Efficiency of the Foreign Currency Options Market*

by

Ariful Hoque†
Felix Chan
And
Meher Manzur
School of Economics and Finance
Curtin University of Technology

Abstract

This paper provides a new test of the efficiency of the currency option markets for four major currencies – British Pound, Euro, Swiss Frank and Japanese Yen vis-à-vis the U.S. dollar. The approach is to simulate trading strategies to see if the well-accepted no arbitrage condition of put-call parity (PCP) holds in a trading environment. Augmented Dickey-Fuller and Philips-Perron tests are used to check for the presence of unit roots in the data, followed by a formal econometric analysis. The results indicate that the most currency option prices do not violate the PCP conditions, when transaction costs are allowed for.

Keywords: foreign currency options, lower boundary conditions, put-call parity, conditional variance, transaction costs.

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†Correspondence to: Assoc Prof Meher Manzur, Curtin Business School, GPO Box U1987, Perth WA 6845 Australia, Email: Meher.Manzur@cbs.curtin.edu.au

1. Introduction

One of the greatest innovations in financial markets has been the options on currencies. These were designed not as substitutes for forward or futures contracts, but as an additional and potentially more versatile financial vehicle that can offer significant opportunities and advantages to those seeking protection or investment from changes in exchange rates. Since 1982, foreign currency options have been offered in a number of exchange and dealer markets. The major currency options markets include Philadelphia, Montreal, Vancouver, and Amsterdam stock exchanges. U.S. dollar denominated options on the British pound, Canadian dollar, Deutsche mark, Japanese yen, and Swiss franc trade in the first three markets. Philadelphia Stock Exchange (PHLX), the predominant currency options exchange market, also trades option contracts on Euro introduced in 1999. The Amsterdam Exchange trades options on the U.S. dollar denominated in Euro as well as options on the Euro denominated in U.S. dollars. Finally, the London Stock Exchange trades British pound options. The currency options trading have been exploding over the recent years (see BIS, 2006).

Efficiency is the key factor for the functioning and the development of financial markets. It can be investigated either by means of model-based tests or by testing no-arbitrage relationships that must hold among financial assets. Given that the approach involves a test of the market efficiency and of the option pricing model specification, most empirical research rests on the definition of market efficiency as the absence of arbitrage opportunities. The two fundamental no-arbitrage conditions that have to hold for the market efficiency are the lower boundary conditions and the put-call parity (PCP). The lower boundary condition essentially states that the value of an option can never be less than its intrinsic value. For a call option, intrinsic value is the greater of the excess of the asset price over the strike price and zero. For a put option, it is the greater of the excess of the strike price over the asset price and zero. The PCP is a no-arbitrage relationship that must hold between the prices of a European call and a European put written on the same underlying currency and having the same strike and time to expiration.

Merton (1973) derived the rational lower boundary conditions for option prices with respect to underlying stock prices which must be satisfied in order to prevent dominance or arbitrage possibilities. The existence of dominant asset means that with a zero investment position, one can derive non-negative (not necessarily constant) returns under all states of the world. Studies by Galai (1978), Bhattacharya (1983), and Halpern and Turnbull (1985) examined boundary conditions for equity options. Galai (1978) expanded and tested the call boundary conditions of Merton (1973). In his study, rational boundaries for the price of an option are derived, based on three assets: the option itself, its underlying stock, and a risk-free bond. He examined daily closing prices of the Chicago Board Options Exchange (CBOE). The hypothesis that simultaneous closing prices are within theoretical boundaries, was rejected.

The PCP relationship was originally developed by Stoll (1969) and later on extended and modified by Merton (1973) to account for European stock options with continuous dividend streams. A number of studies have empirically tested the PCP theorem for individual stock option markets and index option markets. The major studies include, but not limited to: Gould and Galai (1974); Geske and Roll (1984); Klemkosky and Resnick (1979); Evnine and Rudd (1985); Gray (1989); Tylor (1990); Finucane (1991); Francfurter and Leung (1991); Brown and Easton (1992); Easton (1994); Kamara and Miller (1995); Wagner et al. (1996); Broughton et al. (1998); Mittnick and Rieken (2000); Bharadwaj and Wiggins (2001); Garay et al. (2003). The results of these studies are mixed; a vast majority of them tend to reject PCP. For a more recent study, see Ghosh and Ghosh (2005).

This paper provides a systematic analysis of the efficiency issues in options markets for major currencies including Euro. Empirical tests are carried out to simulate trading strategies for exploiting lower boundary conditions and the well-known put-call parity (PCP) relationship violations. It is important to point out that diagnostic tests were seldom reported in any of the previous studies on testing for PCP using the linear regression approach. Consequently, it is difficult to assess the validity of these results as the presence of issues such as serial correlation, heteroscedasticty and ARCH error can lead to bias inferences. Apart from using the newly-created options on Euro, a major attraction of this paper is that it will provide a general framework to accommodate these potential problems arising from linear regression analysis and thus, providing a general robust parametric framework for testing PCP. Furthermore, spot market bid-ask spread have been used in this study as a measure of transaction costs, which makes this paper more distinctive.

The paper is organized as follows. Section 2 gives the research methodology and the data used in this study, followed by the preliminary and more formal analysis in sections 3 and 4, respectively. Section 5 contains empirical results with transaction costs. The last section concludes the paper.

2. Methodology and Data

We start with Table 1, which presents the notations and definitions of the variables used in this study.

[Insert Table 1 here]

The no-arbitrage lower boundary conditions for foreign currency options market are based on Budurtha and Courtadon (1986). Violations of these conditions imply that the stream of future cash flows promised by the option could have been bought at a cost lower than the option price. It means the lower bounds are the minimum option prices to ensure the absence of arbitrage opportunities. Since both put and call option is sold by one party, an arbitrage opportunity would result if it is profitable to purchase an option and then exercise. The restrictions of this arbitrage activity on option prices can be stated as:

$$C_t \geq \left(S_t e^{R_t^d T} - X_t e^{R_t^f T} - TTC_t \right), \tag{1}$$

for calls, and

$$P_t \geq \left(X_t e^{R_t^d T} - S_t e^{R_t^f T} - TTC_t \right), \tag{2}$$

for puts. Note that the conditions in inequalities (1) and (2) are quite general since they do not rely on a specific pricing model. Based on the above principles, we have the following testable lower boundary conditions for calls and puts, respectively.

$$\Pi_{Cj} = \left(S_{tj} e^{R_t^f T} - X_{tj} e^{R_t^d T} - C_{tj} - TTC_{tj} \right) \tag{3}$$

$$\Pi_{Pj} = \left(X_{tj} e^{R_t^d T} - S_{tj} e^{R_t^f T} - P_{tj} - TTC_{tj} \right) \tag{4}$$

where,

Π_{Cj} arbitrage profit when call price is less than its intrinsic value;

Π_{Pj} arbitrage profit when put price is less than its intrinsic value.

The lower boundary conditions essentially state that the value of an option can never be less than its intrinsic value. Thus for the efficiency tests involving lower boundary condition, we have $\Pi_{ij} \leq 0$, where *i* = C (call) and P (put); *j* = BP (British pound), SF (Swiss franc), JP (Japanese yen), and EC (Euro).

The put-call parity (PCP) condition states that there exists a deterministic relationship between put and call prices, irrespective of the investor demand for the option, if both options are purchased on the same currency and have the same exercise price and expiration date. The PCP relationship is based on the arbitrage principle. If this relationship is violated, an arbitrage opportunity arises, indicating a mispricing. For example, a long-hedge or conversion strategy would involve buying the foreign currency, writing a call, buying an equivalent put, and borrowing the present value of the exercise price. If arbitrage opportunity dose not exist, the present value of long-hedge strategy should be

$$\left(C_t + X_t e^{-R_t^d T} - P_t - S_t e^{-R_t^f T} - TTC_t\right) \leq 0 \tag{5}$$

Conversely, a short-hedge or reversal strategy could be used by writing a put, buying a call, shorting the foreign currency, and lending an amount equivalent to the present value of the exercise price. At no arbitrage opportunity, the present value of short-hedge strategy should be

$$\left(P_t + S_t e^{-R_t^f T} - C_t - X_t e^{-R_t^d T} - TTC_t\right) \leq 0 \tag{6}$$

In an efficient option market, these two strategies should not yield any profit. The testable PCP conditions then become:

$$\Psi_{Lj} = \left(C_{tj} + X_{tj} e^{-R_{tj}^d T} - P_{tj} - S_{tj} e^{-R_{tj}^f T} - TTC_{tj}\right), \quad \text{and} \tag{7}$$

$$\Psi_{Sj} = \left(P_{tj} + S_{tj} e^{-R_{tj}^f T} - C_{tj} - X_{tj} e^{-R_{tj}^d T} - TTC_{tj}\right), \tag{8}$$

where Ψ_L and Ψ_S is the arbitrage profit under long-hedge (conversion) and short-hedge (reversal) strategy, respectively, when options market is not efficient. Thus, testing all above PCP conditions is equivalent to testing the hypothesis that the foreign currency option market is efficient when $\Psi_{ij} \leq 0$, where i = L(Long) and S (Short), and j is same as discussed in lower boundary conditions above.

For PCP, we also employ a more formal statistical analysis. By dropping transaction costs terms and rearranging equations (7) and (8), we have the following PCP regression equations:

$$C_{tj} - P_{tj} = \lambda_0^{'} + \lambda_1 \left(S_{tj} e^{-R_t^f T} - X_{tj} e^{-R_t^d T} \right) - TTC_{tj} + \varepsilon_{tj} \qquad (9)$$

$$P_{tj} - C_{tj} = \lambda_0^{'} + \lambda_1 \left(X_{tj} e^{-R_t^d T} - S_{tj}^B e^{-R_t^f T} \right) - TTC_{tj} + \eta_{tj} \qquad (10)$$

Under the null hypothesis that PCP is valid, coefficients λ_0 and λ_1 in equations (9) and (10) should be 0 and 1, respectively. Since equations (9) and (10) are analogous, only equation (9) is tested under formal statistical analysis.

However, it is likely that C_{tj} and P_{tj} are $I(1)$, and hence, they can be non-stationary variables. This implies that the OLS estimates in equation (9) are likely to be spurious. In order to overcome this potential problem, consider equation (9) in two consecutive time periods, that is,

$$C_{tj} - P_{tj} = \lambda_0^{'} + \lambda_1 \left(S_{tj} e^{-R_t^f T} - X_{tj} e^{-R_t^d T} \right) - TTC_{tj} + \varepsilon_{tj} \ \textit{and}$$
$$C_{(t-1)j} - P_{(t-1)j} = \lambda_0^{'} + \lambda_1 \left(S_{(t-1)j} e^{-R_{(t-1)j}^f T} - X_{(t-1)j} e^{-R_{(t-1)j}^d T} \right) - TTC_{tj} + \varepsilon_{(t-1)j} \, .$$

Take the difference between the two equations yields

$$\Delta C_{tj} - \Delta P_{tj} = \lambda_0 + \lambda_1 \left(\Delta S_{tj} e^{-R_{fj}^B T} - \Delta X_{tj} e^{-R_{dj}^A T} \right) + u_{tj} \qquad (11)$$

where Δ denotes the difference operator such that $\Delta C_{tj} = C_{tj} - C_{(t-t)j}$, $\Delta TTC_{tj} = \lambda_0$ and $u_{tj} = \Delta \varepsilon_{tj}$. Equation (11) can now be estimated consistently as all variables are now most likely to be stationary.

Muller et al. (1990) present a statistical analysis of four exchange spot rates against the U.S. dollar with several millions of intra-day data for a period over 3 years (March 1986 to March 1989). Their main results indicate that the distributions of price changes become more leptokurtic with decreasing-time intervals, and autocorrelation coefficients of price changes show that intra-day data suffers from considerable heterokedasticity. In order to accommodate potential autocorrelation and conditional heteroscedesticity, equation (11) needs to be augmented further as follows:

$$Y_{tj} = \lambda_0 + \lambda_1 \left(\Delta S_{tj} e^{-R_{fj}^{B} T} - \Delta X_{tj} e^{-R_{dj}^{A} T} \right) + \sum_{i=1}^{p} \phi_i Y_{(t-i)j} + \sum_{i=1}^{q} \theta u_{(t-i)j} + u_{tj} \qquad (12)$$

where $Y_{tj} = \Delta C_{tj} - \Delta P_{tj}$. Failing to accommodate issues such as serial correlation and heteroscedasticity would lead to biased and inconsistent inference for λ_0 and λ_1 as shown in the formal analysis section. The choice of the lag order, *p* and *q*, will be driven by the results of the diagnostic tests and various information criteria.

In the presence of GARCH(r, s) error proposed by Bollerslev (1986), u_t in equation (12) is further decomposed to

$$u_t = \eta_t \sqrt{h_t}, \qquad \eta_t \sim iid(0,1)$$

$$h_t = \omega + \sum_{i=1}^{r} \alpha_i u_{t-i}^2 + \sum_{i=1}^{s} \beta_i h_{t-i} \ ,$$

with $\omega > 0$, $\alpha_i \geq 0$ and $\beta_i \geq 0$ to ensure $h_t > 0$. Once the presence of GARCH error is confirmed by the LM test of Bollerslev (1986), the lag order, *r* and *s*, will be determined by further diagnostic tests and various information criteria as suggested in Bollerslev (1986).

The Data

The tests of the lower boundary conditions and PCP are based on data for the following four currency options – the British pound, the Euro, the Japanese yen, and the Swiss franc. All data are obtained from DATASTREAM database, and provided in a separate appendix, available on request. The data consist of daily closing prices for each option traded on the PHLX, daily spot exchange rates, and daily Eurocurrency interest rates for the period. Option on Euro started trading December 2000. The data set for all currencies, therefore, includes the options trading period from January 2001 to March 2006. There is some inconsistency in data (due to recording error in the database) for the Japanese yen from January 2001 to end of March 2001 and consequently, these are excluded from the sample. The total number of put-call pairs from the observations of daily prices across all four currencies is 5377. The expiration dates of options are within 90 days during the sample period. If the expiration month has 5 Fridays, the options expire on the third Friday, otherwise second Friday of the expiration month. The Eurocurrency interest rates are used to determine daily domestic and foreign bond prices, respectively. The option contract size is £31250, €62500, ¥6250000, and SF62500 and for British pound, Euro, Japanese yen and Swiss franc, respectively. For transaction costs, the spread between bid and ask exchange rates for all currency against U.S. dollar are also obtained for the sample period.

4. Preliminary Analysis

This section provides a descriptive analysis of the data. First, we look at the lower boundary conditions, followed by the PCP conditions. The lower boundary conditions are examined for 5377 put-call pairs with same maturity under the assumption of zero transaction costs $\left(i.e.\ TTC_t = 0\right)$ and the summary of the results are reported in Tables 2 and 3, for calls and puts, respectively.

Table 2 reveals that for all currencies there were 156 violations of the lower boundary condition for calls, representing 2.9 per cent of the total number of observations. As can be seen, about 60 per cent of the violations come from calls maturing in 60 days or less (a total of 90 violations of which 46 are for maturity less than 30 days, and 44 are between 30 & 60 days). Also note that most of the violations (151 out of 156) are for calls having been in-the-money (ITM).

[Insert Table 2 here]

The violations of the lower boundary condition for puts are presented in Table 3. Compared to the results in Table 2, it is clear the number and frequency of violations for puts are less than those for calls. For all currencies, we have 25 violations that represent only 0.46 per cent of the total number of observations. All of the violations (25 out of 25) occurred for puts that were in-the-money (ITM). Note that using a slightly different sample (British pound, Japanese yen, German marks and Swiss franc) Shastri and Tandon (1985) found that 2.68 per cent and 3.01 per cent violations of lower boundary condition for calls and puts, respectively. While our results for call options (Table 2) are very similar to that study, our results for put

[Insert Table 3 here]

options (Table 3) appear different. In general, the results indicate that the violations in the lower boundary conditions are minimal. This finding, however, needs to be interpreted with caution, because the data do not allow for transaction costs and other possible pitfalls as indicated in Bhattacharya (1983).

We now turn to PCP validity tests under the assumption of zero transaction costs $(i.e.\ TTC_t = 0)$. Table 4 reports results for calls. As can be seen, the PCP violations are apparently due to overpricing of calls, resulting in profits from arbitrage opportunity under conversion strategy. For all currencies, the results indicate that violations account for 38.14 per cent of total cases, with an average profit of $135.38. Calls on Japanese yen are found to be the most profitable ($200.85) while those on Swiss franc were the least profitable ($89.79). For frequency of violations by option maturity (in days), we observed that for all currencies, 72.26 per cent [(16.01+11.55) ÷ (16.01+11.55+10.58)} x 100] of total PCP violations are for calls that have fewer than 60 days to maturity. The results given in the bottom panel of the table are discussed subsequently.

Table 5 contains the results for put options. It can be seen that the PCP violations are due to overpricing of puts, resulting in arbitrage profit through reversal strategy. As can be seen, violations account for 61.86 per cent for all currencies with an average arbitrage profit of $205.87. Note that both the size of violations and average profit due to overpricing of puts are higher than those for calls as reported in Table 4. The most profitable ($252.34) and least profitable ($124.38) put options are on Euro and British pound, respectively. From option life point of view (frequency of violations by option maturity in days), 64.79 per cent [(17.91+22.17) ÷ (17.91+22.17+21.78)} x 100] of total PCP violations are found for options with less than 60 days of maturity.

[Insert Table 4 here]

[Insert Table 5 here]

The PCP results in the bottom panels of Tables 4 and 5 can be discussed in terms of the moneyness of puts and calls, that is, in terms of whether an option is in ITM (in-the-money), ATM (at-the-money), and OTM (out-of-the-money). The moneyness is defined by the ratio of spot rate and exercise price. Table 6 presents the frequency of violations based on the moneyness. As can be seen that for the total number of observations, overpricing of put option account for 61.86 per cent, while the remainder is for calls. Thus, the put options tend to be more overpriced relative to call options over the sample period. Also the violations are mostly detected for either ITM or OTM options. It implies that options pricing volatility is relatively low for ATM. It becomes progressively higher as an option moves from ATM to either ITM or OTM.

[Insert Table 6 here]

Overall, the results in this section do not lend support to the PCP condition. For all currencies in the sample and both calls and puts taken together, the PCP violations account for about half the cases. In what follows, we make further investigations into this issue using more formal methods.

5. Econometric Analysis of PCP

We start with the time series properties of the data. Table 7 shows the descriptive statistics of the variables. As can be seen, the mean and median values are very close and the skewness is nearly zero for most of the data series. However, Jarque-Bera (JB) normality test reject the approximately normal distribution assumption. This implies that the distribution of the data used in this paper is not normal.

The contemporary time series literature pays special attention to the issue of stationary versus non-stationary variables. The standard Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests are applied to examine whether a unit root is present in the data series. The ADF test accommodates serial correlation and time trend by explicitly specifying the autocorrelation structure. The PP test accommodates heteroscedasticity and autocorrelation using non-parametric method. As shown in Phillips and Perron (1988) that the PP test has better power than ADF under a wide range of circumstances and hence more appropriate to use for the time series data analysed in this paper.

[Insert Table 7 here]

ADF and PP unit root tests are run on levels and first differences of the variables. The test results on levels are given in Table 8. The call and put price for all currencies reject the null hypothesis of unit root significantly, under both ADF and PP (British pound only PP test) tests. The strike price of Swiss franc also rejects unit root at a high level of significance. However, interest rates, strike prices (except Swiss franc), and spot rates failed to reject the unit root at the conventional levels of significance. Thus, the results in this table tend to be mixed.

[Insert Table 8 here]

ADF and PP tests on the first difference of variables are then conducted and results are presented in Table 9. The reported t-statistics in the table reject the null hypothesis of unit root for all first differenced variables at less than 1% levels of significance. The next step is to perform the regression analysis with the first difference of the variables for all currencies using equation (11) (reproduced below):

$$\Delta C_{tj} - \Delta P_{tj} = \lambda_0 + \lambda_1 \left(\Delta S_{tj} e^{-R_{fj}^B T} - \Delta X_{tj} e^{-R_{dj}^A T} \right) + u_{tj}.$$

In order to obtain valid inferences from λ_0 and λ_1, two diagnostic tests have been conducted for each currencies, namely, the LM test for serial correlation and ARCH effects. Identification of these problems allow us to make the appropriate adjustment using equation (12) and hence obtain consistent estimates for λ_0 and λ_1 . The results are reported in Table 10 as 'After'. The impacts of these problems on the estimates of λ_0 and λ_1 can be found in Table 10 as 'Before'. The 'After' results in Table 10 indicate that for all currencies, the hypothesis of an intercept (λ_0) of 0 cannot be rejected at any standard significance level, except for euro. Since, $\left(\Delta S_t B_t^* - \Delta X_t B_t\right) = 0$ at ATM, a zero intercept can be interpreted as saying that the call and put price is the same when both options are ATM. The estimated slopes for all currencies, λ_1, are positive and significantly less than one. Consequently, the econometric results, consistent with those in the previous section, tend to reject PCP for our sample.

[Insert Table 9 here]

[Insert Table 10 here]

It is to be noted that a positive slope indicates that calls are overpriced relative to puts. The estimated differences between the theoretical and the empirical regressions can be as seen as follows:

$$\hat{u}_t = \Delta C_t - \Delta P_t - \left(\Delta S_t B_t^* - \Delta X_t B_t\right) = \lambda_0 + \lambda_1 \left(\Delta S_t B_t^* - \Delta X_t B_t\right) - \left(\Delta S_t B_t^* - \Delta X_t B_t\right)$$
$$= \lambda_0 + \left(\lambda_1 - 1\right)\left(\Delta S_t B_t^* - \Delta X_t B_t\right) \ . \qquad (13)$$

The relationship between slope coefficient and relative call overpricing is plotted in Figure 1 ($\hat{u}_t$ plotted on the vertical axis against intrinsic value of calls on the horizontal axis). Note that the scales on the axis are different for different currency due to their exchange rate to the US dollar.

[Insert Figure 1 here]

A slope $\left(\lambda_1\right)$ being less than 1 suggests that the extent of relative call overpricing decreases as calls (puts) get deeper into (out of) the money, that is, an increasing value of $\left(\Delta S_t B_t^* - \Delta X_t B_t\right)$. The relationship is reversed when put is in-the-money. The relative put overpricing increases as puts (calls) get deeper into (out of) the money, as decreasing the value of $\left(\Delta S_t B_t^* - \Delta X_t B_t\right)$.

Note that as can be seen from in Figure 1B, the relative over or under pricing of call or put options on Euro is hardly detectable, unlike other currencies. A possible explanation for this is that the Euro option market is in its early stage of development. The sample under investigation begins in January 2001 which is about one month after the Euro option started trading. Consequently, the agents may still be on a learning stage as to how to price this new financial instrument to compete with the global option market.

6. Empirical Results with Transaction Costs

In the foregoing analysis, the transaction costs were ignored. The results lose real-world appeal if the transaction costs are not allowed for. Several studies confirm that the transaction costs are far from negligible, and the larger are these costs, the wider are the bands within which options prices can oscillate without signaling arbitrage opportunities. Studies which ignore transaction costs may overestimate the degree of option market inefficiency (See Galai, 1978; Bhattacharya, 1983, among others). In what follows, we extend the study introducing transaction costs in the analysis. As violations of lower boundary conditions were marginal (see Tables 2 and 3), data analysis including the transaction costs in this section is done for PCP conditions only.

It is well-known that the option contract sizes and transaction costs vary across markets and currencies and it is not easy to standardize the data that apply to all currency across markets. The level of transaction costs also varies across investor types; institutional investors typically face lower costs compared to their retail counterparts. Transaction costs in option trading include all charges associated with executing a trade and maintaining a position. Items such as, brokerage commissions, fees for exercise and/or assignment, exchange fees, SEC fees, and so on, as part of transactions are typically negligible, but the bid-ask spread in the option market represents the actual significant measure of the transaction costs. Unfortunately, a reliable series of option market bid-ask quotes is not available for our sample. Consequently, we use the spot foreign exchange market spread as a crude proxy for the option market bid-ask spread for our purpose. This approach of using the spot market spread, while not perfect, is consistent with other studies (see, for example, El-mekkaoui and Flood, 1998). Further, as Bhattacharya (1983) points out, not all transactions occur at the bid or ask price; a significant percentage occurs within the bid/ask spread. There is also no information on whether a trade was initiated by a buy order or a sell order. As a compromise, we handle this problem by using the mean of bid/ask spread as the proxy of transaction costs.

With transaction costs (mean bid/ask spread) included, we have $TTC_t \neq 0$ in equations (7) and (8) for conversion and reversal strategy, respectively. The results are given in Table 11 for conversion strategy. In the top-half of this table, the summary of results with no transaction costs are reproduced for comparison purposes, from Table 4. As can be seen, for British pound, in the presence of transaction cost $\left(i.e.\ TTC_t \neq 0\right)$, the mean profit ($73.54) is calculated as the mean bid-ask spread ($18.90) taken away from the mean profit in absence of transaction costs ($92.44). The reduction in mean profit has now lowered the number of violation (288 for the British pound), and consequently, the violation in percent (calculated as the number of violations divided by sample size). Similar calculations apply for entries in Table 11. It is clearly observed that the inclusion of transaction costs decreases the percentage of PCP violations substantially for all currencies. For all currencies taken together, almost two-thirds of the violations (38.14 percent without transaction costs versus 14.13 percent with transaction costs) have disappeared due to introduction spot market bid and ask spread.

Table 12 is the Table 11 version for the reversal strategy and all information regarding the PCP violations in absence of transaction costs $\left(i.e.\ TTC_t = 0\right)$ are obtained from Table 5. As can be seen, with transaction costs included, the PCP violations have decreased substantially for put options written on all four currencies. Evidently, for all currencies taken together, the transaction costs have wiped out almost three-fourths of the PCP violations under the reversal strategy. Thus, the results in Tables 11 and 12 highlight the significance of the transaction costs in testing the efficiency of option markets.

[Insert Table 11 here]

7. Conclusion

This paper has examined the market efficiency for major currency options including the Euro. A total of 5377 daily put-call pairs are included in the sample from January 2001 to March 2006 option trading period. The analysis is conducted in two steps: first, the two fundamental no-arbitrage conditions, namely, the lower boundary condition and the put-call parity (PCP) condition are examined in a descriptive manner; second, more formal regressions analysis is performed for PCP. We have also allowed for the spot market bid-ask spread to represent transaction costs in the analysis.

Preliminary investigations for no-arbitrage lower boundary conditions indicate that there are only 2.9 per cent and 0.46 per cent violations in pricing of calls and puts, respectively. It is observed that the most of the currency option prices are within their rational boundaries. Preliminary analysis also indicates that, under no-arbitrage PCP conditions tests, 38 per cent and 62 per cent of violations are due to overpricing of calls and puts, respectively. Moreover, an average arbitrage profit under reversal strategy (for overpricing of put) is higher than conversion strategy (for overpricing of call). The results suggest that the put options tend to be more overpriced relative to call options over the sample period. Most of these violations are from options contract period of less than 60 days. One interpretation of this result is that currency options markets may not be efficient at the shorter ends. Also the violations are mostly detected for either in-the-money (ITM) or out-of-the-money (OTM) options.

[Insert Table 12 here]

The econometric analysis tends to reject PCP. The results indicate that calls are overpriced relative to puts. It is also observed that the relative call overpricing decreases as calls (puts) get deeper into (out of) the money. The relationship is reversed when put is overpriced. However, this relationship is not noticeable for options on Euro. The reason for this may be the fact that the Euro and the options on Euro have both been introduced only recently. The traders may still be in the learning process in pricing this relatively new instrument. On the whole, the results from both preliminary and more formal analysis provide a mixed picture for PCP.

The transaction costs have been a thorny issue in this topic of options market efficiency. While there is no disagreement as to the fundamental role of transaction costs in estimating PCP from trading data, accurate data are almost impossible to get a hand on. In this study we have used the spread between the bid and ask quotes in the spot foreign exchange rates as a crude proxy for transaction costs. As expected, the PCP violations have decreased substantially in the presence spot market spread imitating transaction costs. This result tends to confirm the notion that derivative pricing may be less determined by theoretical arbitrage relationships than by the possibility of practical implementation in a given market.

Overall, the results indicate that the lower boundary condition and PCP do not hold up tightly. The results also suggest that the options mispricing is relatively low for ATM. It becomes progressively higher as an option moves from ATM to either ITM or OTM. There are no readily available explanations for these results. In future research, the analysis can be extended further by considering the factors associated with the mispricing of options. One approach is to focus on ε_{tj} and η_{tj} in equations (9) and (10) and decompose them to examine the effects of relevant issues including the simultaneity of spot and option prices, depth of market, search costs, execution lag and so on.

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Table 1: Notations and Descriptions of the Variables

Variables	Notations	Descriptions
Call Price	C_t	Call price in domestic currency at time ***t***.
Put Price	P_t	Put price in domestic currency at time ***t***.
Spot price	S_t	Spot price in domestic currency at time ***t*** for one unit of foreign currency.
Strike price	X_t	Option exercise price in domestic currency at time ***t*** for one unit of foreign currency.
Interest rate	R_t^d	Domestic currency risk-free interest rate at time ***t***.
	R_t^f	Foreign currency risk-free interest rate at time ***t***.
Option life	T	Expiration time of the option.
Transaction cost	TTC_t	Total transaction costs at time ***t*** that include costs for all transactions involve either buying or selling options.

Table 2: Violations of Lower Boundary Conditions Call Option

$$\Pi_{Cj} = \left(S_{tj} e^{R_t^f T} - X_{tj} e^{R_t^d T} - C_{tj} \right)$$

	All Currency	Currency			
		British pound	Euro	Japanese yen	Swiss franc
No. of observations	5377	1359	1359	1300	1359
No. of violations	156	6	34	49	67
% violations	2.90%	0.44%	2.50%	3.77%	4.93%
Frequency of violations by option maturity in days					
≤ 30	46	1	12	3	30
31 – 60	44	0	6	19	19
61 – 90	66	5	16	27	18
Frequency of violations by ratio of S to X					
>1 (ITM)	151	6	34	44	67
=1 (ATM)	0	0	0	0	0
1> (OTM)	5	0	0	5	0

Table 3: Violations of Lower Boundary Conditions for Put option

$$\Pi_{Pj} = \left(X_{tj} e^{R_t^d T} - S_{tj} e^{R_t^f T} - P_{tj} \right)$$

	All Currency	Currency			
		British pound	Euro	Japanese yen	Swiss franc
No. of observations	5377	1359	1359	1300	1359
No. of violations	25	3	15	6	1
% violations	0.46%	0.22%	1.10%	0.46%	0.07%
Frequency of violations by option maturity in days					
≤ 30	9	1	6	2	0
31 – 60	4	0	3	0	1
61 – 90	12	2	6	4	0
Frequency of violations by ratio of X to S					
>1 (ITM)	25	3	15	6	1
=1 (ATM)	0%	0	0	0	0
1> (OTM)	0%	0	0	0	0

Table 4: PCP Violations Under Conversion Strategy

$$\Psi_{Lj} = \left(C_{tj} + X_{tj} e^{-R_{tj}^{d} T} - P_{tj} - S_{tj} e^{-R_{tj}^{f} T} \right)$$

	All Currency	Currency			
		British pound	Euro	Japanese yen	Swiss franc
Mean ($)	135.38	92.44	175.42	200.85	89.79
No. of observations	5377	1359	1359	1300	1359
No. of violations	2051	645	548	404	454
Violations (%)	38.14	47.46	40.32	31.08	33.41
Frequency of violations by option maturity in days					
≤ 30	16.01%	130	197	285	249
31 – 60	11.55%	222	179	92	128
61 – 90	10.58%	293	172	27	77
Frequency of violations by ratio of S to X					
>1 (ITM)	15.14%	272	214	151	177
=1 (ATM)	0.04%	2	0	0	0
1< (OTM)	22.96%	371	334	253	277

Table 5: PCP Violations Under Reversal Strategy

$$\Psi_{Sj} = \left(P_{tj} + S_{tj} e^{-R_{tj}^{f} T} - C_{tj} - X_{tj} e^{-R_{tj}^{d} T} \right)$$

	All Currency	Currency			
		British pound	Euro	Japanese yen	Swiss franc
Mean ($)	205.87	124.38	252.34	205.27	229.13
No. of observations	5377	1359	1359	1300	1359
No. of violations	3326	714	811	896	905
% violations	61.86%	52.54%	59.68%	68.92%	66.59%
Frequency of violations by option maturity in days					
≤ 30	17.91%	331	264	156	212
30 – 60	22.17%	237	280	344	331
61 – 90	21.78%	146	267	396	362
Frequency of violations by ratio of X to S					
>1 (ITM)	25.24%	272	344	415	326
=1 (ATM)	0.11%	5	0	0	1
<1 (OTM)	36.51%	437	467	481	578

Table 6: Moneyness Test for PCP
(in percentage)

Moneyness	Conversion	Reversal	Total
ITM / OTM	15.14	25.24	40.38
ATM	0.04	0.11	0.15
OTM / ITM	22.96	36.51	59.47
Total	38.14	61.86	100

Table 7: Descriptive Statistics of Variables

Currency	Statistical measures	Variables				
		Call price	Put price	Strike price	Spot rate	Interest rate
British pound	Mean	1.40	1.74	165.06	1.65	4.39
	Median	1.41	1.68	164.00	1.64	4.42
	Skewness	0.01	0.55	-0.02	-0.02	-0.37
	Kurtosis	3.37	3.38	1.57	1.57	2.66
	JB	7.90*	77.04*	115.49*	115.18*	38.02*
Euro	Mean	1.19	1.34	109.71	1.09	3.00
	Median	1.22	1.28	115.00	1.15	2.69
	Skewness	-0.09	3.69	-0.23	-0.23	0.68
	Kurtosis	3.28	37.66	1.58	1.58	2.05
	JB	6.69*	71102.44*	126.47*	126.61*	154.45*
Swiss franc	Mean	1.01	0.83	73.77	0.72	1.55
	Median	0.91	0.81	73.50	0.75	1.22
	Skewness	25.75	0.54	24.66	-0.31	0.83
	Kurtosis	669.29	4.19	631.39	1.81	2.35
	JB	25288077*	147.41*	22497256*	102.17*	181.44*
Japanese yen	Mean	1.03	0.87	86.67	0.01	0.24
	Median	0.98	0.81	85.5	0.01	0.21
	Skewness	1.87	2.78	-0.04	0.03	1.38
	Kurtosis	11.49	26.26	2.12	2.18	5.26
	JB	4876.89*	32393.99*	43.38*	37.94*	719.18*
U.S. dollar	Mean					2.91
	Median					2.88
	Skewness					0.12
	Kurtosis					1.76
	JB					90.66*

Note: The Jarque-Bera (JB) statistic follows a chi-square distribution with 2 degree of freedom. The critical value of the chi-square distribution is 5.99 at the 5% level of significance. The statistical significance level at 5% is denoted by *.

Table 8: Unit Root Tests on Level of Variables

Currency	Test	Variables				
		Interest rate	Call price	Put price	Strike price	Spot rate
British pound	ADF	-2.14	-2.27	-9.69***	-1.082	-1.09
	PP	-2.15	-15.19***	-14.51***	-1.13	-1.09
Euro	ADF	-2.02	-9.13***	-9.18***	-1.01	-0.88
	PP	-2.01	-20.80***	-17.49***	-0.95	-0.92
Japanese yen	ADF	-1.46	-9.55***	-7.06***	-2.24	-1.63
	PP	-1.39	-11.77***	-21.03***	-2.14	-1.63
Swiss franc	ADF	-2.13	-10.58***	-7.88***	-9.04***	-1.14
	PP	-2.11	-19.78***	-13.99***	-19.89***	-1.24
US dollar	ADF	-1.53				
	PP	-1.49				

Note: The t-statistics are presented in the table. The critical values for the tests are -3.43, -2.86, -2.56 at the 1%, 5%, and 10%, respectively. *, **, and *** denotes 10%, 5% and 1% level of significance, respectively.

Table 9: Unit Root Tests on First Difference of Variables

Currency	Test	Variables				
		Interest rate	Call price	Put price	Strike price	Spot rate
British pound	ADF	-32.84	-10.67	-17.14	-41.29	-37.25
	PP	-32.77	-113.07	-94.64	-41.42	-37.25
Euro	ADF	-39.19	-16.73	-16.86	-46.70	-39.95
	PP	-39.20	-171.93	-163.16	-48.42	-39.96
Japanese yen	ADF	-35.33	-45.83	-26.83	-39.21	-37.46
	PP	-35.31	-59.81	-124.60	-39.23	-37.45
Swiss franc	ADF	-37.54	-15.78	-27.98	-15.74	-40.76
	PP	-37.57	-388.49	-77.97	-477.44	-40.77
US dollar	ADF	-35.62				
	PP	-35.68				

Note: The t-statistics are presented in the table. The critical values for the tests are -3.43, -2.86, -2.56 at the 1%, 5%, and 10%, respectively.

Table 10: Regression Test for PCP

Currency	λ_0		λ_1		Auto-correlation	GARCH
	Before	After	Before	After		
British Pound	0.0014 (0.105)	0.0028 (0.013)	0.5802 (0.0169)	0.6265 (0.0102)	ARMA(0,2)	GARCH(2,2)
Euro	0.0058 (0.013)	0.0016 (0.007)	0.5813 (0.0170)	0.5890 (0.0054)	ARMA(2,2)	GARCH(1,1)
Swiss franc	0.0051 (0.064)	0.0018 (0.013)	0.4032 (0.0162)	0.3995 (0.0100)	ARMA(0,1)	GARCH(2,1)
Japanese yen	0.00004 (0.0007)	-0.0002 (0.0002)	0.4282 (0.0180)	0.5033 (0.0043)	ARMA(0,1)	GARCH(1,1)

Note: The test of H_0: $\lambda_0 = 0$ and $\lambda_1 = 1$. The number in parentheses below the coefficient estimates are standard errors. For brevity, the results for λ_0 are multiplied by 1,000. See text for interpretation of 'Before' and 'After'.

Table 11: PCP Violations in Presence of Transaction Costs (TC)
Under Conversion Strategy

$$\Psi_{Lj} = \left(C_{tj} + X_{tj} e^{-R_{tj}^{d} T} - P_{tj} - S_{tj} e^{-R_{tj}^{f} T} - TTC_{tj} \right)$$

		All currency	British pound	Euro	Japanese yen	Swiss franc
	Sample size	5377	1359	1359	1300	1359
Absence of transaction cost $TTC_{tj} = 0$	Mean profit ($)	135.38	92.44	175.42	200.85	89.79
	No. of violations	2051	645	548	404	454
	Violations (%)	38.14	47.46	40.32	31.08	33.41
Presence of transaction cost $TTC_{t} \neq 0$	Mean bid-ask spread ($)	23.05	18.90	29.65	26.03	17.60
	Mean profit ($)	112.34	73.54	145.77	174.82	72.19
	No. of violations	760	288	158	125	189
	Violation (%)	14.13	21.19	11.62	9.62	13.90

Note: All information for $TTC_{tj} = 0$ are obtained from Table 4. Mean profit and bid-ask spread for TC (transaction cost) are calculated per option contract size.

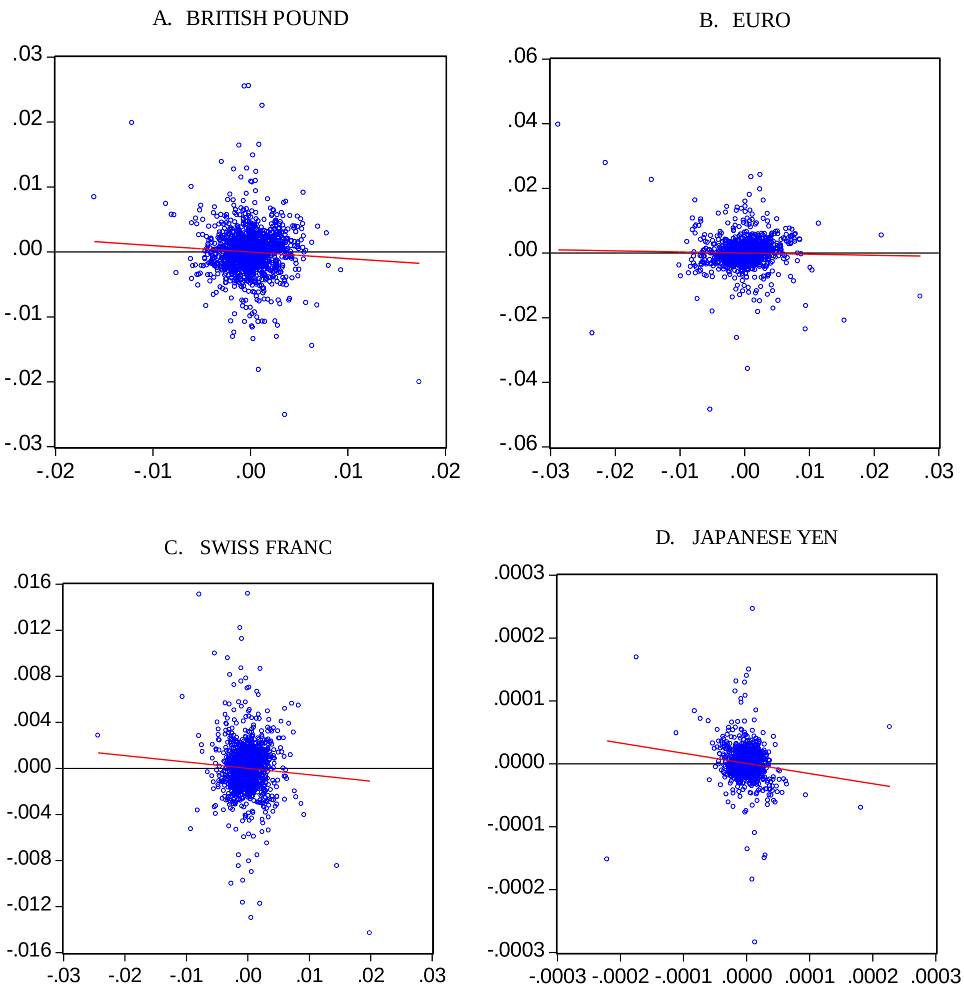
Table 12: PCP Violations with Presence of Transaction Costs (TC) Under Reversal Strategy

$$\Psi_{Sj} = \left(P_{tj} + S_{tj} e^{-R_{tj}^{f} T} - C_{tj} - X_{tj} e^{-R_{tj}^{d} T} - TTC_{tj} \right)$$

		All currency	British pound	Euro	Japanese yen	Swiss franc
	Sample size	5377	1359	1359	1300	1359
Absence of transaction cost $TTC_{tj} = 0$	Mean profit ($)	205.87	124.38	252.34	205.27	229.13
	No. of violations	3326	714	811	896	905
	Violations (%)	61.86	52.54	59.68	68.92	66.59
Presence of transaction cost $TTC_{t} \neq 0$	Mean bid-ask spread ($)	23.99	18.74	34.10	26.13	16.97
	Mean profit ($)	181.89	105.64	218.24	179.14	212.16
	No. of violations	967	223	185	335	224
	Violation (%)	17.98	16.41	13.61	25.77	16.48

Note: All information for $TTC_{tj} = 0$ are obtained from Table 5. Mean profit and bid-ask spread for TC (transaction cost) are calculated per option contract size.

Fig. 1: Scatter Plots for Regression Tests of Put-Call Parity



$\hat{\varepsilon}$ *Versus* $\left(\Delta S_t B_t^* - \Delta X_t B_t\right)$