EFFECT OF BEAM-COLUMN JOINT STIFFNESS ON THE DESIGN OF BEAMS

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ABSTRACT

The infrastructure should be design in such a way that ensures safe, serviceable, durable and economic construction. For the beam design, the joints between beam and column is traditionally considered a rigid connection which actually not fully rigid and therefore accounted an extra moment leading to structural over-design and subsequently higher cost of beam construction. This paper proposed a theoretical model for moment and deflection considering the actual stiffness of beam-column joint rather than traditional concept of rigid connection. A concentrated moving load is applied on the beam with three different support conditions such as fixed both end, simply supported and propped cantilever. This proposed model is then verified theoretically considering known boundary conditions. Results showed that the proposed theoretical model for moment and deflection of beam perfectly captured the existing beam equations with that specific support conditions and the cost of the beam construction can be reduced due to considering the actual beam-column joint stiffness.

KEYWORDS

Semi rigid joint, rotational stiffness, moment, deflection

INTRODUCTION

Traditionally, the connections between beam and column in a structure is assuming either fully rigid or pinned which is not the actual behaviour according to the previous investigations (Johnson and Hope-Gill 1972; Cabrero and Bayo 2005). Rigid joints does not allow any rotations between the connected members while pinned joints are characterised by the free rotational movement between the connected elements and prevents the transmission of bending moments. The intermediate behaviour which is non-zero and non-infinite stiffness is called semi-rigid (Jaspart 2000). Figure 1 illustrated the classification of joints according to their rotational stiffness.

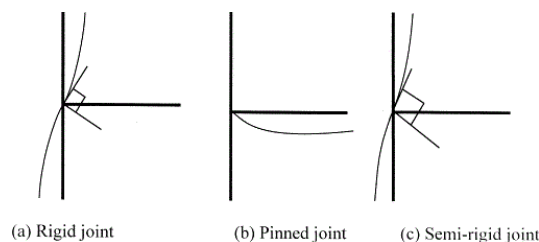(a) Rigid joint   (b) Pinned joint   (c) Semi-rigid joint
Figure 1. Different joints according to rotational stiffness (Jaspart 2000)

The study conducted by Johnson and Hope found that the pin joints are very unpredictable while the rigid joints are often too stiff in relation to their strength, and are expensive. Therefore, an ideal semi rigid joint is required to consider for the structural design with a large rotational capacity and a predictable flexural strength. Many studies (See 1983; Fathelbab 1987) have shown that the

connection stiffness has a considerable impact on the load-displacement behaviour of the structure and the considerations of semi rigid connection in building frame can reduce the beam size, the overall building height and cladding costs (Barnard 1970). This approach is not only advantageous in terms of material savings but also providing a lateral stiffness for sway frames and these advantage may even increase if the joints are designed as semi rigid in both major and minor axis of the column (Gil et al. 2013).

In 2005, Cabrero and Bayo (Cabrero and Bayo 2005) proposed a design method for semi-rigid joints incorporating the design examples for demonstrating the applicability of the proposed method. Their study concluded that the semi-rigid design is the most cost effective solution when it is compared with the traditional pinned and rigid joints. In order to obtain the optimal design, the rotational stiffness and moment resistance is required to analyse (Simões 1996). Among the various methods (empirical, analytical, mechanical, numerical and experimental) of establishing moment-rotation curves for semi rigid joints, the so-called mechanical component method is the most commonly used method in practice. It estimates the properties of the materials and joints and allows calculating the rotational stiffness and moment resistance of the joint (Faella et al. 1999). Although the joint behaviour through experimental test can provide the most accurate knowledge, but this technique is too expensive in everyday design practice and is usually reserved for research purposes only (Díaz et al. 2011). Due to the lack of appropriate design methods, models and tools for joint rotational stiffness, this study focused on the theoretical model for moment and deflection in case of semi rigid joint.

THEORETICAL MODEL FOR BENDING MOMENT

In contrast with the traditional concept of rigid connection between beams and columns, this study considers a partial rigid joint between them. The stiffness of the joint is the function of bending moment and deflection of the beam. The analytical model is developed for the moment and deflection considering a moving point load (P) on the beam as shown in Figure 2. Considering the joint-stiffness K_a and K_b in the support point 'a' and 'b' respectively, the slope of the beam at two supports can be presented by the definition of stiffness.

$$\theta_a = \frac{M_a}{K_a} \text{ and } \theta_b = \frac{M_b}{K_b}$$

where M_a and M_b are the corresponding bending moments at two supports.
The tangent of slope can also be calculated by

$$\tan \theta_a = \frac{t_{b/a}}{L} \text{ and } \tan \theta_b = \frac{t_{a/b}}{L}$$

As long as θ_a and θ_b are very small, $\tan \theta_a = \theta_a$ and $\tan \theta_b = \theta_b$
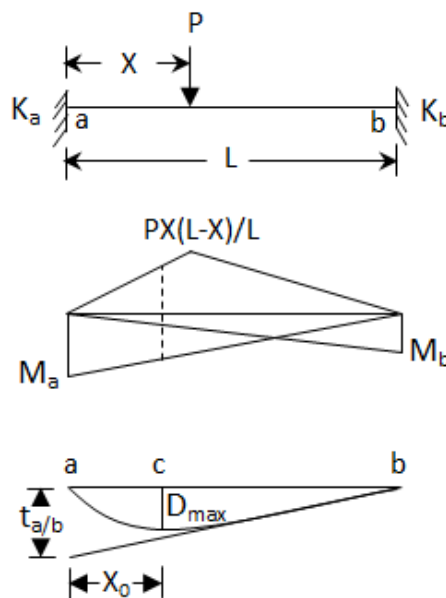

Figure 2. Moment and deflection for the semi rigid beam

From the above correlations and using area-moment theorem,

$$\frac{M_a}{K_a} = \frac{1}{L}\left[\frac{1}{EI}(Area)_{ba} \times \overline{X_b}\right] \tag{1a}$$

$$\frac{M_b}{K_b} = \frac{1}{L}\left[\frac{1}{EI}(Area)_{ab} \times \overline{X_a}\right] \tag{1b}$$

where, EI is the flexural rigidity of the beam. Solving Eqs. (1a) and (1b)

$$M_a = \frac{PX(L-X)}{L}f_1 \tag{2a}$$

$$M_b = \frac{PX(L-X)}{L}f_2 \tag{2b}$$

Eqs. (2a) and (2b) represents the general equations for moment at point 'a' and 'b' respectively where the factors f_1 and f_2 mainly depends on the stiffness of joint, beam stiffness and the location of the load.

$$f_1 = \frac{\frac{n}{6}\left(1+\frac{X}{L}\right)-\left(2-\frac{X}{L}\right)\left(m+\frac{n}{3}\right)}{\frac{n}{6}-6\left(m+\frac{1}{3}\right)\left(m+\frac{n}{3}\right)} \tag{3a}$$

$$f_2 = \frac{\frac{n}{6}\left(2-\frac{X}{L}\right)-n\left(1+\frac{X}{L}\right)\left(m+\frac{1}{3}\right)}{\frac{n}{6}-6\left(m+\frac{1}{3}\right)\left(m+\frac{n}{3}\right)} \tag{3b}$$
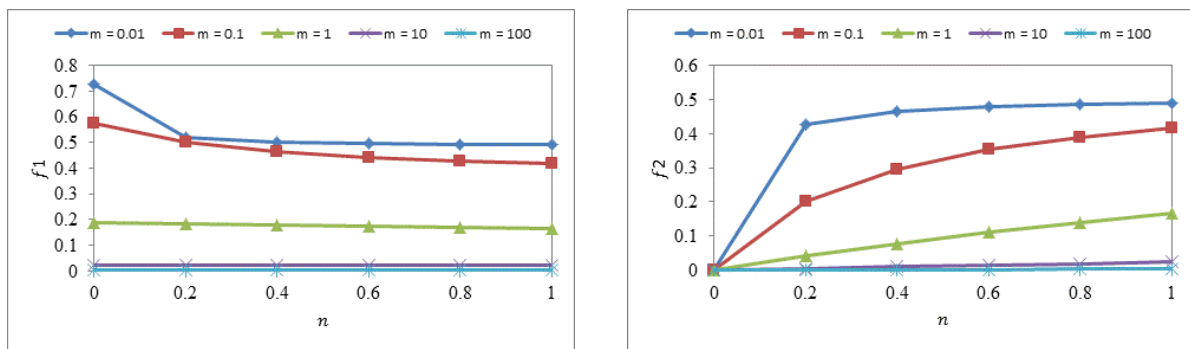
where,

Support stiffness ratio, $\qquad\qquad n = \dfrac{K_b}{K_a} \tag{4a}$

$$m =$$

Beam to support stiffness ratio, $\tag{4b}$

$$\frac{\left(\frac{EI}{L}\right)}{K_a}$$

For centre point loading, the variations of f_1 and f_2 with n and m has been plotted in Figure 3. The beam to support stiffness ratio (m) has been considered from 0.01 to 100 which indicates the support stiffness K_a is ranged from 100 to 1/100 times of beam stiffness.

(a) Variations of f_1 with n and m $\qquad\qquad$ (b) Variations of f_2 with n and m

Figure 3. Factors f_1 and f_2 for center point loading (i.e., $\frac{X}{L} = 0.5$)

For the identical support stiffness ($n = 1$) and a very small value of 'm' which is basically the rigid support condition, the factors f_1 and f_2 are both 0.5. Putting the magnitude in equation 2(a) and 2(b), the rigid end moment can be obtained.

THEORETICAL MODEL FOR DEFLECTION

Location of the Maximum Deflection

Before developing the theoretical model for the magnitude of maximum deflection, it is important to model the location of maximum deflection (X_0). By definition,

$$\theta_{ac} = \theta_a - \theta_c \tag{5a}$$

Since the maximum deflection occurs at point 'c' (Figure 2) and therefore, $\theta_c = 0$. The area-moment theorem can also be used for determining θ_{ac} as below.

$$\theta_{ac} = \frac{1}{EI}(Area)_{ac} \tag{5b}$$

Substituting the values of θ_a and θ_c and equating Eqs. (5a) and (5b), the location of maximum deflection can be determined from the quadratic Eq. (6).

$$\frac{X_0}{L} = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a} \tag{6}$$

The coefficient of a, b and c in Eq. (6) can be determined from the following equations depending on the point of load application and the location of the maximum deflection.

if, $X_0 \leq X$

$$a = 1 + \frac{X}{L}(f_1 - f_2)$$

$$b = -2\left(\frac{X}{L}\right)f_1$$

$$c = -\frac{1}{3}\left(\frac{X}{L}\right)\left[\left(2 - \frac{X}{L}\right) - (2f_1 + f_2)\right]$$

if, $X_0 \geq X$

$$a = \left(1 - \frac{X}{L}\right)(f_1 - f_2) - 1$$

$$b = 2\left[1 - \left(1 - \frac{X}{L}\right)f_1\right]$$

$$c = -\frac{1}{3}\left(1 - \frac{X}{L}\right)\left[\left(2 - \frac{X}{L}\right) - (2f_1 + f_2)\right] - \frac{X}{L}$$

Magnitude of Maximum Deflection

Theoretically the slope at the point of maximum deflection is zero (Figure 2) indicating

$$D_{max} = t_{a/c} = \frac{1}{EI}(Area)_{ac} \times \overline{X_a} \tag{7}$$

when, $X_0 \leq X$

$$D_{max} = \left[\frac{PL^3}{3EI}\left(\frac{X_0}{L}\right)^3\left(1 - \frac{X}{L}\right)\right] - \left[\frac{PL^3}{6EI}\left(\frac{X_0}{L}\right)^2\left(\frac{X}{L}\right)\left(1 - \frac{X}{L}\right)\left[3f_1 - 2\frac{X_0}{L}(f_1 - f_2)\right]\right] \tag{8a}$$

Eq. (8a) can be rearranged as below

$$\frac{EID_{max}}{PL^3} = \left[\frac{1}{3}\left(\frac{X_0}{L}\right)^3\left(1 - \frac{X}{L}\right)\right] - \left[\frac{1}{6}\left(\frac{X_0}{L}\right)^2\left(\frac{X}{L}\right)\left(1 - \frac{X}{L}\right)\left[3f_1 - 2\frac{X_0}{L}(f_1 - f_2)\right]\right] \tag{8b}$$

when, $X_0 \geq X$

$$D_{max} = \left[\frac{PL^3}{6EI}\left(\frac{X}{L}\right)\left[2\left(\frac{X}{L}\right)^2\left(1 - \frac{X}{L}\right) + 3\left(1 - \frac{X_0}{L}\right)\left(\frac{X_0}{L} - \frac{X}{L}\right)\left(\frac{X_0}{L} + \frac{X}{L}\right) + \left(2\frac{X}{L} + \frac{X_0}{L}\right)\left(\frac{X_0}{L} - \frac{X}{L}\right)^2\right]\right]$$
$$- \left[\frac{PL^3}{6EI}\left(\frac{X}{L}\right)\left(\frac{X_0}{L}\right)^2\left(1 - \frac{X}{L}\right)\left[3\left(1 - \frac{X_0}{L}\right)f_1 + \frac{X_0}{L}(f_1 + 2f_2)\right]\right] \tag{9a}$$

Similarly,

$$\frac{EID_{max}}{PL^3} = \left[\frac{1}{6}\left(\frac{X}{L}\right)\left[2\left(\frac{X}{L}\right)^2\left(1-\frac{X}{L}\right) + 3\left(1-\frac{X_0}{L}\right)\left(\frac{X_0}{L}-\frac{X}{L}\right)\left(\frac{X_0}{L}+\frac{X}{L}\right) + \left(2\frac{X}{L}+\frac{X_0}{L}\right)\left(\frac{X_0}{L}-\frac{X}{L}\right)^2\right]\right]$$

$$-\left[\frac{1}{6}\left(\frac{X}{L}\right)\left(\frac{X_0}{L}\right)^2\left(1-\frac{X}{L}\right)\left[3\left(1-\frac{X_0}{L}\right)f_1 + \frac{X_0}{L}(f_1+2f_2)\right]\right] \qquad (9b)$$

Eqs. (8a) and (9a) presents the general equations for maximum deflection. The first part of those equations indicates the deflection of the beam due to simply supported action and the second part indicates the reduction of deflection due to semi rigid action. This can simplify as below.

$$D_{max} = D_{ss} - D_{sr} \qquad (10)$$

where,

D_{ss} = deflection due to simply supported action

D_{sr} = reduction of deflection due to semi rigid action

VALIDATION OF THE MODEL

The theoretical model proposed in this study has been validated using three known support conditions (Figure 4) and they are (a) rigid support, (b) simply support and (c) propped cantilever condition. The moment and deflection in the aforementioned condition is well established. The proposed model is validated using the necessary boundary conditions.
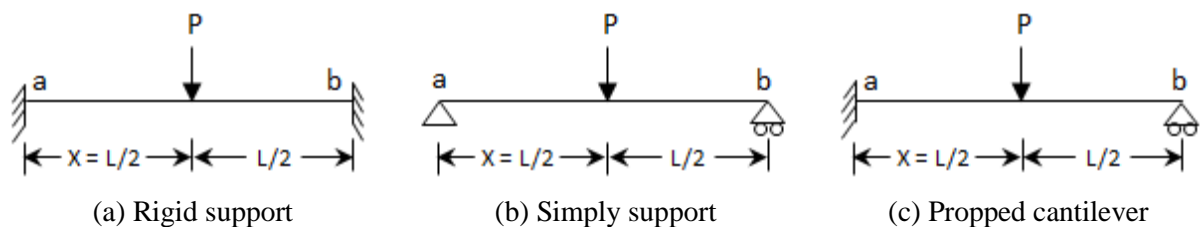

(a) Rigid support   (b) Simply support   (c) Propped cantilever

Figure 4. Different support conditions with centre point loading

Table 1. Moment and deflection calculation for the known condition

Equations	Parameters	Rigid support	Simply support	Propped cantilever
	K_a	$= K_b$ but $\gg \frac{EI}{L}$	$= K_b$ but $\ll \frac{EI}{L}$	$\gg K_b$ and $\gg \frac{EI}{L}$
[Eq. 4a]	n	1	1	Very small
[Eq. 4b]	m	Very small	Very large	Very small
[Eq. 3a]	f_1	$\frac{1}{2}$	0	$\frac{3}{4}$
[Eq. 3b]	f_2	$\frac{1}{2}$	0	0
[Eq. 2a]	M_a	$\frac{PL}{8}$	0	$\frac{3PL}{16}$
[Eq. 2b]	M_b	$\frac{PL}{8}$	0	0
[Eq. 6]	$\frac{X_0}{L}$	$\frac{1}{2}$	$\frac{1}{2}$	$1-\frac{1}{\sqrt{5}}$
[Eq. 8a]	D_{max}	$\frac{1}{192}\left(\frac{PL^3}{EI}\right)$	$\frac{1}{48}\left(\frac{PL^3}{EI}\right)$	$\frac{1}{48\sqrt{5}}\left(\frac{PL^3}{EI}\right)$

Based on the known boundary conditions, Table 1 provides the end moment, location of maximum deflection and the magnitude of maximum deflection. The results obtained from the theoretical model are similar to the traditional equations for moment and deflection.

CONCLUSIONS

This study provides a theoretical model for moment and deflection incorporating the rotational support stiffness from which the following conclusions are drawn:

1. The theoretical model for moment and deflection can explain the intermediate behaviour of pinned and rigid joint condition and can easily introduce in everyday design practice.
2. The semi rigid joint between beam and column provides a lower moment at the end of the beam when it is compared with fully rigid joint. This lower design moment can reduce the required section modulus of the beam and subsequently a potential cost savings may achieve.
3. The results obtained from the model for the specific boundary conditions comply with the well-known equations of simple mechanics.

FUTURE RECOMMENDATIONS

A basic approach has taken to establish the fundamentals of beam-column joint stiffness from which the further research could continue on the following areas:

1. Experimental investigation on the behaviour of beam when it is semi-rigidly connected with column.
2. Further research need to establish the theoretical model for semi-rigidly connected continuous beam.

ACKNOWLEDGMENTS

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