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The End of Al-qa`ida: Rationality, Survivability and Risk Aversion

Abstract:

Along with such factors as declining support, the dismantling of the leadership group and transition into legitimate political engagement there is another factor that is relevant to the decline and fall of terrorist groups. This is the group's risk aversion. This aspect of the terrorist group's profile, which shapes the way in which it makes decisions under conditions of risk, relates crucially to its evolutionary stability and longevity. In fact, it may play a more important role than 'rationality'. Terrorist groups must compete for a share of the available 'political influence' and violent groups do so by competing for a share of 'inflicted fatalities'. Differential rates of accumulation determine the differential significance of some individuals and groups over others. Terrorist groups with logarithmic utility functions will exhibit longevity because logarithmic utility maximisation naturally equates with the maximisation of the growth rate of the share of inflicted fatalities and political influence. Terrorist groups with non-logarithmic utility functions will see their significance decline. The study of risk aversion has implications for the prediction from microeconomic theory of which terrorist groups will survive in the long run. The analysis presented herein computes a value of 0.56 for Al-Qa`ida's relative risk aversion coefficient. This value is too low to be consistent with logarithmic utility.

Key Words: Terrorism, Political Influence, Fatalities, Al-Qa`ida, Risk Aversion, Utility Maximisation, Logarithmic Utility, Evolutionary Stability

JEL Classification: H56, D74, D81

INTRODUCTION

There can be found in the literature two comprehensive discussions of the decline and demise of terrorist groups. These discussions are Martha Crenshaw's (1991) *How Terrorism Declines* and Audrey Kurth Cronin's (2006) *How Al-Qaida Ends*. The point of these discussions is to highlight the complex interplay of factors that contribute to the demise of terrorist groups. Cronin (2006, pp.17-32) lists no fewer than seven 'decline factors': (1) capture or killing of the leader; (2) failure to transition to the next generation; (3) achievement of the group's aims; (4) transition to a legitimate political process; (5) undermining of popular support; (6) repression; and (7) transition from terrorism to other forms of violence. It is the purpose of this paper to extract from an economic analysis of terrorism an additional 'decline factor' to contribute to Cronin's (2006) list. This decline factor is risk aversion and, in particular, a

degree or level of risk aversion that is either too high or too low to be consistent with the maximisation of the growth rate of the terrorist group's share of influence within the terrorism context.

Research into evolutionary processes has been a substantial and growing research program within economics and the mathematical sciences for a number of years. Peace scientists will be familiar with the importance that game theoretical analysis plays in both the economic analysis of conflict and the biological evolutionary analysis of conflict and strategic interaction. Research undertaken by Konrad and Morath (2012), Leininger (2009) and Eaton and Eswaran (2003) are relevant examples from this particular literature. Within other parts of economics, evolutionary analysis, survivability and longevity have been intensively studied. One contribution that has not yet attracted the attention of those interested in peace, conflict and strategy is Blume and Easley's (1992) analysis of rationality and 'fitness' (survivability). Written in the context of asset market behaviours, Blume and Easley's analysis shows that 'rationality' is not necessarily a criterion for survivability. One 'behavioural rule' that was found to be 'fit' was the maximisation of the expected growth rate of wealth share accumulation.

Imagine some total, which may be growing larger over time, being divided between individuals. Blume and Easley argue that one of the behavioural rules that ensure the survivability of an individual is the maximisation of the expected growth rate of his cumulative share. Within the terrorism context, one can also imagine some relevant total that is divided between all of the operational terrorist groups. This could be something abstract such as 'political influence' or its wealth equivalent. However, the economic analysis of terrorism is probably best advanced by choosing a more tangible unit of analysis. This could be the number of supporters, minutes of major network news coverage or the amount of human tragedy that is inflicted. The latter is the most direct outcome of terrorism and there are strong arguments to be made in favour of its use as a unit of analysis. Terrorist groups express by word and deed a desire to inflict as many injuries and fatalities as possible and Cronin (2006, p.11) argues that the terrorist group, "... may have an innate compulsion to act—for example, it may be driven to engage in terrorist attacks to maintain support, to shore up its organisational integrity or even to foster its continued existence⁵." The direct and indirect 'payoffs' associated with the infliction of injuries and fatalities upon others are important to terrorist groups. If this proposition is accepted it becomes a logical analytical step to consider not only the importance of inflicted injuries and fatalities but also the importance of a terrorist group's accumulated fatalities and the expected growth rate of its cumulative share of the human tragedy that is inflicted by the aggregate of all terrorist groups. If this is important for the existence of the terrorist group, as Cronin (2006, p.11) rightly intuitively, then Blume and Easley's criterion for survivability is applicable to the terrorism context. A terrorist group exhibits evolutionary stability, fitness or survivability that maximises the growth rate of its expected share of cumulative human tragedy.

Risk aversion is central to the analysis of this survivability criterion because the level or degree of risk aversion exhibited by a terrorist group may or may not be consistent with the 'growth rate fitness criterion'. Although the growth rate fitness criterion is not necessarily a 'rational' criterion, it is consistent with a particular type of utility maximisation. Economic

⁵ Cronin (2006, p.11) notes terrorism researcher Bruce Hoffman's observation that Al-Qaida is like a shark that must keep swimming to survive.

theory makes use of four popular specifications of the expected utility function. These are quadratic, power, exponential and logarithmic. It so happens that maximising a logarithmic expected utility function is the same thing, more or less, as the maximisation of the expected growth rate (Blume and Easley 1992, p.11). The purpose of an expected utility function is to model the process of decision-making under conditions of risk. The essence of this decision-making process is the ordering of preferences or the arranging from best to worst of the available opportunities. The ordering is undertaken on the basis of expected utility and the decision-maker chooses the opportunity that has the highest expected utility. The extent to which the decision-maker dislikes the risks inherent in the risky opportunities from which he will choose is reflected in the value for the risk aversion coefficient (a technical aspect of the specification of the expected utility function). Certain values of the risk aversion coefficient are consistent with particular utility functions and, therefore, with the growth rate fitness criterion.

The purpose of this paper is to present an analysis of the measurement of the risk aversion coefficients of Al-Qa`ida terrorists. This measurement is undertaken by building upon the mean-variance approach introduced into the analysis of terrorism by Phillips (2009; 2011). First, the zero-beta equilibrium model constructed by Phillips (2011) may be used to compute a 'market price of terrorism risk'. This is the ratio of the premium above the zero-beta portfolio (measured in terms of expected fatalities) generated by a benchmark combination of attack methods to the risk (the standard deviation of the divergence of expected fatalities from actual fatalities). Another way to think of the market price of risk is as the reward (in additional expected fatalities) for bearing an additional unit of risk. By analysing the attack method combinations chosen by the Al-Qa`ida terrorist group and exploiting the relationship between these choices and the market price of risk, the relative risk aversion coefficient for the Al-Qa`ida terrorist group may be computed. The analysis will show whether the choices of the Al-Qa`ida terrorist group have been consistent with the growth rate fitness criterion. The results have implications for the survivability of the group.

II. RISK AVERSION

Although most discussions of risk aversion in the economics literature are concerned with decisions involving wealth and consumption of material goods, the discussion presented in this section will consider utility from the perspective of the terrorist. The terrorist or terrorist group has much in common with the standard economic agent of orthodox economic theory. This is evident in many economic analyses of terrorist behaviour (see Arce and Sandler (2005), Frey and Luechinger (2003) and Sandler and Arce (2003; 2007)). The terrorist agent or group attempts to maximise some objective function subject to constraints. All that is required to commence an analysis of terrorist behaviour within this framework is the definition of the good that conveys utility upon the terrorist agent. This may be political influence (or a monetary equivalent) or, perhaps, something more tangible such as fatalities caused, resources, supporters or media coverage. In making choices from the available opportunities, the decisions of the terrorist are assumed to obey the usual axioms:

1. Completeness (or Comparability). Given two alternatives, A and B, the terrorist agent or group must be able to state: (1) A is preferred to B ($A \succ B$); or (2) B is preferred to A ($B \succ A$); or (3) the agent or group is indifferent between A and B.

2. Transitivity. Given three alternatives, A, B and C and if the terrorist agent or group prefers A to B and B to C, the agent or group prefers A to C: $(A \succ B)$ and $(B \succ C)$ implies $(A \succ C)$.
3. Non-satiation. More of any good—political influences, fatalities caused, resources, supporters, media coverage—is preferred to less.
4. Convexity. The rate at which the terrorist agent is willing to give up some good in exchange for another while remaining indifferent after the exchange is decreasing.
5. Certainty Equivalent. For every gamble there is an amount of political influence (or some such good) such that the terrorist group is indifferent between the gamble and the certainty equivalent.

When decisions are made under conditions of risk and uncertainty, the standard von Neumann and Morgenstern (NM) expected utility framework is applied to the behavioural analysis. It is not surprising that the behaviour of terrorists or terrorist groups has been analysed within this framework. After all, almost any choice under risk and uncertainty—including choices made by terrorist groups—can be viewed as a decision involving gambles or lotteries. The analysis of terrorist behaviour can easily be undertaken within an expected utility framework in which terrorist agents or groups make decisions between or among gambles or lotteries. The NM expected utility framework enables the economist to predict that the terrorist agent or group will always choose the gamble or lottery with the highest expected utility. This is, of course, entirely different from saying that the agent or group will choose the gamble or lottery with the highest expected value.

The analysis of terrorist decision making under conditions of risk and uncertainty utilising the tools of NM expected utility theory can generate important insights into terrorist behaviour even if the analysis employs ‘hypothetical’ values. The analysis proceeds by assuming that the terrorist group faces the choice between two gambles. The gambles are, in this case, two types of terrorist attack, A and B. The payoffs, in terms of fatalities are presented in Table One. NM expected utility analysis involves assigning weights to outcomes. The weighting function is the NM expected utility function. The weighted value of an expected outcome is the utility of the outcome. NM expected utility is computed by multiplying the weighted value (utility) by the probability that the outcome will occur. Once again, this is different from computing the simple expected value of an outcome. The outcome with a higher expected value may not be the outcome with the higher expected utility.

Table One Hypothetical Attack Fatalities and Probabilities

| Terrorist Attack Type A | | Terrorist Attack Type B | |
|-------------------------|-------------|-------------------------|-------------|
| Fatalities | Probability | Fatalities | Probability |
| 100 | 0.05 | 100 | 0.07 |
| 50 | 0.05 | 50 | 0.09 |
| 1 | .90 | 0 | 0.84 |

Assume, for example, that a terrorist group is characterised by a logarithmic utility function where utility is derived from the number of fatalities F generated by a particular terrorist attack.

$$U(F) = \ln(F) \tag{1}$$

For the data contained in Table One, the utility of 100 fatalities is equal to $\ln(F) = 4.60$, the utility of 50 fatalities is 3.91 and the utility of one or zero fatalities is zero. The NM expected utility is computed for attack types A and B as follows:

$$E(U)_A = 4.60(0.05) + 3.91(0.05) + 0(0.90) = 0.4255$$

$$E(U)_B = 4.60(0.07) + 3.91(0.09) + 0(0.84) = 0.6739$$

The terrorist group is assumed to maximise NM expected utility. Therefore, the group would choose attack type B. This type of analysis may be quite useful when probabilities may be assigned to the fatalities (or whatever the object of analysis may be) associated with particular attack methods deployed by terrorist groups. For various specifications of terrorist groups' utility functions it is possible to determine the attack methods (or combinations of attack methods) that generate the highest utility. At this level of analysis, conclusions that may be drawn for terrorist behaviour are not far reaching but understanding which attack methods have the highest utility for a range of 'plausible' utility function specifications is certainly far from trivial.

Interestingly, the type of analysis outlined in this section has not been explored in great detail by those defence economists who have studied terrorist behaviour. NM expected utility analysis has usually been restricted to situations where it has not been necessary to specify the exact form—power, logarithmic, quadratic and so on—of the utility function. For example, consider the utility analysis undertaken by Enders and Sandler (2002) where a terrorist group's NM expected utility is:

$$E(U) = \pi U(W^S) + (1 - \pi) U(W^F) \quad (2)$$

Where $U(w)$ is the NM utility function and w^S and w^F are the net wealth equivalent measures over two states, success (S) and failure (F). These net wealth equivalent measures over states S and F may be defined as:

$$W^S = w + g^\ell(R^\ell) + g^t(R^t, e_1)$$

$$W^F = w + g^\ell(R^\ell) - f(R^t, e_2) \quad (3)$$

Equations (3) indicate that the gain from a successful incident includes the group's current assets net of current earnings (w), the monetary equivalent net gains from legal activities (g^ℓ) and monetary equivalent net gains from the terrorist activities (g^t). The resources devoted to both types of activities are denoted by (R^ℓ) and (R^t) and (e_1) and (e_2) represent the environmental factors that influence the amount of gains or losses incurred during successful or failed attacks. A resource constraint may be imposed as follows:

$$R = R^t + R^{\ell} \quad (4)$$

The terrorist group is assumed to maximise expected utility $U(w)$ subject to this resource constraint. This analysis generates important insights when the effects of government policies on the environmental factors and the resource constraints are analysed within a comparative statics framework. Most obviously, actions by governments and their agencies to reduce the terrorist group's total resources will reduce terrorist activities (Enders and Sandler 2002, p.149). These results can be generated without specifying an exact form for $U(w)$. Similar approaches have been taken by Landes (1978) and Sandler, Tschirhart and Cauley (1983). The number of investigations that have explicitly defined and deployed a NM expected utility function as the basis for an investigation of terrorist behaviour is very small.

In general, those authors who deploy NM expected utility analysis have recognised the importance of terrorist groups' risk preferences to the choices that those terrorist groups might make under conditions of risk and uncertainty. The choices that terrorist groups make under conditions of risk and uncertainty depend upon the preference for risk that characterises the group. There are three possibilities: (1) risk aversion; (2) risk neutrality; or (3) risk seeking. Formally, these three possibilities imply that the second derivative of the NM expected utility function is (1) negative; (2) zero; (3) positive, respectively. It is usual in microeconomic theory to describe risk averse individuals as individuals who will not accept a fair gamble. A risk averse individual will prefer \$1000 with a probability of 1 to a gamble with a 50-50 chance of \$2000 or \$0. To be enticed to select the gamble, the risk averse individual must be offered a gamble with an expected utility that is at least as great as the certainty equivalent. This is equivalent to saying that a risk averse individual will demand a risk premium for bearing the risk associated with the gamble. The risk averse terrorist group demands a premium in the form of additional expected fatalities or casualties the riskier the planned terrorist activity happens to be.

The 'classical' measures of risk aversion may be applied to the analysis of terrorist behaviour under conditions of risk and uncertainty. The second derivative of the utility function is negative for a risk averse terrorist group. This, on its own, does not say very much about the choices that the terrorist group will make. The terrorist group's preferences for risk may change as its number of inflicted fatalities changes. The terrorist group may engage in more, less or the same amount of risky terrorist attacks as the group's inflicted fatalities change. The relationship between the fatalities inflicted by a terrorist group and its engagement in risky attack methods is described by the group's coefficient of absolute risk aversion. In a similar manner, the terrorist group may increase, decrease or maintain the same weighting accorded to risky attack methods in its attack method combination as the group's inflicted fatalities change. The relationship between a terrorist group's inflicted fatalities and the changes in the weightings it accords to particular attack methods in its attack method combination is described by the group's coefficient of relative risk aversion. Measures of these coefficients were derived by Pratt (1964).

The Arrow-Pratt measures of absolute and relative risk aversion provide much more precision than naïve measures of risk aversion such as the second derivative of the utility function. The Arrow-Pratt measures may be presented formally as:

$$R_A(W) = -\frac{U''(W)}{U'(W)} \quad (5)$$

$$R_R(W) = -W \frac{U''(W)}{U'(W)} \quad (6)$$

$R_A(W)$ is the Arrow-Pratt measure of absolute risk aversion and $R_R(W)$ is the Arrow-Pratt measure of relative risk aversion. These measures have a very desirable property. Although Equations (5) and (6) are specified in terms of wealth, which is presumably measured in dollars, the numerical estimates of equation (5) and equation (6) do not depend upon the units in which the utility is measured. This is a significant advance over the naïve measure of risk aversion computed simply by taking the second derivative of the utility function, $U''(\cdot)$. These measures are particularly suitable to the analysis of terrorist behaviour precisely because the measures do not depend on the units in which utility is measured. The measures are able to cope with ‘non-standard’ units such as units of political influence, inflicted fatalities and so on.

Whereas the exact functional form for a terrorist group’s NM expected utility function has not always been explicitly stated in studies of terrorist behaviour, specifying an exact functional form has the significant advantage of allowing the economist to determine both how much a terrorist group with a particular utility function dislikes risk and how the terrorist group’s preferences may change with changes in the number of fatalities its attacks generate (the absolute and relative risk aversion coefficients). The first derivative of Equation (5) provides a measure of how the absolute number of risky terrorist attacks changes as inflicted fatalities change (that is, how absolute risk aversion changes with changes in inflicted fatalities). The first derivative of Equation (6) provides a measure of how the weighting accorded to risky attack methods in the group’s attack method combination changes as inflicted fatalities changes (that is, how relative risk aversion changes with changes in inflicted fatalities).

Consider the terrorist group with the utility function depicted by Equation (1): $U(F) = \ln(F)$.

The first derivative of the logarithmic utility function is $U'(F) = \frac{1}{F}$ and the second derivative is $U''(F) = -F^{-2}$. The Arrow-Pratt measures of risk aversion are therefore:

$$R_A(F) = -\frac{U''(F)}{U'(F)} = -\frac{-F^{-2}}{F^{-1}} \quad (7)$$

$$R_R(F) = -F \frac{U''(F)}{U'(F)} = -F \frac{-F^{-2}}{F^{-1}}$$

For this utility function, the relative risk aversion is 1 and absolute risk aversion is F^{-1} . As the generated fatalities of the terrorist group changes, the terrorist group characterised by the logarithmic utility function displays the following behaviour. First, the absolute number of

risky terrorist attacks in which it is engaged increases (decreases) as the inflicted fatalities of the terrorist group increases (decreases). Second, the weighting (percentage) accorded to particular attack methods in the terrorist group's attack method combination remains constant as the inflicted fatalities of the terrorist group increases or decreases (the weightings accorded to particular attack methods in the group's combination is independent of the inflicted fatalities of the group). This is expressed formally by the derivatives of the Arrow-Pratt risk aversion measures:

$$R_A'(F) = -F^{-2} \tag{8}$$

$$R_R'(F) = 0$$

Studies contained within the literature of defence economics have not usually been very precise in defining the functional forms of terrorist groups' utility functions and, as a consequence, the Arrow-Pratt measures of risk aversion and other important tools of utility analysis lay underutilised by defence economists. Sandler, Tschirhart and Cauley's (1983, p.44) statement, "Some ambiguity cannot be removed until more empirical information concerning the functional specifications is available," still holds true today. Indeed, in many ways, Sandler, Tschirhart and Cauley's (1983) paper remains one of a very small number of investigations that have applied utility analysis to the analysis of terrorist behaviour and taken steps toward the analysis of the implications of risk preferences for terrorist behaviour under risk and uncertainty. Sandler, Tschirhart and Cauley (1983) take a traditional microeconomic approach to utility analysis and apply it to terrorism whereas Phillips (2009) approaches the problem by the introduction of mean-variance methods to the analysis of terrorist choice in an attempt to generate computable results and short-cut a full NM expected utility analysis.

Sandler, Tschirhart and Cauley (1983) generate a number of important insights by analysing a terrorist utility function stated as:

$$U = U(L, D, \Pi; C^*) \tag{9}$$

Where L is a measure of gain from a legal activity, D is the demand(s) against the government as a result of an illegal activity; Π is the probability that the demands are met and C^* is the most recent concession made by the government in response to terrorist demands (Sandler, Tschirhart and Cauley (1983, pp.39–40)). Of particular interest, is Sandler, Tschirhart and Cauley's (1983, p.45) Equation (21), which is used to analyse the effect of changes in government concessions to terrorist groups during negotiations:

$$\partial D^* / \partial \mu = \left[\sigma N' U_D \Pi - U_{\Pi \Pi} - \lambda \sigma N'' \right] \Big|_{H_T} \tag{10}$$

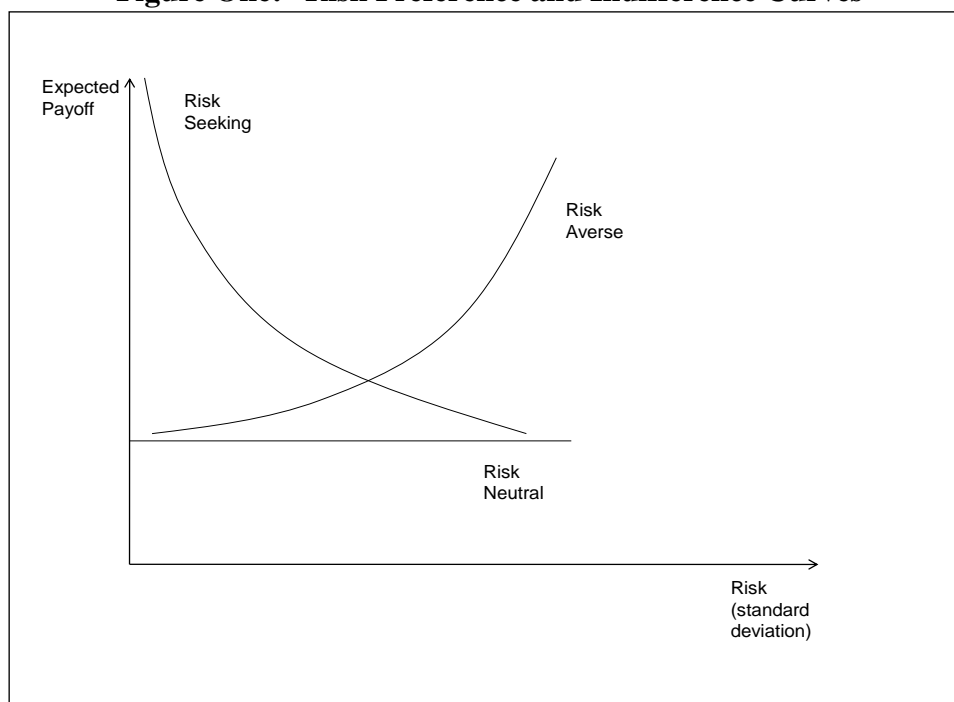
Using this equation, Sandler, Tschirhart and Cauley (1983) draw conclusions regarding the relationship between the mean and variance of government concessions and terrorist's demands. In Equation (10), a terrorist group is risk-loving if $U_D \Pi$ is negative and risk averse

if the term is positive. The implications of this analysis are, according to Sandler, Tschirhart and Cauley (1983, p.45):

1. A greater variance will increase rather than decrease the demands of risk loving terrorist agents. To reduce terrorism, governments should choose little variance in their concessions when dealing with a risk loving group; and
2. Since the opposite tendency is true for risk averse terrorist agents, governments should adopt a high variance strategy in granting concessions in order to curb terrorism.

These results flow logically from the utility analysis presented by Sandler, Tschirhart and Cauley (1983). However, it is easier to analyse these choices when a mean-variance analysis is deployed. The indifference curve geometry that is deployed in mean-variance analysis illustrates very clearly the tradeoffs that an economic agent makes between return and risk (standard deviation or variance of returns) depending on whether the agent is risk averse, risk seeking or risk neutral. These indifference curves are sketched in Figure One. Within this theoretical context, it is very easy to see that the risk seeking agent will demand more return as risk diminishes. The opposite is true for the risk averse individual. Although quadratic utility ensures that mean-variance preference orderings will be consistent with 'full' NM expected utility preference orderings, it is possible to find other specifications for the functional form of the terrorist groups' utility function that generate preference orderings that almost completely agree with the preference orderings made on the basis of mean and variance. There are several common functional forms that may describe a terrorist groups' utility function. Each of these is based on the assumption that terrorist agents dislike risk and will only bear greater amounts of risk if they are compensated with higher expected returns. The negative second derivatives of these utility functions indicate risk aversion. However, the type of risk aversion exhibited by terrorist groups with these utility functions will vary.

Figure One: Risk Preference and Indifference Curves



Four types of utility function are common in financial economics: (1) quadratic; (2) exponential; (3) logarithmic; and (4) power. Assuming once more that terrorist groups derive utility from the number of fatalities that an attack method (or combination of attack methods) generates, these utility functions may be expressed in terms of fatalities. Each of the four utility functions is presented in Table Two, along with information concerning the type of absolute and relative risk aversion that characterises each function.

Table Two (HARA)⁶ Utility Functions (Expressed in Terms of Fatalities, F) and Risk Aversion

| Function | Formal Expression | $A(F)$ | $R(F)$ |
|-------------|-----------------------------------|-----------------------------------|----------|
| Quadratic | $\gamma_0 F - \gamma_1 F^2$ | Increasing Absolute Risk Aversion | - 1 |
| Exponential | $-\frac{1}{\gamma} e^{-\gamma F}$ | γ | 0 |
| Logarithmic | $\ln(F)$ | Decreasing Absolute Risk Aversion | 1 |
| Power | $\frac{1}{1-\gamma} F^{1-\gamma}$ | Decreasing Absolute Risk Aversion | γ |

In the power utility function, which is very popular in financial economics, $\frac{1}{1-\gamma} F^{1-\gamma}$, γ is the ‘curvature parameter’ which controls (1) risk aversion (aversion to a stream of fatalities that varies over time); (2) aversion to a stream of fatalities that varies across states of nature; and (3) precautionary hoarding. Only the power utility function closely ties these three properties together (Cochrane 2001, p.15). The type of risk aversion exhibited by these functions is revealed by the first derivatives of the relevant Arrow-Pratt measures of risk aversion. Both the logarithmic and power utility functions exhibit decreasing absolute risk aversion. The quadratic utility function exhibits increasing absolute risk aversion. The logarithmic and power utility functions exhibit constant relative risk aversion (although, it should be noted, that the logarithmic utility function requires a relative risk aversion coefficient equal to 1). The quadratic utility function exhibits increasing relative risk aversion.

The specification of the functional form for the utility functions of terrorist groups must begin with an analysis of the applicability of quadratic, logarithmic and power utility functions to terrorist behaviour. To push beyond the work of Landes (1978), Sandler, Tschirhart and Cauley (1983) and Enders and Sandler (2002) it is necessary to explore the functional forms of terrorist utility functions more intensively. In so doing, the aim must be to estimate the parameters of terrorist utility functions (especially the risk aversion parameters) and produce computable (rather than theoretical or logical) results where possible. Such an undertaking is not easy. However, the research program that has been established by those economists who first applied rational actor models to the analysis of terrorism must be completed in order to

⁶ See Carpenter (2000), especially p.2314 for a discussion of HARA utility and risk aversion.

increase the power of the comparative statics analysis, remove the ambiguity noted by Sandler, Tschirhart and Cauley (1983, p.44) and, perhaps, enable a more dynamic analysis with computable results. The remainder of this paper presents an analysis that promises to take defence economics a few steps closer towards achieving this goal. Building on the rational actor frameworks and, especially, the mean-variance analysis of Phillips (2009; 2011), (1) an estimate of the (relative) risk aversion coefficient of the Al-Qa`ida terrorist group is computed; and (2) the utility functions most likely to characterise Al-Qa`ida group are explored.

III. THE RISK AVERSION OF THE AL-QA`IDA TERRORIST GROUP

In this section, the relative risk aversion coefficient of the Al-Qa`ida terrorist group is computed by exploiting the relationship that exists between the aggregate risk-reward ratio exhibited by a benchmark combination of attack methods and the choices of attack method combinations made by the Al-Qa`ida group. The most difficult part of the analysis is to compute the aggregate risk-reward ratio for terrorist attacks. This is the ratio of the premium of a benchmark combination of attack methods above a ‘riskless’ attack method combination to the variance of the payoffs generated by the benchmark combination. Fortunately, Phillips (2011) has provided the fundamental elements of this analysis by computing the expected return (measured in fatalities) generated by a ‘zero-beta’ combination of attack methods. The zero-beta combination exhibits no sensitivity to the benchmark combination of terrorist attack methods. The premium (in terms of expected fatalities) generated by the benchmark combination of attack methods above the zero-beta combination provides the essential inputs for determining the market price of risk for terrorist activities. With this value in hand, the relationship between the aggregate risk-reward ratio and the make-up of Al-Qa`ida’s attack method portfolio provides a mechanism with which to compute Al-Qa`ida’s relative risk aversion coefficient.

There is a ‘membrane’ that holds together the aggregate relationship between the return and risk associated with terrorist attacks and the characteristics of the attack method combinations that make up the ‘portfolios’ of individual terrorist groups. This relationship is stated by Friend and Blume (1975, p.903) as:

$$\alpha_k = \frac{E(r_m - r_f)}{\sigma_m^2} \cdot \frac{1}{C_k} \quad (11)$$

Where α_k is the proportion of the terrorist group k’s attack method combination that is constituted by the risky benchmark combination, σ_m^2 is the variance of the returns (measured in terms of fatalities) generated by the risky benchmark combination, r_m and r_f are the return on the benchmark combination of attack methods and the return on the zero-beta combination of attack methods respectively. The parameter C_k is the Arrow-Pratt measure for relative risk aversion (Equation (6)). If the aggregate risk-reward ratio, $\frac{E(r_m - r_f)}{\sigma_m^2}$, and the

proportion, α_k , are known, C_k can be estimated directly from Equation (11)). This will provide an estimate for the Arrow-Pratt measure of relative risk aversion (which may be interpreted as a point estimate of the ‘fatalities elasticity’ of the marginal utility of fatalities generated by terrorist operations). Time-variation in the underlying distributions may generate alternative values in future research and the following analysis should be interpreted with this in mind.

In most major respects, the computation of the market price of risk that describes terrorist attack method combinations has been completed by Phillips (2011). Using data collected by the RAND Corporation in the MIPT Database, Phillips (2011) computes the expected return (measured in terms of fatalities) and risk (variance of returns) for an equally-weighted combination of terrorist attack methods. Each of the ten attack methods identified by the RAND Corporation is assigned a 10 percent weighting in the ‘market’ or ‘benchmark’ combination and the portfolio techniques utilised by Phillips (2009) are deployed to compute the expected return and risk of the benchmark combination. Because there is no such thing as a ‘riskless’ attack method, Phillips (2011) uses instead a zero-beta attack method combination that is risk-free in the sense that its expected returns do not vary systematically with the benchmark combination. The aggregate risk-reward ratio is easily computed as ratio of the premium (in terms of higher expected fatalities) generated by the benchmark combination above the zero-beta combination to the variance of the benchmark combination’s returns.

The analysis presented by Phillips (2011) deserves some comment. First, in constructing a zero-beta equilibrium model of terrorist attack method choices, Phillips (2011) does not rely on quadratic utility or a normal distribution. Although such assumptions will guarantee a direct match of the mean-variance preference ordering with the full EU preference ordering (and consistency with the NM axioms), empirical work shows that mean-variance preference orderings provide a close approximation of full EU analysis with the added advantage of generating computable results (see Elton et al. (2003, p.232); Kroll, Levy and Markowitz (1984); Levy and Markowitz (1979); Meyer (1987) and, for a discussion of theoretical considerations, especially the utilisation of mean-variance orderings for portfolio problems and the selection of the optimal choice, Baron (1977, p.1690-1692)). Second, the choice of an ‘equally weighted’ benchmark (rather than some other weighting scheme) will serve its purpose if it reflects the systematic relationship between fatalities and terrorist attack methods. There is no reason to suppose that a different benchmark would do a better job of capturing the systematic trade-off between risk and return so far as it pertains to terrorist operations. The market price of risk computed by Phillips (2011) should be a reasonable place to commence an analysis of the kind attempted here.

Following Phillips (2011), an equally-weighted benchmark combination of attack methods—armed attacks, arson, assassination, hostage, bombing, hijacking, kidnapping, other, unconventional, unknown—is constructed and the expected return and risk computed using the formulas for portfolio return and variance:

$$E(F_p) = \sum_{i=1}^n x_i E(F_i) \tag{12}$$

$$\sigma_p^2 = \sum_{i=1}^n x_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{\substack{j=1 \\ i \neq j}}^n x_i x_j \rho_{ij} \sigma_i \sigma_j \quad (13)$$

Where $E(F_p)$ is the expected payoff (measured in fatalities, F) of the attack method combination (portfolio) P, x_i attack method i's weighting in the portfolio P, and σ_p^2 is the variance of the attack method 'portfolio' P. When each of the ten attack methods is assigned a weighting of 10 percent in the benchmark combination, the expected return of the benchmark combination (measured in terms of fatalities) is 1.8116 fatalities per attack. The standard deviation of the expected returns of the benchmark combination is 2.71 fatalities per attack. The expected return on the zero-beta combination that has zero sensitivity to the fluctuations in the expected returns of the equal-weighted benchmark is 0.4157 fatalities per attack. This data enables the 'aggregate risk-reward ratio' for terrorist attack methods to be computed.

Using part of Equation (11), $\frac{E(r_m - r_f)}{\sigma_m^2}$, the aggregate risk-reward ratio or 'market price of risk' for terrorist attack methods is:

$$\frac{E(r_m - r_f)}{\sigma_m^2} = \frac{1.8116 - 0.4157}{2.71^2} = 0.1900$$

Risky attack methods have a risk premium of 0.1900 fatalities per unit of risk (variance). This is an important result. As terrorist agents bear more units of risk—higher variance of expected fatalities generated by attack methods or combinations of attack methods—they can expect to generate 0.1900 additional fatalities per unit of risk. An attack method combination that bears no 'systematic' risk (no sensitivity to the benchmark combination of attack methods) will generate 0.4157 fatalities per attack. As the terrorist group assumes more risk by shifting resources to riskier combinations of attack methods, the number of expected fatalities increases. The terrorist group is rewarded with an additional 0.1900 fatalities per unit of risk.

The Al-Qa`ida terrorist group is characterised by a particular 'portfolio' of attack methods. This portfolio is the combination of attack methods that have characterised its history of terrorist attacks. To determine the structure of Al-Qa`ida's attack method combination, the data available from the University of Maryland's Global Terrorism Database (GTD) is used. The Al-Qa`ida 'dataset' covers the period from 1998 (the August 7th attack in Nairobi, Kenya) to the latest recorded attack on November 6th 2007 (Ghalibiyah, Iraq) and contains all of the necessary information: fatalities, target type, attack type. More information is available from the GTD but this is all that is required to construct Al-Qa`ida's combination or portfolio of attack methods for comparison with the benchmark. The payoffs generated by Al-Qa`ida's portfolio of attack methods (measured in terms of fatalities) are displayed in Table Three along with the mean return generated by each attack per year and the standard deviation of the returns generated by each attack method since 1998. The relationship between return and risk is clear.

Table Three Al-Qa`ida's 'Portfolio' Returns 1998 to 2007

| Attack Type | Bombing | Armed Assault | Arson | Assassination | Hostage (Kidnapping) | Unconventional | Unknown |
|--------------------|------------|---------------|-------|---------------|----------------------|----------------|---------|
| 1998 | 117.5 (2) | 0 | 0 | 0 | 0 | 0 | 0 |
| 1999 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2000 | 19 (1) | 0 | 0 | 0 | 0 | 0 | 0 |
| 2001 | 0 | 0 | 0 | 0 | 0 | 403 (4) | 0 |
| 2002 | 13.75 (12) | 0 | 0 | 3 (1) | 1 (1) | 0 | 12 (1) |
| 2003 | 7.47 (21) | 1.60 (5) | 0 | 15 (1) | 0 (1) | 0 | 0 |
| 2004 | 27.18 (16) | 10.66 (3) | 0 | 0 | 0 | 0 | 0 |
| 2005 | 8.50 (4) | 0.50 (2) | 0 (1) | 2 (1) | 0 | 0 | 0 |
| 2006 | 4.25 (4) | 5 (2) | 0 | 11 (1) | 0 | 0 | 0 |
| 2007 | 8.55 (9) | 2 (2) | 0 | 0 | 0 | 0 | 0 |
| Mean | 20.62 | 1.976 | 0 | 3.1 | 0.10 | 40.3 | 1.2 |
| STDEV | 35.05937 | 3.438 | 0 | 5.404 | 0.3162 | 127.43 | 3.794 |

Notes: The figures in the columns report the number of fatalities per attack for each year. For example, the GTD Database records four attacks on September 11th 2001 generating a total of 1,612 fatalities or 403 fatalities per attack. The numbers in parentheses identify the number of incidents that took place during the year. Where a 'zero' is input into a cell in the table (and there are no accompanying numbers in parentheses) there were no incidents of that particular attack type in the year. The mean is the mean number of fatalities per year.

During the period 1998 to 2007, Al-Qa`ida executed 95 attacks. The combination or portfolio of attack methods that characterises Al-Qa`ida may be computed by determining the weighting of each attack method in these total of 95 attacks. The portfolio of attack methods deployed by Al-Qa`ida is heavily weighted towards bombing and armed assaults. The portfolio consists of: (1) bombing 72.63 percent; (2) armed assault 14.73 percent; (3) arson (incendiary) 1.05 percent; (4) assassination 4.21 percent; (5) hostage (kidnapping) 2.10 percent; (6) unconventional 4.21 percent; (7) unknown 1.05 percent. The Al-Qa`ida group has not engaged in any hijacking or 'other' attacks, which are the two additional attack method types listed by RAND-MIPT. On the one hand, the attack method combination deployed by the Al-Qa`ida terrorist group is quite well diversified. Almost all of the attack method types identified by RAND-MIPT are contained in Al-Qa`ida's portfolio. On the other hand, the portfolio is heavily concentrated in just two attack methods: bombing and armed assault. Interestingly, bombing has been (historically across all terrorist groups and incidents) a relatively high risk attack method.

To start to determine what this means for the risk aversion that characterises the Al-Qa`ida terrorist group it is necessary to compare Al-Qa`ida's portfolio to the benchmark equal-weighted combination of all attack types identified by RAND-MIPT and the historical return and risk across the aggregate data (including attacks by all other terrorist individuals and groups) and the zero-beta combination of attack methods computed by Phillips (2011). The expected returns and risk for each of ten attack method types identified by RAND-MIPT are listed in Table Four. Historically, across all incidents since 1968, unconventional attacks,

hostage taking and bombing have exhibited the highest risk and highest returns. Armed assaults (and the other attack methods) have exhibited lower returns and lower risk. Upon first analysis, it seems as though the Al-Qa`ida terrorist group's combination of attack methods is concentrated in one of the more risky attack methods. However, the relatively high weighting accorded to armed assaults—a less risky activity—offsets the volatility in expected payoffs that would be expected from solely focussing on risky bombing operations. The relatively low weighting in higher risk activities such as hostage taking and unconventional attack implies that the Al-Qa`ida terrorist group is sensitive to very high risk operations (or does not believe that the expected payoffs compensate for the variance).

Table Four: Aggregate (Benchmark) Returns (Fatalities) 1968 to 2007

| | Armed Assaults | Arson | Assassination | Hostage | Kidnapping | Bombing | Hijacking | Other | Unconventional | Unknown |
|--------------|----------------|--------|---------------|----------|------------|---------|-----------|--------|----------------|---------|
| VAR | 1.261 | 0.565 | 0.150 | 135.798 | 0.113 | 28.311 | 14.816 | 3.756 | 576.281 | 16.028 |
| STDEV | 1.122 | 0.7519 | 0.3877 | 11.65324 | 0.335 | 5.3207 | 3.8491 | 1.9379 | 24.0058 | 4.0035 |
| Mean | 1.296 | 0.322 | 1.045 | 3.620 | 0.393 | 4.604 | 1.566 | 0.473 | 3.883 | 0.915 |

The equal-weighted or benchmark combination of all attack methods generates an expected return of 1.8116 fatalities per attack with a risk (standard deviation) of 2.71 fatalities. Al-Qa`ida's combination has generated an actual or realised return of 17.10 fatalities per attack. The expected return of Al-Qa`ida's attack method combination must be computed using Phillips' (2011) equilibrium equation for the relationship between expected fatalities and beta (systematic) risk and the entire RAND-MIPT dataset of terrorist attacks and fatalities. That is, once the proportions of each attack method in Al-Qa`ida's combination have been determined, it is necessary to apply these proportions or weightings to the aggregate historical fatalities time-series data in order to compute series of expected returns for Al-Qa`ida's attack method combination and the sensitivity (beta risk) of these expected returns to the equal-weighted combination provides the necessary inputs for Equation (14):

$$E(F_p) = E(F_{zero}) + \beta_p (E(F_e) - E(F_z)) \quad (14)$$

Where $E(F_{zero})$ is the expected fatalities generated by the zero-beta combination of attack methods, β_p measures the sensitivity of a combination of attack methods to a combination constituted by all attack methods (equal weighted) and $E(F_e)$ is the expected fatalities from this equal weighted combination. Beta can be computed using Equation (15):

$$\beta_i = \frac{\sigma_{i, equal-weighted}}{\sigma_{equal-weighted}^2} \quad (15)$$

The numerator in Equation (15) is the covariance of the payoffs of a combination of attack methods with the payoffs of the equally weighted combination of attack methods and the denominator is the variance of the payoffs of the equally weighted combination. A Beta of zero will be generated by a combination of attack methods that generates a series of payoffs that has a covariance of zero with the equally weighted benchmark combination. Applying Al-Qa`ida's portfolio weightings to the aggregate data (of which Al-Qa`ida's attacks are merely a part) and computing the beta for Al-Qa`ida's attack method combination for the

period 1998 to 2007 yields a result of 0.3247. The equilibrium expected return for Al-Qa`ida's attack method combination (using Equation (14)) is:

$$\begin{aligned} E(F_p) &= E(F_{zero}) + \beta_p (E(F_e) - E(F_z)) \\ &= 0.4157 + 0.3247(1.8116 - 0.4157) \\ &= 0.88 \end{aligned}$$

Al-Qa`ida's attack method combination is considerably more risky than the zero-beta combination computed by Phillips (2011). A combination whose expected fatalities do not systematically vary with the volatility exhibited by the equal-weighted benchmark combination is highly weighted towards kidnapping, assassination and arson. Almost the entire zero-beta combination consists of these attack methods and, in fact, exhibits negative weightings for the riskier attack methods of bombing, armed assault and unconventional attacks (see Phillips (2011)). Al-Qa`ida's attack method combination is concentrated in bombing operations and armed assaults (and, to a lesser degree, assassinations and unconventional attacks). The result is a riskier attack method combination but one that is rewarded with a higher expected return than the zero-beta (zero systematic risk) combination. Al-Qa`ida's attack method combination is, however, much less risky than the equal-weighted benchmark combination and, consequently, generates a lower expected return of 0.88 fatalities per attack compared with 1.8116 for the equal-weighted benchmark combination of attack methods.

Putting all this together, it is now possible to determine the relative risk aversion coefficient that characterises the Al-Qa`ida terrorist group. All attack method combinations can be formed by combinations of two fundamental combinations: (1) the zero-beta combination; and (2) the equal-weighted benchmark combination. Given expected returns (1) for Al-Qa`ida's attack method combination, (2) the zero-beta combination; and (3) the equally-weighted benchmark combination it is a straightforward matter to utilise this fact—that all attack method combinations can be formed by combinations of two fundamental combinations—to determine the weightings in both the ('riskless') zero-beta combination and the (risky) equally-weighted combination that characterises a 'synthetic' Al-Qa`ida attack method combination. Solving for x in the following equation yields the sought after result:

$$\begin{aligned} E(F_{Al-Qa`ida}) &= xE(F_{zero-beta}) + (1-x)E(F_{benchmark}) \\ 0.88 &= x(0.4157) + (1-x)(1.8116) \\ 0.88 &= 0.4157x + 1.8116 - 1.8116x \\ x &= 0.66 \end{aligned}$$

A synthetic portfolio that reflects the expected return and risk of the Al-Qa`ida terrorist group requires an allocation of 66 percent of attack method combination to the ('riskless' or zero-systematic risk) zero-beta combination and 34 percent of its attack method combination to the risky equal-weighted benchmark combination of attack methods. With this final piece of information, it is possible to deploy Equation (11) to determine the relative risk aversion coefficient for the Al-Qa`ida terrorist group:

$$\alpha_k = \frac{E(r_m - r_f)}{\sigma_m^2} \cdot \frac{1}{C_k}$$

$$0.34 = 0.1900 \cdot \frac{1}{C_k}$$

$$C_k = 0.56$$

The relative risk aversion coefficient of the Al-Qa`ida terrorist group is 0.56. This value is determined by establishing a relationship between the proportion of the terrorist group's attack method combination that is allocated to the risky equal-weighted benchmark combination and the market price of risk (the reward that terrorist groups can expect for bearing an additional unit of risk). In the finance literature, the general view is that households or investors exhibit a relative risk aversion coefficient of between 1 and 5 (see, for example, Barro (2006)). There are many reasons why the terrorist group's risk aversion coefficient cannot be compared with the average investor (for one thing, these two agents are choosing from different choice sets) but such a comparison does indicate that Al-Qa`ida's relative risk aversion is very low. Referring to Table Two it would appear that several conclusions can be reached. If 0.56 is a reasonable estimate of Al-Qa`ida's relative risk aversion, this particular terrorist group is (1) less risk averse than would be the case if Al-Qa`ida was characterised by a logarithmic utility function; (2) more risk averse than would be the case if Al-Qa`ida was characterised by either a quadratic or exponential utility function.

IV. CAN AL-QA`IDA SURVIVE?

There are, as mentioned previously, several common specifications of the utility function: (1) quadratic; (2) exponential; (3) logarithmic; and (4) power. The properties of the Arrow-Pratt measures for absolute and relative risk aversion that characterise each of these utility functions permit the defence economist to investigate the trade-offs that terrorist agents make between risk and return and the utility that they generate from particular actual or potential outcomes. Such an analysis, even it is purely theoretical or hypothetical, can shed much needed light upon the decision-making processes underlying terrorist activity. By taking steps to compute a numerical result for the relative risk aversion coefficient for a terrorist group, the analysis presented in this paper permits the economist to go beyond the purely logical or hypothetical analysis of terrorist behaviour and begin building a store of numerical results. A relative risk aversion coefficient of 0.56 permits the economist to focus his efforts more intensively and, in the process, (1) make more informed decisions about the type of utility function that may characterise the Al-Qa`ida terrorist group; (2) determine how much utility the Al-Qa`ida terrorist group derives from particular numbers of fatalities; and (3) draw preliminary conclusions about the potential longevity of the Al-Qa`ida terrorist group. Each of these is addressed in turn in this section.

It is clear from the information presented in Table Two that a relative risk aversion coefficient of 0.56 is not consistent with some forms of utility functions. The quadratic, exponential and logarithmic utility functions require a relative risk aversion of -1 , 0 and 1 respectively. If the estimate of 0.56 is correct, Al-Qa`ida is characterised by a relative risk aversion coefficient that is too high to be consistent with the quadratic and exponential utility functions and too low to be consistent with the logarithmic utility function. Of the four functions under consideration, this leaves only the power utility function as a possibility. If Al-Qa`ida can be

said to be characterised by the power utility function, a few important conclusions follow. First, Al-Qa`ida will exhibit decreasing absolute risk aversion and will be observed to engage in a higher (lower) number of risky terrorist activities as the number of fatalities the group causes increases (decreases). Second, Al-Qa`ida will exhibit a tendency to maintain as constant (at 34 percent) the weighting it accords to risky attack methods as the number of fatalities increases or decreases. By monitoring the number of fatalities accruing to Al-Qa`ida's terrorist attacks, government agencies may be able to infer Al-Qa`ida's future involvement in risky terrorist attacks.

The utility that the Al-Qa`ida terrorist group has derived from the fatalities its attacks have caused may be computed using the power utility function with a risk aversion (or curvature) parameter equal to 0.56. The utility that Al-Qa`ida has generated from the total number of fatalities each year since 1998 is reported in Table Five. Not surprisingly, Al-Qa`ida generated higher utility during 2001 than in other year for the period under consideration.

Table Five: The Utility from Fatal Attacks Generated by Al-Qa`ida 1998 to 2007

| Year | Total Number of Fatalities | Utility Derived from Power Utility Function |
|-------------|-----------------------------------|--|
| 1998 | 235 | 4.860965 |
| 1999 | 0 | 0 |
| 2000 | 19 | 1.607327 |
| 2001 | 1612 | 11.3421 |
| 2002 | 181 | 4.333425 |
| 2003 | 180 | 4.322874 |
| 2004 | 467 | 6.575853 |
| 2005 | 37 | 2.15507 |
| 2006 | 38 | 2.180507 |
| 2007 | 79 | 3.008906 |

The numbers reported in Table Five are not cardinal NM expected utility numbers. They are actual utility numbers and are ordinal only. The only conclusion that can be drawn directly from Table Five is that the power utility function assigns higher utility numbers to higher numbers of fatalities. However, if probabilities are assigned to the fatalities that can be expected to be generated by particular attack methods, it is possible to construct a 'matrix' of expected utilities for Al-Qa`ida and, thereby, to undertake a full NM expected utility analysis of Al-Qa`ida's choices. It should be the case that the mean-variance technique suggested by Phillips (2009) will generate preference orderings that agree very closely with any preference ordering generated by full NM expected utility analysis. This is an interesting task for future research.

One of the more potentially significant and far-reaching implications of the analysis presented in this paper concerns the implications of the results for the longevity of the Al-Qa`ida terrorist group. While work in the area of evolutionary dynamics is still evolving, certain results that have been reported in the economics literature have implications for the longevity of economic agents that are characterised by particular types of utility functions. The computation of a value of 0.56 for Al-Qa`ida's relative risk aversion coefficient, points the defence economist from quadratic and logarithmic utility. This is important because it can be shown that the type of utility function that characterises a terrorist group also has implications

for its longevity. Reaching such a conclusion is a simple matter of extending some interesting results of orthodox microeconomic theory to defence economics and the study of terrorist behaviour. In particular, Blume and Easley (1992) have examined the evolutionary dynamics of asset markets and determined that 'natural selection' favours those individuals who maximise the expected growth rate of wealth share accumulation. The implications for the analysis of terrorist behaviour follow from considering whether particular terrorist groups behave in a manner that is likely to maximise the expected growth rate of their generated fatalities.

If the utility of a terrorist group is a function of the number of fatalities it generates through terrorist attacks, the way in which terrorist groups choose among attack method combinations (with different expected payoffs in terms of fatalities) is tied to its generated fatalities over time. An implication of the analysis presented in this paper is that terrorist agents learn about the success of particular attack method combinations by observing the numbers of fatalities that are generated by particular attacks over time. In this way, the aggregate terrorist attack data is relevant to individual terrorist groups and the behaviour of individual terrorist groups feeds back into the aggregate terrorist attack statistics. Blume and Easley's (1992) analysis, once applied analogously to terrorist behaviour, shows that the defence economist must consider the terrorist group's accrued fatalities over time. Blume and Easley (1992) show that, in the context of asset markets, the maximisation of expected growth rate of wealth share accumulation is the dominant 'natural selection' strategy. Importantly, maximising logarithmic utility is consistent with this strategy.

Although the full implications for the evolutionary stability of Al-Qa`ida's attack method selection cannot be explored in this paper, some preliminary conclusions can be reached and the path cleared for some extraordinarily exciting research. The results presented in this paper imply that Al-Qa`ida is more likely to be characterised by a power utility function, for example, than a logarithmic utility function. The relative risk aversion coefficient computed for Al-Qa`ida of 0.56 is too low to be consistent with a logarithmic utility function. If the Al-Qa`ida terrorist group's utility is a function of the fatalities that its attacks generate, maximising the expected utility of fatalities through the selection of 'appropriate' combinations of attack methods will affect the terrorist group's accumulation fatalities over time. The terrorist groups that maximise the expected growth rate of their share of total fatalities will 'survive'. Such groups will attract an increasing share of fatalities relative to other groups. Since maximising the expected growth rate of fatalities is the same as maximising the expected logarithmic utility of fatalities, terrorist groups characterised by logarithmic utility functions will survive. The results presented in this paper represent preliminary evidence that suggests that Al-Qa`ida is not such a group.

V. CONCLUSIONS

To generate more than logical results from the application of expected utility analysis to terrorist behaviour it is necessary to reach some conclusions regarding the specification of the utility functions that characterise terrorist groups. Similarly, the importance of terrorists' risk preferences has been noted numerous times in the defence economics literature. In this paper, an attempt was made to compute a value for the relative risk aversion coefficient that characterises the Al-Qa`ida terrorist group. Using the mean-variance approach and the equilibrium conditions that follow when all agents base their choices on the mean-variance

criterion, it was shown that the beta coefficient for Al-Qa`ida is consistent with a synthetic portfolio that assigns a weighting in risky terrorist activities of 34 percent and a weighting of 66 percent in the zero-beta (zero systematic risk) combination. By exploiting a relationship between an individual terrorist group's allocation to risky attack methods and the market price of risk (the additional number of fatalities that can be expected by bearing an additional unit of risk), a value for Al-Qa`ida's relative risk aversion coefficient was computed. Al-Qa`ida is characterised by a relative risk aversion coefficient of 0.56.

The determination of a value for the relative risk aversion coefficient for a terrorist group is a positive step. However, the possession of a precise (or even approximate) value for Al-Qa`ida's relative risk aversion coefficient can be utilised to generate further insights. In particular, a relative risk aversion coefficient of 0.56 is only consistent with particular types of utility functions. This insight permits the defence economist to generate numerical (rather than logical) results by applying the utility function to possible payoffs generated by particular attack methods and combinations of attack methods and permits a full expected utility analysis with computable results. The determination of the utility generated by particular numbers of fatalities is complemented by results concerning the terrorist group's engagement in risky terrorist activities as the number of fatalities generated by the group's activities increases or decreases. Al-Qa`ida may be expected to increase the number of risky terrorist attacks as the fatalities it generates increases. The weighting of risky attack methods in Al-Qa`ida's attack method combination may be expected to remain constant as the number of fatalities generated by the group increases. As fatalities mount, Al-Qa`ida will launch more attacks but the composition of its attack method combination will not change.

In addition to all of these insights that may be generated by exploring the relative risk aversion of a terrorist group, the consideration of the rate of accumulation of fatalities by terrorist groups over time and the implications of particular utility functions for the longevity of particular 'fatalities generating' activities promises to add another level of sophistication to the economic analysis of terrorism. Maximising the growth rate of accumulation of fatalities is a strategy that is evolutionarily stable (Blume and Easley 1992). Terrorist groups that maximise their expected growth rate share of total fatalities accruing to terrorist activities will eventually dominate all other groups. Such a strategy is consistent with the behaviour of a terrorist group that is characterised by logarithmic utility and maximises the expected value of the logarithm of fatalities. The calculation of a value for Al-Qa`ida's relative risk aversion coefficient of 0.56 is too low to be consistent with logarithmic utility. Examining the full implications of this and the many other issues that emerge throughout this paper will make for interesting future research.

References

- Arce M., D.G. and Sandler, T. 2005, "Counterterrorism: A Game-Theoretic Analysis," *Journal of Conflict Resolution*, 49, No.2, pp.183-200.
- Baron, D.P. 1977, "On the Utility Theoretic Foundations of Mean-Variance Analysis," *Journal of Finance*, 32, pp.1683-1697.
- Barro, R.J. 2006, "Rare Disasters and Asset Markets in the Twentieth Century," *Quarterly Journal of Economics*, 121(3), pp.823-866.
- Blume, L. and Easley, D. 1992, "Evolution and Market Behaviour," *Journal of Economic Theory*, 58, pp.9-40.

- Carpenter, J.N. 2000, "Does Option Compensation Increase Managerial Risk Appetite?" *Journal of Finance*, 55, pp.2311-2331.
- Cochrane, J.H. 2001, *Asset Pricing Theory*, Princeton University Press, Princeton, NJ.
- Crenshaw, M. 1991, "How Terrorism Decline," *Terrorism and Political Violence*, 3, pp.69-87.
- Cronin, A.K. 2006, "How Al-Qaida Ends," *International Security*, 31, pp.7-48.
- Eaton, B.C. and Eswaran, M. 2003, "The Evolution of Preferences and Competition: A Rationalisation of Veblen's Theory of Invidious Comparisons," *Canadian Journal of Economics*, pp.832-859.
- Elton, E.J., Gruber, M.J., Brown, S.J. and Goetzmann, W.N. 2003, *Modern Portfolio Theory and Investment Analysis*, John Wiley & Sons, New York, New York.
- Enders, W. and Sandler, T. 2002, "Patterns of Transnational Terrorism, 1970–1999: Alternative Time Series Estimates," *International Studies Quarterly*, 46, pp.145-165.
- Friend, I. and Blume, M.E. 1975, "The Demand for Risky Assets," *American Economic Review*, 65(5), pp.900-922.
- Frey, B.S. and Luechinger, S. 2003, "How to Fight Terrorism: Alternatives to Deterrence," *Defence and Peace Economics*, 14, No.4, pp.237-249.
- Konrad, K. and Morath, F. 2012, "Evolutionarily Stable In-Group Favouritism and Out-Group Spite in Intergroup Conflict," *Journal of Theoretical Biology*, 306, pp.61-67.
- Kroll, Y., Levy, H. and Markowitz, H.M. 1984, "Mean-Variance Versus Direct Utility Maximisation," *Journal of Finance*, 39, pp.47-61.
- Landes, W.M. 1978, "An Economic Study of U.S. Aircraft Hijackings 1961–1976," *Journal of Law and Economics*, 21, No.1, pp.1-31.
- Leininger, W. 2009, "Evolutionarily Stable Preferences in Contests," *Public Choice*, 140, pp.341-356.
- Levy, H. and Markowitz, H.M. 1979, "Approximating Expected Utility by a Function of Mean and Variance," *American Economic Review*, 69, pp.308-317.
- Meyer, J. 1987, "Two-Moment Decision Models and Expected Utility Maximisation," *American Economic Review*, 77, No.3, pp.421-430.
- Pape, R.A. 2003, "The Strategic Logic of Suicide Terrorism," *American Political Science Review*, 97, No.3, pp.343-361.
- Phillips, P.J. 2005, "The 'Price' of Terrorism," *Defence and Peace Economics*, 16, No.6, December, pp.403-414.
- Phillips, P.J. 2009, "Applying Portfolio Theory to the Analysis of Terrorism," *Defence and Peace Economics*, 20, No. 3, June, pp.193-213.
- Phillips, P.J. 2011, "Terrorists' Equilibrium Choices When No Attack Method is Riskless," *Atlantic Economic Journal*, 39, pp.129-141.
- Pratt, J.W. 1964, "Risk Aversion in the Small and in the Large," *Econometrica*, 32 (1), pp.122-136.
- Sandler, T. and Arce M., D.G. 2003, "Terrorism and Game Theory," *Simulation and Gaming*, 34, No.3, pp.319-337.
- Sandler, T. and Arce, D.G. 2007, "Terrorism: A Game Theoretic Approach," *Handbook of Defense Economics*, 2, Edited by T. Sandler and K. Hartley, 2007, pp.775-813.
- Sandler, T. and Hartley, K. (Editors) 2007, *Handbook of Defense Economics*, 2, North-Holland, Amsterdam, The Netherlands.
- Sandler, T., Tschirhart, J.T. and Cauley, J. 1983, "A Theoretical Analysis of Transnational Terrorism," *American Political Science Review*, 77, No.1, pp.36-54.