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Application of Generalized Intervention Analysis Model

Xiaohua HU[1*], Zhongwei Zhang[2]

[1]School of mathematics and statistics, Hainan Normal University
[2]School of Agricultural, Computational and Environmental Sciences, Faculty of Health, Engineering and Science, University of Southern Queensland, Australia

*Corresponding author: Xiaohua HU
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Abstract **Original Research Article**

In this paper, the intervention analysis model has been generalized in the sense that its intervention function either being a jumping function used to describe the continuing intervention or a unit impulse function used to describe the transient intervention has been replaced with a unified function, $\delta \exp(-\lambda(t-T))$, which represents an intervention process, δ denotes the amplitude (or strength) of the impact of intervention events at intervention time, λ denotes the decay factor of continuing process. Applying the generalized model to predict the Gross Domestic Product (GDP) of Hainan province from 1989 to 2016, as a Government intervention was introduced in 2010, results in a 60% of improvement in terms of the Root Mean Square Error (RMSE), when compared with the original model.
Keywords: intervention event, jump function, unit impulsive function, generalized intervention analysis model, Hainan GDP.
JEL Classification: C53.

INTRODUCTION

The invention analysis model was initially proposed in 1975 by Statistics Prof. Box and Tiao of the University of Wisconsin, US, in a journal paper (Box [1]), which was primarily applied to quantitatively analyze and discuss the effects on economics, which caused by the sudden changes of economic policies or occurring events. Enforcement of economic polices and intervention of sudden events occurring, such as political events, natural disasters and economic policies all called interventions, must have some influences on economic developments, even ultimately restructure the economics (Feng [2]).

An intervention variable is the main variable in the intervention analysis model, there are two types of intervention variables: one is continuous variable and another is transient variable. The continuous variable is keeping on influence after the moment of its occurring. A continuous variable is described by a jumping function, as follows:

$$S_t^T = \begin{cases} 0, \text{before the intervention event} & (t \le T) \\ 1, \text{after the intervention event} & (t > T) \end{cases} \quad (1)$$

While the transient variable has some effect at the moment of its occurring, for which a unit impulsive function can be used to represent,

$$P_t^T = \begin{cases} 1, \text{before the intervention event} & (t = T) \\ 0, \text{after the intervention event} & (t \ne T) \end{cases} \quad (2)$$

Neither the continuous intervention variable in (1) nor the transient intervention variable in (2) has indicated it as an intervention process and its effect amplitude.

Here, we would introduce a time related intervention function as represented

$$H_t^T = \begin{cases} 0, \text{prior to intervention event} \quad (t < T) \\ \delta e^{-\lambda(t-T)}, \text{at time of and after intervention event } (t \geq T) \end{cases} \quad (3)$$

where λ and δ are constants, and called the amplitude (or strength) of the intervention event, the decay factor, respectively, at the T moment.

It is apparent that H_t^T in (3) is more reasonable in describing the intervention while a sudden event occurred. That is to say δ is the amplitude (or strength) of intervention at the time T ; λ is the speed of intervention decaying.

1, if λ in (3) is large, and $\delta = 1$, then the intervention decay fast, and intervention duration is short, then (3) should approach to (2).

2, if λ in (3) is small, and $\delta = 1$, the the intervention last long, (3) should be closer to (1).

In other words, S_t^T in (1) and P_t^T in (2) are special cases of H_t^T in (3). In fact, the intervention variables in (1) and (2) only describe two extreme cases, namely, long-term intervention of the same intensity and instantaneous intervention. In a general way, the intervention of unexpected events is often a dynamic process, there is strong or weak for the intervention, there is short and long for the process. Obviously, (1) and (2) are two extreme cases of (3), and idealized. Consequently, (3) has generalized (1) and (2), to be more practicable in real. The intervention analysis model of continuous variable is

$$X_t = \frac{\omega B^b}{1 - \theta_1 B - \theta_2 B - \ldots - \theta_r B^r} S_t^T \quad (4)$$

The intervention analysis model of transient variable is

$$X_t = \frac{\omega}{1 - \theta_1 B - \theta_2 B - \ldots - \theta_r B^r} P_t^T \quad (5)$$

The generalized intervention model is

$$Z_t = \frac{\omega B^b}{1 - \theta_1 B - \theta_2 B - \ldots - \theta_r B^r} H_t^T \quad (6)$$

where $\omega, b, r, \theta_i, i = 1, \ldots, r,$ are the parameters to be estimated, B is the lag operator.

As the youngest province and the biggest economic special zone in China after 1989, Hainan province has achieved tremendously progress, in terms of its GDP figure, since 1989 to 2009, with her special geographic location and pleasant climate conditions. On Jan 4, 2010, the State Council of China issued a Governmental document, which aims to turn Hainan island (or Hainan province) into a world tourist attraction. This policy, considered as a sudden intervention event, has significantly changed the economic development of Hainan province, in particular the GDP figures. Since then, Hainan has come into a completely new era of economic development. In this study, we are interested in the amplitude of intervention and focus on the continuing influences of the intervention.

There are many literatures in this field. For example, Gao[3], has studied how the increase of investment effect on the Hainan's GDP data; Kong[16] applied GM(1, 1) and the Principle Component Regression model to forecast Hainan's GDP trend; In addition, Zhang[5], attempted to forecast the GDP of Xinjiang Autonomy District, China, based on the intervention analysis model; Guo [6] has case studied the structure and the economic increases of Hainan industry. Finally, Yang [7] has applied the intervention analysis regression to predict the GDP data of whole China, etc. Common to all these studies is the use of the original intervention model, to our best knowledge, no studies has used the intervention analysis model on the Hainan's GDP, in particular, no studies at all which take advantage of the generalized intervention analysis model on Hainan's GDP.

MODELING AND EMPIRICAL ANALYSIS

Here GDP is denoted by GDP_t, and t is another time variable, representing the calender year, $t = 1, 2, \ldots, 28$, which represents the year 1989, 1990, ..., 2016. GDP_t time plot is shown in Figure 1, sample data from Hainan Bureau of Statistics (www.stats.hainan.gov.cn) is shown in Table 1. Due to the sudden event occurred at year 2010, it's noticeable there was an accelerated increasing rates in GDP_t data, which was clearly higher than the previous increasing rates since that year.

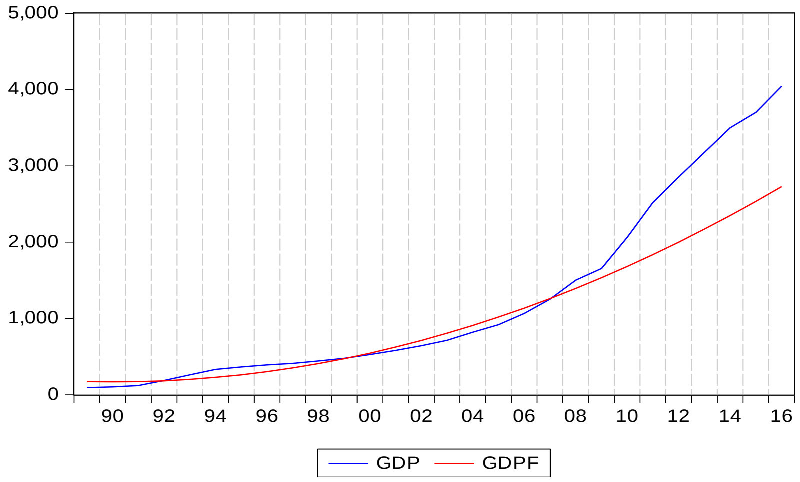

Fig-1: GDP_t and its trend extrapolation plot

Table-1: Hainan Province GDP data from 1989 to 2016 (in 100 million Chinese yuan)

year	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998
GDP	91.32	102.42	120.52	184.92	260.41	331.98	363.25	389.68	411.16	442.13
year	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
GDP	476.67	526.68	579.17	642.73	713.96	819.66	918.75	1065.67	1254.17	1503.06
year	2009	2010	2011	2012	2013	2014	2015	2016		
GDP	1654.21	2064.5	2522.66	2855.54	3177.56	3500.72	3702.76	4044.51		

Effect sequence and Regression model of Intervention

The GDP_t data of Hainan province prior to 2009 can be regressed by a quadratic function, $t = 1, \ldots, 21$

$$GDP_t = 183.53 - 15.55t + 3.8t^2 \qquad (7)$$

where the fitting goodness: $R^2 = 0.97$. Use (7) to extrapolate the GDP_t data after year 2010, $t = 22, \ldots, 28$, see Figure 1. these data can be regarded as the actual GDP_t without the intervention.

Let Z_t be the effect sequences that are the differences between the extrapolated GDP_t data and the actual GDP_t data after year 2010. Table 2 is the effect sequence of intervention after year 2010.

Table-2: Effect sequences of Intervention after 2010

t	22	23	24	25	26	27	28
Z_t	382.6736	685.2776	854.997	1006.252	1151.042	1167.108	1315.28

Here by the use of the original model (4) and EViews 8.0, on Z_t in Table 2, we can get the estimate parameters as:

$$Z_t = 0.744 Z_{t-1} + 379.214 S_t^T$$

$$Z_t = \frac{\hat{\omega}}{1-\hat{\theta}B} S_t^T = \frac{379.214}{1-0.744B} S_t^T \qquad (8)$$

When $T = 20$,(or 2010 year)

$$\hat{\omega} = 379.214, \hat{\theta} = 0.744, R^2 = 0.956, RSE = 11513.87$$

Again, by the use of the generalized intervention model (6), these parameters become

$$Z_t = 0.425 Z_{t-1} + 79.513 e^{-(-0.082)t}$$

$$Z_t = \frac{1}{1-\hat{\theta}B} H_t^T = \frac{79.513}{1-0.425B} e^{0.082t} \qquad (9)$$

When $T = 20$

$$\hat{\lambda} = -0.082, \hat{\delta} = 79.513, \hat{\theta} = 0.425, \ R^2 = 0.975, RSE = 6358.149$$

Obviously, the RSE (6358.149) of (9) is smaller than that (11513.87) in (8), the R^2 (0.975) of (9) is bigger than that (0.956) in (8).

When $T = 2000$ year, $\hat{\delta} = 79.513$, and $\hat{\lambda} = -0.082$, it indicates that (3) is better than (1), when it comes to describe the intervention.

Purified Sequence and Fitting Model

We calculate the purified sequence. The purified sequence is a sequence that it is eliminated the effects of intervention, or it is the difference between the actual GDP_t and the intervened effect value Z_t .

By the use of (8), we get

$$y_t = GDP_t - \frac{\hat{\omega}}{1-\hat{\theta}B} S_t^T = GDP_t - \frac{379.214}{1-0.744B} S_t^T, \quad T = 22, \quad t = 1,2,...,28 \qquad (10)$$

Similarly, by the use of (9), it becomes

$$x_t = GDP_t - \frac{1}{1-\hat{\theta}B} H_t^T = GDP_t - \frac{79.513}{1-0.425B} e^{0.082t}, \quad T = 22, t = 1,2,...,28 \qquad (11)$$

Prior to Jan, 2010, there was no intervention, the first 21 GDP_t data (1654.21) in Table 1 should be the net data. The GDP_t figure of year 2010, however, was recorded at the end of year, so the GDP_t data at that year was effected by the intervention. The purified GDP_t figure should be simplified as the average value of the GDP_t data of year 2009 and 2010. or 1859.3. The purified sequence after 2010 is shown in Table 3.

By the use of (4), (8) and (10), the fitting model, y_t , is

$$\hat{y}_t = 173.557 - 13.357t + 3.738t^2 \qquad (12)$$

where $R^2 = 0.9915, RSE = 151404.3$, By the use of (6), (9) and (11), the fitting model, x_t , is

$$\hat{x}_t = 176.744 - 14.303t + 3.784t^2 \qquad (13)$$

where $R^2 = 0.9917, RSE = 148888.7$, Compare R^2 and RSE from (12) with that from (13). Obviously (13) is better than (12).

Table-3: The purified sequences after 2010(based on (1) and (3))

t	22	23	24	25	26	27	28
y_t	1859.3	1858.6	1982.2	2148.5	2355.7	2471.5	2749.1
x_t	1859.3	1829.8	1985.1	2182.1	2398.2	2496.4	2730.6

Intervention Analysis Modeling

By use of (4), (8), (10) and (12), we can obtain an intervention analysis model, $GDP1_t$,is

$$GDP1_t = \hat{y}_t + \frac{\hat{\omega}}{1-\hat{\theta}B} S_t^T$$

$$GDP1_t = 173.557 - 13.357t + 3.738t^2 + \frac{379.214}{1-0.744B} S_t^T \qquad (14)$$

Where

$$S_t^T = \begin{cases} 0, \text{prior to } 2010\ (t < 22) \\ 1, \text{after and at year } 2010\ (t \geq 22) \end{cases} \qquad (15)$$

After the use of (14) and (15), the predicted GDP_t are shown in Table 4.

The Root Mean Squared Error ($RMSE$) between the Actual GDP_t and the predicated data $GDP1_t$ is 96.96, denoted by $RMSE_1$. GDP_t And $GDP1_t$ time plot as Figure 2.

Table-4: Predicated Data by use of (14) and (15)

year	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998
$GDP1_t$	163.9	161.7	167.1	179.9	200.2	228.0	263.2	305.9	356.1	413.8
year	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
$GDP1_t$	479.0	551.6	631.7	719.3	814.4	916.9	1026.9	1144.5	1269.4	1401.9
year	2009	2010	2011	2012	2013	2014	2015	2016		
$GDP1_t$	1541.8	1689.2	2508.1	2879.8	3205.4	3498.61	3769.7	4026.1		

On the other hand, if we use (6), (9), (11) and (13), then the intervention analysis model, $GDP2_t$,is

$$GDP2_t = \hat{x}_t + \frac{1}{1-\hat{\theta}B} H_t^T$$

$$GDP2_t = 176.744 - 14.303t + 3.78t^2 + \frac{1}{1-0.425B} H_t^T \qquad (16)$$

Where

$$H_t^T = \begin{cases} 0, \quad \text{prior to } 2010\ (\text{ie.t} < 22) \\ 79.513e^{-(-0.082)t}, \quad \text{after and at year } 2010\ (\text{ie.t } \geq 22) \end{cases} \qquad (17)$$

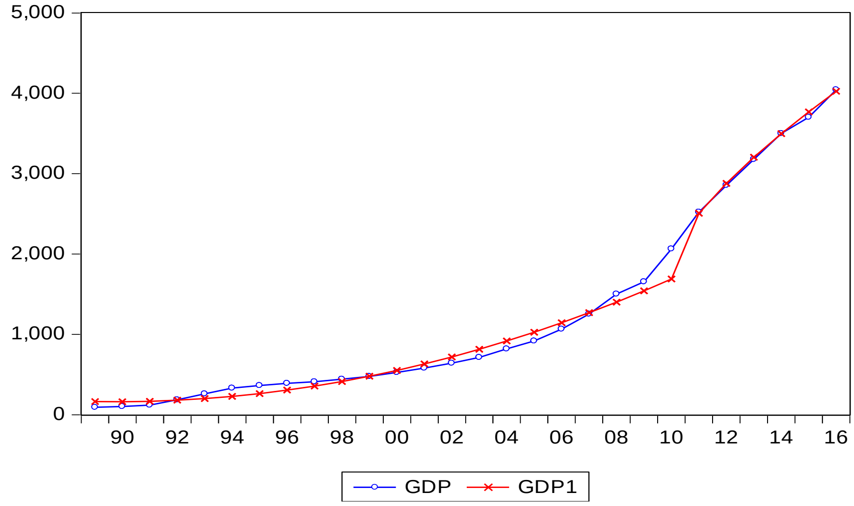

Fig-2: GDP_t **and** $GDP1_t$ **time plot**

After the use of (16) and (17), the predicted GDP_t are shown in Table 5.

Table-5: Predicated Data by use of (16) and (17)

year	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998
$GDP2_t$	166.2	163.2	167.8	180.0	199.8	227.1	262.0	304.5	354.5	412.1
year	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
$GDP2_t$	477.3	550.0	630.3	718.2	813.6	916.6	1027.2	1145.4	1271.1	1404.4
year	2009	2010	2011	2012	2013	2014	2015	2016		
$GDP2_t$	1545.2	1693.6	2542.5	2883.6	3179.83	3465.5	3755.6	4057.0		

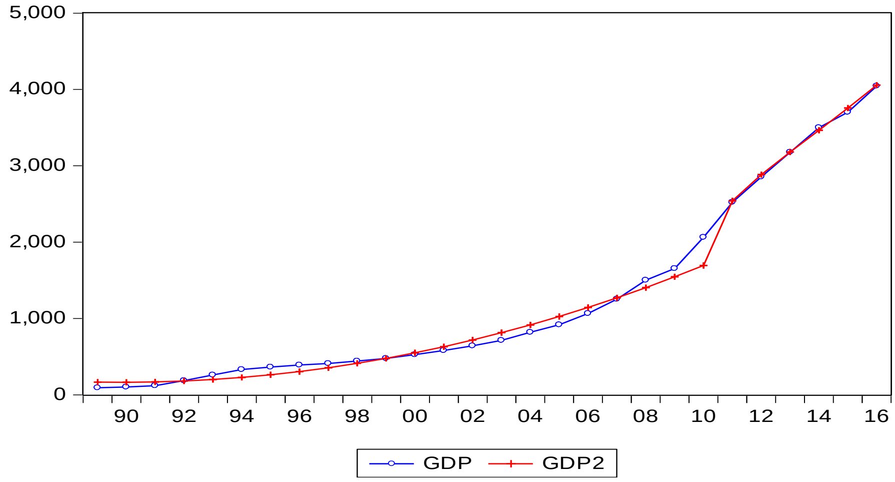

Fig-3: GDP_t **and** $GDP2_t$ **time plot**

The $RMSE$ between the Actual GDP_t and the predicated data $GDP2_t$ is 38.77, denoted by $RMSE_2$. GDP_t And $GDP2_t$ time plot as Figure 3.

It is clearly that (16) and (17) is better than (14) and (15), in terms of the $RMSE$ values, and the Improvement Percentage is as follows:

$$\text{Improvement Percentage} = \frac{RMSE_1 - RMSE_2}{RMSE_1} = \frac{96.96 - 38.77}{96.96} = 60\%$$

CONCLUSION

In this paper, we have generalized the intervention analysis model from the theoretical aspect. The generalized model has been applied to fit the Hainan province GDP data from 1989 to 2016, compared with that of using the original model. We have established two models represented in (14),(15) and (16),(17). From the results (16) and (17) of generalized intervention analysis model (3) and (6), we can clear see there was a significant effect on Hainan GDP due to the introduction of Government intervention policy in 2010. Moreover, the amplitude of the intervention was $\delta = 79.513$, and the decay factor is $\lambda = -0.082$. Improvement Percentage is 60%.

The generalized intervention analysis model has shown the superiority over the original model in terms of the $RMSE$ values.

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