF of demand
O	The option price per order that the retailer paid to a wholesaler for option orders

C	Production cost per order
E	Exercise price per order that wholesaler paid to a retailer when retailer returns goods under a put option agreement
H	Holding cost of orders (until end of sales season) which is surplus on the retailer side
B	Buyback price that the wholesaler paid to a retailer when the retailer returns goods under the buyback agreement
Q_{CS}	Order quantity in a supply chain that is centralized
Q_{PO}	Order quantity in the put option contract
M	Option order quantity in the put option contract
Q_{BB}	Order quantity in buyback contract
Π	Profit function of variable

Also, we set the symbols "CS," "BB," and "PO," to represent the centralized supply chain model, buyback contract model, and put option contract model, respectively (Fig. 1). We also use the following assumption in the current study.

Assumption 1

The retail price should be greater than the wholesaler price; otherwise, the retailer will not deal with the wholesaler.

$P > w.$

Assumption 2

The wholesaler's price should be greater than the production cost per order; otherwise, the wholesaler would face disadvantages and will not supply the products.

$w > c.$

Assumption 3

In the buyback contract, a summation of the wholesaler price and holding cost per order should be greater than the buyback price; otherwise, the retailer does not concern about surplus inventory or goods and they will get the inventory risk neutral.

$w + h > b.$

Assumption 4

In a buyback contract, the retail price should be greater than the buyback price; otherwise, the retailer does not have any interest in selling goods to customers and may prefer to return goods to wholesalers.

$P > b.$

Assumption 5

In the put option contract, the summation of the wholesaler price and option price should be greater than the exercise price; otherwise, the retailer does not concern about surplus inventory or goods and they will get the inventory risk neutral.

$w + o > e.$

Assumption 6

In the put option contract, the retail price should be greater than the exercise price; otherwise, the retailer does not have any interest in selling goods to customers and may prefer to return goods to wholesalers.

$P > e.$

Centralized supply chain

In a centralized supply chain, the supply chain's members, the retailer, and the wholesaler collaborate to maximize the benefit of the SC. Thus, their economic order quantities are equal. The expected profit function is formulated as follows for the whole chain:

$$E\left[\Pi_{CS}\right] = E\left[P\min\{Q_{CS}, D(S_{CS}, p)\} - cQ_{CS} - \alpha\frac{S_{CS}^2}{2}\right]. \tag{1}$$

The first part is income from the sale of the goods, the next part is the cost of procuring the products, and the last part is the cost of advertising, and promoting products, in brief, the cost of sales effort variable (Fig. 1).

Proposition 1 *For the centralized supply chain, the optimal order quantity and optimal sales effort are* $Q_{CS}^* = \frac{S_{CS}}{P}F^{-1}\left(\frac{P-c}{P}\right)$ *and*

$$S_{CS}^* = \frac{(P-c)}{\alpha P}F^{-1}\left(\frac{P-c}{P}\right) - \frac{1}{\alpha}\int_0^{F^{-1}\left(\frac{P-c}{P}\right)} F(x)dx,$$

respectively.

Proposition 1 demonstrates that the expected profit function of the centralized SC is concave, regarding the sales effort and order quantity. It is obvious that if

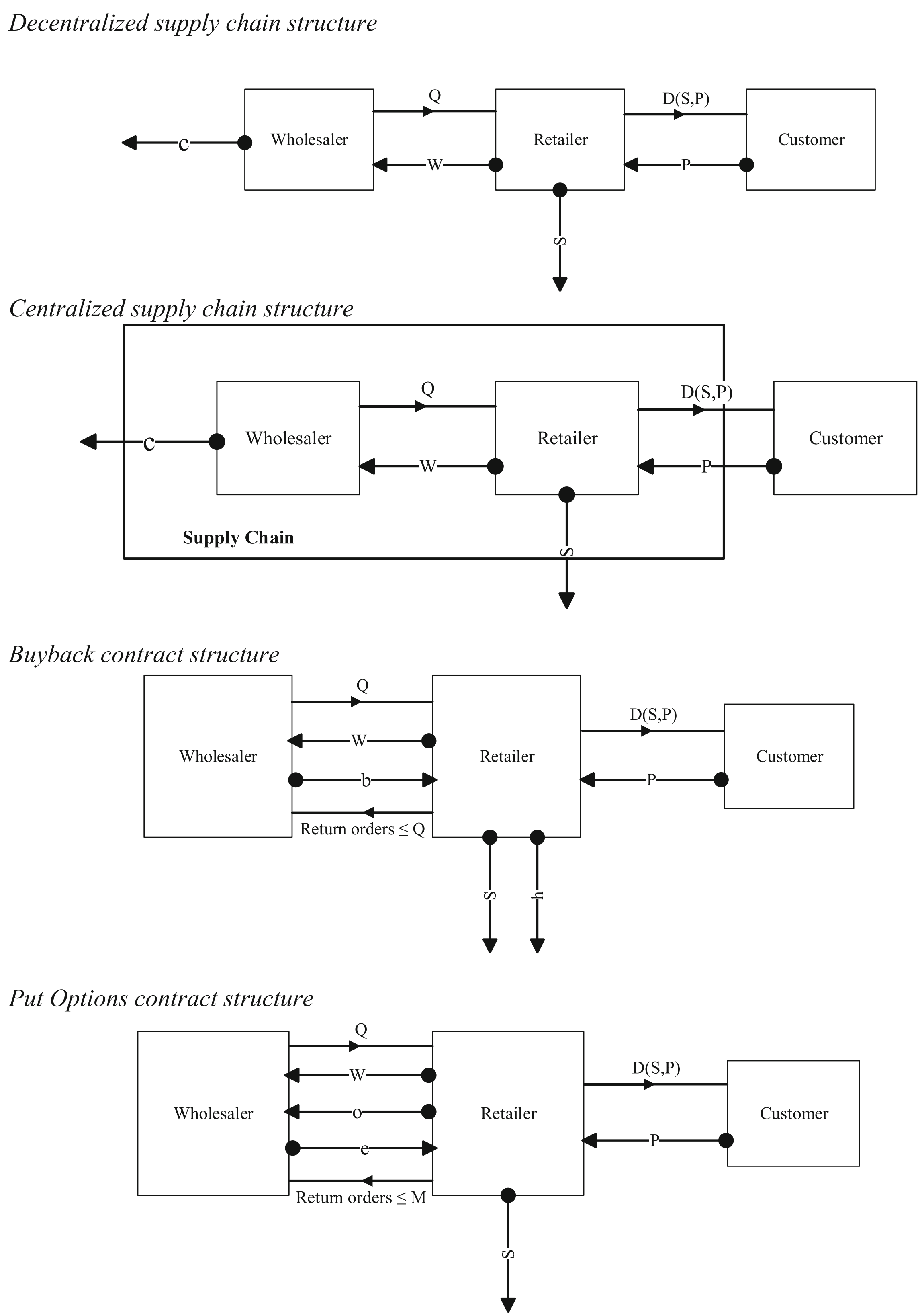


Fig. 1 Put options contract structures

advertising, promoting products, and sales effort increase, the amount of demand will increase, as also the order quantity. Therefore, Proposition 1 has shown that optimal order quantity and optimal sales effort have direct relations. Also, they have been influenced by price and the production cost per order (Appendix D for Proof of Proposition).

Decentralized supply chain with buyback contract

The buyback agreement gets this permission to the retailer to return surplus goods to a wholesaler when their demand is less than the amount of order quantity. Thus, the retailer can hold surplus orders until the end of the sales season. After that, the wholesaler takes back surplus orders and pays the buyback price per order which is returned from a retailer. By the buyback contract in the decentralized supply chain, we can reduce the inventory risk of a retailer which will increase their profits as well as the profit of the supply chain. Holding cost per order (this holding cost belongs to stocking surplus goods for the extra period after-sales season at the retailer side. This is an offer that the retailer gets to the wholesaler in this contract) and buyback price indicated by "h" and "b," respectively. If the wholesaler offers the buyback contract after that retailer will obtain optimal decision variables to increase their profit. The profit function of the retailer is

$$E\left[\Pi_{BB}\right] = E\left[P\min\{Q_{BB}, D(S_{BB}, p)\} - wQ_{BB} + (b-h)(Q_{BB} - D(S_{BB}, p))^{+} - \alpha\frac{S_{BB}^{2}}{2}\right]. \tag{2}$$

In Eq. 2, the first part is income from sales of the products, and the second part is the cost of purchasing goods from a wholesaler. The third part is revenue from orders that are returned to the wholesaler, and the last part is the cost of sales effort on the retailer side (Fig. 1).

Proposition 2 *For the decentralized supply chain formulated with a buyback contract, the optimal order quantity and optimal sales effort are* $Q_{BB}^{*} = \frac{S_{BB}}{P}F^{-1}\left(\frac{P-w}{P+h-b}\right)$ *and*

$$S_{BB}^{*} = \frac{(P-w)}{\alpha P}F^{-1}\left(\frac{P-w}{P+h-b}\right) - \frac{P+h-b}{\alpha P}\int_{0}^{F^{-1}\left(\frac{P-w}{P+h-b}\right)} F(x)dx,$$

respectively.

Proposition 2 has shown that the profit function of a retailer is concave concerning the order quantity and sales effort when formulated with a buyback contract. Therefore, Proposition 2 shows if the optimal sales effort increases, the optimal amount of order quantity will increase as well. In addition, the production cost per order, retail price, and buyback agreement parameters affect optimal decision variables (Appendix D for Proof of Proposition).

Corollary 1 *If the buyback price increases, the optimal order quantity will increase, and if the holding cost per order increases, the optimal order quantity will decrease*

The high buyback price was offered by a wholesaler to a retailer under the buyback agreement. It causes the retailer's revenue to increase from returning surplus orders. As a result, the retailer prefers to increase their optimal order quantity. Corollary 1 shows that if the buyback price increases, the difference between it and the summation of holding cost and the wholesaler price has decreased. Thus, the retailer will be less concerned about the surplus inventory and their optimal order quantity will increase. In the following, another important parameter that influences the retailer's optimal order quantity is the holding cost per order. While the holding cost is increased, the cost of surplus orders on the retailer side will increase and the retailer will avoid raising this cost. Therefore, Corollary 1 has shown that by increasing this cost, the amount of optimal order quantity will decrease (Yang et al. 2017) have studied the same approach with this Corollary to make proof the sales effort variable in the call option contract is less than this variable in the centralized supply chain. While in this research the sales effort variable is one of the decision variables. Also, that particular approach is employed for proofing that by increasing "h," S_{BB}^{*} and Q_{BB}^{*} will decrease and by increasing "b" S_{BB}^{*} and Q_{BB}^{*} will increase (Appendix C for Proof of Corollary).

Decentralized supply chain with a put option contract

The number of goods that the retailer returns to the wholesaler should not exceed the option order quantity that the retailer has obtained. Indeed, the products returned to the wholesaler by a retailer can be only equal to or less than the number of option orders that have been set before and the retailer will not be allowed to return more products. While the wholesaler offers put option contract, the retailer after determining the initial order quantity should be obtained their option order quantity. Then they could pay an options price to the wholesaler per order which is determined as an option order. If the retailer proceeds to pay the options price per option order to the wholesaler (i.e., if there will be surplus goods on the retailer side), the wholesaler will pay the exercise price to the retailer per order returned to them. In this segment, the retailer intends to maximize the profits, and their expected profit function is formulated as follows:

$$E[\Pi_{PO}] = E\left[P\min\{Q_{PO}, D(S_{PO}, p)\} - wQ_{PO} - oM + e\min\left\{M, (Q_{PO} - D(S_{PO}, p))^{+}\right\} - \alpha\frac{S_{PO}^2}{2}\right]. \tag{3}$$

The above equation differs from previous models in the third and fourth parts. The third part is the cost of the option order that the retailer pays to the wholesaler and the fourth part is the income of orders which has been returned to the wholesaler by a retailer at the end of the sales season (Fig. 1).

Proposition 3 *For the decentralized supply chain formulated with the put option contract, the optimal initial order quantity, optimal option order quantity, and optimal sales effort are* $Q_{PO}^{*} = \frac{S_{PO}}{P}F^{-1}\left(\frac{P-w-o}{P-e}\right)$, $M^{*} = \frac{S_{PO}}{P}\left[F^{-1}\left(\frac{P-w-o}{P-e}\right) - F^{-1}\left(\frac{o}{e}\right)\right]$, *and*

$$S_{PO}^{*} = \frac{(P-w-o)}{\alpha P}F^{-1}\left(\frac{P-w-o}{P-e}\right) - \frac{P-e}{\alpha P}\int_{0}^{F^{-1}\left(\frac{P-w-o}{P-e}\right)} F(x)dx + \frac{o}{\alpha P}F^{-1}\left(\frac{o}{e}\right) - \frac{e}{\alpha P}\int_{0}^{F^{-1}\left(\frac{o}{e}\right)} F(x)dx.$$

respectively.

Proposition 3 indicates that the retailer's profit function under the put option agreement regarding the sales effort and order quantity and options order quantity is concave. Also, it is shown that the order quantity is greater than the option order quantity and both of these variables are influenced by the sales effort directly. It is obvious, if a retailer's sales effort increases, the amount of their order will increase. In addition, the put option contract parameters also affect the decision variables of the problem (Appendix D for Proof of Proposition).

Corollary 2 *If the exercise price increases and* $\frac{F^{-1}\left(\frac{o_{PO}}{e} - \frac{1}{2e^2}\right)}{F^{-1}\left(\frac{P-w-o_{PO}}{P-e} + \frac{P-w-o_{PO}}{2(P-e)^2}\right)} > \frac{(P-w-o_{PO})e^2}{(P-e)^2}$ *the optimal order quantity will increase and if the option price increases and* $\frac{F^{-1}\left(\frac{P-w-o_{PO}}{P-e} - \frac{1}{2(P-e)}\right)}{F^{-1}\left(\frac{o_{PO}}{e} + \frac{1}{2e}\right)} + 1 > \frac{P}{e}$ *the optimal order quantity will decrease.*

If the put option agreement is offered by the wholesaler to the retailer and the wholesaler provides the high exercise price to the retailer, then the retailer may prefer to increment the order quantity as well as the option order quantity to increase their revenue of return goods. Corollary 2 shows that by increasing the exercise price, the difference between it and the summation of the option price and the wholesaler price has decreased. Also, the retailer may less concerned about the surplus goods and their optimal orders may increase. The option price is another important parameter that affects the optimal option order quantity and the optimal initial order quantity of the retailer. It is clear that increasing the option price causes the cost of the retailer to procure the option orders will increment and the retailer intends to reduce this. Hence, Corollary 2 has shown that the optimal initial order quantity may be decreasing in the option price (Appendix C for Proof of Corollary).

Comparison and discussion of buyback and put option contracts

In Appendix A, we compare the buyback and put option contracts in two different cases. In the first case, we assume that the buyback price in the buyback agreement and the exercise price in the put option contract is equal. After coordinating the supply chain under these contracts we can obtain the relationship between the option price and the holding cost for surplus goods. We compare these parameters to each other; thus, a relationship is obtained that if these parameters satisfy it, We will conclude the put option contract is more suitable than a buyback contract to reduce inventory risk.

On the other hand, we have assumed in Case 2 that the holding cost and the option price are equal, and we have performed the above steps on the buyback price and the exercise price. Finally, we find a relation between these two parameters that if it's satisfied, as a result, the put option Contract is better than a buyback contract to reduce the inventory risk of the retailer.

Numerical study and its discussion

We extracted this numerical data from a real example of a cosmetics SC which includes a wholesaler and a retailer who sells cosmetic products to the end customer. We focused on one of the products provided by this supply chain which is a sort of baby shampoo. In this supply chain, the put option and buyback agreement are offered by the wholesaler to the retailer.

This is in the context that the wholesaler offers these two contracts to the retailer in three different conditions: (1) The equality of the option price and the holding cost and inequality of the buyback price and the exercise price; (2) Inequality of the option price and holding cost and equality of the exercise price and the buyback price; and (3) The equality of the option price and holding cost, as well as the equality of the exercise price and the buyback price.

The main purpose of this project is to help to choose a suitable contract for the retailer because we have decided to use the relationships obtained in this research to guide the retailer of this chain in choosing the best contract in different parametric conditions. Therefore, the data that we use to investigate the two proposed contracts are as follows: $P = 22$, $w = 11$, $c = 5$. The demand of the retailer demonstrated by "D" follows the continuous distribution which is uniform that demonstrates U [50,150]. The retailer of this chain is a distribution and sales company of cosmetics products and the project of choosing the best contract has been done in this company. We obtained a profit chart for two contracts that offer and compare them under three conditions in two levels. First, we compare the decision variables of this problem, and second, the amount of profit is compared.

The comparison of holding cost and option price when $e = b = \psi$

In this section, the option price is compared with the holding cost per order under the condition that the exercise price is equal to the buyback price. We have shown in Theorem 1 that the holding cost per order in the buyback contract is greater than the option price per order in the put option contract when the buyback price in the buyback contract and exercise price in the put option contract are equal to each other. Therefore, in this part, we have represented that the theorem happens and Fig. 2 proves what we obtained before. To continue, if the exercise price or buyback price increases, the difference between the holding cost and the option price will increase too.

The comparison of optimal sales effort when $e = b = \psi$

Figure 2 shows that the optimal sales effort in the put option and buyback agreement is not much different from each other. Even in some parts, they are equal to each other. Given the relationship in Corollary 3 and 4, that may happen where the sales effort of the put option contract is greater than the buyback contract's sales effort. Hence, in this chart, when the buyback price or the exercise price is $\psi > 13$, the amount of the effort to sell in the option contract will be greater than the amount of this variable in the buyback contract.

The comparison of optimal order quantity when $e = b = \psi$

Based on propositions 2 and 3, it can be realized that in each of the contracts, the order quantity was directly related to the sales effort variable. For this reason, we can conclude that if the sales effort variable increases, the order quantity will increase. Also, if it decreases the order quantity will decrease too. Even in Fig. 2, it is shown that if the amount of the exercise price parameter or the buyback price gets more than 13, the number of orders in the put option contract will become more than the order quantity in the buyback contract. As in the previous section that the sales effort variable in the put option contract was greater than the sales effort variable in the buyback contract under the same condition.

The comparison of retailer's profit when $e = b = \psi$

According to the data in this numerical example that is given from a cosmetic company, the amount of the retailer's benefit under the put option agreement is greater than the amount of their benefit under the buyback agreement. Therefore, it can be concluded that in this cosmetic SC, it will be better for the retailer if the wholesaler of cosmetic products offers the put option and buyback contract to the retailer and if the retailer chooses the put option agreement. However, the retailer of a supply chain should set exercise price parameters in the put option contract in a smaller amount. So, it is more profitable for the retailer.

The comparison of the supply chain's profit when $e = b = \psi$

The profit of the supply chain under the buyback agreement is greater than the amount of its benefit under the put option agreement. Therefore, the benefit of the retailer under the put option agreement is greater than the profit of the retailer under the buyback agreement. It can be concluded that if the retailer chooses the put option contract, it may be more profitable for the retailer compared to when the retailer chooses the buyback contract. Nevertheless, the result shows that for the chain the buyback agreement is more profitable than the put option contract. Over time, it will make the supply chain stronger and it is better for all supply chain members.

The comparison of buyback price and exercise price when $o = h = \varphi$

According to Theorem 2, if the holding cost in the buyback contract is equal to the option price, then the parameter of the exercise price will be slightly greater than the parameter of the buyback price. In the following, we can find out from Fig. 2 that if the option price or holding cost per order increases, the difference between the exercise price and buyback price will increase too.

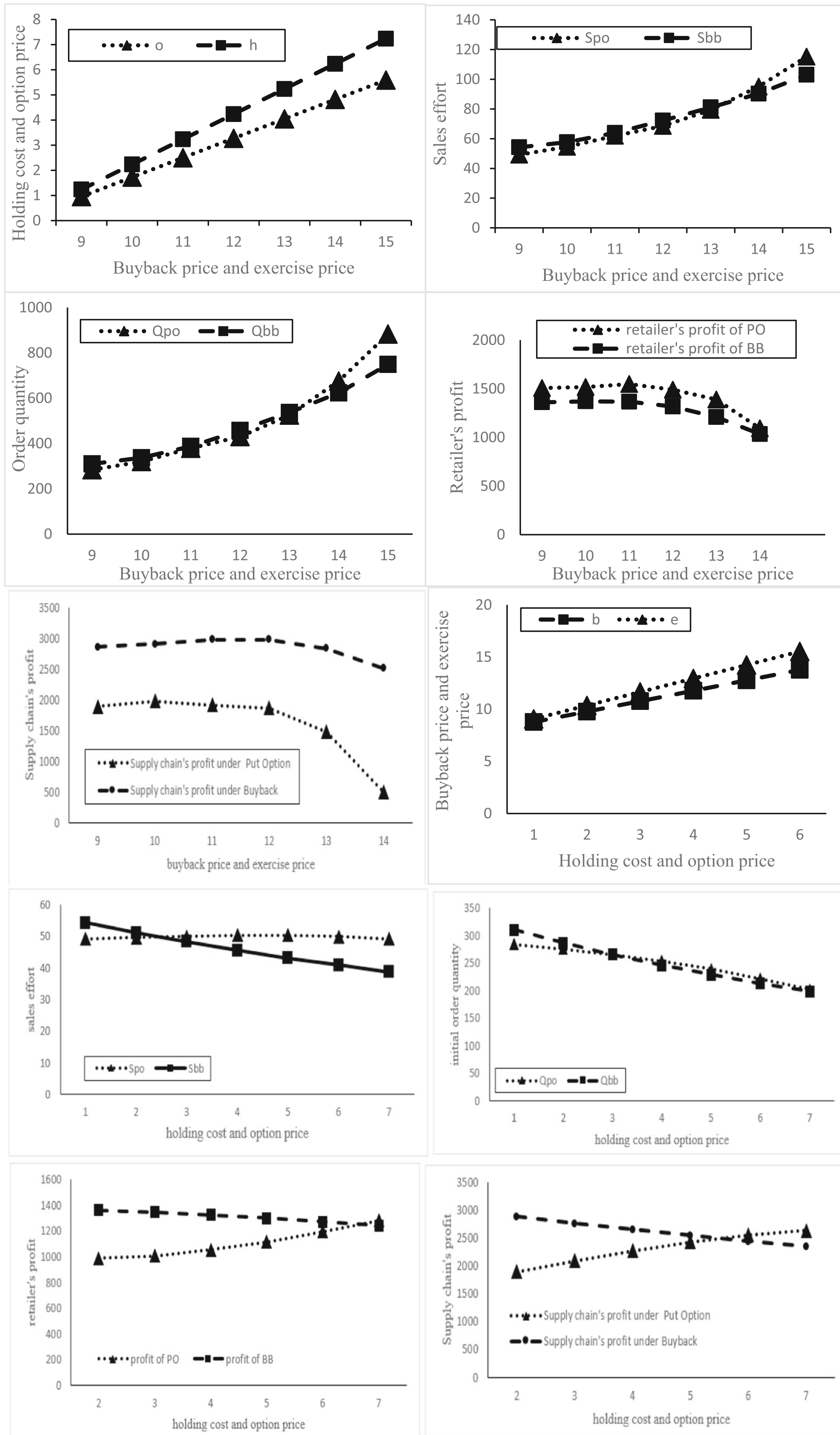


Fig. 2 The comparisons

The comparison of optimal sales effort when $o = h = \varphi$

If the option price raises in the put option contract, the amount of sales effort variable is almost constant and will not change significantly. If the holding cost per order in the buyback contract increases, the amount of sales effort variable in this contract will decrease. Due to the sensitivity of the sales effort variable concerning the holding cost and option price, we can realize that it is possible that each of the sales effort variables, can be more than the other. According to the relationship obtained in Corollary 6, by the mean value theorem. If the chart of the sales effort variable in the put option contract is higher than the chart of this variable in the buyback contract, the relationship of Corollary 6 will satisfy.

The comparison of optimal order quantity when $o = h = \varphi$

If the exercise price is equal to the buyback price, the order quantity in both contracts will be almost equal. Also, it can be realized that according to propositions 2 and 3, the relevance between the sales effort variable and the order quantity variable is direct. Even the intersection of the two graphs of sales effort is approximately equal to the intersection of the order quantity variable graphs in Fig. 2.

The comparison of retailer's profit when $o = h = \varphi$

If the holding cost per order is equal to the option price, the buyback contract will be more profitable than the put option contract for a retailer. Therefore, the retailer of the cosmetic supply chain, in the case of choosing the buyback contract gains more profit than the case the retailer chooses the put option contract.

The comparison of the supply chain's profit when $o = h = \varphi$

If the holding cost per order is equal to the option price, it may be that the buyback agreement will be more profitable than the put option contract for the supply chain. Therefore, it can be induced that if the retailer chooses the buyback agreement, it may gain more benefit for both retailer and chain than the time the retailer chooses the put option contract.

The comparison of optimal sales effort when $o = h = \varphi$ and $e = b = \psi$

If the parameters of the put option contract and the buyback contract are equal, the probability that the sales effort variable in the buyback contract is greater than this variable in the put option contract is high. Also, it can be seen that with the reduction of the parameters of these two contracts, the sales effort variable of the put option contract decreases more than the sales effort variable of the buyback contract, and the difference between the two variables increases. The reason for this issue is when the parameters of the put option contract decrease, then the retailer reduces their order quantity and increases the option order quantity and then the sales effort variable will decrease. On the other hand, by increasing the parameters of these two contracts, the sales effort variable of the put option contract increases more than this variable in the buyback contract (Appendix B).

The comparison of optimal order quantity when $o = h = \varphi$ and $e = b = \psi$

The very similarity of the behavior of the two graphs in Appendix B shows that the sales effort variables and the order quantity are directly related, and all the results obtained in the previous section in this section will be true.

The comparison of retailer's profit when $o = h = \varphi$ and $e = b = \psi$

Appendix B shows that if the parameters of the put option and the buyback agreement are equal, the amount of the retailer's profit when choosing the put option contract will be greater than when choosing the buyback contract. Therefore, it can be concluded that when the parameters of the two contracts have the same amount, the put option contract will be more suitable than a buyback contract to reduce the risk of surplus inventory. Thus, it is a good idea that the retailer chooses the put option contract when the parameters of the buyback contract and the put option contract are equal. when both contracts are offered to the retailer of this cosmetics supply chain, a retailer should choose the put option contract because it will be more profitable for the retailer.

The comparison of supply chain's profit when $o = h = \varphi$ and $e = b = \psi$

Appendix B illustrates that if the parameters of the put option and the buyback contract are equal, the amount of the supply chain's profit when choosing the buyback contract will be greater than when choosing the put option contract. The benefit of the retailer under the put option agreement is greater than the profit of the retailer under the buyback agreement, so it can be concluded that if the retailer chooses the put option contract, it may be more profitable for a retailer than the retailer chooses the buyback contract. Nonetheless, the buyback contract is more profitable than the put option contract for the chain, and over time will make the chain stronger and it is better for a retailer.

The impact of option and exercise price on the sales effort variable in a put option contract

If the exercise price increases, the amount of sales effort variable will increase in some areas of the chart, and in other areas, the chart will decrease. This is logical because, in Corollary 2, it has been proven that in certain situations and by establishing a relationship, the sales effort variable is increasing whit respect to this parameter, and in the other conditions sales effort variable is decreasing concerning this parameter. Also, if the option price increases, sometimes the amount of sales effort variable will decrease, and occasionally will decrease. This is logical because, in Corollary 2, it has been shown that in a certain condition and by satisfying a relationship, the sales effort variable decreases by the options price parameter, and in the other situation sales effort variable is increasing by the options price parameter (Appendix B).

The impact of buyback price and holding cost on the sales effort variable in the buyback contract

If the buyback price raises, thus, the optimal sales effort of the buyback contract will increase, and if the parameter of holding cost per order increases, the optimal sales effort of this contract will decrease. This has been proven in Corollary 1, and it has been shown that if the sales effort variable in the buyback contract increases, the amount of the order quantity in this contract will increase, and if it decreases, the amount of the order quantity will detract (Appendix B).

Implications and conclusion

In the case of comparing the put option and the buyback agreement, if the parameter of the option price and holding cost is equal to each order. This study found that the exercise price in the put option contract will be greater than the buyback price. Furthermore, it is more likely that the decision variables in the buyback contract will be greater than the decision variables of the put option contract. Finally, it is more likely that the retailer gave more benefit under the buyback agreement than the time the retailer chooses the put option contract. Therefore, it can be concluded that if the retailer chooses the buyback agreement in this situation, can gain more benefits.

This matter proves that the contract performs the task of reducing the risk of surplus inventory well, which is more than the case where the retailer chooses the put option contract. Additionally, if the parameter of the exercise price and buyback price be equal then we obtained that the holding cost in the buyback agreement will be greater than the option price. In addition, the decision variables in the put option contract will be greater than the decision variables of the buyback contract. Besides, the benefit of the retailer under the put option agreement is more than the time the retailer chooses the buyback contract. Therefore, it can be concluded if the retailer chooses the put option contract in this situation then the contract will gain more profit for the retailer. The supply chain's profit under this choice is less than the time the retailer chooses the buyback contract. Hence, it is better for all members of the supply chain if the retailer uses the buyback contract.

Next, if the parameter of the put option contract and buyback contract be equal then we obtained that the decision variables in the buyback contract will be greater than the decision variables of the put option contract. The benefit of the retailer under the put option contract is more than the time the retailer chooses the buyback contract. Therefore, it can be concluded that if the retailer chooses the put option contract in the situation the option price and holding cost are equal. As a result, exercise price and buyback price are equal too. This contract can gain more benefit for the retailer versus of buyback contract If the retailer under this condition chooses the buyback contract, it can help SC to gain more profit and become stronger. Hence, it is better for all members of the supply chain.

In this research, we considered a cosmetic SC which includes a wholesaler and a retailer. The demand in this chain is affected by retail price and sales effort variables. First, we provided a model of centralizing SC and founding out the decision variables. In the following, we represented two contracts, buyback and put option contracts, then we obtained the optimal variables of the contracts. Both contracts which are provided have the same effects on the supply chain. Therefore, we compared these contracts with each other in three different cases. In all comparative cases of these two contracts, due to the existence of a sales effort variable as a decision variable in the problem and the dependence of the order quantity variable on the sales effort variable. It needed to examine the sales effort variable first. In each case, after comparing the sales effort variable of buyback and put option contracts, we found that if the sales effort variable for one contract is greater than the sales effort variable of another contract, then it applies to the order quantity of these contracts as well.

In the comparisons made in this research, because the relationship of the sales effort variable in the put option contract has two parts, we cannot easily compare it with the sales effort variable of the buyback contract. For this purpose, the mean value was used to compare two functions of the sales effort. The innovation of this article was to compare the two contracts in a situation where the sales effort variable is the decision variable and is dependent on the order quantity variable. This matter made it difficult to compare the two contracts and this problem was solved by using the mean value theorem in the case of the demand function following the uniform distribution function. Ultimately, the retailer's expected benefit is compared in the two modes of comparison between the two contracts.

This research has tried to provide all the assumptions in the option contract as well as the buyback contract. Further assumptions can be added to this study until this model brings the research closer to the reality of corporations. For instance, the demand function in this research is stochastics and depends on the price and the efforts to sell but to obtain some mathematical relations and proofs about the mean value theorem. It is considered that the demand distribution function follows a uniform distribution function. This research can be accomplished by the normal distribution function, whose cumulative distribution function has an almost linear behavior. Also, another distribution whose cumulative distribution function has a linear relationship can be investigated.

In this study, the demand function depends on the retail price parameter, which can be converted into a decision variable. A new assumption can be added to demand then we can decide the retail price in that situation according to the dependence of the other decision variables on the retail price. Hence, the model will be closer to reality. Next, in this study, the cost of sales effort variable on the retail side has been calculated, which according to the research in the literature, it can be considered that this cost is intended for wholesale or supply chain suppliers. It is one of the incentive policies wholesalers offer to a retailer.

Lastly, put option and buyback contracts have been presented and these contracts have been compared with each other. According to the third sort of options contract, which is a bilateral option contract, and in this contract there are both assumptions to reduce the risk of surplus inventory and shortage risk. We can present a new contract that is mixed with an options contract and a buyback contract. In fact, in terms of structure, it is similar to the bilateral option contract and we can compare the two contracts.

Appendix A: comparison and discussion of buyback and put option contracts

Case 1

The put option contract and the buyback contract are similar in terms of performance. Hence, their parameters have the same effect on the chain. The exercise price in the put option agreement and the buyback price in the buyback agreement are paid by the wholesaler to the retailer for surplus goods. In this part, we assumed that they are equal to each other and we will compare the put option contract and the buyback contract based on this assumption.

Theorem 1 *Given* $e = b = \psi$, *we have* $h > o$.

Theorem 1 shows while we assume the buyback price is equal to the exercise price, the holding cost of surplus orders is greater than the option price. This situation happens when the supply chain has been coordinated via these two contracts.

Proof of Theorem 1 *According to the SC coordination requirement when the decentralized supply chain has been formulated by the buyback contract. We should set* $Q^*_{BB} = Q^*_{CS}$ *and* $S^*_{BB} = S^*_{CS}$, *thus,* $\frac{S_{CS}}{P}F^{-1}\left(\frac{P-c}{P}\right) = \frac{S_{BB}}{P}F^{-1}\left(\frac{P-w}{P+h-b}\right)$. *Indeed, we get* $\frac{P-c}{P} = \frac{P-w}{P+h-b}$, *as a result, we obtain* $h = \frac{(c+b-w)P-cb}{P-c}$, *which is a positive value. On the other side, we also establish the coordination condition when the decentralized supply chain has been formulated with the put option contract,* $Q^*_{PO} = Q^*_{CS}$ *and* $S^*_{PO} = S^*_{CS}$, *thus,* $\frac{S_{CS}}{P}F^{-1}\left(\frac{P-c}{P}\right) = \frac{S_{PO}}{P}F^{-1}\left(\frac{P-w-o}{P-e}\right)$. *Then, we can set* $\frac{P-c}{P} = \frac{P-w-o}{P-e}$, *finally, we reach to* $o = \frac{(c+e-w)P-ce}{P}$, *that is a positive value same as "h."*

Due to the $e = b = \psi$, we have $h = \frac{(c+\psi-w)P-c\psi}{P-c}$ and $o = \frac{(c+\psi-w)P-c\psi}{P}$, as a result $\Delta_{h,o} = \frac{(c+\psi-w)P-c\psi}{P-c} - \frac{(c+\psi-w)P-c\psi}{P} = \frac{c[(c+\psi-w)P-c\psi]}{P(P-c)} > 0$, therefore, we prove that $h > o$.

Corollary 3 *Optimal decision variables of the put options contract are greater than that variable in the buyback contract if "h" and "o" satisfy* $h \geq \frac{o(P-\psi)}{P-w-o}$.

Corollary 3 shows that if the holding cost parameter in a buyback contract is greater than the relationship which is dependent on the option price and includes other parameters in two contracts. Therefore, the decision variable in put option contracts is greater than that variable in the buyback contract.

Proof* of Corollary *3 *According to Corollary 1 and 2, it can be concluded that the relation of the optimal sales effort level in the put option contract and the buyback agreement,* $S^*_{PO} = \frac{(P-w-o)}{\alpha P}F^{-1}\left(\frac{P-w-o}{P-e}\right) - \frac{P-e}{\alpha P}\int_0^{F^{-1}\left(\frac{P-w-o}{P-e}\right)} F(x)dx + \frac{o}{\alpha P}F^{-1}\left(\frac{o}{e}\right) - \frac{e}{\alpha P}\int_0^{F^{-1}\left(\frac{o}{e}\right)} F(x)dx, S^*_{BB} = \frac{(P-w)}{\alpha P}F^{-1}\left(\frac{P-w}{P+h-b}\right) - \frac{P+h-b}{\alpha P}\int_0^{F^{-1}\left(\frac{P-w}{P+h-b}\right)} F(x)dx$ *are increasing in an argument of* $F^{-1}()$ *in the close form of optimal order quantity. Note that the relation of optimal sales effort level in the put option agreement is influenced by* *optimal option and initial order quantity. Hence, the relation is consists of two parts, the first term.*

$$S^*_{PO1} = \frac{(P-w-o)}{\alpha P} F^{-1}\left(\frac{P-w-o}{P-e}\right) - \frac{P-e}{\alpha P} \int_0^{F^{-1}\left(\frac{P-w-o}{P-e}\right)} F(x)dx$$

is obtained by the initial order quantity, and the second term $S^*_{PO2} = \frac{o}{\alpha P} F^{-1}\left(\frac{o}{e}\right) - \frac{e}{\alpha P} \int_0^{F^{-1}\left(\frac{o}{e}\right)} F(x)dx$ *is obtained by the option order quantity. Thus, it can be proved that if* $\frac{P-w-o}{P-\psi} = \frac{P-w}{P+h-\psi}$ *and we reach to* $h = \frac{o(P-\psi)}{P-w-o}$ *then if it satisfies, therefore,* $S^*_{PO1} = S^*_{BB}$ *and in the following* $S^*_{PO} = S^*_{PO1} + S^*_{PO2} > S^*_{BB}$.

That means if the put option agreement coordinates the supply chain. The optimal sales effort of the retailer is greater than when the chain has been coordinated by a buyback contract. Finally, we have considered the optimal order quantity has a direct relation with the optimal sales effort then it can be deduced $Q^*_{PO} > Q^*_{BB}$. Also, if $\frac{P-w-o}{P-\psi} > \frac{P-w}{P+h-\psi}$ then it changes to $h > \frac{o(P-\psi)}{P-w-o}$ then if it satisfies,$S^*_{PO1} > S^*_{BB}$ and definitely $S^*_{PO} > S^*_{BB}$. Hence, as a previous part, we can conclude$Q^*_{PO} > Q^*_{BB}$.

Corollary 4 *If* $h < \frac{o(P-\psi)}{P-w-o}$ *and* $\frac{F^{-1}\left(\frac{P-w}{P+h-\psi} + \frac{o(P-\psi)-h(P-w-o)}{2(P-\psi)(P+h-\psi)}\right)}{F^{-1}\left(\frac{o}{2\psi}\right)} < \frac{(P-\psi)(P+h-\psi)}{(P-\psi)-h(P-w-o)\psi}$ *optimal sales effort and amount of optimal order in put option contract are greater than that variable for buyback contract.*

Corollary 3 shows that if the holding cost parameter in a buyback contract is greater than the relationship which is dependent on the option price and includes other parameters in two contracts. Therefore, the decision variable in put option contracts such as optimal sales effort variable and optimal order quantity is greater than that variable for buyback contract.

Proof* of Corollary *4 *Based on proof of Corollary 3, it can be concluded that if parameters of two contracts satisfy* $h < \frac{o(P-\psi)}{P-w-o}$, *then the argument of sales effort will have* $\frac{P-w-o}{P-\psi} > \frac{P-w}{P+h-\psi}$ *and we can due to the ascending behavior of the optimal sales effort function we can deduce* $S^*_{PO1} < S^*_{BB}$. *To continue we must calculate the difference between* S^*_{PO1} *and* S^*_{BB} *then find out the result of this part we should compare that with* S^*_{PO2}.

To compare the result $S^*_{BB} - S^*_{PO1}$ with the amount of the second part of the sales effort equation for the put option contract, the mean value theorem should be used. The mean value theorem proves that it $f()$ is a continuous function in the range $[a, b]$ and derivable in the range (a, b), then at least there is one coordinating point such as $c \in (a, b)$ that $f\prime(c) = \frac{f(b)-f(a)}{b-a}$.

The functions of sales effort in the option and the buyback contract follow $G(u) = uF^{-1}(u) - \int_0^{F^{-1}(u)} F(x)\,dx$ then according to Corollary 1 we have $\frac{\partial G(u)}{\partial u} = F^{-1}(u) \geq 0$. To obtain the value of "c" for the sales effort function, it can be considered that if the demand function follows a continuous uniform distribution function $D \sim U(a, b)$, then we have $u = \frac{x-a}{b-a}$ and it has found out that $F^{-1}(u) = u(b-a) + a$.

We put the previous relationship into $G(u) = uF^{-1}(u) - \int_0^{F^{-1}(u)} F(x)\,dx$ and reach to $G(u) = \frac{u^2(b-a)}{2} + ua - \frac{a^2}{2(b-a)}$. For obtaining the amount of "c" we should be used to $G\prime(c) = \frac{G(u_2)-G(u_1)}{u_2-u_1}$.

In the following we calculate,

$$\frac{G(u_2) - G(u_1)}{u_2 - u_1} = \frac{\frac{u_2^2(b-a)}{2} + u_2 a - \frac{a^2}{2(b-a)} - \frac{u_1^2(b-a)}{2} - u_1 a + \frac{a^2}{2(b-a)}}{u_2 - u_1} = \frac{(u_1+u_2)}{2}$$

$(b-a) + a$, then it is obvious $c = \frac{u_1+u_2}{2}$ or in other words the amount of "c" is the middle value of the two arguments of sales effort function.

Finally, to compare the two variables of sales effort in buyback contract and put option contract due to $S^*_{BB} - S^*_{PO} = S^*_{BB} - S^*_{PO1} - S^*_{PO2}$ we reached to. $\frac{o(P-\psi) - h(P-w-o)\psi}{(P-\psi)(P+h-\psi)} F^{-1}\left(\frac{P-w}{P+h-\psi} + \frac{o(P-\psi)-h(P-w-o)}{2(P-\psi)(P+h-\psi)}\right)$, to subtract the first part of sales effort in the put option contract from the amount of sales effort of the buyback contract and we have $\frac{o}{\psi} F^{-1}\left(\frac{o}{2\psi}\right)$ for the second part of sales effort variable in a put option contract. As a result, it is obvious if $\frac{o(P-\psi) - h(P-w-o)\psi}{(P-\psi)(P+h-\psi)} F^{-1}\left(\frac{P-w}{P+h-\psi} + \frac{o(P-\psi)-h(P-w-o)}{2(P-\psi)(P+h-\psi)}\right) < \frac{o}{\psi} F^{-1}\left(\frac{o}{2\psi}\right)$, then $S^*_{PO} > S^*_{BB}$ hence, we can conclude $Q^*_{PO} > Q^*_{BB}$.

Case 2

As in Case 1, in this part, we compared the buyback and the put option agreement. Meanwhile, we consider the option price parameter in the put option contract which is paid to the wholesaler for the option orders and the holding cost imposed on the retailer by the surplus products in the buyback contract are equal. Eventually, we will operate like Case 1.

Theorem 2 *Given* $o = h = \varphi$, *we have* $e > b$.

In Theorem 2, it is proved that if the holding cost and the option price for the surplus products are equal, the exercise price parameter in the put option contract would be greater than the buyback price in the buyback contract. It happens while the supply chain is coordinated by these two contracts.

Proof of Theorem 2 *Regarding the condition of the supply chain coordination when the decentralized SC has been modeled by the buyback contract, as a Theorem 1, we should adjust* $Q^*_{BB} = Q^*_{CS}$ *and* $S^*_{BB} = S^*_{CS}$*; thus,* $\frac{S_{CS}}{P}F^{-1}\left(\frac{P-c}{P}\right) = \frac{S_{BB}}{P}F^{-1}\left(\frac{P-w}{P+h-b}\right)$*, and we get* $\frac{P-c}{P} = \frac{P-w}{P+h-b}$*. As a result, we reach,* $b = \frac{(w+h-c)P-ch}{P-c}$ *which is a positive value. On the other hand, we also establish the coordination requirement when the decentralized supply chain has been formulated with the put option contract,* $Q^*_{PO} = Q^*_{CS}$ *and* $S^*_{PO} = S^*_{CS}$*; thus,* $\frac{S_{CS}}{P}F^{-1}\left(\frac{P-c}{P}\right) = \frac{S_{PO}}{P}F^{-1}\left(\frac{P-w-o}{P-e}\right)$ *then we can get* $\frac{P-c}{P} = \frac{P-w-o}{P-e}$*, finally, we obtain* $e = \frac{(w+o-c)P}{P-c}$ *that is a positive value like "b." Due to the* $o = h = \varphi$*, we have* $b = \frac{(w+\varphi-c)P-c\varphi}{P-c}$ *and* $e = \frac{(w+\varphi-c)P}{P-c}$*, as a result* $\Delta_{e,b} = \frac{(w+\varphi-c)P}{P-c} - \frac{(w+\varphi-c)P-c\varphi}{P-c} = \frac{c\varphi}{(P-c)} > 0$*. Therefore, we prove that* $e > b$*.*

Corollary 5 *A put options contract is more suitable than a buyback contract to reduce inventory risk if " e" and "b" satisfy* $e \geq \frac{b(P-w-\varphi)+\varphi(w+\varphi)}{P-w}$*.*

In Corollary 5, we represent that if the exercise price parameter in the put option contract is greater than the relationship which is dependent on the buyback price. Hence, the optimal order quantity is greater than the optimal order quantity in a buyback agreement. As a result, in this situation, the put option contract is more suitable than the buyback agreement to detract from the retailer's inventory risk.

Proof* of Corollary *5 *Similar to Corollary 3, we can conclude the relationship of the optimal sales effort in the put option contract and the buyback agreement.* $S^*_{PO} = \frac{(P-w-o)}{\alpha P}F^{-1}\left(\frac{P-w-o}{P-e}\right) - \frac{P-e}{\alpha P}\int_0^{F^{-1}\left(\frac{P-w-o}{P-e}\right)} F(x)dx + \frac{o}{\alpha P}F^{-1}\left(\frac{o}{e}\right) - \frac{e}{\alpha P}\int_0^{F^{-1}\left(\frac{o}{e}\right)} F(x)dx$, $S^*_{BB} = \frac{(P-w)}{\alpha P}F^{-1}\left(\frac{P-w}{P+h-b}\right) - \frac{P+h-b}{\alpha P}\int_0^{F^{-1}\left(\frac{P-w}{P+h-b}\right)} F(x)dx$ *are increasing in an argument of* $F^{-1}()$ in the relationship of the optimal order quantity.

According to Corollary 3, that the relationship between optimal sales effort in the put option agreement is influenced by the optimal order and options quantity. In fact, the relationship consists of two terms, the first term $S^*_{PO1} = \frac{(P-w-o)}{\alpha P}F^{-1}\left(\frac{P-w-o}{P-e}\right) - \frac{P-e}{\alpha P}\int_0^{F^{-1}\left(\frac{P-w-o}{P-e}\right)} F(x)dx$ is obtained by the initial order quantity, and the second term $S^*_{PO2} = \frac{o}{\alpha P}F^{-1}\left(\frac{o}{e}\right) - \frac{e}{\alpha P}\int_0^{F^{-1}\left(\frac{o}{e}\right)} F(x)dx$ is obtained by the option order quantity. Therefore, it can be proved that if $\frac{P-w-\varphi}{P-e} = \frac{P-w}{P+\varphi-b}$ and we reach to $e = \frac{b(P-w-\varphi)+\varphi(w+\varphi)}{P-w}$ then if it satisfies, therefore, $S^*_{PO1} = S^*_{BB}$ and in the following $S^*_{PO} = S^*_{PO1} + S^*_{PO2} > S^*_{BB}$.

That means, the optimal sales effort of the retailer when the chain has been coordinated by put option agreement is greater than when the chain has been coordinated by a buyback contract. Finally, we have considered that the amount of optimal order has a direct relationship with the amount of optimal sales effort then it can be deduced $Q^*_{PO} > Q^*_{BB}$.

If $\frac{P-w-\varphi}{P-e} > \frac{P-w}{P+\varphi-b}$, then it changes to $e > \frac{b(P-w-\varphi)+\varphi(w+\varphi)}{P-w}$ then if it satisfies. $S^*_{PO1} > S^*_{BB}$ and definitely $S^*_{PO} > S^*_{BB}$ hence as a previous part, we can conclude $Q^*_{PO} > Q^*_{BB}$.

Corollary 6 *Optimal sales effort and optimal order quantity in put option contract are greater than that variable for buyback contract if parameters satisfy* $e < \frac{b(P-w-\varphi)+\varphi(w+\varphi)}{P-w}$ *and* $\frac{b(P-w-\varphi)-e(P-w)+\varphi(w+\varphi)}{(P+\varphi-b)(P-e)}F^{-1}\left(\frac{P-w}{P+\varphi-b} + \frac{b(P-w-\varphi)-e(P-w)+\varphi(w+\varphi)}{2(P+\varphi-b)(P-e)}\right) < \left(\frac{\varphi}{e}\right)F^{-1}\left(\frac{\varphi}{e}\right)$*.*

Corollary 6 demonstrates that if the exercise price is greater than the relationship which is dependent on the buyback price and involves other parameters that there are in buyback and put option contracts. Therefore, an optimal sales effort variable in the put option contract is greater than optimal sales effort in the buyback contract. Hence according to a direct relationship between optimal decision variables we can conclude $Q^*_{PO} > Q^*_{BB}$.

Proof* of Corollary *6 *Based on proof of Corollary 5, it can be concluded that if the parameters of two contracts satisfy* $e < \frac{b(P-w-\varphi)+\varphi(w+\varphi)}{P-w}$*, then the argument of sales effort will have* $\frac{P-w-\varphi}{P-e} > \frac{P-w}{P+\varphi-b}$ *and due to the ascending behavior of the optimal sales effort function, we can conclude* $S^*_{PO} < S^*_{BB}$*. In following we must calculate the difference between* S^*_{PO1} *and* S^*_{BB} *then we should compare that with* S^*_{PO2}*.*

To compare the result $S^*_{BB} - S^*_{PO1}$ with the amount of the second part of the sales effort equation for the put option contract, according to Corollary 4 we can use the mean value theorem and find the amount of "c" for this part. We've proved that $c = \frac{u_1+u_2}{2}$ or in other words the amount of "c" is the middle value of the two arguments of sales effort function. eventually, to compare the two variables of sales effort in buyback contract and put option contract according to the $S^*_{BB} - S^*_{PO} = S^*_{BB} - S^*_{PO1} - S^*_{PO2}$, we reached to.

$$\frac{b(P-w-\varphi)-e(P-w)+\varphi(w+\varphi)}{(P+\varphi-b)(P-e)}F^{-1}\left(\frac{P-w}{P+\varphi-b}+\frac{b(P-w-\varphi)-e(P-w)+\varphi(w+\varphi)}{2(P+\varphi-b)(P-e)}\right),$$

to subtract the first part of sales effort in the put option contract from the amount of sales effort of the buyback contract then we have $\frac{o}{\psi}F^{-1}(\frac{o}{2\psi})$ for the second part of sales effort variable in a put option contract. As a result, it is obvious if $\frac{b(P-w-\varphi)-e(P-w)+\varphi(w+\varphi)}{(P+\varphi-b)(P-e)}F^{-1}\left(\frac{P-w}{P+\varphi-b}+\frac{b(P-w-\varphi)-e(P-w)+\varphi(w+\varphi)}{2(P+\varphi-b)(P-e)}\right)$ $<(\frac{\varphi}{e})F^{-1}(\frac{\varphi}{e})$, then $S^*_{PO} > S^*_{BB}$ hence, we can conclude $Q^*_{PO} > Q^*_{BB}$.

Case 3

The put option contract and the buyback contract are similar in terms of performance, so their parameters have the same effect on the supply chain. Hence, the exercise price and the buyback price in the buyback contract that is paid by the wholesaler to the retailer for surplus goods, in this part we assumed that they are equal to each other and on the other hand the option price in the put option contract and holding cost that costs for orders which are surplus when sales season become to over in the buyback contract again in this part we assumed that they are equal then we will compare the put option contract and the buyback contract based on this assumption.

Theorem 3 *Given* $o = h = \varphi$ *and* $e = b = \psi$ *we have* $\frac{P-w}{P+h-b} > \frac{P-w-o}{P-e} > \frac{o}{e}$.

In Theorem 3, it is proved that if the option price and the holding cost for the surplus products are equal then the exercise price parameter in the put option contract and the buyback price in the buyback contract be equal in value. The argument of sales effort variables in two contracts satisfy $\frac{P-w}{P+h-b} > \frac{P-w-o}{P-e} > \frac{o}{e}$.

Proof of Theorem 3 *According to Proposition 3, it is clear to prove that* $\frac{P-w-o}{P-e} > \frac{o}{e}$ *because* $\frac{P-w-o}{P-e}$ *is an argument of* Q^*_{PO} *and* $\frac{o}{e}$ *is an argument of* $Q^*_{PO} - M^*_{PO}$. *In continuing due to* $o = h = \varphi$ *and* $e = b = \psi$, *we have* $A = \frac{P-w}{P+\varphi-\psi}$ *and* $B = \frac{P-w-\varphi}{P-\psi}$, *then we can reach* $\Delta_{A,B} = \frac{P-w}{P+\varphi-\psi} - \frac{P-w-\varphi}{P-\psi}$ $= \frac{\varphi(w+\varphi-\psi)}{(P+\varphi-\psi)(P-\psi)}$. *Therefore, based on Assumption 3 and Assumption 5 we can conclude that* $\Delta_{A,B} > 0$, as a result $\frac{P-w}{P+h-b} > \frac{P-w-o}{P-e}$.

Corollary 7 *If* $\frac{F^{-1}\left(\frac{P-w}{P+\varphi-\psi}+\frac{\varphi(w+\varphi-\psi)}{2(P-\psi)(P+\varphi-\psi)}\right)}{F^{-1}(\frac{\varphi}{2\psi})} < \frac{(P-\psi)(P+\varphi-\psi)}{\psi(w+\varphi-\psi)}$ *optimal sales effort and optimal order quantity in put option contract are greater than that variable for buyback contract.*

Corollary 7 shows that if the information represents in Theorem 3 is established in two decentralized contracts, we can find out a relationship that satisfies the throw problem's parameters thereupon decision variables in the put option contract becomes greater than the buyback contract's decision variables.

***Proof* of Corollary 7** *Based on proof of Theorem 3, the argument of optimal sales effort function will have* $\frac{P-w}{P+h-b} > \frac{P-w-o}{P-e} > \frac{o}{e}$ *and due to the ascending behavior of the optimal sales effort function, we can deduce* $S^*_{PO2} < S^*_{PO1} < S^*_{BB}$. *In following we must calculate the difference between* S^*_{PO1} *and* S^*_{BB} *then for finding out the result of this part we should compare that with* S^*_{PO2}. *We should compare the result of* $S^*_{BB} - S^*_{PO1}$ *which has a negative amount with the amount of the second part of the optimal sales effort equation for the put option contract, as previous proof we can use the mean value theorem.*

The functions of optimal sales effort in the put option and the buyback contract follow $G(u) = uF^{-1}(u) - \int_0^{F^{-1}(u)} F(x)\,dx$ then based on Corollary 1 we reached to $\frac{\partial G(u)}{\partial u} = F^{-1}(u) \geq 0$. To obtain the value of "c" for the sales effort function, like before if the demand function follows a continuous uniform distribution function $D \sim U(a,b)$, then we have $u = \frac{x-a}{b-a}$ and it has found out that $F^{-1}(u) = u(b-a) + a$ therefore, we reach to $G(u) = \frac{u^2(b-a)}{2} + ua - \frac{a^2}{2(b-a)}$. For obtaining the amount of "c" we can use $G\prime(c) = \frac{G(u_2)-G(u_1)}{u_2-u_1}$.

In the following we calculate.

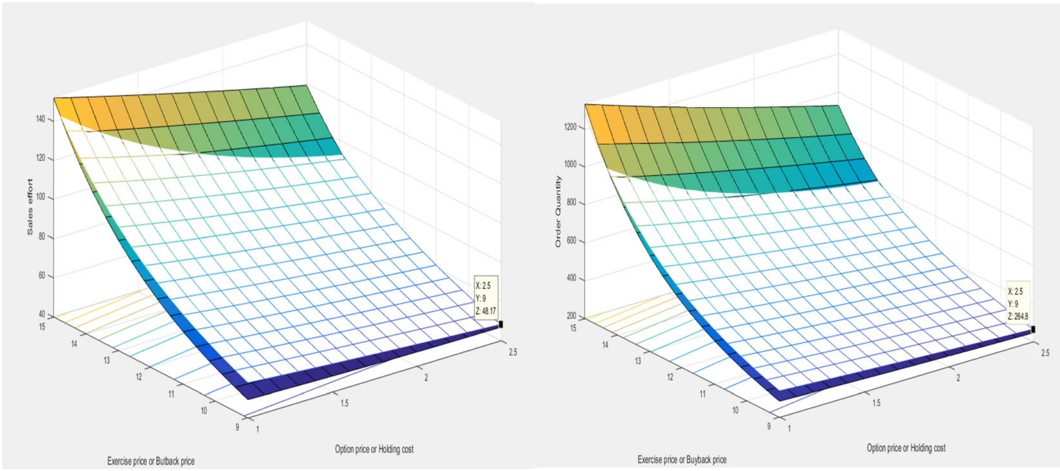
$\frac{G(u_2)-G(u_1)}{u_2-u_1} = \frac{\frac{u_2^2(b-a)}{2}+u_2a-\frac{a^2}{2(b-a)}-\frac{u_1^2(b-a)}{2}-u_1a+\frac{a^2}{2(b-a)}}{u_2-u_1} = \frac{(u_1+u_2)}{2}(b-a)+a$, then it is obvious $c = \frac{u_1+u_2}{2}$ or in other words the amount of "c" is the middle value of the two arguments of optimal sales effort function. Finally, to compare the two variables of sales effort in buyback contract and put option contract due to $S^*_{BB} - S^*_{PO} = S^*_{BB} - S^*_{PO1} - S^*_{PO2}$ we reached to

$$\frac{\varphi(w+\varphi-\psi)}{(P-\psi)(P+\varphi-\psi)}F^{-1}\left(\frac{P-w}{P+\varphi-\psi}+\frac{\varphi(w+\varphi-\psi)}{2(P-\psi)(P+\varphi-\psi)}\right)$$

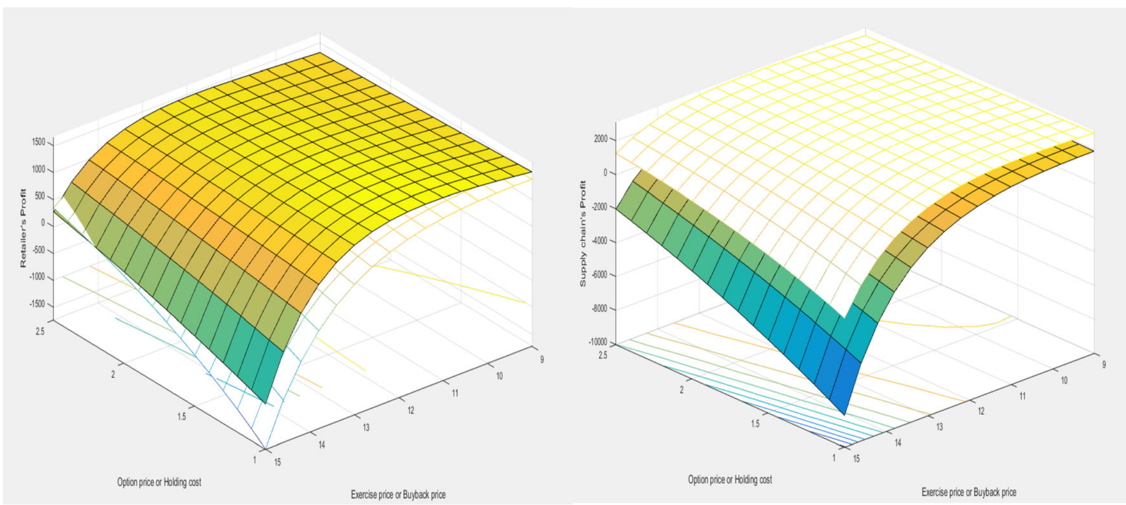
, to subtract the first part of sales effort in the put option contract from the amount of sales effort of the buyback contract and we have $\frac{\varphi}{\psi}F^{-1}(\frac{\varphi}{2\psi})$ for the second part of sales effort variable in a put option contract. As a result, it is clear if $\frac{\varphi(w+\varphi-\psi)}{(P-\psi)(P+\varphi-\psi)}F^{-1}\left(\frac{P-w}{P+\varphi-\psi}+\frac{\varphi(w+\varphi-\psi)}{2(P-\psi)(P+\varphi-\psi)}\right)$ $< \frac{\varphi}{\psi}F^{-1}(\frac{\varphi}{2\psi})$, then $S^*_{PO} > S^*_{BB}$. Hence, we can conclude $Q^*_{PO} > Q^*_{BB}$.

Appendix B: the comparison

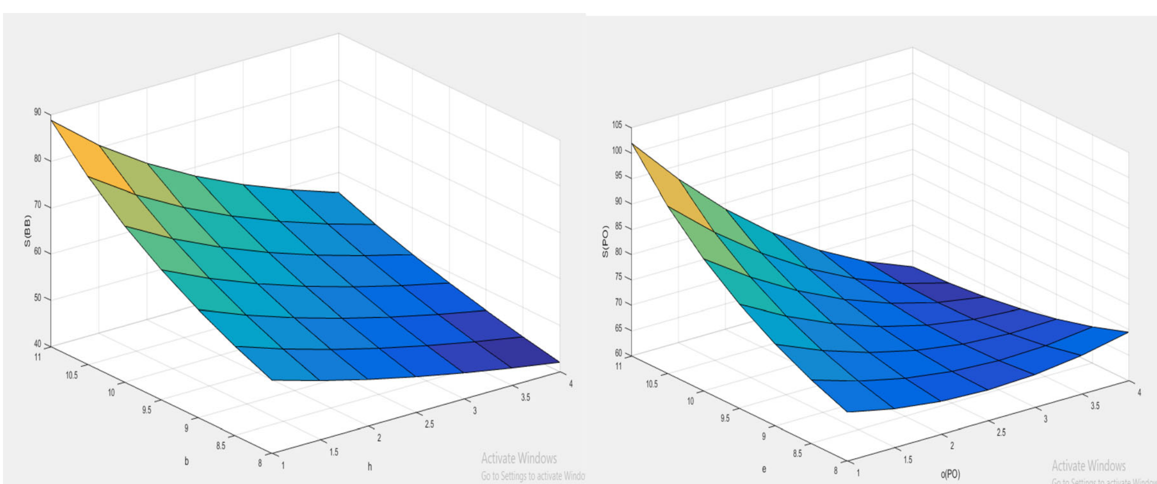
The comparison of the sales effort (left) and order quantity (right).



The comparison of the retailer's profit (left) and the supply chain's profit (right).



The impact of *e* and *o* on sales effort variable in the put option contract (left), and The impact of *b* and *h* on sales effort variable in buyback contract (right).



Appendix C: proof of Corollary

***Proof* of Corollary 1** *The buyback contract's optimal order quantity* $Q_{BB}^{*}=\frac{S_{BB}}{P}F^{-1}\left(\frac{P-w}{P+h-b}\right)$*. In addition to the buyback contract parameters, it depends on the variable of the optimal sales effort* $S_{BB}^{*}=\frac{(P-w)}{\alpha P}F^{-1}\left(\frac{P-w}{P+h-b}\right)-\frac{P+h-b}{\alpha P}\int_{0}^{F^{-1}\left(\frac{P-w}{P+h-b}\right)}F(x)dx$*, for the relationship of the optimal sales effort, we have* $G(u)=uF^{-1}(u)-\int_{0}^{F^{-1}(u)}F(x)dx$*, according to* $\frac{\partial G(u)}{\partial u}=F^{-1}\ (u)\geq 0$*, that is increasing in "u." If* $b=b_0$ *and* $u_0=\frac{P-w}{P+h-b_0}$ *by increasing the buyback price, we set* $b=b_0+\delta_1$ *and* $u_1=\frac{P-w}{P+h-(b_0+\delta_1)}$*, we reach to* $\Delta u_{1,0}=\frac{\delta_1(P-w)}{(P+h-b_0)(P+h-b_0+\delta_1)}>0$*,* δ_1 *is a positive value. Thus, as the buyback price increases, the optimal sales effort will increase as well as the amount of optimal order quantity.*

In the following, to prove that the optimal order quantity is decreasing in the holding cost for surplus products. We have $h=h_0$ and $u_0=\frac{P-w}{P+h_0-b}$, after that if the holding cost increases, we can adjust $h=h_0+\delta_2$ and $u_2=\frac{P-w}{P+(h_0+\delta_2)-b}$, then we find $\Delta u_{2,0}=-\frac{\delta_2(P-w)}{(P+h_0-b)(P+(h_0+\delta_2)-b)}<0$, while δ_2 is positive. Hence, it can be concluded that by increasing "h," S_{BB}^{*} and Q_{BB}^{*} will decrease.

***Proof* of Corollary 2** *The put option contract's optimal order quantity* $Q_{PO}^{*}=\frac{S_{PO}}{P}F^{-1}\left(\frac{P-w-o}{P-e}\right)$*. In addition, to put option contract parameters, it depends on the optimal sales effort* $S_{PO}^{*}=\frac{(P-w-o)}{\alpha P}F^{-1}\left(\frac{P-w-o}{P-e}\right)-\frac{P-e}{\alpha P}\int_{0}^{F^{-1}\left(\frac{P-w-o}{P-e}\right)}F(x)dx+\frac{o}{\alpha P}F^{-1}\left(\frac{o}{e}\right)-\frac{e}{\alpha P}\int_{0}^{F^{-1}\left(\frac{o}{e}\right)}F(x)dx$*, for the relationship of the optimal sales effort, we have* $G(u)=uF^{-1}(u)-\int_{0}^{F^{-1}(u)}F(x)dx$*, according to Corollary 1, that is increasing in "u." Sales effort variable in put option contract involves two parts. One of them is* $S_{PO1}^{*}=\frac{(P-w-o)}{\alpha P}F^{-1}\left(\frac{P-w-o}{P-e}\right)-\frac{P-e}{\alpha P}\int_{0}^{F^{-1}\left(\frac{P-w-o}{P-e}\right)}F(x)dx$ *and another one is* $S_{PO2}^{*}=\frac{o}{\alpha P}F^{-1}\left(\frac{o}{e}\right)-\frac{e}{\alpha P}\int_{0}^{F^{-1}\left(\frac{o}{e}\right)}F(x)dx$*. The argument* S_{PO1}^{*} *is* $u_1=\frac{P-w-o}{P-e}$ *and the argument of* S_{PO2}^{*} *is* $u_2=\frac{o}{e}$*. We find out* $\frac{\partial u_1}{\partial e}=\frac{P-w-o}{(P-e)^2}>0$ *and according to* $\frac{\partial S_{PO1}}{\partial u_1}>0$*; therefore, as a result* $\frac{\partial S_{PO1}}{\partial e}=\frac{\partial S_{PO1}}{\partial u_1}\times\frac{\partial u_1}{\partial e}>0$*, then the first part of optimal sales effort* S_{PO1}^{*} *is increasing in* e*. Also, we reach* $\frac{\partial u_2}{\partial e}=-\frac{1}{e^2}<0$ *and we have* $\frac{\partial S_{PO2}}{\partial u_2}>0$ *hence* $\frac{\partial S_{PO2}}{\partial e}=\frac{\partial S_{PO2}}{\partial u_2}\times\frac{\partial u_2}{\partial e}<0$*, then the second part of optimal sales effort* S_{PO2}^{*} *is decreasing in* e. As it turned out, the first part of the sales effort variable has incremental behavior relative to the exercise price parameter and the second part has declining behavior. Therefore, the sales effort variable relative to this parameter has both incremental and decreasing behavior. we must use the mean value theorem to find out the impact of exercise price on the sales effort variable. The mean value theorem proves that if $f(\ )$ is a continuous function in the range $[a,b]$ and derivable in the range (a,b), then at least there is one point such as $c\in(a,b)$ that $f\prime(c)=\frac{f(b)-f(a)}{b-a}$. The functions of optimal sales effort in the put option contract follow $G(u)=uF^{-1}(u)-\int_{0}^{F^{-1}(u)}F(x)\,dx$ then based on Corollary 1 we reached to $\frac{\partial G(u)}{\partial u}=F^{-1}(u)\geq 0$. To obtain the value of "c" for the sales effort function, like before if the demand function follows a continuous uniform distribution function $D\sim U(a,b)$, then we have $u=\frac{x-a}{b-a}$ and it has found out that $F^{-1}(u)=u(b-a)+a$ therefore, we reach to $G(u)=\frac{u^2(b-a)}{2}+ua-\frac{a^2}{2(b-a)}$. For obtaining the amount of "c" we can use $G\prime(c)=\frac{G(u_1)-G(u_0)}{u_1-u_0}$. In the following we calculate $\frac{G(u_1)-G(u_0)}{u_1-u_0}=\frac{\frac{u_1^2(b-a)}{2}+u_1a-\frac{a^2}{2(b-a)}-\frac{u_0^2(b-a)}{2}-u_0a+\frac{a^2}{2(b-a)}}{u_1-u_0}=\frac{(u_1+u_0)}{2}(b-a)+a$, then it is obvious $c=\frac{u_1+u_2}{2}$ or in other words the amount of "c" is the middle value of the two arguments of optimal sales effort function. Based on the mean value theorem (MVT) we can obtain $\frac{\Delta S_{PO1}^{*}}{\frac{P-w-o_{PO}}{(P-e)^2}}=F^{-1}\left(\frac{P-w-o_{PO}}{P-e}+\frac{P-w-o_{PO}}{2(P-e)^2}\right)$ and $\frac{\Delta S_{PO2}^{*}}{\frac{-1}{e^2}}=F^{-1}\left(\frac{o_{CO}}{e}-\frac{1}{2e^2}\right)$. If $\Delta S_{PO1}^{*}>|\Delta S_{PO2}^{*}|$ then we must calculate

$$\frac{P-w-o}{(P-e)^2}F^{-1}\left(\frac{P-w-o}{P-e}+\frac{P-w-o}{2(P-e)^2}\right)>\frac{1}{e^2}F^{-1}\left(\frac{o_{CO}}{e}-\frac{1}{2e^2}\right)$$

, then defiantly sales effort variable and order quantity are increasing in exercise price.

For proving the impact of option price on sales effort variable in continue. We reach to $\frac{\partial u_1}{\partial o}=-\frac{1}{P-e}<0$ and according to $\frac{\partial S_{PO1}}{\partial u_1}>0$ hence as a result $\frac{\partial S_{PO1}}{\partial o}=\frac{\partial S_{PO1}}{\partial u_1}\times\frac{\partial u_1}{\partial o}<0$, then the first part of optimal sales effort S_{PO1}^{*} is decreasing in o. also We obtain $\frac{\partial u_2}{\partial o}=\frac{1}{e}>0$ and we have $\frac{\partial S_{PO2}}{\partial u_2}>0$ hence $\frac{\partial S_{PO2}}{\partial o}=\frac{\partial S_{PO2}}{\partial u_2}\times\frac{\partial u_2}{\partial o}>0$, then the second part of optimal sales effort S_{PO2}^{*} is increasing in o. As it turned out, the first part of the sales effort variable has declining behavior relative to the option price parameter and the second part has incremental behavior. Hence, the sales effort variable relative to this parameter has both incremental and decreasing behavior. We must use the mean value theorem to obtain the effect of option price on the sales effort variable. Based on the mean value theorem we can obtain $\frac{\Delta S^*_{PO1}}{-\frac{1}{P-e}} = F^{-1}\left(\frac{P-w-o_{PO}}{P-e} - \frac{1}{2(P-e)}\right)$ and $\frac{\Delta S^*_{PO2}}{\frac{1}{e}} = F^{-1}\left(\frac{o_{PO}}{e} - \frac{1}{2e}\right)$. If $|\Delta S^*_{PO\,1}| > \Delta S^*_{PO\,2}$ then we must calculate

$$\frac{1}{P-e}F^{-1}\left(\frac{P-w-o_{PO}}{P-e} - \frac{1}{2(P-e)}\right) > \frac{1}{e}F^{-1}\left(\frac{o_{PO}}{e} - \frac{1}{2e}\right)$$

. This defiant sales effort and order quantity variable are decreasing in the option price.

Appendix D: proof of propositions

Proof of Proposition 1 *First, to prove the concavity of the expected profit function of centralized supply chain regarding the sales effort and the order quantity, the Hessian matrix should be considered as follows*:

$$\begin{bmatrix} \frac{\partial^2 E[\Pi_{CS}]}{\partial Q^2_{CS}} & \frac{\partial^2 E[\Pi_{CS}]}{\partial Q_{SC}\partial S_{SC}} \\ \frac{\partial^2 E[\Pi_{CS}]}{\partial S_{SC}\partial Q_{SC}} & \frac{\partial^2 E[\Pi_{CS}]}{\partial S^2_{CS}} \end{bmatrix} = \begin{bmatrix} -\frac{P^2}{S_{CS}}f\left(\frac{PQ_{CS}}{S_{CS}}\right) & \frac{P^2 Q}{S^2_{CS}}f\left(\frac{PQ_{CS}}{S_{CS}}\right) \\ \frac{P^2 Q}{S^2_{CS}}f\left(\frac{PQ_{CS}}{S_{CS}}\right) & -\frac{P^2 Q^2}{S^3_{CS}}f\left(\frac{PQ_{CS}}{S_{CS}}\right) - \alpha \end{bmatrix}$$

with respect to variables equal to zero. $\frac{\partial E[\Pi_{CS}]}{\partial Q_{CS}} = P - PF\left(\frac{PQ_{CS}}{S_{CS}}\right) - c$ is the first-order partial derivative of the centralized supply chain's expected profit function Q_{CS}. If we set this relation to zero, the optimal order quantity will be obtained $Q^*_{CS} = \frac{S_{CS}}{P}F^{-1}\left(\frac{P-c}{P}\right)$. Also, $\frac{\partial E[\Pi_{CS}]}{\partial S_{CS}} = -\int_0^{\frac{PQ_{CS}}{S_{CS}}} F(x)dx + \frac{PQ_{CS}}{S_{CS}}F\left(\frac{PQ_{CS}}{S_{CS}}\right) - \alpha S_{CS}$ is the first-order partial derivative of the centralized supply chain's expected profit function concerning S_{CS}. If we set this relation to zero, the optimal sales effort level will be obtained $S^*_{CS} = \frac{(P-c)}{\alpha P}F^{-1}\left(\frac{P-c}{P}\right) - \frac{1}{\alpha}\int_0^{F^{-1}\left(\frac{P-c}{P}\right)} F(x)dx$.

Proof of Proposition 2 *First, to prove the concavity of the retailer expected profit function in the decentralized supply chain with buyback contract regarding the sales effort and the order quantity, the Hessian matrix should be considered as follows:*

$$\begin{bmatrix} \frac{\partial^2 E[\Pi_{BB}]}{\partial Q^2_{BB}} & \frac{\partial^2 E[\Pi_{BB}]}{\partial Q_{BB}\partial S_{BB}} \\ \frac{\partial^2 E[\Pi_{BB}]}{\partial S_{BB}\partial Q_{BB}} & \frac{\partial^2 E[\Pi_{BB}]}{\partial S^2_{BB}} \end{bmatrix} = \begin{bmatrix} -\frac{P}{S_{BB}}(P+h-b)f\left(\frac{PQ_{BB}}{S_{BB}}\right) & \frac{(P+h-b)PQ_{BB}}{S^2_{BB}}f\left(\frac{PQ_{BB}}{S_{BB}}\right) \\ \frac{(P+h-b)PQ_{BB}}{S^2_{BB}}f\left(\frac{PQ_{BB}}{S_{BB}}\right) & -\frac{(P+h-b)PQ^2_{BB}}{S^3_{BB}}f\left(\frac{PQ_{BB}}{S_{BB}}\right) - \alpha \end{bmatrix}$$

Then, according to the object of the model which is to maximize the benefit of the supply chain. Therefore, this object should be cocave. According to $\frac{\partial^2 E[\Pi_{CS}]}{\partial Q^2_{CS}} = -\frac{P^2}{S_{CS}}f\left(\frac{PQ_{CS}}{S_{CS}}\right) < 0, \frac{\partial^2 E[\Pi_{CS}]}{\partial S^2_{CS}} = -\frac{P^2 Q^2}{S^3_{CS}}f\left(\frac{PQ_{CS}}{S_{CS}}\right) - \alpha < 0$ and $\left(\frac{\partial^2 E[\Pi_{CS}]}{\partial Q^2_{CS}} \times \frac{\partial^2 E[\Pi_{CS}]}{\partial S^2_{CS}}\right) - \left(\frac{\partial^2 E[\Pi_{CS}]}{\partial S_{SC}\partial Q_{SC}} \times \frac{\partial^2 E[\Pi_{CS}]}{\partial Q_{SC}\partial S_{SC}}\right) = \frac{P^2\alpha}{S_{SC}}f\left(\frac{PQ_{SC}}{S_{SC}}\right) > 0$ afterward, it can be concluded that the expected profit function of the centralized SC $E[\Pi_{CS}] = E[P\min\{Q_{CS}, D(S_{CS},p)\} - cQ_{CS} - \alpha\frac{S^2_{CS}}{2}] = (P-c)Q_{CS} - S_{CS}\int_0^{\frac{PQ_{CS}}{S_{CS}}} F(x)dx - \alpha\frac{S^2_{CS}}{2}$, is concave in S_{CS} and Q_{CS}.

Optimal decision variables in this model can be obtained when we set the relation of first-order partial derivative

Then, according to the object of the model which is to maximize the benefit of the retailer. Therefore, this object should be cocave. According to $\frac{\partial^2 E[\Pi_{BB}]}{\partial Q^2_{BB}} = -\frac{P}{S_{BB}}(P-b+h)f\left(\frac{PQ_{BB}}{S_{BB}}\right) < 0, \frac{\partial^2 E[\Pi_{BB}]}{\partial S^2_{BB}} = -\frac{(P+h-b)PQ^2_{BB}}{S^3_{BB}}f\left(\frac{PQ_{BB}}{S_{BB}}\right) - \alpha < 0$ and $\left(\frac{\partial^2 E[\Pi_{BB}]}{\partial Q^2_{BB}} \times \frac{\partial^2 E[\Pi_{BB}]}{\partial S^2_{BB}}\right) - \left(\frac{\partial^2 E[\Pi_{BB}]}{\partial S_{BB}\partial Q_{BB}} \times \frac{\partial^2 E[\Pi_{BB}]}{\partial Q_{BB}\partial S_{BB}}\right) = \frac{P\alpha}{S_{BB}}(P+h-b)f\left(\frac{PQ_{BB}}{S_{BB}}\right) > 0$, hence, it can be concluded that the retailer expected profit function in the decentralized SC with buyback contract $E[\Pi_{BB}] = E[P\min\{Q_{BB}, D(S_{BB},p)\} - wQ_{BB} + (b-h)(Q_{BB} - D(S_{BB},p))^+ - \alpha\frac{S^2_{BB}}{2}] = (P-w)Q_{BB} - S_{BB}\int_0^{\frac{PQ_{BB}}{S_{BB}}} F(x)dx + (b-h)\frac{S_{BB}}{P}\int_0^{\frac{PQ_{BB}}{S_{BB}}} F(x)dx - \alpha\frac{S^2_{BB}}{2}$, is concave in S_{BB} and Q_{BB}.

Amount of optimal order quantity and optimal sales effort for a retailer when a wholesaler offers a buyback contract to the retailer can be obtained by setting the relation of first-order partial derivative concerning these variables equal to zero.

$\frac{\partial E[\Pi_{BB}]}{\partial Q_{BB}} = P - PF\left(\frac{PQ_{BB}}{S_{BB}}\right) - w \quad +(b-h)F\left(\frac{PQ_{Bb}}{S_{BB}}\right)$ is the first-order derivative of the expected profit function of the retailer Q_{BB}. If we set this relation to zero, the optimal order quantity will be obtained.

$$Q^*_{BB} = \frac{S_{BB}}{P}F^{-1}\left(\frac{P-w}{P+h-b}\right).$$

also, $\frac{\partial E[\Pi_{BB}]}{\partial S_{BB}} = -\int_0^{\frac{PQ_{BB}}{S_{BB}}} F(x)dx + \frac{PQ_{BB}}{S_{BB}}F\left(\frac{PQ_{BB}}{S_{BB}}\right) + \frac{(b-h)}{P}\int_0^{\frac{PQ_{BB}}{S_{BB}}} F(x)dx + \frac{(h-b)Q_{BB}}{S_{BB}}F\left(\frac{PQ_{BB}}{S_{BB}}\right) - \alpha S_{BB}$ is the first-order derivative of the expected profit function of the retailer regarding S_{BB}. If we set this relation to zero, the optimal sales effort level will be obtained.

$$S^*_{BB} = \frac{(P-w)}{\alpha P}F^{-1}\left(\frac{P-w}{P+h-b}\right) - \frac{P+h-b}{\alpha P}\int_0^{F^{-1}\left(\frac{P-w}{P+h-b}\right)} F(x)dx..$$

Proof of Proposition 3 *First, to prove the concavity of the retailer expected profit function in the decentralized supply chain with put option contract regarding the sales effort, option order quantity, and the initial order quantity, the Hessian matrix should be considered as follows:*

$$A = \begin{bmatrix} \frac{\partial^2 E[\Pi_{PO}]}{\partial Q^2_{PO}} & \frac{\partial^2 E[\Pi_{PO}]}{\partial Q_{PO}\partial M_{PO}} & \frac{\partial^2 E[\Pi_{PO}]}{\partial Q_{PO}\partial S_{PO}} \\ \frac{\partial^2 E[\Pi_{PO}]}{\partial M_{PO}\partial Q_{PO}} & \frac{\partial^2 E[\Pi_{PO}]}{\partial M^2_{PO}} & \frac{\partial^2 E[\Pi_{PO}]}{\partial M_{PO}\partial S_{PO}} \\ \frac{\partial^2 E[\Pi_{PO}]}{\partial S_{PO}\partial Q_{PO}} & \frac{\partial^2 E[\Pi_{PO}]}{\partial S_{PO}\partial M_{PO}} & \frac{\partial^2 E[\Pi_{PO}]}{\partial S^2_{PO}} \end{bmatrix} =$$

$$\begin{bmatrix} -\frac{P(P-e)}{S_{PO}}f\left(\frac{PQ_{PO}}{S_{PO}}\right) - \frac{eP}{S_{PO}}f\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right) & \frac{eP}{S_{PO}}f\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right) & \frac{PQ_{PO}(P-e)}{S^2_{PO}}f\left(\frac{PQ_{PO}}{S_{PO}}\right) + \frac{eP(Q_{PO}-M)}{S^2_{PO}}f\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right) \\ \frac{eP}{S_{PO}}f\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right) & -\frac{eP}{S_{PO}}f\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right) & -\frac{eP(Q_{PO}-M)}{S^2_{PO}}f\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right) \\ \frac{PQ_{PO}(P-e)}{S^2_{PO}}f\left(\frac{PQ_{PO}}{S_{PO}}\right) + \frac{e(P-1)}{S_{PO}}F\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right) + \frac{eP^2(Q_{PO}-M)}{S^2_{PO}}f\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right) & -\frac{e(P-1)}{S_{PO}}F\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right) - \frac{eP^2(Q_{PO}-M)}{S^2_{PO}}f\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right) & -\frac{PQ^2_{PO}(P-e)}{S^3_{PO}}f\left(\frac{PQ_{PO}}{S_{PO}}\right) - \frac{eP^2(Q_{PO}-M)^2}{S^3_{PO}}f\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right) - \alpha \end{bmatrix}.$$

Then, according to the object of the model which is to maximize the benefit of the retailer. Therefore, this object should be cocave. According to $A_{Minor1} = \frac{\partial^2 E[\Pi_{PO}]}{\partial Q^2_{PO}} = -\frac{P}{S_{PO}}(P-e)f(\frac{PQ_{PO}}{S_{PO}}) - \frac{eP}{S_{PO}}f(\frac{P(Q_{PO}-M)}{S_{PO}}) < 0$

$$A_{Minor2} = \left[\frac{P(P-e)}{S_{PO}}f(\frac{PQ_{PO}}{S_{PO}})\right] \times \left[\frac{eP}{S_{PO}}f(\frac{P(Q_{PO}-M)}{S_{PO}})\right] > 0$$

and $\det[A] = -\frac{2e^2P^3Q_{PO}M(P-e)}{S^5}f\left(\frac{PQ_{PO}}{S_{PO}}\right)f\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right)f\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right) - \frac{2\alpha e^2P^2}{S^2}f\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right)f\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right) - \frac{\alpha eP^2(P-e)}{S^2}f\left(\frac{PQ_{PO}}{S_{PO}}\right)f\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right) - \frac{e^2P^2(P-e)(P-1)(Q-M)}{S^4}f\left(\frac{PQ_{PO}}{S_{PO}}\right)f\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right)F\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right) < 0$ hence, it can be concluded that the expected profit function of the retailer in the decentralized SC with put option contract $E[\Pi_{PO}] = E[P\min\{Q_{PO}, D(S_{PO},p)\} - wQ_{PO} - oM + e\min\{M, (Q_{PO} - D(S_{PO},p))^+\} - \alpha\frac{S^2_{PO}}{2}] = (P-w)Q_{PO} \quad -oM - S_{PO}\int_0^{\frac{PQ_{PO}}{S_{PO}}} F(x)dx + \frac{eS_{PO}}{P}\int_{\frac{P(Q_{PO}-M)}{S_{PO}}}^{\frac{PQ_{PO}}{S_{PO}}} F(x)dx - \alpha\frac{S^2_{PO}}{2}$, is concave in $Q_{PO}M_{PO}$ and S_{PO}.

The first-order derivative of the expected profit function of the retailer in M_{PO} is $\frac{\partial E[\Pi_{PO}]}{\partial M} = -o + eF\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right)$ and the first-order derivative of the retailer's profit function in Q_{PO} is $\frac{\partial E[\Pi_{PO}]}{\partial Q_{PO}} = P - PF\left(\frac{PQ_{PO}}{S_{PO}}\right) - w + eF\left(\frac{PQ_{PO}}{S_{PO}}\right) - eF\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right)$. If we set these equations equal to zero and combine these to each other, the order quantity and options order quantity will obtain as $Q^*_{PO} = \frac{S_{PO}}{P}F^{-1}\left(\frac{P-w-o}{P-e}\right)$ and $M^* = \frac{S_{PO}}{P}\left[F^{-1}\left(\frac{P-w-o}{P-e}\right) - F^{-1}\left(\frac{o}{e}\right)\right]$.

Finally, for finding the optimal sales effort of retailer we should set the first-order derivative of expected profit function regarding sales effort variable.

$\frac{\partial E[\Pi_{PO}]}{\partial S_{PO}} = -\int_0^{\frac{PQ_{PO}}{S_{PO}}} F(x)dx + \frac{PQ_{PO}}{S_{PO}}F(\frac{PQ_{PO}}{S_{PO}}) + \frac{e}{P}\int_{\frac{P(Q_{PO}-M)}{S_{PO}}}^{\frac{PQ_{PO}}{S_{PO}}} F(x)dx - \frac{eQ_{PO}}{S_{PO}} F(\frac{PQ_{PO}}{S_{PO}}) + e\frac{P(Q_{PO}-M)}{S_{PO}}F\left(\frac{P(Q_{PO}-M)}{S_{PO}}\right) - \alpha S_{BB}$ equal to zero, accordingly we can obtain to

$$S^*_{PO} = \frac{(P-w-o)}{\alpha P}F^{-1}\left(\frac{P-w-o}{P-e}\right) - \frac{P-e}{\alpha P}\int_0^{F^{-1}\left(\frac{P-w-o}{P-e}\right)} F(x)dx + \frac{o}{\alpha P}F^{-1}\left(\frac{o}{e}\right) - \frac{e}{\alpha P}\int_0^{F^{-1}\left(\frac{o}{e}\right)} F(x)dx$$

as the optimal sales effort level.

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Code availability Not applicable.

Declarations

Conflict of interest The authors declare that we do not have any possible conflicts of interest.

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