Ultimate strength of continuous composite beams in combined

bending and shear

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**Abstract** 

The contributions of the concrete slab and composite action to the vertical shear strength of

continuous steel-concrete composite beams are ignored in current design codes, which result

in conservative designs. This paper investigates the ultimate strength of continuous composite

beams in combined bending and shear by using the finite element analysis method. A three-

dimensional finite element model has been developed to account for the geometric and

material nonlinear behaviour of continuous composite beams. The finite element model is

verified by experimental results and then used to study the effects of the concrete slab and

shear connection on the vertical shear strength. The moment-shear interaction strength of

continuous composite beams is also investigated by varying the moment/shear ratio. It is

shown that the concrete slab and composite action significantly increase the ultimate strength

of continuous composite beams. Based on numerical results, design models are proposed for

the vertical shear strength and moment-shear interaction of continuous composite beams. The

proposed design models, which incorporates the effects of the concrete slab, composite action,

stud pullout failure and web shear buckling, are compared with experimental results with

good agreement.

*Keywords*: Bending; Composite beams; Finite element analysis; Shear; Ultimate strength.

## Nomenclature

$A_{ m ec}$	effective shear area of concrete
$b_{ m f}$	width of the top flange of steel beam
$d_{ m w}$	depth of the web of steel beam
$D_{\rm c}$	total depth of the concrete slab
$d_{\rm s}$	diameter of the head of headed stud shear connector
$E_{\mathrm{c}}$	Young's modulus of concrete
$E_{ m s}$	Young's modulus of steel
$f_{ m c}^{'}$	cylinder compressive strength of concrete
$f_{ m ct}$	concrete tensile strength
$f_{ m sy}$	yield strength of steel
$f_{ m su}$	ultimate strength of steel
G	shear modulus of cracked concrete
$G_{\mathrm{c}}$	elastic shear modulus of uncracked concrete
$h_{\rm c}$	height of shear connector
$h_{ m r}$	rib height of profiled steel sheeting
$M_{\mathrm{u}}$	ultimate moment capacity of composite beam in combined bending and shear
$M_{ m uo}$	ultimate moment capacity of composite section in pure bending
$T_{ m p}$	pullout capacity of stud shear connectors
$t_{ m w}$	thickness of steel web
$V_{\rm c}$	shear contribution of the concrete slab
$V_{\circ}$	ultimate shear strength of non-composite section in pure shear

- $V_{\rm s}$  ultimate shear strength of the steel web
- $V_{\rm slab}$  shear capacity of the concrete slab
- $V_{\rm u}$  Ultimate shear strength of composite beam in combined bending and shear
- $V_{100}$  ultimate shear strength of composite section in pure shear
- $\alpha$  moment/shear ratio,  $\alpha = M/V$
- $\alpha_{\rm m}$  exponent in the strength interaction formulas
- $\alpha_{y}$  exponent in the strength interaction formulas
- $\alpha_{\rm w}$  reduction factor for slender web
- $\beta$  degree of shear connection
- $\beta_1$  shear strength factor for concrete
- $\beta_2$  shear strength factor considering composite action
- parameter used to define stress-strain curve for concrete
- $\varepsilon_{\rm c}$  strain in concrete
- $\varepsilon_{\rm c}^{'}$  strain in concrete corresponding to  $f_{\rm c}^{'}$
- $\varepsilon_{\rm max}$  maximum direct strain
- $\varepsilon_{\rm s}$  strain in steel
- $\varepsilon_{\mathrm{su}}$  ultimate strain in steel
- $\varepsilon_{\rm sy}$  yield strain in steel
- *v* Poisson's ratio
- $\sigma_{\rm c}$  compressive stress in concrete
- $\sigma_{\rm s}$  stress in steel
- $\varphi$  reduction factor

#### 1. Introduction

Continuous steel-concrete composite beams have been widely used in multistory buildings and bridges. In a composite beam, shear connectors are welded to the top flange of the steel beam to achieve composite action between the concrete slab and the steel beam. The composite action significantly improves the strength and stiffness performance of composite beams, which depends on the degree of shear connection. Continuous composite beams under applied loads are often subjected to combined actions of bending and vertical shear. In current design codes such as AS 2327.1 [1], Eurocode 4 [2] and LRFD [3], the ultimate moment capacity of a continuous composite beam comprises the contributions from the concrete slab and composite action. However, design codes completely ignore these contributions to the vertical shear strength of a continuous composite beam, assuming that the web of the steel beam resists the entire vertical shear. This assumption obviously leads to conservative designs, as pointed out by Johnson and Anderson [4].

Experiments indicated that the concrete slab and composite action significantly increase both the flexural and shear strengths of continuous composite beams. Johnson and Willmington [5] reported that the longitudinal steel reinforcement in the concrete slab increases the vertical shear strength and stiffness of continuous composite beams. The ultimate strength and failure modes of continuous composite beams have been studied experimentally by Hamada and Longworth [6]. They concluded that if other failure modes are prevented by appropriate provisions, continuous composite beams would mainly fail by the crushing of concrete in the sagging moment regions and local buckling of the bottom steel flange in the hogging moment regions. Tests undertaken by Ansourian [7] demonstrated that the concrete slab of a continuous composite beam in combined bending and shear could carry about 20% of the total vertical shear force.

The concrete slab also makes a significant contribution to the vertical shear strength of simply supported composite beams with and without web openings. Porter and Cherif [8] have undertaken tests on simply supported composite plate girders under shear loads. They proposed an equation for the vertical shear strength, which includes contributions from both the concrete slab and the steel plate girder. Tests of short-span composite plate girders [9,10,11] showed that the vertical shear strength of a composite plate girder designed with adequate shear connectors is significantly higher than that of a steel plate girder alone. Design equations were proposed for the vertical shear strength of composite plate girders, which comprises a contribution from the concrete slab [9,10]. However, the effect of composite action was not considered in these equations for the vertical shear strength. The moment-shear interaction behaviour of composite beams with web openings has been investigated by Clawson and Darwin [12]. Darwin and Donahey [13] proposed an interaction equation for the strength design of composite beams with web openings.

Many researchers have undertaken the nonlinear analyses of continuous composite beams. Yam and Chapman [14] presented a numerical method for the inelastic analysis of continuous composite beams. A three-dimensional bar element has been formulated by Razaqpur and Nofal [15] for modeling shear connectors in composite beams. The stiffness properties of the bar element were defined by a shear-slip relationship derived from experimental data. Salari et al. [16] presented a composite beam element for the nonlinear analysis of composite beams. A distributed spring model was used to simulate shear connectors. A finite element program has been developed by Sebastian and McConnel [17] for the nonlinear analysis of continuous composite beams. Moreover, Fabbrocino et al. [18] proposed a specific kinematic model for the analysis of continuous composite beams. Liang et al. [19] has undertaken nonlinear finite element analyses on simply supported composite beams in combined bending and shear. They

proposed design models, which incorporate contributions from the concrete slab and composite action, for the vertical shear strength and strength interaction of simply supported composite beams.

In this paper, the ultimate flexural and shear strengths of continuous composite beams in combined bending and shear are investigated by using the finite element method. A three-dimensional finite element model is presented and verified by corresponding experimental results. The verified finite element model is then employed to investigate the contributions of the concrete slab and shear connection to the vertical shear strength and to study the moment-shear interaction strengths of continuous composite beams. Based on results obtained from the nonlinear finite element analysis, design models for the vertical shear strength and moment-shear interactions are proposed for the design of continuous composite beams. The accuracy of the proposed design models is established by comparisons with existing experimental results.

#### 2. Finite element analysis

#### 2.1 General

The present study utilized the finite element program ABAQUS version 6.3 [20] to quantify the interaction strengths of continuous composite beams in combined bending and shear. A three-dimensional finite element model has been developed to simulate the geometric and material nonlinear behaviour of continuous composite beams. The four-node doubly curved general shell elements with reduced integration were used to model the concrete slab, the flanges and the web of the steel beam. Discrete stud shear connectors were modeled by using

3D beam elements. A typical finite element mesh used for the analysis of continuous composite beams is shown in Fig. 1. The present study employed the von Mises yield criterion in the nonlinear analysis to treat material plasticity with five integration points through the element thickness.

#### 2.2 Concrete

### 2.2.1 Concrete in compression

The elastic-plastic material behaviour of compressive concrete with strain softening was modeled by the equation suggested by Carreira and Chu [21]. The equation is expressed by

$$\sigma_{c} = \frac{f_{c} \gamma \left(\varepsilon_{c} / \varepsilon_{c}^{'}\right)}{\gamma - 1 + \left(\varepsilon_{c} / \varepsilon_{c}^{'}\right)^{\gamma}}$$
(1)

where  $\sigma_{\rm c}$  is the compressive stress in concrete,  $\varepsilon_{\rm c}$  is the strain in concrete,  $f_{\rm c}^{'}$  is the cylinder compressive strength of concrete,  $\varepsilon_{\rm c}^{'}$  is the strain corresponding to  $f_{\rm c}^{'}$ , and  $\gamma$  is given by

$$\gamma = \left| \frac{f_{\rm c}'}{32.4} \right|^3 + 1.55 \tag{2}$$

where  $f_c$  is in N/mm<sup>2</sup>. The strain  $\varepsilon_c$  is taken as 0.002. Fig. 2 shows the stress-strain curve for concrete with a compressive strength of 35 N/mm<sup>2</sup>. In the nonlinear finite element modeling, it was assumed the stress-strain behaviour of concrete in compression is linear elastic up to 0.4  $f_c$ . Beyond this point, it was in the plastic regime. The failure ratio option was used to define the failure surface of the concrete. The ratio of the ultimate biaxial compressive stress

to the ultimate uniaxial compressive stress was taken as 1.16. The ratio of the uniaxial tensile stress to the uniaxial compressive stress at failure was taken as 0.0836.

#### 2.2.2 Concrete in tension

The stress-strain relationship for concrete in tension assumes that the tensile stress increases linearly with an increase in tensile strain up to concrete cracking. After concrete cracking, the tensile stress decreases linearly to zero as the concrete softens. The bond between the concrete and reinforcing bars was simulated approximately by the tension stiffening model, which defines the stress-strain relationship for concrete in tension after cracking. For heavily reinforced concrete slabs, the total strain at which the tensile stress is zero is usually taken as ten times the strain at failure in the tension stiffening model. However, it has been found that this value is not adequate for concrete slabs in composite beams. In the present study, a total strain of 0.1 was used for reinforced concrete slabs in composite beams, as suggested by Liang et al. [19].

#### 2.2.3. Shear retention

The shear retention model assumes that the shear stiffness of open cracks reduces linearly to zero as the crack opening increases. In the shear retention model, the reduction in shear modulus due to concrete cracking was expressed as a function of direct strain across the crack. The shear modulus of cracked concrete is defined as  $G = \varphi G_c$ , where  $G_c$  is the elastic shear modulus of uncracked concrete and  $\varphi$  is a reduction factor, which is given by

$$\varphi = (1 - \varepsilon_{c} / \varepsilon_{max}) \qquad \text{for } \varepsilon_{c} < \varepsilon_{max}$$
 (3)

$$\varphi = 0$$
for  $\varepsilon_{\rm c} \ge \varepsilon_{\rm max}$  (4)

where  $\varepsilon_{\rm c}$  is the direct strain across the crack. Parameters  $\varepsilon_{\rm max} = 0.005$  and  $\varphi = 0.95$  were used in the present study to define the shear retention of concrete [19].

#### 2.3 Shear connection

The shear connection between the concrete slab and the steel beam by means of welded studs is discrete in nature [22]. To simulate the discrete behaviour of stud shear connectors, a three-dimensional beam element was used in the present study. It was assumed that shear connectors connected the middle plane of the concrete slab and the top flange of the steel beam. The cross-sectional area of the beam element was modified to make it equivalent in both strength and stiffness to the actual stud shear connector in a continuous composite beam. The material stress-strain behaviour of shear connectors was defined by a trilinear relationship as illustrated in Fig. 3. Pin jointed truss elements with an effective stiffness were employed in place of studs to transfer direct stress from the concrete slab to the top flange of the steel beam.

#### 2.4 Structural steel and reinforcement

Structural steel sections and reinforcing bars were modeled in the nonlinear analysis as an elastic-plastic material with strain hardening. A tri-linear stress-strain curve was used for steel sections and reinforcing bars in both compression and tension, as shown in Fig. 3. Steel reinforcing bars in a concrete slab were modeled as smeared layers with a constant thickness in the shell elements. The thickness of a reinforcement layer was calculated as the area of a

reinforcing bar divided by the spacing of the reinforcing bars. The Rebar Layer option was used to define the reinforcement in a concrete slab, in which the top and bottom longitudinal and transverse reinforcing bars were modeled by four layers. Each layer was defined by the cross-sectional area of the reinforcing bar, spacing, distance from the mid surface of the concrete slab, material property name, angle to the reference axis and the reference axis.

#### 2.5 Comparison with experimental results

The two-span continuous composite beam (CTB3) tested to failure by Ansourian [7] has been analysed by using the finite element model developed in this study and the results are calibrated against corresponding experimental data. The span of the continuous composite beam was 4.5 m and the loads were applied to the midspan of the beam. Fig. 4 shows the cross section details of the continuous composite beam. In half of the continuous composite beam,  $13 \times 45$  elements were used for the concrete slab,  $2 \times 45$  elements for the flange of the steel beam and  $2 \times 45$  elements for the web, as shown in Fig. 1. In the finite element model, stiffeners were welded to the web of the steel beam at the three support positions. Material properties used in the analysis of the continuous composite beam are given in Table 1. The load-deflection curve of the composite beam predicted by the finite element model is compared with corresponding experimental data in Fig. 5. It is seen from Fig. 5 that the finite element model predicts very well the initial stiffness and the ultimate strength of the continuous composite beam. The ultimate load obtained by the present study is 97.2% of the experimental value. The nonlinear finite element analysis confirmed the experimental observation that the continuous composite beam failed by crushing of the top concrete slab at midspan and local buckling of the bottom steel flange at the interior support. Therefore, it can be concluded that the finite element model developed is reliable and conservative in predicting the ultimate strength of continuous composite beams.

#### 3. Effect of shear connection

The vertical shear strength of a composite beam is a function of the degree of shear connection. To quantify the contributions of composite action and the concrete slab to the vertical shear strength, a two-span continuous composite beam with a span of 0.625 m and with various degrees of shear connection has been analysed. This continuous deep composite beam is a non-flexural member where the shear load is transferred to the supports by a strutand-tie model, as shown by Liang et al. [23, 24]. The continuous composite beam was shortened version of the one (CTB3) tested by Ansourian [7]. In Ansourian's beam, the degree of shear connection in sagging and hogging moment regions was 1.6 and 1.3, respectively. The full composite action with a degree of shear connection of 1.0 in the hogging moment region was considered in the present study. The degree of shear connection was determined by modifying the cross-sectional area of the stud shear connectors. Material properties given in Table 1 were used in the analysis of the continuous composite beam.

The vertical shear strengths of the continuous composite beam with various degrees of shear connection obtained from the nonlinear finite element analysis are shown in Fig. 6. The shear strength of the composite beam  $(V_{uo})$  was normalized to the shear strength of the noncomposite beam  $(V_o)$  with zero degree of shear connection. It is seen from Fig. 6 that the vertical shear strength generally increases with an increase in the degree of shear connection. However, due to the concrete cracking in the hogging moment region, the full shear connection does not necessarily yield the maximum vertical shear strength, as indicated in

Fig. 6. The maximum shear strength is achieved for the continuous composite beam with  $\beta = 0.8$ . Furthermore, it can be observed from Fig. 6 that the degree of shear connection has lesser effect on the ultimate shear strength of the continuous composite beam at the interior support than that at the exterior support.

#### 4. Moment-shear interaction

The ultimate strength of continuous composite beams under combined actions of bending and shear depends on the moment/shear ratio ( $\alpha$ ). The finite element model developed herein has been used to investigate the interaction strengths of continuous composite beams with various moment/shear ratios. The span of the continuous composite beam (CTB3) tested by Ansourian [7] was shortened to give different moment/shear ratios while other conditions of the composite beam were unchanged. The moment/shear ratios used in the analysis were 2.25, 1.768, 1.286, 0.964, 0.643, 0.482 and 0.322 m, which were determined by the applied sagging moment and vertical shear at the exterior support. Material properties given in Table 1 were used for all cases.

The load deflection curves of continuous composite beams with the moment/shear ratios of 2.25 and 1.768 are shown in Fig. 7. The moment/shear ratios of these two beams are so high that the ultimate strength of these beams was governed by the moment capacity of the composite section and the effect of the vertical shear could be neglected. It can be seen from Fig. 7 that the continuous composite beams possess a very ductile behaviour. The finite element model developed herein predicted very well the post ultimate load behaviour of continuous composite beams. Fig. 8 shows the load-deflection responses of three continuous composite beams with the moment/shear ratios of 1.286, 0.964 and 0.643. It is noted that the

moment/shear ratios of these three beams are intermediate. The interaction between flexural and shear strengths of these continuous composite beams is most critical. It can be observed from Fig. 8 that the ductility of these composite beams is reduced with a decrease in the moment/shear ratio.

Fig. 9 presents the load deflection curves of continuous composite beams with the moment/shear ratios of 0.482 and 0.322. Because of the low moment/shear ratios, the ultimate strength of these two continuous composite beams was governed by the vertical shear capacity of the composite section. The beams failed in vertical shear, which is a very brittle failure mode as indicated in Fig. 9. The effect of moment/shear ratios on the ultimate loads of continuous composite beams is illustrated in Fig. 10. It can be observed that the ultimate load of continuous composite beams in combined bending and shear decreases with an increase in the moment/shear ratio. This is justified by the fact that for the same composite section, increasing the span of a continuous composite beam will reduce the load carrying capacity of the composite beam.

The ultimate flexural and shear strengths of continuous composite beams with various moment/shear ratios can be calculated from the ultimate loads and reaction forces predicted by the nonlinear finite element analysis. Fig. 11 shows the sagging moment-shear interaction curve for continuous composite beams. The maximum vertical shear strength in the sagging moment region was used in Fig. 11. The hogging moment-shear interaction curve is presented in Fig. 12. It can be seen from Figs. 11 and 12 that the ultimate moment capacity of composite beams is almost not affected by the vertical shear when the moment/shear ratio is high. However, when the moment/shear ratio is low, the vertical shear significantly reduces the ultimate flexural strength of continuous composite beams. The reduction in the ultimate

moment capacity of composite beams due to high vertical shear is most critical in the hogging moment regions. It can be seen from Fig. 12 that when the vertical shear approaches to the ultimate shear capacity of the composite section, the flexural strength of the continuous composite beam at this critical section approaches to zero.

#### 5. Design models for vertical shear strength

Experiments and nonlinear finite element analyses have confirmed that the concrete slab and composite action contribute significantly to the vertical shear strength of continuous composite beams. These contributions are included in the proposed design models for the vertical shear strength of continuous composite beams to achieve economical designs. If no shear connection is provided between the concrete slab and the steel beam, the two components will work independently to resist the vertical shear. As a result, the superposition principle can be applied to the vertical shear strength of the non-composite section, which is expressed by

$$V_{o} = V_{c} + V_{s} \tag{5}$$

where  $V_{\rm c}$  is the contribution of the concrete slab, and  $V_{\rm s}$  is the shear capacity of the web of the steel beam. Tests indicated that the pullout failure of stud shear connectors in composite beams might occur [9]. This failure mode may reduce the shear resistance of the concrete slab. As a result, the contribution of the concrete slab  $V_{\rm c}$  should be taken as the lesser of the shear strength of the concrete slab  $V_{\rm slab}$  and the pullout capacity of stud shear connectors  $T_{\rm p}$ . The shear strength of the concrete slab is proposed as

$$V_{\text{slab}} = \beta_1 \left( f_{\text{c}}^{\, \cdot} \right)^{1/3} A_{\text{ec}} \tag{6}$$

where  $\beta_1$  is equal to 1.31,  $f_c$  is the compressive strength of the concrete (N/mm²), and  $A_{ec}$  is the effective shear area of the concrete slab. The effective shear area of a solid slab can be taken as  $A_{ec} = (b_f + D_c)D_c$ , where  $b_f$  is the width of the top flange of the steel beam and  $D_c$  is the total depth of the concrete slab. For a composite slab with profiled steel sheeting placed perpendicular to the steel beam,  $A_{ec}$  can be taken as  $(b_f + h_r + D_c)(D_c - h_r)$ , in which  $h_r$  is the rib height of the profiled steel sheeting. The factor  $\beta_1$  in Eq. (6) accounts for the effects of longitudinal steel reinforcement in tension in the hogging moment region and of the continuity of the continuous concrete slab at the top of the steel beam. For concrete slabs in simply supported composite beams, the factor  $\beta_1$  can be taken as 1.16, as suggested by Liang et al. [19].

The pullout capacity of stud shear connectors in composite beams with solid slabs can be expressed as

$$T_{\rm p} = [\pi(d_{\rm s} + h_{\rm c}) + 2s]h_{\rm c}f_{\rm ct} \quad \text{(pair studs)}$$
 (7)

$$T_{\rm p} = \pi (d_{\rm s} + h_{\rm c}) h_{\rm c} f_{\rm ct} \quad \text{(single stud)}$$
 (8)

where  $d_s$  is the head diameter of the stud,  $h_c$  is the total height of the stud, s is the transverse spacing of studs, and  $f_{ct}$  is tensile strength of concrete (N/mm<sup>2</sup>). The pullout capacity of stud shear connectors in composite slabs incorporating profiled steel sheeting should be determined using the effective pullout failure surface in Eqs. (7) and (8).

The shear capacity of the web of the steel beam can be determined by [25]

$$V_{\rm s} = 0.6\alpha_{\rm w} f_{\rm vw} d_{\rm w} t_{\rm w} \tag{9}$$

where  $f_{\rm yw}$  is the yield strength of the steel web (N/mm<sup>2</sup>),  $d_{\rm w}$  is the depth of the steel web,  $t_{\rm w}$  is the thickness of the steel web, and  $\alpha_{\rm w}$  is the reduction factor for slender webs in shear buckling. For stocky steel webs without shear buckling, the reduction factor  $\alpha_{\rm w}$  is taken as 1.0.

It is noted that Eq. (5) is for non-composite sections without incorporating the effect of composite actions. In order to take advantage of composite actions, a design model for the vertical shear strength of continuous composite beams with any degree of shear connection is proposed as

$$V_{uo} = V_o \left( 1 + \beta_2 \sqrt{\beta} \right) \qquad (0 \le \beta \le 1)$$
 (10)

where  $V_{\rm uo}$  is the ultimate shear strength of the composite beam in pure vertical shear,  $\beta_2$  is equal to 0.295 for sections at the exterior support and 0.092 for sections at the interior support of a composite beam, and  $\beta$  is the degree of shear connection. It should also be noted that pullout failure of stud shear connectors leads to the damage of composite action. If this occurs, the ultimate shear strength of the damaged composite beam ( $V_{\rm uo}$ ) should be taken as  $V_{\rm o}$  for safety.

#### 6. Design models for strength interaction

It can be seen from the moment-shear interaction curves shown in Figs. 11 and 12 that high vertical shear forces significantly reduce the ultimate sagging and hogging moment capacities of continuous composite beams. Interaction equations are used in AS 2327.1 [1] and Eurocode 4 [2] to account for the effect of vertical shear on the ultimate moment capacity of composite beams. However, current design codes allow only the shear strength of the steel web to be considered in the interaction equations. To determine the ultimate strength of continuous composite beams under combined actions of bending and vertical shear, design models for strength interactions are proposed here as

$$\left(\frac{M_{\rm u}}{M_{\rm uo}}\right)^{\alpha_{\rm m}} + \left(\frac{V_{\rm u}}{V_{\rm uo}}\right)^{\alpha_{\rm v}} = 1$$
(11)

where  $M_{\rm u}$  and  $V_{\rm u}$  are the ultimate flexural and shear strengths of the continuous composite beam in combined bending and shear respectively, and  $M_{\rm uo}$  is the ultimate moment capacity of the composite section in pure bending. The exponents  $\alpha_{\rm m}$  and  $\alpha_{\rm v}$  are equal to 5.0 for sagging moment regions. For hogging moment regions,  $\alpha_{\rm m}=0.6$  and  $\alpha_{\rm v}=6.0$  can be used in design. Figs. 13 and 14 show the comparison of the proposed design interaction equations with the results obtained from the nonlinear finite element analysis. It appears from these figures that the proposed design equations agree very well with the numerical results.

In design, the rigid plastic analysis method [26] can be used to determine the ultimate moment capacity ( $M_{\rm uo}$ ) of a composite section in accordance with design codes. Any point ( $M_{\rm u}$ ,  $V_{\rm u}$ ) on the moment-shear interaction curve corresponds to the applied moment/shear ratio that

defines the load path. This means that the ultimate moment/shear ratio  $(M_{\rm u}/V_{\rm u})$  is equal to  $\alpha$ . If the applied moment and shear at a cross section is known, the ultimate moment and shear capacities of a continuous composite beam in combined bending and shear can be determined by solving the only unknown in Eq. (11).

#### 7. Design model calibration

The proposed design models for the vertical shear strength and the strength interaction of continuous composite beams under bending and shear are calibrated with existing experimental results presented by Ansourian [7] in this section. In the theoretical prediction, the vertical shear strength of composite sections in a continuous composite beam was calculated by using Eqs. (5) to (10). The full shear connection was considered for all beams. The simple rigid plastic method using the yield stress of the steel section usually underestimates the ultimate moment capacity of a composite section, as indicated by the experiment [7]. To predict the real ultimate strength of composite beams, a factor of 1.04 was applied to  $M_{\rm uo}$  for considering the strain hardening of steel sections in the comparison. The ultimate moment capacity (  $M_{\rm uo}$  ) of composite sections given by Ansourian [7] based on the simple rigid plastic method was used. The theoretical predictions are compared with experimental data in Table 2 for continuous composite beams in the sagging moment regions. It can be seen from the table that the mean theoretical ultimate strength is 95.2% of that of the experimental results. It can be concluded that the proposed design formulas predict very well the ultimate strength of continuous composite beams under the combined actions of bending and shear.

#### 8. Conclusions

The effects of the concrete slab and composite action on the vertical shear strength and on the interaction strength of continuous composite beams under combined bending and shear have been investigated by using the finite element method in this paper. A three-dimensional finite element model has been developed for the geometric and material nonlinear analysis of continuous composite beams with any degrees of shear connection. The moment-shear interaction behaviour of continuous composite beams has been presented and discussed. Design models for the vertical shear strength and for the flexural-shear interaction strengths have been proposed for design of continuous composite beams in combined bending and shear.

Numerical results have demonstrated that the concrete slab makes a significant contribution to the vertical shear strength of continuous composite beams. In addition, the vertical shear strength of composite beams generally increases with an increase in the degree of shear connection. The proposed design models for the vertical shear strength take into account the effects of the concrete slab, the degree of shear connection, the pullout failure of stud shear connectors and the shear capacity of the steel web. The proposed design models for strength interaction allow for the flexural and shear strengths of continuous composite beams with any degree of shear connection under any combined actions of bending and shear to be determined. Design formulas proposed have been verified by experimental results and are suitable for inclusion in design codes for continuous composite beams.

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# **Figures and Tables**

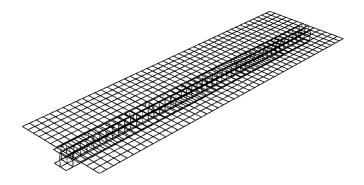


Fig. 1 Typical finite element mesh for composite beams

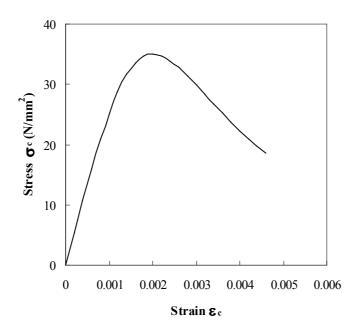


Fig. 2 Stress-strain curve for concrete in compression

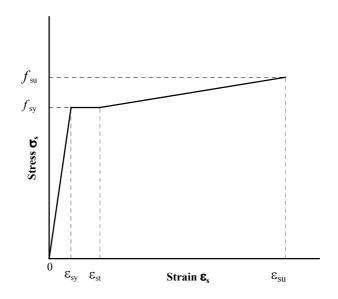


Fig. 3 Stress-strain curve for steel with strain hardening

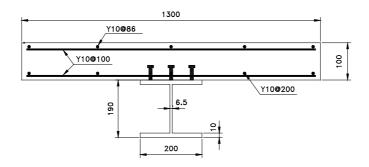


Fig. 4 Cross-section of continuous composite beam

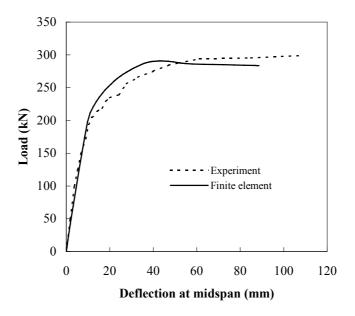


Fig. 5 Comparison of finite element modeling with experimental results

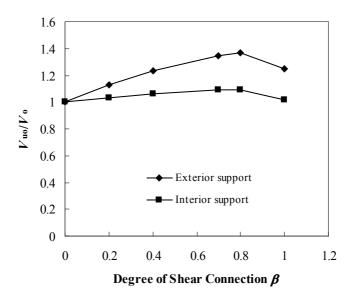


Fig. 6 Effect of the degree of shear connection on the vertical shear strength

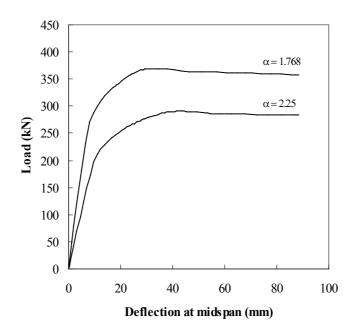


Fig. 7 Load-deflection curves of continuous composite beams

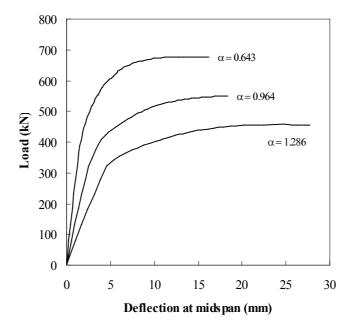


Fig. 8 Load-deflection curves of continuous composite beams

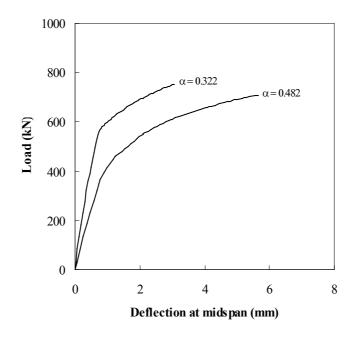


Fig. 9 Load-deflection curves of continuous composite beams

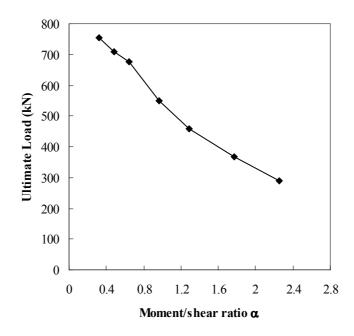


Fig. 10 Effect of moment/shear ratios on the ultimate load of continuous composite beams

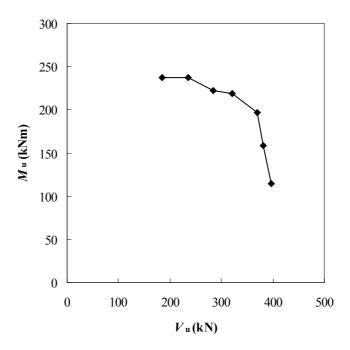


Fig. 11 Sagging moment-shear interaction of continuous composite beams

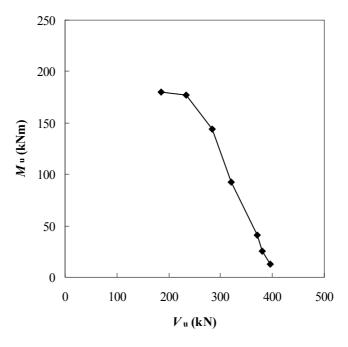


Fig. 12 Hogging moment-shear interaction of continuous composite beams

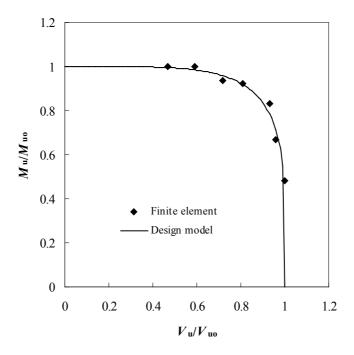


Fig. 13 Design model for sagging moment-shear interaction of continuous composite beams

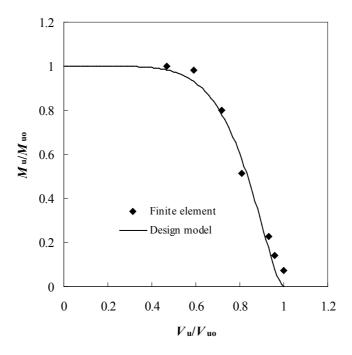


Fig. 14 Design model for hogging moment-shear interaction of continuous composite beams

Table 1. Material properties used in the analysis of continuous composite beams

Material	Property	Value	
Structural steel	Yield stress, $f_{sy}$ ( N/mm <sup>2</sup> )		
	Flange	220	
	Web	235	
	Ultimate strength, $f_{su}$ ( N/mm <sup>2</sup> )	200	
	Flange	390	
	Web	411	
	Young's modulus, $E_s$ (N/mm <sup>2</sup> )	200000	
	Poisson's ratio, $\nu$	0.3	
	Ultimate strain, $\varepsilon_{\rm su}$	0.25	
Reinforcing bar	Yield stress, $f_{sy}$ ( N/mm <sup>2</sup> )	430	
	Ultimate strength, $f_{su}$ ( N/mm <sup>2</sup> )	533	
	Young's modulus, $E_s$ ( N/mm <sup>2</sup> )	200000	
	Poisson's ratio, $\nu$	0.3	
	Ultimate strain, $\varepsilon_{\rm su}$	0.25	
Concrete	Compressive strength, $f_{c}^{'}$ ( N/mm <sup>2</sup> )	35	
	Tensile strength, $f_{ct}$ ( N/mm <sup>2</sup> )	2.926	
	Young's modulus, $E_c$ ( N/mm <sup>2</sup> )	29876	
	Poisson's ratio, $\nu$	0.15	
	Ultimate compressive strain, $\varepsilon_{cu}$	0.0045	
Stud shear connector	Number of studs	84	
	Number of rows	3	
	Yield stress, $f_{sy}$ (N/mm <sup>2</sup> )	435	
	Ultimate strength, $f_{su}$ ( N/mm <sup>2</sup> )	565	
	Young's modulus, $E_s$ ( N/mm <sup>2</sup> )	200000	
	Poisson's ratio, <i>v</i>	0.3	
	Ultimate strain, $\varepsilon_{\rm su}$	0.25	

Table 2. Comparison of theoretical predictions with experimental results

Composite Beams	M <sub>u.test</sub> (kNm)	V <sub>u.test</sub> (kN)	$M_{\rm u.theory}$ (kNm)	V <sub>u.theory</sub> (kN)	$\frac{M_{ m u.theory}}{M_{ m u.test}}$
CTB1	166	122	157.8	116	0.95
CTB2	184	155	169.8	143	0.923
CTB3	250	200	225	180	0.9
CTB4	217	170	217	170	1.0
CTB5	232	190	223.5	183	0.963
CTB6	254	208	247.3	202.5	0.974
Mean					0.952