MATHEMATICAL REFLECTION APPROACH TO INSTRUMENTAL VARIABLE ESTIMATION METHOD FOR SIMPLE REGRESSION MODEL

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ABSTRACT

The measurement errors problem is endemic in many econometric studies, and one of the oldest known statistical problems. Instrumental variable (IV) method is one of the popular solutions adopted to deal with the mismeasured variables in statistical and econometric analyses. This paper proposes an efficient IV estimator to the parameters of the simple regression model where both variables are subject to measurement errors. The proposed IV is defined using simple mathematical transformation of the manifest independent variable (mismeasured variable). The proposed method is straightforward, and easy to implement. The theoretical superiority of the proposed estimator over the existing IV based estimators due to Wald (1940), Bartlett (1949), and Durbin (1954) is established by analytical comparison and geometric expositions. Simulation based numerical comparisons of the proposed estimator with four different existing estimators are also included.

Keywords: Simple regression model, Error-in-Variables model, Instrumental variable.

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1- INTRODUCTION

The measurement errors problem is a very old problem, and it has been considered by a host of authors since the late nineteenth century. This problem is seldom taken into fully account, although it has very serious consequences on the statistical inference.

In reality the measurement errors in data are inevitable and exist in almost all the applied fields. Linnet (1993) states, “It is rare that one of the measurement methods is without error.” The motivation of proposed methods in the literature of measurement error is to eliminate, or at least reduce implications of the measurement error on the estimator of parameters. The measurement error problem it is often given prominence in econometrics texts, for example Judge et al (1980), Stock et al (2003), Hill et al (2008), Wooldridge (2010), but it is rarely included in statistical texts (Gillard, 2005). This problem has been studied in depth by some authors such as Fuller (2006), Cheng and van Ness (1999), Casella and Berger (1990), Sprent (1969), Dunn (2004), and Kendall and Stuart (1973, chap. 29). They concentrated on the maximum likelihood principle and summarize correction formulae for measurement error models based on assumptions of additional information.

However despite all these efforts the challenge is still looming, since Riggs et al. (1978) stated that no one method of estimating the true slope is the best method under all circumstances. Cheng and van Ness (1999) stated that some users object to the use of these side conditions and prefer other methods of approaching the measurement error model. This is common, for example, in the econometrics literature.

The alternative method which was pioneered to overcome the measurement error problem since 1920 is the instrumental variables approach (see Goldberger (1972) for a historical review). This approach provides supplementary information to make the parameters identifiable (cf Cheng and van Ness, 1999, p.93). This method is one of the popular solutions adopted to deal with the mismeasured variables in statistical and econometric analyses by Wald (1940), Durbin (1954), and Sargan (1958). Most recently, Chen et al. (2014) reviewed and investigated the existing errors-in-variables estimation methods and their applications in finance research. Almeida et al. (2010) have argued and presented an alternative instrumental method to deal with measurement error problems.

Instrumental variable (IV) technique requires defining an IV that is uncorrelated with the model error but highly correlated with the independent variable. Wald (1940) suggested to use -1 and +1 for values less than or greater than the median of the manifest variable, Bartlett proposed to divide the values in three equal groups and use the first and third groups, and Durbin used the ranks of the values to define the IV. In each of the method there is loss of information (for not using actual values and dropping some of the data points), and there are different formulae to find the sum of squares error, and hence lead to different mean sum of square error, making the analysis incomparable.

The IV method has been used for studying the natural and quasi-natural experiments such as Miguel et al. (2004) studied the weather shocks to identify the effect of changes in economic growth on civil conflict. Angrist and Krueger (2001) pointed out that instrumental variables have been widely used to reduce bias from omitted variables in estimates of causal relationships in randomized experiments such as the effect of schooling on earnings. They have presented a survey of the history and uses of instrumental variable technique. Cheng and van Ness (1999) stated that the instrumental variable method suits all kinds of regression with random regressors for which the explanatory variables are correlated with the errors. Bowden and Turkington (1981), and Martens (2006) introduced the details of general treatment of instrumental variables and their applications and limitations.

It is worth noting that the greatest drawback of IV approach is how or where to find valid instrumental variable, which it is not easy to obtain. Therefore, this paper proposes an instrumental variable which is easier to obtain in practice to estimate the parameters of bivariate errors-in-variables model. The proposed instrumental variable is defined using reflection of the observed values of the independent variable. The proposed modified method uses the reflection of the manifest values of the independent variable to define IV estimator. The using of the reflections of the observed values of the independent variable in defining the IV method provides a much better estimator of the slope and intercept parameters. It also reduces the mean sum of squares error. The analysis of variance and regression inferences based on the reflections have much better statistical properties than any other form of the IV estimator (Saqr and Khan, 2012).

In the next section the measurement error regression model is introduced. Section 3 covers the existing estimation methods for the measurement error model. The proposed modified estimator based on the reflections of the observed values of the independent variable is provided in Section 4. The superior properties of the modified estimator are discussed in Section 5. A simulation study is presented which compares the proposed estimator with five different existing estimators are provided in Section 6, and some concluding remarks are given in Section 7.

2- MEASUREMENT ERROR MODELS

In the conventional notation, let ξ_j denote the true measurement on the independent variable. This is also called the latent independent variable. In the presence of measurement error the actual observations are different from ξ_j. Let x be the observable, or manifest variable of the independent variable. When the true value of the latent variable ξ_j is observed, the commonly used classical simple linear regression model is represented by

$$\eta_j = \beta_{0\xi} + \beta_{1\xi}\xi_j + e_j, \quad j = 1, 2, \ldots, n, \qquad (1)$$

where η_j is the jth realisation of the latent dependent variable, ξ_j is the fixed jth value of the independent variable, and e_j is the equation error for $j = 1, 2, \ldots, n$. It is assumed that the equation error e_j is independently distributed with constant but unknown variance, that is, $e_j \sim N(0, \sigma_e^2)$.

If there is error in the independent variable, the actual observed value, x_j, is not the `true' value of the independent variable. The observed value of the independent variable contains measurement error given as

$$x_j = \xi_j + \delta_j, \quad j = 1, 2, \ldots, n, \qquad (2)$$

where δ_j is the measurement error, and is assumed to be distributed as $N(0, \sigma_\delta^2)$. Note that, unlike ξ_j, x_j is a random variable which is assumed to be distributed as $N(\mu_x, \sigma_x^2)$. The model with the fixed ξ_j is called the functional model, and the model with the random or stochastic x is called the structural model.

The simple regression model with measurement error in the independent variable can be expressed as

$$\eta_j = \beta_{0\xi} + \beta_{1\xi} x_j + v_j, \qquad j = 1, 2, \ldots, n, \qquad (3)$$

where $v_j = e_j - \beta_{1\xi}\delta_j$. Note in equation (1) ξ_j and e_j are independent, but in equation (3), x_j and v_j are not independent. So the application of least squares method is not valid for the models with measurement error. Thus, unlike for the model in (1), the validity of the estimator of the slope and intercept of the model in (3) is not obvious. However, Fuller (2006, p. 3) assumes that δ_j, ξ_j and e_j are mutually independent for the estimation of the parameters. It also assumes that the reliability ratio, $k_{x\xi} = \sigma_x^{-2}\sigma_\xi^2$ is known, where σ_x^2 is the variance of the manifest variable x_j, and σ_ξ^2 is the variance of the latent variable ξ_j.

3- THE LEAST SQUARES ESTIMATOR OF PARAMETERS

The ordinary least squares (OLS) estimator of the regression parameters for the functional model are

$$\hat{\beta}_{1\xi} = \frac{S_{\xi\eta}}{S_{\xi}^{2}}, \text{and } \hat{\beta}_{0\xi} = \overline{\eta} - \hat{\beta}_{1\xi}\overline{\xi}, \tag{4}$$

where

$$S_{\xi\eta} = \frac{1}{n-1}\sum_{j=1}^{n}(\xi_j - \overline{\xi})(\eta_j - \overline{\eta}), \qquad S_{\xi}^{2} = \frac{1}{n-1}\sum_{j=1}^{n}(\xi_j - \overline{\xi})^2, \tag{5}$$

in which $\overline{\eta} = \frac{1}{n}\sum_{j=1}^{n}\eta_j$ and $\overline{\xi} = \frac{1}{n}\sum_{j=1}^{n}\xi_j$. The estimators of slope and intercept parameters are well known to be the best linear unbiased estimators if there is no measurement error in the variables.

The sampling distribution of the estimator of the regression parameters is given by

$$\begin{pmatrix}\hat{\beta}_{0\xi}\\ \hat{\beta}_{1\xi}\end{pmatrix} \sim N_2\left[\begin{pmatrix}\beta_{0\xi}\\ \beta_{1\xi}\end{pmatrix}, \sigma_e^2\begin{pmatrix}\frac{1}{n}+\frac{\overline{\xi}^2}{S_{\xi}^{2}} & \frac{-\overline{\xi}}{S_{\xi}^{2}}\\ \frac{-\overline{\xi}}{S_{\xi}^{2}} & \frac{1}{S_{\xi}^{2}}\end{pmatrix}\right]. \tag{6}$$

The unbiased estimator of the error variance σ_e^2 is given by

$$\hat{\sigma}_e = (n-2)^{-1}SSE_e = S_e^2,$$

where $$SSE_e = \sum_{j=1}^{n}(\eta_j - \hat{\eta}_j)^2,$$

in which $\hat{\eta}_j = \hat{\beta}_{0\xi} + \hat{\beta}_{1\xi}\xi_j$ is the estimated value of η_j. Also, $\sigma_e^{-2} SSE_e$ follows a χ^2 distribution with $(n-2)$ degrees of freedom.

In the presence of measurement error, the x values are observed instead of ξ_j, then the least squares method yields the estimator of the slope as

$$\hat{\beta}_{1x} = \frac{S_{x\eta}}{S_x^2}, \text{and } \hat{\beta}_{0x} = \overline{\eta} - \hat{\beta}_{1x}\overline{x}. \tag{7}$$

It can be easily shown that $\hat{\beta}_{1x}$ is a biased estimator of $\beta_{1\xi}$. Also, the above estimator is not a consistent estimator of $\beta_{1\xi}$.

Note that the regression parameters are different for the model with the manifest variable than the model with the latent variable. Even though the aim is to estimate and test $\beta_{0\xi}$ and $\beta_{1\xi}$, in reality one may end up estimating and testing β_{0x} and β_{1x} if one fully relies upon x, and over looks the presence of the measurement error.

4- INSTRUMENTAL VARIABLE (IV) ESTIMATOR

In the presence of measurement error in the independent variable the IV estimator for the regression parameters is defined as

$$\hat{\boldsymbol{\beta}} = (z'x)^{-1} z'\eta, \tag{8}$$

where $\hat{\boldsymbol{\beta}} = (\hat{\beta}_0, \hat{\beta}_1)'$ is the vector of estimator of the intercept and slope parameters of the model

where

$$x = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{pmatrix} \text{and } z = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_n \end{pmatrix},$$

in which z_j's are the values of the second row of the instrumental variable z. The selection of the values of z_j's require that it is highly correlated with the independent variable but uncorrelated with the model errors. The variance-covariance of the above estimator vector is given by

$$\text{var}(\hat{\boldsymbol{\beta}})=\sigma_\delta^2(\text{z}'\text{x})^{-1}(\text{z}'\text{z})(\text{z}'\text{x})^{-1}. \tag{9}$$

Obviously the value of the estimator and the variance depend on the choice of z (see Johnson, 1972). For instance, the Wald method, as suggested by Maddala (1988), defines z by assigning z_j to be -1 or +1 depending upon if x_j is smaller or larger than the median value of the manifest variable. The estimator of slope under this choice of IV is

$$\hat{\beta}_{IW} = \frac{\bar{\eta}_2 - \bar{\eta}_1}{\bar{x}_2 - \bar{x}_1},$$

where $\bar{\eta}_1$ is the mean of η -values associated with the values of x less than its median, and $\bar{\eta}_2$ is for the mean values larger than the median value of η. Bartlett (1949) followed the same selection criterion of z_j's but suggested the exclusion of the middle $1/3$ of the values, and his estimator is based on the lower and upper $1/3$ of the values of x and the associated $\eta's$. The estimator is expressed as

$$\hat{\beta}_{1B} = \frac{\bar{\eta}_3 - \bar{\eta}_1}{\bar{x}_3 - \bar{x}_1},$$

where $\bar{\eta}_1$ is the mean of η-values associated with the smallest $1/3$ of the values of x, and $\bar{\eta}_3$ is that for the largest $1/3$. Durbin (1954) proposed to use the rank of x as z_j's. His method yields the following estimator of the slope parameter

$$\hat{\beta}_{1D} = \left[\sum_{j=1}^{n} j\eta_j\right] / \left[\sum_{j=1}^{n} jx_j\right],$$

but does not define the estimator of the intercept.

The IV method of estimation of the regression parameters does not require any strict assumptions such as the ratio of error variances is known. But the actual estimator depends on how the IV is defined, as the definition of z affects both the estimator and its variance. In general, the available methods of defining IV causes a significant loss of sample information (data) either by replacing the observed values of the independent variable by -1 or +1, or exclusion of some data, or due to ranking of data.

5- PROPOSED IV ESTIMATOR OF SLOPE

The idea of the proposed estimator of slope is based on using the reflection variable of the manifest independent variable as IV variable. The proposed IV variable is obtained by reflecting all values of the manifest independent variable about the unfitted regression line. This is essentially done by a transformation of the observed values of the independent variable to their reflection on the Euclidean plane. In the conventional notation, the reflection of the manifest independent variable $x_j = \xi_j + \delta_j$ (with measurement error δ_j) for $j = 1, 2, \ldots, n$, can be defined as

$$x^* = x \cos 2\psi + (\eta - \hat{\beta}_{0x}) \sin 2\psi, \qquad (10)$$

where $\hat{\beta}_{0x}$ is the least squares estimate of the intercept parameter, ψ is the angle measure defined as $\psi = \arctan \hat{\beta}_{1x}$ in which $\hat{\beta}_{1x}$ is the least squares estimate of the slope parameter in the manifest model, and *Cos*, *Sin* are the usual trigonometric cosine and sine functions respectively. For the definition of reflection points on the Cartesian plane readers may see (Vaisman 1997, p. 164-169; Saqr and Khan, 2012).

The proposed reflection method requires to compute the reflection of all data points, and the use of the transformed values of x , i.e. x^*, in defining the IV to fit the regression line of η. The IV estimator of the slope parameter under the proposed modified method is

$$\hat{\beta} = (z_r{}'x)^{-1} z_r{}'\eta, \text{and } \hat{\beta}_{1R} = \frac{S_{x^*\eta}}{S_x^2},$$

where

$$Z_r = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ x_1^* & x_2^* & \cdots & x_n^* \end{pmatrix}, \quad S_{x^*x} = S_x^2, \quad and \quad S_{x^*x} = \sum_{j=1}^{n} (x_j^* - \bar{x}^*)(x_j - \bar{x}).$$

The proposed estimator of the slope parameter of the simple regression model using IV based on the reflection of x is

$$\hat{\beta}_{1R} = \frac{S_{x^*\eta}}{S_x^2}. \tag{11}$$

From (11), it is easy to show that $S_{xy} = S_{\xi y}$ and $S_x^2 = S_\xi^2 + S_\delta^2$.

It can be found that

$$S_{x^*y} - S_{xy} = SSE_x \sin 2\psi, \tag{12}$$

where ψ is as defined in equation (10), and SSE_x is the sum of squares error for the manifest model. The above result follows from the fact that

$$x_j^* - x_j = x_j \cos 2\psi + (\eta_j - \hat{\beta}_{0x}) \sin 2\psi - x_j$$

$$= x_j (\cos 2\psi - 1) + \eta_j \sin 2\psi - \hat{\beta}_{0x} \sin 2\psi$$

$$= -x_j (2\sin^2 \psi) + \eta_j \sin 2\psi - \bar{\eta} \sin 2\psi + \bar{x}\, 2\sin^2 \psi$$

$$= (\eta_j - \bar{\eta}) \sin 2\psi - (x_j - \bar{x}) 2\sin^2 \psi, \tag{13}$$

where x_j^* is the reflection of x_j. Multiplying both sides of the above equation by η_j and taking sum over j, yields

$$\sum (x_j^* - x_j)\eta_j = \sum (\eta_j - \bar{\eta})\eta_j \sin 2\psi - \sum (x_j - \bar{x})\eta_j\, 2\sin^2 \psi$$

$$S_{x^*\eta} - S_{x\eta} = S_\eta^2 \sin 2\psi - S_{x\eta} 2\sin^2\psi$$

$$\frac{S_{x^*\eta} - S_{x\eta}}{\sin 2\psi} = SST - SSR_x = SSE_x \,, \qquad (14)$$

where $S_\eta^2 = SST$ is the sum of squares total, SSR_x is the sum of squares regression, and SSE_x is the sum of squares error for the regression of η on x . Note that $\frac{2\sin^2\psi}{\sin 2\psi} = \tan\psi = \hat{\beta}_{1x}$.

Then using equation (10), it can be written as

$$\hat{\beta}_{1\xi} = \frac{S_{\xi\eta}}{S_\xi^2} = \frac{S_{x\eta}}{S_\xi^2} = \frac{S_{x^*\eta} - SSE_x \sin 2\psi}{S_x^2 - S_\delta^2}$$

$$\hat{\beta}_{1R} = \frac{S_{x^*\eta}}{S_x^2} = \frac{S_{x\eta} + SSE_x \sin 2\psi}{S_\xi^2 + S_\delta^2} = \frac{S_{\xi\eta} + SSE_x \sin 2\psi}{S_\xi^2 + S_\delta^2} .$$

Let λ^* be the ratio of the vertical error variance σ_v^2 and horizontal error variance σ_δ^2, that is $\lambda^* = \frac{\sigma_v^2}{\sigma_\delta^2}$.

Based on the assumption $\lambda^* = \frac{S_{x^*\eta}}{S_x \sin 2\psi}$, then

$$\hat{\beta}_{1R} = \frac{S_{x^*\eta}}{S_x^2} = \frac{S_{x^*\eta} - S_{\xi\eta}}{S_x^2 - S_\xi^2} \qquad (15)$$

$$S_{x^*\eta}(S_x - S_\xi) = S_x^2(S_{x^*\eta} - S_{\xi\eta})$$

which leads to $S_{x^*\eta}S_\xi^2 = S_{\xi\eta}S_x$, and finally simplification yields

$$\frac{S_{x^*\eta}}{S_x^2} = \frac{S_{\xi\eta}}{S_\xi^2}, \text{hence } \hat{\beta}_{1R} = \hat{\beta}_{1\xi}. \qquad (16)$$

6- GEOMETRIC EXPLANATION OF THE PROPOSED ESTIMATOR

The presence of measurement error in the independent variable and its impact on the estimator of the slope as well as how the proposed method 'treats' the measurement error can be explained by graphs. The graphical representation also explains how the actual estimator of the slope is recovered by the new method. Figure 1 represents the sum of squares and sum of products associated with the definition of the estimators of slope both for the latent and manifest variables. This graph represents the presence of measurement error in the independent variable as well as the two estimators of the slope parameter. On the other hand Figure 2 displays the same along with that of the reflection of the manifest variable and three estimators of the slope parameter.

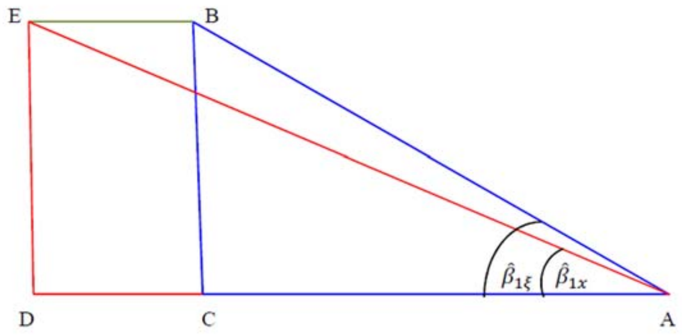

Figure 1: Graph representing the sum of squares and products in the presence of measurement error in the independent variable.

From Figure 1, the true estimator of the slope when the latent variable is available, that is, $\hat{\beta}_{1\xi}$ is represented by the *tan* of $\angle BAC$ of Δ *ABC*. In the absence of the values of the latent variable this is unavailable. But for the manifest variable one can find the estimator of the slope to be $\hat{\beta}_{1x}$ which is represented by the $\tan$ of $\angle DAE$ of ΔADE. Note that here *DC* (or equivalently *BE*) represents the sum of squares of measurement error (S_{δ}^{2}). Furthermore, under the assumptions of $E[\eta\delta]=0$ and $E[\xi\delta]=0$, we have $BC=DE$ or $S_{\xi\eta}=S_{x\eta}$. Finally, $\hat{\beta}_{1\xi}=\frac{S_{\xi\eta}}{S_{\xi}^{2}}=\frac{BC}{AC}$, and $\hat{\beta}_{1x}=\frac{S_{xy}}{S_{x}^{2}}=\frac{ED}{AD}$.

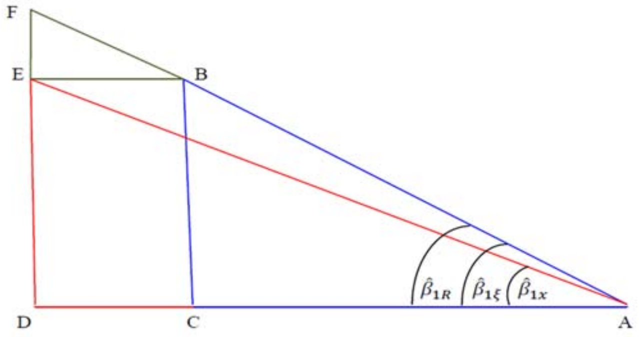

Figure 2: Graph representing the sum of squares and products when the measurement error in the independent variable is 'treated' by reflection.

The introduction of the reflection of the manifest variable changes ΔADE of Figure 1 to ΔADF in Figure 2. In fact the main difference between the two Figures is that Figure 2 has the small ΔBEF added to Figure 1. This triangle represents the effect of the reflection of the manifest variable. From Figure 2 the estimates of the slope are

$$\hat{\beta}_{1x} = \frac{S_{x\eta}}{S_x^2}\left(=\frac{DE}{DA}\right) \tag{17}$$

$$\hat{\beta}_{1\xi} = \frac{S_{\xi\eta}}{S_\xi^2}\left(=\frac{BC}{AC}\right) \tag{18}$$

$$\hat{\beta}_{1R} = \frac{S_{x^*\eta}}{S_x^2}\left(=\frac{FD}{AD}\right). \tag{19}$$

Since the $\tan$ of $\angle BAC$ represents the estimator $\hat{\beta}_{1\xi}$ and $\tan$ of $\angle DAF$ represents $\hat{\beta}_{1R}$, then $\hat{\beta}_{1\xi} = \hat{\beta}_{1R}$ because $\angle BAC = \angle DAF$.

7- SIMULATION STUDY

In this section, simulated data are used when both the dependent and independent variables are subject to measurement error. This study reveals that the performance of proposed estimator (RIV) is better than OLS estimator and other estimators proposed by Wald (1940) $(Two-g)$, Bartlett (1949) $(Thr-g)$, and Durbin (1954) (Dur). Here calculations are based on the generated values of variables for preselected values of $\beta_0 = 0$, $\beta_1 = 1$, latent variable $\xi \sim N(0,36)$, $\sigma_\delta^2 = 16$, and $\sigma_e^2 = 9$. The simulation is based on 10,000 replications using MATLAB software.

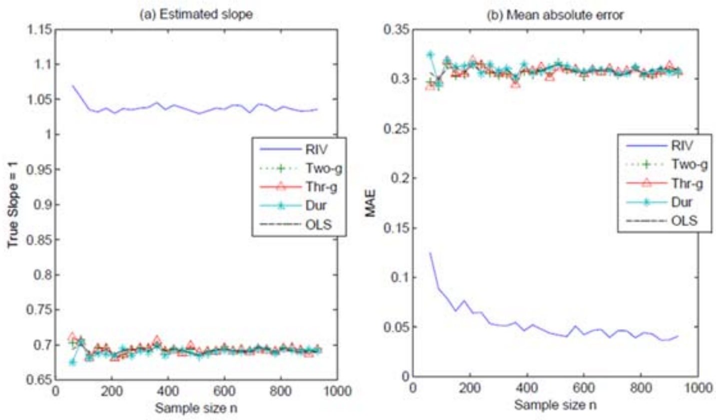

Figure 3: Graphs of the estimated slope (a) and the mean absolute error (b) for five different estimators, proposed instrumental variable estimator (RIV), Wald's estimator $(Two-g)$, three-group estimator $(Thr-g)$, Durbin estimator (Dur), and ordinary least squares estimator (OLS).

To show the behavior of the above estimators we selected samples of size $30, 60, 90, 120, \cdots, 1000$. Then computed values of the estimators from the simulated data and find their means and mean absolute errors (MAE) for each of the five estimators.

Figures 3a and 3b show the estimated slope and the mean absolute error for five different estimators. From the above graph it is evident that the proposed instrumental variable estimator (RIV) is consistently better than the other four estimators. Clearly the *RIV* estimator is much closer to the true value of β_1 than other four estimators. In fact, the proposed *RIV* estimator is consistently closest to the true value of the slope for all sample sizes.

8- CONCLUDING REMARKS

This paper considers the simple regression model with measurement error in both dependent and independent variables. It also proposes a new estimation procedure based on the idea of a new instrumental variable which is defined from reflection of the manifest variable. It compares the existing methods with proposed new method. Unlike, some of the existing methods it does not lose information.

The simulation study demonstrates the fact that the proposed method significantly reduces the mean absolute error than the currently used IV methods. As such, the coefficient of determination of the proposed method is higher than that of the existing IV methods. Surprisingly, the proposed IV method recovers the true estimator of the slope, $\hat{\beta}_{1\xi}$, from the manifest variable and stochastic model even if the true values of the latent independent variable are unobservable. The same comment would apply for the estimator of the intercept.

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REFERENCES

1. Almeida, H., Campello, M., and Galvao, A.F. (2010). Measurement errors in investment equations. *Review of Financial Studies*, 23(9), 3279-3328.
2. Angrist, J., and Krueger, A.B. (2001). Instrumental variables and the search for identification: From supply and demand to natural experiments (No. w8456). National Bureau of Economic Research.
3. Bartlett, M.S. (1949). Fitting a straight line when both variables are subject to error. *Biometrics*, 5(3), 207.
4. Bowden, R.J., and Turkington, D.A. (1981). A comparative study of instrumental variables estimators for nonlinear simultaneous models. *Journal of the American Statistical Association*, 76(376), 988-995.
5. Casella, G. and Berger, R.L. (1990). *Statistical Inference*. Wadsworth and Brooks/Cole, Pacific Grove, CA.
6. Chen, H.Y., Lee, A.C., and Lee, C. F. (2014). Alternative errors-in-variables models and their applications in finance research. *The Quarterly Review of Economics and Finance*.
7. Cheng, C.L. and Van Ness, J.W. (1999). *Statistical regression with measurement Error*. Arnold, London.
8. Dunn, G. (2004). *Statistical Evaluation of Measurement Errors*. (2nd Edition). Arnold, London.
9. Durbin, J. (1954). Errors in variables. *Revue de l'Institut International de Statistique, Review of the International Statistical Institute*, 22(1/3), 23.

10. Fuller, W. A. (2006). *Measurement Error Models*. Wiley Series in Probability and Statistics.
11. Gillard J., T. (2005). Method of moments estimation in linear regression with errors in both variables. Cardiff University School of Mathematics Technical Report, Cardiff, Wales, UK.
12. Goldberger, A. S. (1972). Structural equation methods in the social sciences. *Econometrica: Journal of the Econometric Society*, 979-1001.
13. Hill, R.C., Griffiths, W.E., and Lim, G.C. (2008). *Principles of Econometrics*. (Vol. 5).
14. Hoboken, NJ: Wiley. Johnson, J. (1972). *Econometric Methods*. McGraw Hill Book Company: New York.
15. Judge, G.G., Griffiths, W.E., Carter, R. and Lee, T.C. (1980). *The Theory and Practice of Econometrics*. Wiley, New York.
16. Kendall, M.G. and Stuart, A. (1973). *The Advanced Theory of Statistics*. (Vol.2). Griffin, London.
17. Linnet, K. (1993). Evaluation of regression procedures for methods comparison studies. *Clinical Chemistry* 39, 424-432.
18. Maddala, S., (1988). *Introduction to Econometrics*. (2nd edition). International Inc. Prentice Hall.
19. Martens, E.P., Pestman, W. R., de Boer, A., Belitser, S.V., and Klungel, O.H. (2006). Instrumental variables: application and limitations. *Epidemiology*, 17(3), 260-267.
20. Miguel, E., Satyanath, S., and Sergenti, E. (2004). Economic shocks and civil conflict: an instrumental variables approach. *J Polit Econ*, 112(4), 725-753.
21. Saqr, A., Khan, S. (2012). Reflection method of estimation for measurement error models. *Journal of Applied Probability and Statistics*, 7(2), 71-88.
22. Sargan, J.D. (1958). The estimation of economic relationships using instrumental variables. *Econometrica*, 26(3), 393.
23. Sprent, P. (1969). *Models in Regression and Related Topics*. Matheun and Co Ltd, London.

24. Stock, J.H., and Watson, M.W. (2003). *Introduction to Econometrics*. (Vol. 104). Boston: Addison Wesley.
25. Vaisman, I. (1997). *Analytical Geometry*. Series on University Mathematics.
26. Wald, A. (1940). The fitting of straight lines if both variables are subject to error. *The Annals of Mathematical Statistics*, 11(**3**), 284-300.
27. Wooldridge, J.M. (2010). *Econometric Analysis of Cross Section and Panel Data*. MIT press.