Infiltration Parameters from Surface Irrigation Advance and Run-off Data

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Abstract

A computer model was developed to employ runoff data in the calculation of the infiltration parameters of the modified Kostiakov equation. The model (IPARM) uses a simple volume balance approach to estimate the parameters from commonly collected field data. Several data sets have been used to verify the procedure. Infiltration parameters were calculated using both advance and runoff data combined and advance data alone. Simulations of each example using SIRMOD were compared to the measured data to identify the possible benefits of the procedure. The inclusion of runoff did not compromise the ability to reproduce the advance curve however the simulations are more capable of reproducing the measured runoff rates and volumes and therefore offer better estimations of the total volume applied to the soil (in one case a reduction in error of the total infiltration from 22% to 1%). This procedure will be of most benefit where the infiltration parameters are expected to represent soil hydraulic characteristics for times greater than the completion of the advance phase. Further analysis has shown that the infiltration parameters are more sensitive to runoff than the advance highlighting the requirement for accurate field measurement and a weighting factor between the advance and runoff errors.

Surface irrigation; Infiltration; Runoff; Optimisation; Simulation; IPARM; Kostiakov; Volume balance

Introduction

Surface irrigation is the oldest yet still the most common form of irrigation throughout the world although it traditionally suffers from many problems such as low efficiency and low uniformity. It is possible to improve the performance of most surface irrigation systems through the implementation of optimal management practices such as selection of correct inflow rates and cut-off times. Identification of the correct management requires the study of the complex interaction between irrigation water and the agricultural soil. Therefore it is fair to infer that a significant obstacle in the path of improving irrigation performance is the difficulty of estimating the infiltration function (Elliott *et al.*, 1983).

A common assumption is that data collected during the advance stage provides sufficient information to determine the hydraulic behaviour of a soil. Two such methods in common use are the two-point method (Elliott and Walker, 1982) and the INFILT optimisation (McClymont and Smith, 1996). Both approaches are based on the simple yet robust combination of the modified Kostiakov equation and the volume balance model.

The flaw of such infiltration from advance schemes is that the soil behaviour may change during the irrigation. Large variability is often noted between the infiltration functions in the same field, which is more noticeable where the water reaches the end of the field early compared to the total inflow time (Scaloppi *et al.*, 1995). Simulations using the estimated infiltration characteristic often provide a good fit to the advance data but commonly result in a poor reproduction of the run-off and recession curves thereby indicating the inadequacy of the infiltration function. It is more beneficial to gain precise knowledge of the total infiltrated volume than just providing an accurate reproduction of the advance data.

Variations in the inflow rate often occur in the early stages of an irrigation event. The volume balance equations usually employ a “step inflow” assumption; that is, the inflow is assumed to reach its final steady rate immediately. Techniques based on advance data alone can be adversely affected by any initial variation of inflow rates (Renault and Wallender, 1996). In most cases any initial inflow variation has little impact on the run-off from the tail end of the field.

Previous studies have indicated the benefits of including the run-off phase in the optimisation. Scaloppi *et al.,* (1995) deduced that it provides a better fit to measured data compared to parameters based on advance or run-off data alone. Also infiltration parameters based entirely on the advance phase are more sensitive to errors in the estimation of surface storage volumes (Renault and Wallender, 1997).

The multilevel calibration technique (Walker, 2005) is one recent example of such a procedure. It uses the advance time, runoff hydrograph and recession time to calculate the infiltration parameters and the value of Manning *n*. The multilevel approach provides a closer fit to the runoff curve than the two point method; however it lacks the same capacity to predict the advance trajectory. The requirement for recession data may be a problem, often water does not drain freely from the field following the conclusion of the irrigation.

The aim of this paper is to present a simplified optimisation scheme that calculates infiltration parameters based on both the advance and storage phases of furrow irrigation. The proposed technique gives improved estimates of the final infiltration rate over those techniques based on the advance only, without the requirement for the irrigation to last long enough to reach a steady run-off rate. Such a technique should provide an infiltration function that is applicable for longer times, that is, for a larger portion of the irrigation time.

Model Development

The volume balance equation (law of conservation of mass) can be used to describe the flow of water longitudinally down the furrow, including the infiltration of water into the soil. To represent the storage phase a run-off term is added to the volume balance equation of the two-point method:

$$Q_O t = V_I + V_S + V_R \tag{1}$$

where Q_O is the steady inflow rate (m^3/min), V_I is the volume infiltrated, V_S is the volume temporarily stored on the soil surface, V_R is the volume of run-off, and t is the time (min).

The modified Kostiakov equation yields the cumulative infiltrated volume as a function of time:

$$Z = kt^a + f_o t \tag{2}$$

where Z is the cumulative infiltration (m^3/m), a and k are fitted constants, f_o is the steady infiltration rate (m^3/min per m length) and t is the period of time (min) that water is ponded on the soil surface (Walker and Skogerboe, 1987). The infiltrated volume in (1) is determined by integration of equation (2) over the wetted length of the field:

$$V_I = (\sigma_{z1} kt^a + \sigma_{z2} f_o t)x \tag{3}$$

where x is the length of the field submerged and σ_{z1} and σ_{z2} are subsurface shape factors. During the advance phase they are defined (Elliott and Walker, 1982) (assuming a power curve advance function) as:

$$\sigma_{z1} = \frac{a + r(1-a) + 1}{(1+r)(1+a)} \tag{4}$$

$$\sigma_{z2} = \frac{1}{(1+r)} \tag{5}$$

where r is the exponent in the power curve advance function:

$$x = pt^r \tag{6}$$

The constants p and r are selected so that the function best matches the advance data (performed by least squares).

Following completion of the advance phase the values of the shape factors change; σ_{z1} is represented by an incomplete gamma function that is approximated by a binomial expression (Scaloppi *et al.*, 1995):

$$\sigma_{Z1} = \lambda^r \left[1 - \frac{ar\lambda}{r+1} + \frac{a(a-1)r\lambda^2}{2!(r+2)} - \frac{a(a-1)(1-2)r\lambda^3}{3!(r+3)} + \frac{a(a-1)(a-2)(a-3)r\lambda^4}{4!(r+4)} - .... + ... \right] \tag{7}$$

and:

$$\sigma_{Z2} = 1 - \frac{r\lambda}{(r+1)} \tag{8}$$

where λ is the ratio of the current time to the complete advance time; consequently both sub-surface shape factors are functions of time.

The surface storage portion of the volume balance is difficult to measure and therefore is usually estimated. One common method is to assume that the average cross sectional area of flow is found by multiplying the upstream cross sectional area A_o by a constant surface shape factor (represented by σ_y) typically assumed to have a value of 0.77 (McClymont and Smith, 1996), giving:

$$V_S = \sigma_y A_o x \tag{9}$$

Once the storage phase commences the surface shape factor becomes a function of time that approaches unity. Scallopi *et al.* (1995) utilized a function that gives the flow depth down the furrow during the advance phase:

$$y = y_o \left(1 - \frac{s}{x}\right)^{\beta} \tag{10}$$

where y is the flow depth, y_o is the upstream flow depth, s is the distance from the upstream end, x is the length of the advance profile at the particular time, and β is a curvature constant. A value for β of 0.25 results in a σ_y of approx. 0.77 for most furrow geometries.

From this equation an expression for the surface shape factor of the storage phase σ_{ys} can be developed for any time during the storage phase:

$$\sigma_{ys} = \sigma_y \frac{X_t}{L}\left(1 - \left(1 - \frac{L}{X_t}\right)^{\frac{1}{\sigma_y}}\right) \tag{11}$$

where σ_y is the shape factor for the advance phase, L is the length of the field and X_t is an imaginary advance distance for the particular time, assuming that the advance can continue unimpeded past the end of the furrow (fig 1). This imaginary distance X_t is calculated from the volume balance equation of the advance phase.

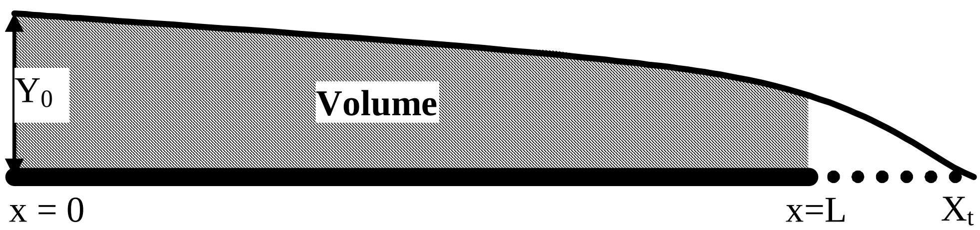

Figure 1 Calculating the surface storage during the storage phase

In this case the volume stored in the furrow is:

$$V_S = \sigma_{ys} A_o L \tag{12}$$

The cross sectional area A_o may be measured, predicted from the inflow using the Manning equation, or calculated from a measured depth of flow. In the latter case the furrow geometry is presented as a power curve with provision for a flat bottom:

$$W = W_B + cy^m \tag{13}$$

where W is the surface width of the flow, W_B is the bottom width and y is the flow depth. The parameters c and m are fitted by the model from measurements of the bottom, middle and top widths and maximum height of the furrow. Flow area is then simply calculated by integrating the flow width over the flow depth.

$$A_o = W_B y_o + \frac{cy_0^{m+1}}{m+1} \tag{14}$$

As well as the measured advance (distances and times), the model requires measurements of the run-off volumes at various times during the storage phase. In field trials it is usual to measure the run-off hydrograph (run-off rate). The run-off volume at any particular time is calculated by the trapezoidal rule, assuming that the run-off hydrograph is linear between each run-off measurement.

The model attempts to minimize the difference between: (i) the calculated and measured advance distances during the advance phase, and (ii) the calculated and measured run-off volumes during the storage phase, by incrementing the parameters of the modified Kostiakov equation. The algorithms for each phase are:

$$SSE_{advance} = \sum_{i=1}^{N_a} \left[x_i - \frac{Q_o t_i}{\sigma_y A_o + \sigma_{z1} k t_i^a + \sigma_{z2} f_o t_i} \right]^2 \tag{15}$$

$$SSE_{runoff} = \sum_{i=1}^{N_r} \left[V_{Ri} - \left(Q_o t_i - \sigma_{ys} A_o L - \sigma_{z1} k t_i^a L - \sigma_{z2} f_o L \right) \right]^2 \tag{16}$$

where SSE is the standard square error, x_i, t_i and V_{Ri} are the measured advance distance, time and run-off volumes, respectively, N_a is the number of advance points, and N_r is the number of run-off volumes. Finally, the objective function is formed by non-dimensionalising and summing the advance and run-off errors, weighted by a predetermined factor.

$$objective\_function = \left(\frac{SSE_{advance}}{\sum_{i=1}^{N_a} x_i^2} \right)^{\frac{1}{2}} + w * \left(\frac{SSE_{runoff}}{\sum_{i=1}^{N_r} V_{ri}^2} \right)^{\frac{1}{2}} \tag{17}$$

The weighting factor, w is included to enable the user to easily change the relative sensitivity of the objective function to the errors of the advance distance or runoff volume.

This objective function can also be expressed in terms of errors in both advance and runoff time. This results in parameter values very similar to those from equation 17 but requires further iterative computations as some of the terms in the model such as the subsurface shape factors (equations 7 and 8) are time dependant.

The optimisation scheme is based on the technique introduced by McClymont and Smith (1996); each of the three parameters (a, k and f_o) is incremented individually until the error reduces no further. Following this the parameters are incremented in the same direction as before but as a group until the error again cannot be reduced further. These two steps are repeated until the objective function is not improved by either the individual or group search. Now the step size is reduced and the whole process repeats.

During the design process it was noticed that occasionally the program jumped to the next smallest step size too quickly or remained incrementing at a particular step size for a long period of time. To overcome these problems the optimisation increases the group step size each time the program loops back to the individual parameter search.

The initial step sizes for the parameter optimisation are selected based on maximum stability combined with minimum execution time. The initial step sizes can be changed but experimentation has found 0.01, 0.0001 and 0.00001 for the step sizes of a, k and f_0 respectively work with most data sets.

Comprehensive tests were carried out to determine the sensitivity of the model to various inputs. These showed that the model is not significantly sensitive to furrow geometry but is influenced by other inputs such as the Manning constant.

Convergence can be compromised by the selection of improper starting estimates. This is overcome by the inclusion of an algorithm to perform an initial rough parameter search. Also, limits have been included to ensure that the parameters do not reach unrealistic values (for example all three parameters are kept positive).

The model has been coded in C++ to create an executable program (IPARM); once it is loaded the user is required to enter input data.

The model requires a number of input measurements. Firstly the advance data in the form of distances and corresponding times, the technique requires a minimum of two advance points. Secondly the run-off data is made up of run-off rates (in l/s) measured at various times during the storage phase (the model is not valid during the depletion and recession phases). Other inputs include field slope, Manning *n* or upstream flow depth, inflow rate and the field length.

Model Verification

Trials of the model have been carried out using a number of data sets, six of which (Table 1) have been selected for discussion. The selected irrigations cover a wide range of soil types, flow rates and irrigation durations.

Table 1: Comparing the selected data sets

Name	**Furrow length (metres)**	**Advance time (minutes)**	**Inflow Time (minutes)**	**Cut off time (minutes)**	**Inflow (L/s)**
Benson	625	199	705	705	0.787
Printz	350	120.5	173	173	3.424
Downs 1	565	424	605	605	3.422
Downs 2	565	520.5	605	605	3.675
Brazil	200	128	300	300	1.270
Merkley	225	38.5	67.2	67.2	2.670

The Benson and Printz trials were carried out in Colorado on clay loam and sandy loam soils, respectively (Walker, 2005). The Benson case study is an example where the advance phase of the irrigation is relatively short compared to the storage phase.

The two Downs case studies represent field measurements from neighboring furrows in the same irrigation on cracking clay soil at Macalister, Darling Downs, Australia

(Dalton *et al.*, 2001). The two data sets are titled Downs1 (Irrigation 2 furrow 3) and Downs2 (Irrigation 2 furrow 2).

The final two data sets, Brazil and Merkley, were sourced from Scaloppi *et al*. (1995). Both of these, in particular the Merkley trial are typical examples of where the advance data does not cover an adequate time to enable the accurate calculation of the infiltration function from the advance data alone.

Infiltration parameters were calculated for each irrigation using the maximum available number of both run-off and advance points for each data set. The optimisation was performed with equal weighting on both the advance and storage phases (see equation 17). To identify the improvement offered by of the proposed technique, the infiltration parameters were also calculated from advance data alone, with results similar to those produced by INFILT (McClymont and Smith, 1996). Values for the upstream flow area were calculated from estimates of Manning *n* and the furrow geometry.

The surface irrigation simulation model SIRMOD (Walker 1999) was then used to simulate the irrigation events using the different sets of infiltration parameters. In each case the values for *a, k* and f_o were entered into the model and the value of the Manning *n* was adjusted to cause the model to predict the end of the advance phase correctly. All simulations were performed by SIRMOD II using the full hydrodynamic model option. SIRMOD produces advance curves and run-off hydrographs that can be compared with the measured data. Other outputs include the total infiltration and total outflow.

Results and Discussion

Model Performance

The IPARM model was able to calculate infiltration functions for all of the case studies (Table 2). The values of the individual parameters *a*, *k* and f_o vary considerably from those calculated using the advance only. However, despite these differences, the cumulative infiltration functions (fig 2.a-f) have similar shapes within each trial. In most cases the two infiltration curves indicate similar infiltrated volumes at times less than the end of the advance phase, in fact the curves cross at a time close to the advance time. The greatest difference between the curves is seen after the end of the advance phase where after the two curves diverge substantially. This difference or error will continue to increase if the curves are extrapolated to greater times. This has enormous implications if the infiltration parameters are to be used in the simulation or management of an irrigation at times greater than the advance time.

Table 2: Estimated Infiltration Parameters

Trial	Data	Kostiakov Parameters		
		a	k	f0
Benson	*Advance*	0	0.00163	0.000070
	Advance + Runoff	0.4553	0.00058	0.000041
Printz	*Advance*	0	0.02624	0.000372
	Advance + Runoff	0.0960	0.01539	0.000477
Downs 1	*Advance*	0	0.04731	0.000320
	Advance + Runoff	0.2932	0.01889	0.000149
Downs 2	*Advance*	0	0.09316	0.000275
	Advance + Runoff	0.1673	0.05037	0.000176
Brazil	*Advance*	0.6831	0.00212	0.000000
	Advance + Runoff	0.3354	0.00463	0.000263
Merkley	*Advance*	0.5449	0.00284	0.000000
	Advance + Runoff	0.2890	0.00264	0.000402

For a number of the trials the value of *a* is reduced to zero when using the advance data only (Table 2). In some cases this may be a limitation of that approach, but in some instances, such as Downs 1 and 2, it may simply be reflecting the cracking nature of the soil. In those cases IPARM also returns very low values for *a*.

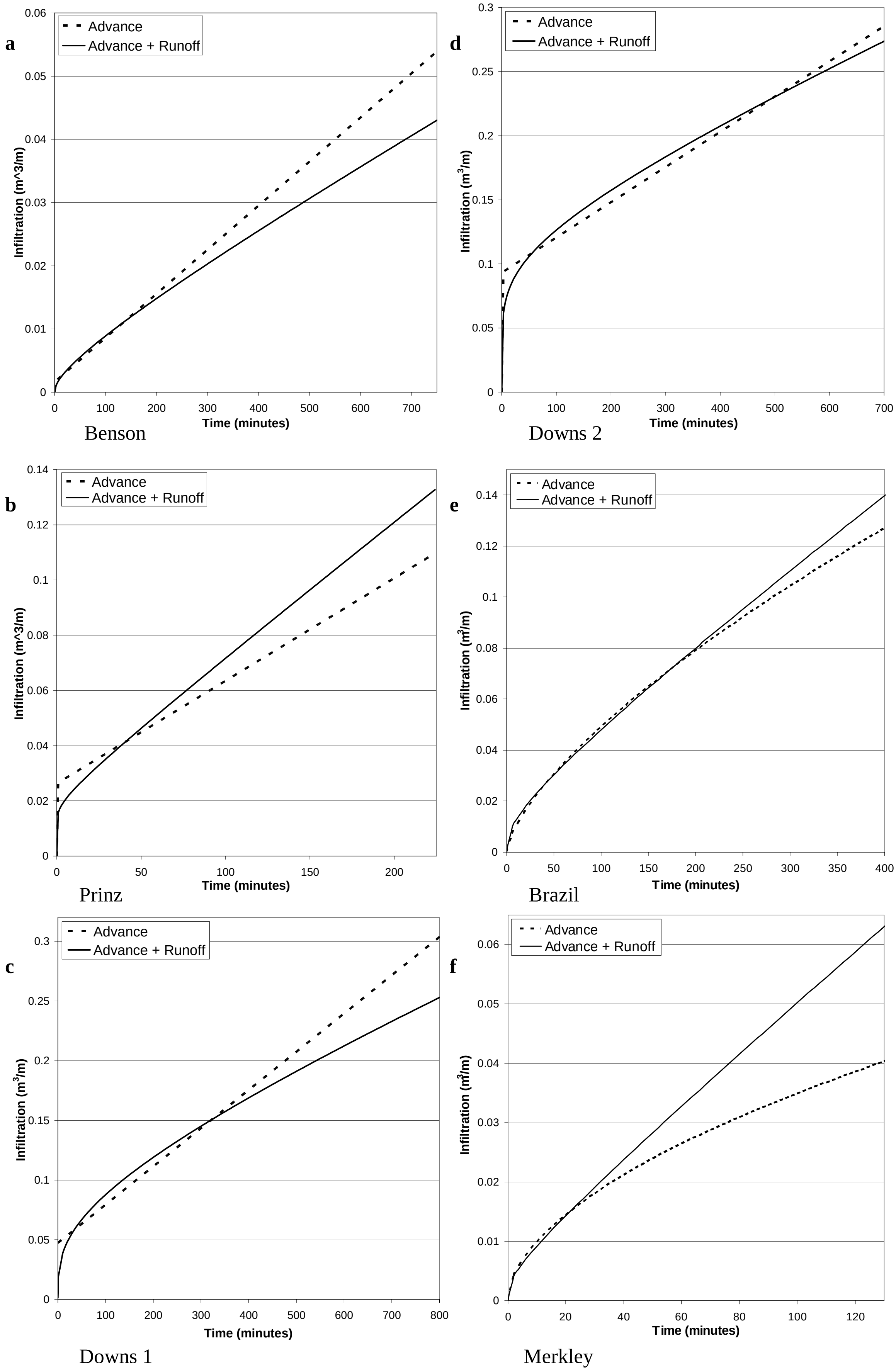

Figure 2 a – f Infiltration functions estimated by using either advance or advance and runoff data.

For each trial both sets of infiltration parameters resulted in similar predictions of the advance trajectories. A calibration of SIRMOD for each parameter set was performed by adjusting the value of Manning n. The resulting *n* values ranged between 0.016 and 0.05 reflecting realistic values. In each example the Manning n in SIRMOD more closely reflected the assumed value for the parameter optimisation where runoff data was used. The example in fig 3 demonstrates the typical behaviour found for all data sets studied. That is the inclusion of run-off data in the calculation of the infiltration parameters does not compromise the ability of the simulations to reproduce the advance data.

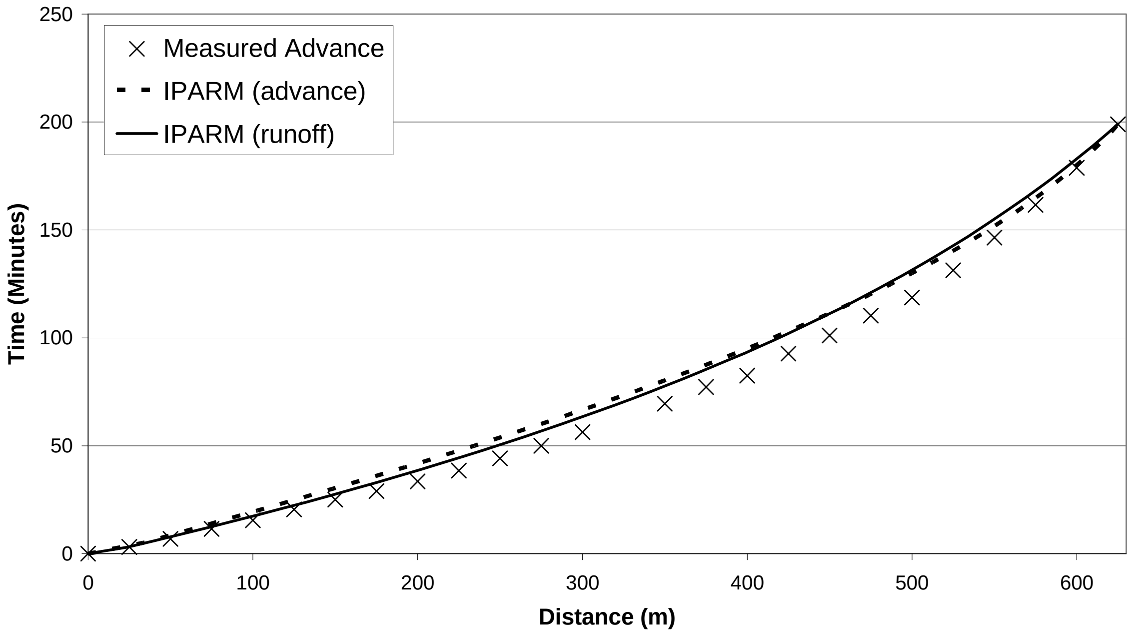

Figure 3 Comparing the measured advance curve of the Benson data with simulated advance curves.

Any differences between the simulations caused by the different infiltration parameters become more apparent in a comparison of the predicted run-off hydrographs (fig 4). The results of the simulations show that in every case (perhaps with the slight exception of *Downs 2*) the run-off volume and shape of the run-off hydrograph is more accurately predicted from infiltration parameters calculated from the advance and run-off data combined rather than the advance alone. Consequently it is concluded that the inclusion of run-off data improves the accuracy of the estimation of the infiltration parameters. When only the advance data is used, in some cases the run-off is over predicted, *e.g. Printz* and in other cases it is under predicted *e.g. Downs 1.*

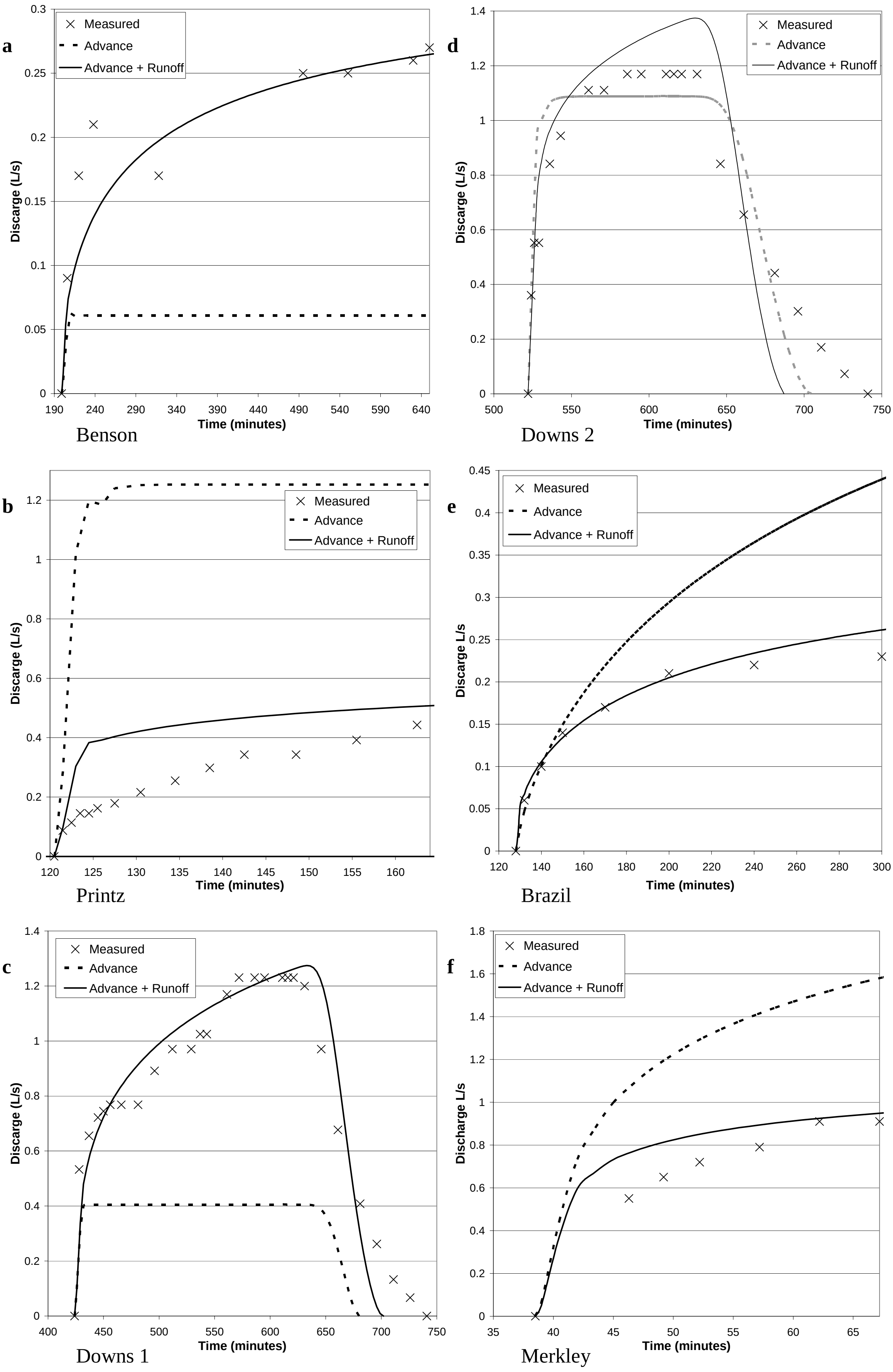

Figure 4 a-f Comparing the measured runoff hydrograph to SIRMOD simulated runoff using infiltration parameters from either advance or advance and runoff data.

Table 3 Summary of results from SIRMOD simulations

Trial	Data	At end of recorded time		Total, at end of irrigation (Italic cells are estimated)			
		Runoff (m^3)	Runoff Error	Runoff (m^3)	Runoff Error	Infiltration mm	Infiltration Error
Benson	Measured	5.84		***7.76***		***26.89***	
	Advance	1.63	-72.1%	2.13	-72.6%	32.81	22.0%
	Advance + Runoff	5.82	-0.3%	8.02	3.4%	26.61	-1.0%
Printz	Measured	0.74		***1.64***		***62.91***	
	Advance	3.01	305.3%	6.41	292.3%	54.05	-14.1%
	Advance + Runoff	1.10	48.6%	2.10	28.7%	62.04	-1.4%
Downs 1	Measured			15.13		96.55	
	Advance			5.72	-62.2%	104.87	8.6%
	Advance + Runoff			15.44	2.0%	96.28	-0.3%
Downs 2	Measured			9.73		109.45	
	Advance			9.62	-1.1%	109.55	0.1%
	Advance + Runoff			10.08	3.6%	109.14	-0.3%
Brazil	Measured	1.97		***2.30***		***128.51***	
	Advance	3.06	55.2%	3.31	44.0%	122.19	-4.9%
	Advance + Runoff	2.08	5.2%	2.34	1.8%	128.25	-0.2%
Merkley	Measured	1.11		***2.20***		***47.60***	
	Advance	2.05	84.1%	4.07	85.1%	37.22	-21.8%
	Advance + Runoff	1.35	21.1%	2.46	11.9%	46.15	-3.1%

Similarly, the error in the predicted run-off volumes is also significant (Table 3), for example, the run-off volume is over predicted by nearly 300% for the *Printz* irrigation, which corresponds to an under prediction of the average infiltrated depth by an estimated 14%.

In summary the inclusion of the run-off data in the identification of the modified Kostiakov parameters gives a greatly improved estimate of the parameters and hence improved cumulative infiltration curve. It retains the ability of the simulations to predict the advance function and it improves the accuracy of run-off and infiltration predictions during later stages of the irrigation.

Number of Run-off and Advance Points

The Merkley data (Tables 4 & 5) were selected for further analysis to determine the sensitivity of the infiltration parameters to the number of run-off and advance points. For this data there was a large difference between the infiltration functions calculated by the two methods.

Table 4: Advance data for Merkley

	Distance (m)	Time (min)
1	25	2.3
2	50	5.4
3	75	8.8
4	100	13.4
5	125	17.6
6	150	22.3
7	175	27.4
8	200	32
9	225	38.5

Table 5: Run-off data for Merkley

	Time (min)	Outflow (L/s)
1	46.3	0.55
2	49.2	0.65
3	52.2	0.72
4	57.2	0.79
5	62.2	0.91
6	67.2	0.91

Varying the number of advance points gave the results shown in Fig. 5. Selecting different data points for parameter calculation has a large impact when only advance data is used. There is a remarkable difference when only the first four advance points are used; the predicted infiltration is much greater. This further illustrates the benefits of using irrigation data spread over longer time periods. The optimisation technique using both advance and run-off points is not particularly sensitive to the selection of advance points indicating that the run-off data moderates the infiltration estimation. This may indicate towards reducing the number of advance points in order to simplify data collection. Recall that the chosen model includes the exponent r (equations 7 and 8) and therefore any use of this technique demands sufficient information to fit the power curve to the advance data.

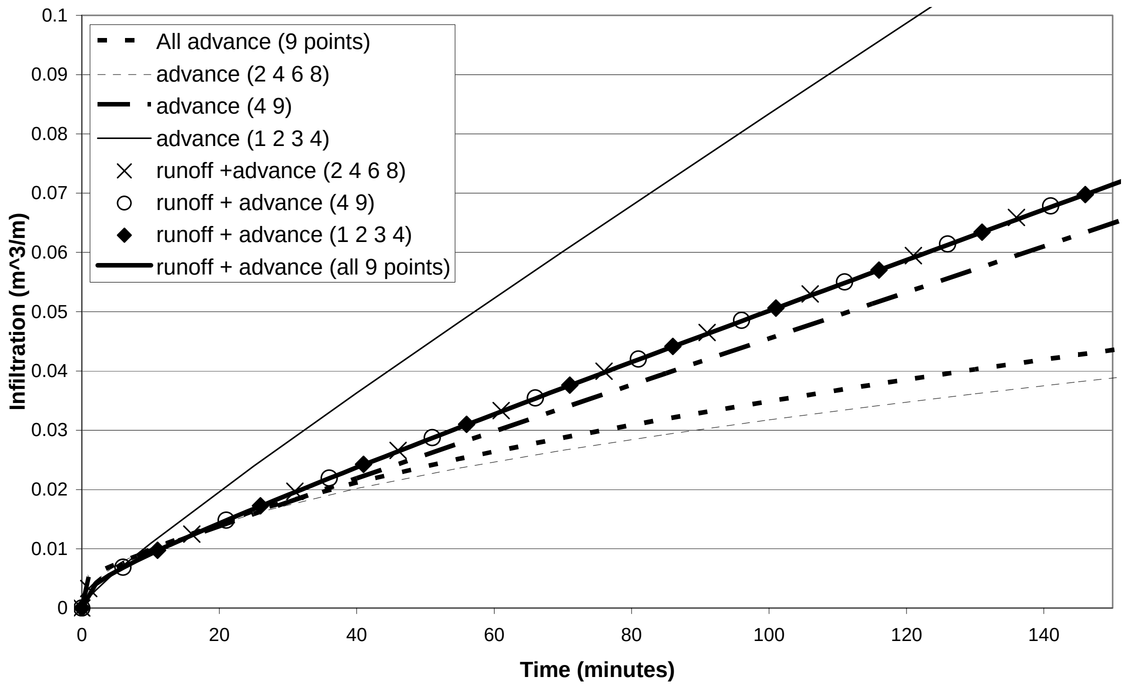

Figure 5 Comparing the two methods and their sensitivity to the correct selection of advance points

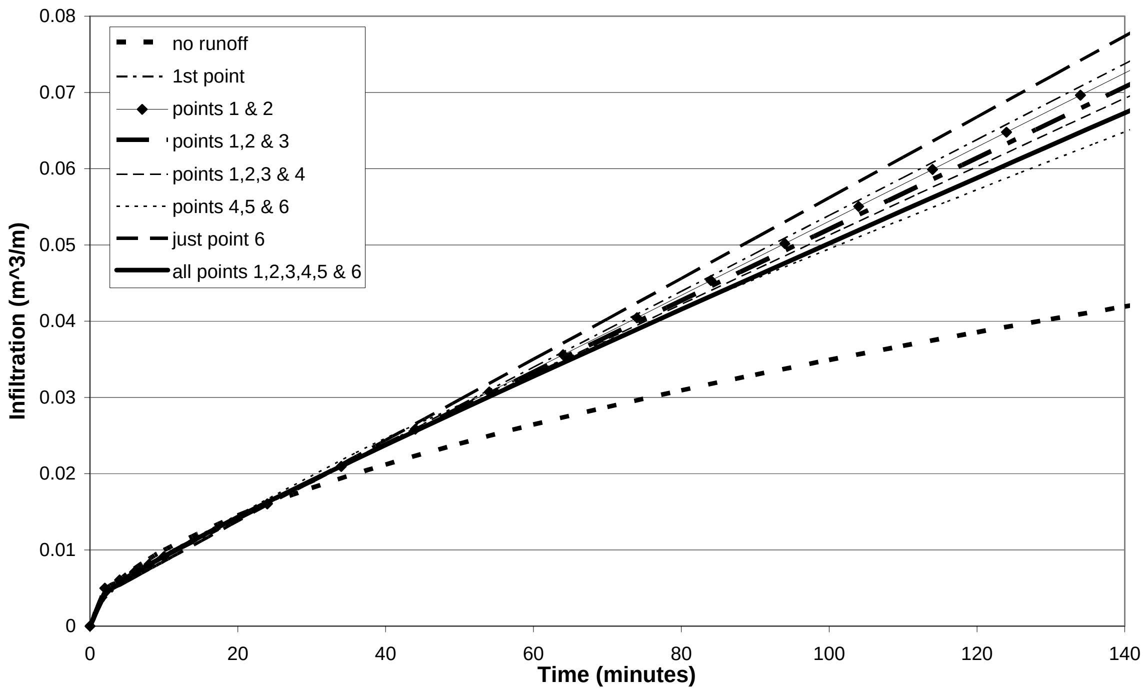

Figure 6 Impact of using different run-off points in the calculation of the infiltration function

The second part of the analysis investigated the effect of changing the selected run-off points used in the model (Fig. 6). Choosing different points does have some impact on the final outcome of the method. Even the use of just the first run-off point results in a far better infiltration function than using no run-off data. Including the first two points is sufficient to provide an accurate estimation of the infiltration curve. When only the steady outflow rate is included, the model produces a poor answer indicating that the shape of the run-off hydrograph is important, not just the final outflow rate. There is little difference between using all six points and only the even numbered points indicating that the number of points is not important as long as the points used represent a significant portion of the storage phase.

Weighting between Run-off and Advance Data

The weighing factor allows the user to change the relative importance of the advance and run-off data in the optimization of the infiltration function. It functions as a multiplier on the sum of errors in the predicted run-off volumes in equation 17. A value of 1 (100%) causes the relative error of the advance (equation 15) and the run-off (equation 16) to be of equal significance. The model will ignore the run-off if the weight is 0 and will ignore the advance data if the weight is given an extremely large value.

A small weighing factor (Fig 7) causes a significant change in the infiltration function; in this case once the weight of the run-off data has reached 5% of the advance data there is no added change by further increasing the importance of the run-off. All tests performed on the Merkley data seem to suggest that the infiltration function is much more sensitive to the run-off than the advance data. The use of both advance and run-off data in the optimisation will reduce the need for high precision advance data providing that the run-off input is accurate. Small errors in the measured run-off will have a significant impact on the infiltration parameters. The objective function operates by minimizing errors in advance distance and run-off volume. An incremental change in the infiltration parameters will cause a relatively modest percentage change in the calculated advance distance. The same change in infiltration will have a magnified effect on the error of run-off volume prediction of the storage phase; a small increase in the infiltration may reduce the run-off volume to zero. The high sensitivity of the optimisation to the run-off part of the volume balance suggests that the weighting should usually take a value equal to or less than 100%.

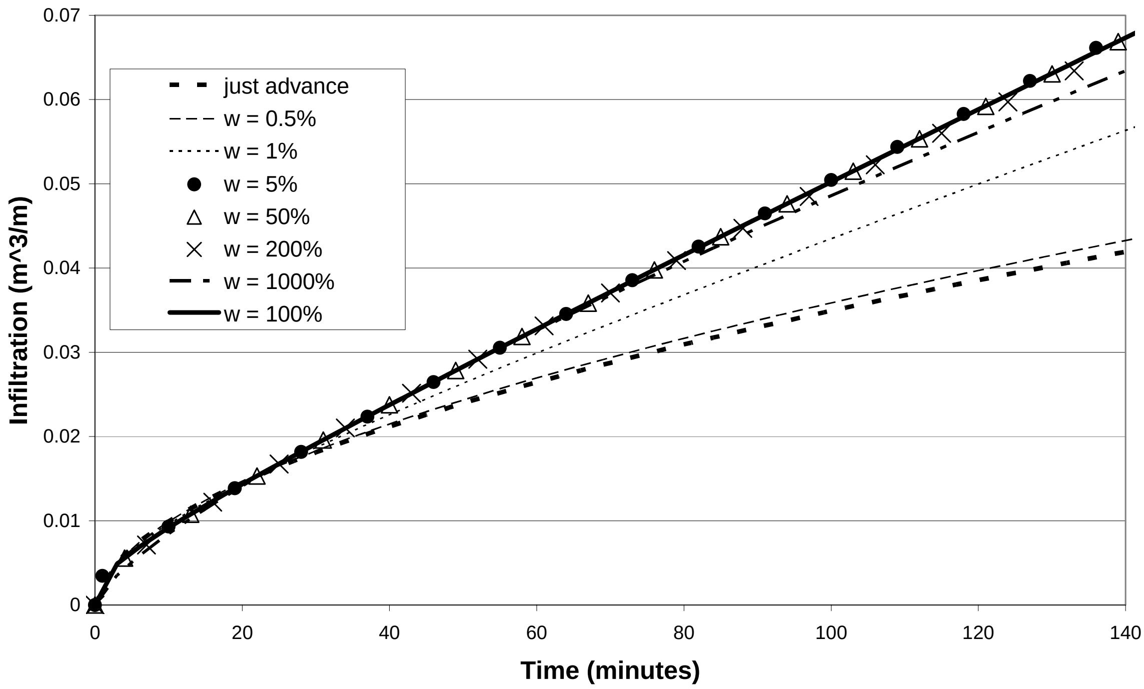

Figure 7 Effect of changing the weight of the run-off error in the Kostiakov parameter optimisation (100% is equal weight, <100% increases importance of the advance data)

Recommendations for the technique

The proposed parameter estimation procedure performed satisfactorily for the case studies presented here, however the volume balance model is based on a number of simplifications that may limit its application in certain conditions. The inflow rate throughout the entire irrigation should be constant with time as relatively small fluctuations may significantly impact on both the advance trajectory and runoff hydrograph. The model is only designed to apply during the storage phase therefore it is only valid to use runoff data collected during the inflow time. Further the location of the runoff measurement should be such that the measurement does not impose a backwater on the flow in the furrow.

Conclusions

The results of this study suggest that infiltration can be calculated more accurately from the combination of advance data and run-off rates measured during the storage phase of an irrigation. Current techniques that depend completely on the advance phase result in infiltration parameters that cannot accurately predict run-off volumes. The use of run-off data enables the extrapolation of the infiltration curve to greater times, which is of particular importance where the advance reaches the end of the furrow early in the irrigation time.

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