Surface-Wave Propagation on a Grounded Dielectric Slab Covered by a High-Permittivity Material

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***Abstract*—A grounded dielectric slab covered by a higher permittivity material would not normally be expected to support surface waves. This conclusion must be modified when the covering material is sufficiently lossy. Assuming a thin slab, approximate analysis shows that the fundamental TM_0 surface wave is able to propagate if the cover loss is high enough. A numerical analysis has verified these conclusions. Propagation of higher order TM and TE modes is found to be possible above cutoff frequencies, which reduce as cover loss is increased. These results are of significance when printed circuit transmission lines such as microstrip or slotline are used as contact sensors, e.g., for moist materials.**

***Index Terms*—Dielectric losses, dielectric measurements, electromagnetic surface waves, microstrip, microwave propagation, moisture measurments, moisture transducers, slotline.**

I. Introduction

In ORDER for a spectral domain analysis of a microstrip transmission line to be highly accurate, it is important to account for signal loss due to leakage into the surface-wave modes on the structure [1]. This is especially true when analyzing microstrip-based sensing heads, where measurement accuracy depends greatly on the quality of the sensor model. It is therefore essential that all surface-wave modes that can be supported by the microstrip background structure be identified in advance, so that the contour of integration can be deformed around the corresponding poles in the spectral plane if necessary [1].

It is in this context that we perform an analysis of the surface-wave modes propagating on a grounded, low-loss dielectric slab covered by a material of infinite extent, as shown in Fig. 1. Significantly, the covering medium is assumed to have a dielectric constant greater than that of the slab, and may be lossy in nature. This choice of material properties reflects measurement situations encountered in practice, for example, in permittivity measurement of moist substances.

II. Theory of Surface Waves

A concise derivation of the characteristic equations for the TM and TE surface wave modes is provided in [2]. A similar formulation is used here, the only difference being that the permittivity of the covering medium is not assumed to be air, but may assume any complex value.

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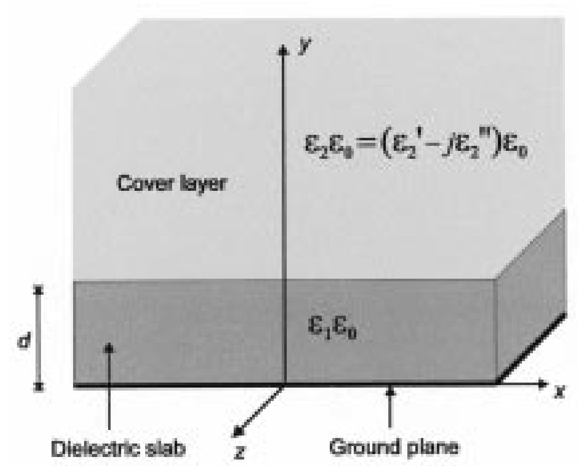

Fig. 1. Grounded dielectric slab covered by superstrate of infinite extent.

For the TM family of surface-wave modes, we obtain the following familiar pair of equations to be solved:

$$hd = \frac{\varepsilon_2}{\varepsilon_1}(k_c d)\tan(k_c d) \tag{1}$$

and

$$(k_c d)^2 + (hd)^2 = (\varepsilon_1 - \varepsilon_2)(k_0 d)^2 \tag{2}$$

where

- d substrate thickness;
- ε_1 substrate permittivity;
- ε_2 complex permittivity of the medium above the slab;
- k_0 free-space wavenumber.

The parameters h and k_c are the cutoff wavenumbers for the top and bottom regions respectively, and are defined as

$$h^2 = \beta^2 - k_2^2 \tag{3}$$

and

$$k_c^2 = k_1^2 - \beta^2 \tag{4}$$

where k_2 and k_1 are the wavenumbers in the top and bottom regions, respectively, and β is the phase coefficient of the wave propagating in the z-direction. ε_1 and ε_2 may be complex (i.e., lossy), which implies that β, k_1, and k_2 may also be complex.

For the TE family of surface-wave modes, (2)–(4) are still applicable and (1) must be replaced by

$$hd = -(k_c d)\cot(k_c d). \tag{5}$$

For the TM and TE modes, respectively, simultaneous solution of (1) and (2) or (2) and (5) allows the propagation characteristics to be determined.

The nonlinear nature of the TM and TE eigenequations usually means that an analytic solution cannot be obtained. However, an approximate solution can be obtained for the TM_0 mode if the substrate is assumed to be thin compared to a wavelength. This enables us to simplify (1) to

$$h \approx \frac{\varepsilon_2}{\varepsilon_1} k_c^2 d \tag{6}$$

assuming $k_0 d \ll 1$.

Substitution into (2) yields a quadratic in h

$$h^2 + \frac{\varepsilon_1}{\varepsilon_2 d} h + k_0^2(\varepsilon_2 - \varepsilon_1) = 0. \tag{7}$$

In order to satisfy the boundary conditions at infinity, the real part of h must be positive [3]. Therefore, the positive root of (7) must be selected. In contrast to an air covered slab, it is found that the TM_0 mode may experience a low-requency cutoff. An approximate analytic expression for the cutoff frequency can be found by separating (7) into real and imaginary parts and setting the real part of h to zero. Assuming that the slab is lossless ($\varepsilon_1'' = 0$), the result is

$$f_c \approx \frac{c\varepsilon_1}{2\pi\varepsilon_2''|\varepsilon_2|^2 d}\sqrt{\varepsilon_2'[\varepsilon_2'(\varepsilon_2' - \varepsilon_1) - (\varepsilon_2'')^2]} \tag{8}$$

where c is the velocity of electromagnetic waves in free space. Propagation down to dc is only possible if no cutoff frequency is present, i.e., when

$$(\varepsilon_2'')^2 > (\varepsilon_2' - \varepsilon_1)\varepsilon_2'. \tag{9}$$

If $\varepsilon_2' > \varepsilon_1$, then (9) can only be satisfied if $\varepsilon_2'' \gg 0$. Hence, TM_0 surface-wave propagation is possible if the covering material is sufficiently lossy.

III. Numerical Solution and Discussion

Further insight into this phenomenon can be gained via a numerical solution of the TM and TE characteristic equations. This verifies that the existence of the TM_0 cutoff frequency is dependent on the amount of loss in the covering material, as shown in Fig. 2. A comparison of computed cutoff frequencies and values estimated from (8) is given in Table I, which shows good agreement.

All higher order TM modes and TE modes have cutoff frequencies that reduce as the amount of loss in the covering material increases. This is illustrated for the TM_1 and TE_1 modes in Fig. 2.

These results have important consequences for the application of microstrip as a contact sensor for moist materials or polar dielectrics such as methanol. The dielectric properties of these substances are highly frequency-dependent and can vary over a wide range. As such, leakage into surface-wave modes may occur at some frequencies but not at others.

As an example, consider a microstrip sensor made out of the slab described in Fig. 2, where $d = 1.27$ mm and $\varepsilon_1 = 10.2$. Now assume that the microstrip sensor is immersed in methanol at a temperature of 25 °C. The normalized attenuation and phase coefficients for the TM_0 mode on this structure are shown in Fig. 3. These results were obtained via numerical analysis, with methanol permittivity estimated using Debye constants from [4]. Methanol has a dielectric constant greater than 10.2 at frequencies below 6.96 GHz. A cutoff frequency exists at $f_c = 2.32$ GHz, below which the TM_0 mode cannot propagate. Above f_c, a proper solution exists. A spectral domain analysis of this transmission line may therefore have a pole on the top Riemann sheet at frequencies above 2.32 GHz, which would have to bypassed by the contour of integration in order to account for leakage into this mode. Below 2.32 GHz, the spectral domain analysis would be somewhat simpler, as no contour deformation would be required.

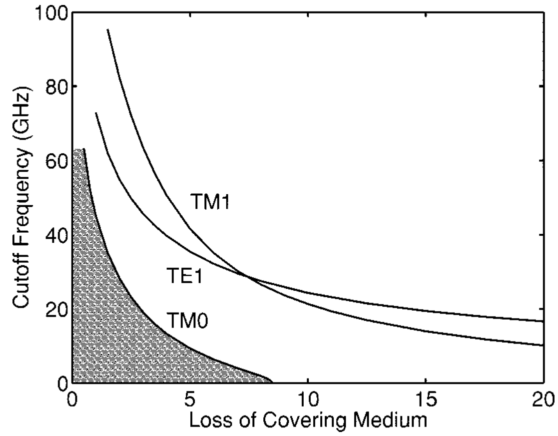

Fig. 2. f_c versus ε_2'' for TM_0, TE_1, and TM_1 surface-wave modes on grounded dielectric slab covered by lossy medium of infinite extent ($\varepsilon_1 = 10.2, \varepsilon_2' = 15$, $d = 1.27$ mm). No propagation is possible in shaded region.

TABLE I
COMPARISON OF NUMERICAL VALUES FOR TM_0 CUTOFF FREQUENCY WITH ESTIMATES FROM (8) FOR $\varepsilon_1 = 10.2, \varepsilon_2' = 15$, and $d = 1.27$ mm

ε_2''	Cutoff Frequency (GHz)	
	Estimate (8)	Numerical
8.485	0	0
5	8.146	9.365
3	16.793	19.196
2	26.741	28.360

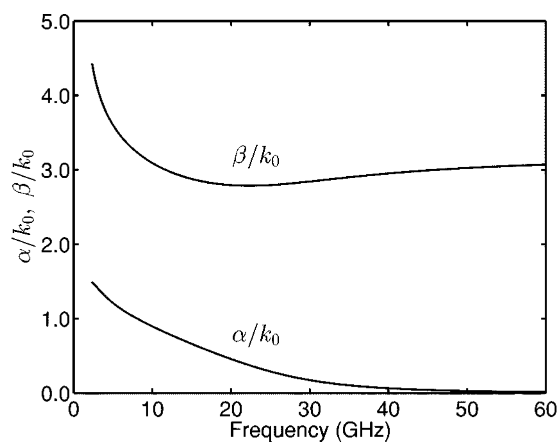

Fig. 3. α/k_0 and β/k_0 versus frequency for TM_0 surface-wave mode on grounded dielectric slab covered by methanol ($\varepsilon_1 = 10.2$, $d = 1.27$ mm, $T = 25\,^\circ$ C).

IV. Conclusion

A numerical analysis of the TM and TE surface-wave characteristic equations is presented for a grounded dielectric slab covered by a nonair medium. It is shown that if the dielectric constant of the covering medium is greater than that of the substrate, then proper surface-wave modes can only be supported by the slab if the covering medium contains some loss. Under these conditions, surface-wave modes will propagate above certain cutoff frequencies. This is a significant result to consider when designing microstrip-based sensors for permittivity measurement applications, as it suggests that below a certain frequency no energy will be lost due to surface wave leakage. This can simplify greatly the spectral domain analysis of such structures, as the absence of surface-wave modes means no poles exist on the top Riemann sheet that will require a deformation of the integration contour.

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