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IMPROVING STATISTICAL INFERENCE WITH UNCERTAIN NON-SAMPLE PRIOR INFORMATION

ShahjahanKhan[1], Muhammed Ashraf Memon[2]
Budi Pratikno[3] and **Rossita M Yunus[4]**
Email address of corresponding author: Shahjahan.Khan@usq.edu.au

[1]School of Agric., Comput. and Environmental Sciences
International Centre for Applied Climate Sciences, Centre for Health Sciences Research, University of Southern Queensland, Toowoomba, Australia
[2]Sunnybank Obesity Centre &SEQS, Suite 9, 259 McCullough Street Sunnybank, Queensland, AUSTRALIA Mayne Medical School, University of Queensland, Australia Faculty of Health Sciences & Medicine, Bond University, Australia
[3]Department of Mathematics and Natural Science Jenderal Soedirman University, Purwokerto, Indonesia
[4]Institute of Mathematical Sciences, University of Malaya Kuala Lumpur, Malaysia

ABSTRACT

In the classical inference, the observed sample data is the only source of information. The Bayesian inferential methods assume prior distribution of the underlying model parameters to combine with sample data. Often non-sample prior information (NSPI) on the value of the model parameters is available from previous studies or expert knowledge which could be used along with the sample data to improve the quality of statistical inference. Obviously the NSPI is not always correct and hence there is uncertainty in the suspected value of the parameter. Any such uncertainty can be removed by conducting an appropriate statistical test, and the quality of statistical inference can be improved by including the outcome of the test in the inferential procedure. This paper provides the underlying methodology to illustrate the process and include an example to demonstrate its application.

KEYWORDS AND PHRASES: Regression model; uncertain non-sample prior information; restricted, preliminary test and shrinkage estimators; bias, relative efficiency, M and score tests, testing after pre-test, power of tests, correlated non-central bivariate chi-square distribution.

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1 INTRODUCTION

Statistical inference uses both sample and non-sample information. Classical inference uses only the sample data for estimation and test of hypotheses. Bayesian methods uses sample data and prior distribution of the model parameters. The notion of inclusion of non-sample prior information (NSPI) on the value of model parameters has been introduced to `improve' the quality of statistical inference. The natural expectation is that the inclusion of additional information would result in a better estimator and test with relevant statistical properties. In some cases this may be true, but in many other cases the risk of worse consequences can not be ruled out.

A number of estimators have been introduced in the literature that uses NSPI and, under particular situation, over performs the traditional exclusive sample information based unbiased estimators when judged by criteria such as the mean square error and squared error loss function.

In many studies the researchers estimate the slope parameter of the regression model. However, the estimation of the intercept parameter is more difficult than that of the slope parameter. This is because the estimator of the slope parameter is required in the estimation of the intercept parameter. Khan et al. (2002) studied the improved estimation of the slope parameter for the linear regression model. They introduced the coefficient of distrust on the belief of the null hypothesis, and incorporated this coefficient in the definition and analysis of the estimators.

In recent time (eg Khan and Pratikno, 2013; Yunus and Khan, 2008, 2010, 2011a,b) several studies used NSPI on the slope of a regression model to test the intercept parameter. Yunus (2010) applied the NSPI in the testing regime using M-test along the line of Humber's M-estimation. Pratikno (2012) studied the parametric test for the intercept parameter using NSPI information on the slope of different regression models.

In general the NSPI on the slope is uncertain and may fall into one of the following three categories: (i) unspecified, no information available, (ii) specified, correct value known, and (iii) specified with uncertainty.

This paper provides alternative estimators and tests of the intercept parameter when NSPI on the slope of the simple linear regression model is available.This include the unrestricted (UE), restricted (RE), preliminary test (PTE) estimators as well as the unrestricted (UT), restricted (RT) and pre-test (PTT) tests of the intercept parameter. Statistical properties of these estimators and tests are investigated both analytically and graphically. Motivation for a real life application of test for the intercept is found in Kent (2009).

Studies in the area of the estimation include Bancroft (1944), Han and Bancroft (1968), Sclove et al. (1972), Saleh and Sen (1978, 1985), Judge and Bock (1978), Stein (1981), Khan (1998, 2003, 2008), Chiouand Saleh (2002), Saleh (2006), Khan and Saleh (1997, 2001, 2005), Salel (2006), Khan et al. (2002, 2005), Hoque et al. (2009). The testing problem has been investigated by Tamura (1965), Saleh and Sen (1978, 1985), Yunus (2010), Yunus and Khan (2008, 2011a,b), Pratikno (2012) and Khan and Pratikno (2013).

The next section introduces the model and definition of the unrestricted estimators of θ and σ^2. The three alternative estimators are defined in Section3 along with their properties. The three tests and their power analyses are provided in Section 4. Some concluding remarks are given in section 5.

2 THE MODEL AND SOME PRELIMINARIES

The *n* independently and identically distributed responses from a linear regression model can be expressed by the equation

$$y = \theta 1_n + \beta x + e, \tag{2.1}$$

where *y* and *x* are the column vectors of response and explanatory variables respectively, $1_n = (1, \ldots, 1)'$ - a vector of *n*-tuple of 1's, θ and β are the unknown intercept and slope parameters respectively and $e = (e_1, \ldots, e_n)'$ is a vector of errors with independent components which is distributed as $N_n(0, \sigma^2 I_n)$. So that $E(e) = 0$ and $E(ee') = \sigma^2 I_n$ where σ^2 is the variance of each of the error component in e and I_n is the identity matrix of order *n*.

Assume that uncertain NSPI on the value of β is available, either from previous study or from practical experience of the researchers or experts. Let the NSPI be expressed in the form of $H_0 : \beta = 0$ which may be true, but not sure. We wish to incorporate both the sample information and the uncertain NSPI in estimating and testing the intercept θ. Following Khan et al (2002) we assign a coefficient of distrust, $0 \le d \le 1$, for the NSPI, that represents the degree of distrust in the null hypothesis.

The *unrestricted* mle of the slope β and intercept θ are given by

$$\tilde{\beta} = (x'x)^{-1} x'y \quad \text{and} \quad \tilde{\theta} = \bar{y} - \tilde{\beta}\bar{x}, \tag{2.2}$$

where $\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j$ and $\bar{y} = \frac{1}{n}\sum_{j=1}^{n} y_j$. The mle of σ^2 is $S_n^{*2} = \frac{1}{n}(y - \hat{y})'(y - \hat{y})$, where $\hat{y} = \tilde{\theta} 1_n + \tilde{\beta} x$. This estimator is biased for σ^2. However, $S_n^2 = \frac{1}{n-2}(y - \hat{y})'(y - \hat{y})$ is unbiased for σ^2. To remove the uncertainty from the NSPI, we perform an appropriate statistical test on $H_0 : \beta = \beta_0$ against $H_a : \beta \ne \beta_0$. Here the appropriate test is given by $L_\nu = S_n^{-1} S_{xx}^{\frac{1}{2}} (\tilde{\beta} - \beta_0)$. Under the H_a, L_ν, follows a non-central Student-t distribution with $\nu = (n-2)$ df and non-centrality parameter $\Delta^2 = \sigma^{-2} S_{xx} (\beta - \beta_0)^2$.

3 ALTERNATIVE ESTIMATORS OF INTERCEPT

In this section we define the alternative estimators of the intercept and investigate its properties.

3.1 The Estimators

The UE, RE and PTE of θ are given by

$$\tilde{\theta}^{\text{UE}} = \bar{y} - \tilde{\beta}\bar{x} \tag{3.1}$$

$$\hat{\theta}^{\text{RE}}(d) = d\tilde{\theta} + (1-d)\hat{\theta}, \quad 0 \le d \le 1 \tag{3.2}$$

$$\begin{aligned}\hat{\theta}^{\text{PTE}}(d) &= \hat{\theta}^{\text{RE}}(d)I(F < F_\alpha) + \tilde{\theta}I(F \ge F_\alpha) \\ &= \tilde{\theta} + \tilde{\beta}\bar{x}(1-d)I(F < F_\alpha)\,. \end{aligned} \tag{3.3}$$

The bias of the estimators are obtained as (cf Hoque et al. 2006)

$$B_1[\tilde{\theta}^{\text{UE}}(d)] = 0 \tag{3.4}$$

$$B_2[\hat{\theta}^{\text{RE}}(d)] = S_{xx}^{-1/2}\bar{x}\sigma(1-d)\Delta \tag{3.5}$$

$$B_3[\hat{\theta}^{\text{PTE}}(d)] = (1-d)\bar{x}\beta G_{3,\nu}(3^{-1}F_\alpha;\Delta^2), \tag{3.6}$$

where $G_{n_1,n_2}(\cdot;\Delta^2)$ is the c.d.f. of a non-central F-distribution with (n_1, n_2) df and non-centrality parameter Δ^2 which is the *departure constant* from the null-hypothesis. Among the three estimators, the UE is the only unbiased estimator.
The mean squared errors (MSE) of the estimators become

$$M_1[\tilde{\theta}^{\text{UE}}]0 = \sigma^2 H \tag{3.7}$$

$$M_2[\hat{\theta}^{\text{RE}}(d)] = \sigma^2\left[d^2H + (1-d)^2 S_{xx}^{-1}\bar{x}^2\Delta^2\right] \tag{3.8}$$

$$\begin{aligned}M_3[\hat{\beta}^{\text{PTE}}(d)] &= \sigma^2 H + S_{xx}^{-1}\sigma^2\bar{x}^2\Big[\Delta^2\Big\{2(1-d)G_{3,v}(3^{-1}F_\alpha;\Delta^2) \\ &-(1-d^2)G_{5,v}(5^{-1}F_\alpha;\Delta^2)\Big\} - (1-d^2)G_{3,v}(3^{-1}F_\alpha;\Delta^2)\Big],\end{aligned} \tag{3.9}$$

where $H = \left\{n^{-1} + S_{xx}^{-1}\bar{x}^2\right\}$.
The relative efficiency of the PTE relative to the UE and RE is

$$\text{RE}[\hat{\theta}^{\text{PTE}}(d):\tilde{\theta}^{\text{UE}}] = H\left[H + S_{xx}^{-1}\bar{x}^2\sigma^2 g(\Delta^2)\right]^{-1} \tag{3.10}$$

and

$$\begin{aligned}\text{RE}[\hat{\theta}^{\text{PTE}}(d):\hat{\theta}^{\text{RE}}(d)] &= \left[d^2H + (1-d)^2\Delta^2 S_{xx}^{-1}\bar{x}^2\right] \\ &\times\left[H + S_{xx}^{-1}\bar{x}^2 g(\Delta^2)\right]^{-1}\end{aligned} \tag{3.11}$$

respectively, where

$$\begin{aligned}g(\Delta^2) &= \Delta^2\{2(1-d)G_{3,v}(3^{-1}F_\alpha;\Delta^2) - (1-d^2)G_{5,v}(5^{-1}F_\alpha;\Delta^2)\} \\ &-(1-d^2)G_{3,v}(3^{-1}F_\alpha;\Delta^2).\end{aligned} \tag{3.12}$$

The bias, MSE and relative efficiency functions of the estimators can be analysed for different values of *d* and Δ and the performances of the estimators could be compared accordingly. Graphs of the relative efficiency of the PTE is given in Figure 1.

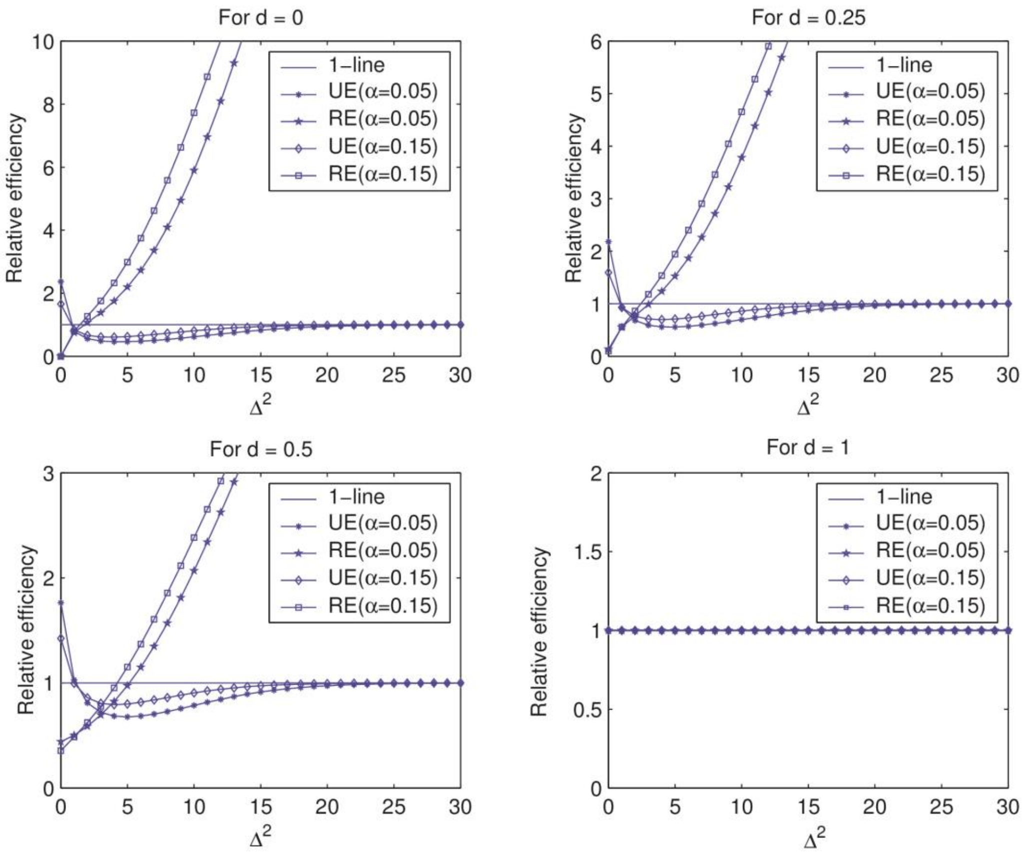

Figure 1: Graph of the relative efficiency of PTE relative to UE and RE against Δ^2.

4 THREE TESTS OF INTERCEPT

In this section we define three alternative tests of the intercept and investigate their properties.

To remove the uncertainty in the NSPI on β, we perform a pretest (PT) on $H_0^*: \beta = \beta_0$ before testing on the intercept. Let ϕ^{PT} be the test function for pretesting $H_0^*: \beta = \beta_0$ (a suspected constant) against $H_a^*: \beta \neq \beta_0$. If the H_0^* is rejected in the PT, then the UT is used to test the intercept, otherwise the RT is used. The appropriate test statistic for the PT is $T^{PT} = S_n^{-1}(\beta - \beta_0)\sqrt{S_{xx}} \sim t_{n-2}$.

4.1 Three Test Statistics

Under the three scenarios on β the UT, RT and PTT for testing $H_0 : \theta = \theta_0$ (known constant) against $H_a : \theta \neq \theta_0$ are defined as follows:

(i) ϕ^{UT} = test function and T^{UT} is the test statistic when β is unspecified,

(ii) ϕ^{RT} = test function and T^{RT} is the test statistic when $\beta = \beta_0$ is specified and

(iii) ϕ^{PTT} = test function and T^{PTT} is the test statistic following a PT on H_0^* when $\beta = \beta_0$ is uncertain.

The test statistics are obtained as

$$T^{UT} = (\tilde{\theta} - \theta_0) / SE(\tilde{\theta}) = \sqrt{n}(\bar{Y} - \tilde{\beta}\bar{X} - \theta_0)\left[S_n^2(1 + S_{xx}^{-1} n\bar{X}^2)\right]^{-\frac{1}{2}} \tag{4.1}$$

$$T^{RT} = (\hat{\theta} - \theta_0) / SE(\hat{\theta})\% = \frac{(\hat{\theta} - \theta_0)}{s_y / \sqrt{n}} = s_y^{-1}\sqrt{n}(\bar{Y} - \theta_0) \sim t_{n-1}, \tag{4.2}$$

where $T^{UT} \sim t_{n-2}$, and $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$. Let us choose a positive number

$\alpha_j, (0 < \alpha_j < 1,$ for j=1,2,3) then let t_{n-2,α_j} be such that $P\left(T^{UT} > t_{n-2,\alpha_1} \hat{U} \theta = \theta_0\right) = \alpha_1,$

$P\left(T^{RT} > t_{n-1,\alpha_2} \hat{U} \theta = \theta_0\right) = \alpha_2$, and

$P\left(T^{PT} > t_{n-2,\alpha_3} \hat{U} \beta = \beta_0\right) = \alpha_3$. Then,the PTT fortesting $H_0 : \theta = \theta_0$ when $\beta = \beta_0$ is uncertain is given by the test function

$$\Phi^{PTT} = \begin{cases} 1, & \text{if } \left(T^{PT} \le t_{n-2,\alpha_3}, T^{RT} > t_{n-1,\alpha_2}\right) \\ & \text{or} \left(T^{PT} > t_{n-2,\alpha_3}, T^{UT} > t_{n-2,\alpha_1}\right); \\ 0, & \text{otherwise.} \end{cases} \tag{4.3}$$

4.2 Properties of the Tests

Let $\{K_n\}$ be a sequence of alternative hypotheses defined as

$$K_n : (\theta - \theta_0, \beta - \beta_0) = \left(\frac{\lambda_1}{\sqrt{n}}, \frac{\lambda_2}{\sqrt{n}}\right) = n^{-1/2}\lambda, \tag{4.4}$$

where $\lambda = (\lambda_1, \lambda_2)$ is a vector of fixed real numbers and θ is the true value of the intercept. Under $K_n, (\theta - \theta_0) \neq 0$ and under $H_0, (\theta - \theta_0) = 0$.

Note that T^{UT} and T^{PT} are correlated, but T^{RT} and T^{PT} are uncorrelated (but not independent). The joint distribution of the T_1^{UT} and T_3^{PT} is $(T_1^{UT}, T_3^{PT})' \sim t_{n-2}$, a bivariate

Student-*t* distribution with *(n-2)*df and correlation coefficient ρ with $Cov(T_1^{UT}, T_3^{PT}) = \frac{(n-2)}{(n-4)}\Sigma$ (cf., Kotz and Nadarajah, 2004).

The power functions of the tests are given by

$$\pi^{UT}(\lambda) = P(T^{UT} > t_{\alpha_1,n-2}\hat{U}K_n)$$

$$= 1 - P\left(T_1^{UT} \le t_{\alpha_1,n-2} - \lambda_1 k^{-1}\right) \quad (4.5)$$

$$\pi^{RT}(\lambda) = P\left(T^{RT} > t_{\alpha_1,n-1}\hat{U}K_n\right)$$

$$= P\left(T_2^{RT} > t_{\alpha_2,n-1} - \sqrt{n}\left((\theta-\theta_0)+(\beta-\beta_0)\bar{X}\right)s_y^{-1}\right)$$

$$= 1 - P\left(T_2^{RT} \le t_{\alpha_2,n-1} - \lambda_1 + \lambda_2 \bar{X} s_y^{-1}\right) \quad (4.6)$$

$$\pi^{PTT}(\lambda) = P\left(T^{PT} \le t_{n-2,\alpha_3}, T^{RT} > t_{n-1,\alpha_2}\right) + P\left(T^{PT} > t_{n-2,\alpha_3}, T^{UT} > t_{n-2,\alpha_1}\right)$$

$$= d_{10}\left\{t_{n-2,\alpha_3} - \lambda_2\sqrt{S_{xx}}[S_n^2 n]^{-\frac{1}{2}}, t_{\alpha_2,n-1} - s_y^{-1}(\lambda_1 + \lambda_2\bar{X}), \rho = 0\right\}$$

$$+ d_{2\rho}\left\{t_{n-2,\alpha_3} - \lambda_2\sqrt{S_{xx}}[S_n^2 n]^{-\frac{1}{2}}, t_{\alpha_1,n-2} - \lambda_1 k^{-1}, \rho \ne 0\right\},$$

$$= d_{10}\left\{t_{n-2,\alpha_3} - \lambda_2\frac{\sqrt{S_{xx}}}{S_n\sqrt{n}}, \%t_{\alpha_2,n-1} - \frac{(\lambda_1 + \lambda_2\bar{X})}{s_y}, \rho = 0\right\}$$

$$+ d_{2\rho}\left\{t_{n-2,\alpha_3} - \lambda_2\frac{\sqrt{S_{xx}}}{S_n\sqrt{n}}, t_{\alpha_1,n-2} - \lambda_1 k^{-1}, \rho \ne 0\right\}, \quad (4.7)$$

where $k = S_n\sqrt{(1+n\bar{X}^2 S_{xx}^{-1})}$, d_{10} and $d_{2\rho}$ are bivariate Student's *t* probability integrals. Here d_{10} is defined as $d_{10} = \int_{-\infty}^{a}\int_{c}^{\infty} f(t^{PT}, t^{RT})dt^{PT}dt^{RT}$,

$$a = \left[t_{n-2,\alpha_3} - \lambda_2\frac{\sqrt{S_{xx}}}{S_n\sqrt{n}}\right] \text{and } c = \left[t_{n-1,\alpha_2} - \frac{\lambda_1 + \lambda_2\bar{X}}{s_y}\right], \text{ and}$$

$d_{2\rho}$ is defined as

$$d_{2\rho}(a,b,\rho) = \frac{\Gamma\left(\frac{\nu+2}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)n\pi\sqrt{1-\rho^2}}\int_a^\infty\int_b^\infty\left[1 + \frac{(x^2+y^2-2\rho xy)}{\nu(1-\rho^2)}\right]^{-\frac{\nu+2}{2}} dxdy,$$

in which $-1 < \rho < 1$ is the correlation coefficient between the T^{UT} , T^{PT} and $b = \left[t_{\alpha_1,n-2} - \lambda_1 / k\right]$.

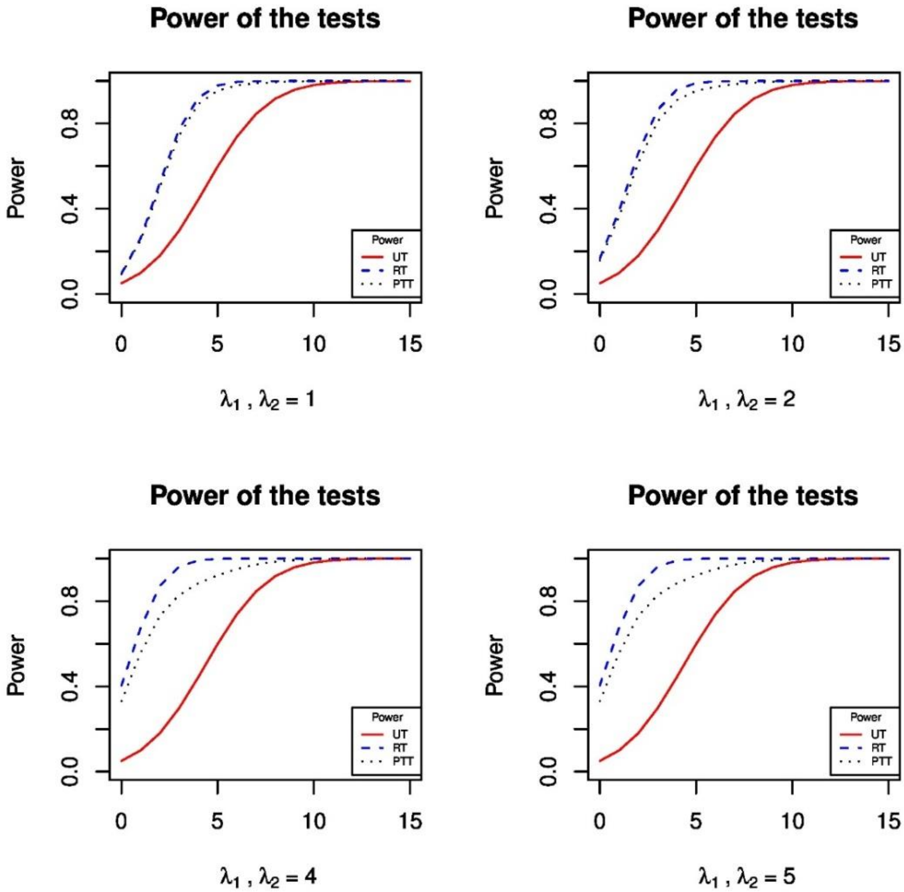

Figure 2: Graphs of the power functions of the UE, RE and PTT for various values of λ_1, and λ_2 with a fixed value of $\rho = 0.1$.

The power curves of the PTT for different values of ρ is provided in Figure 3.

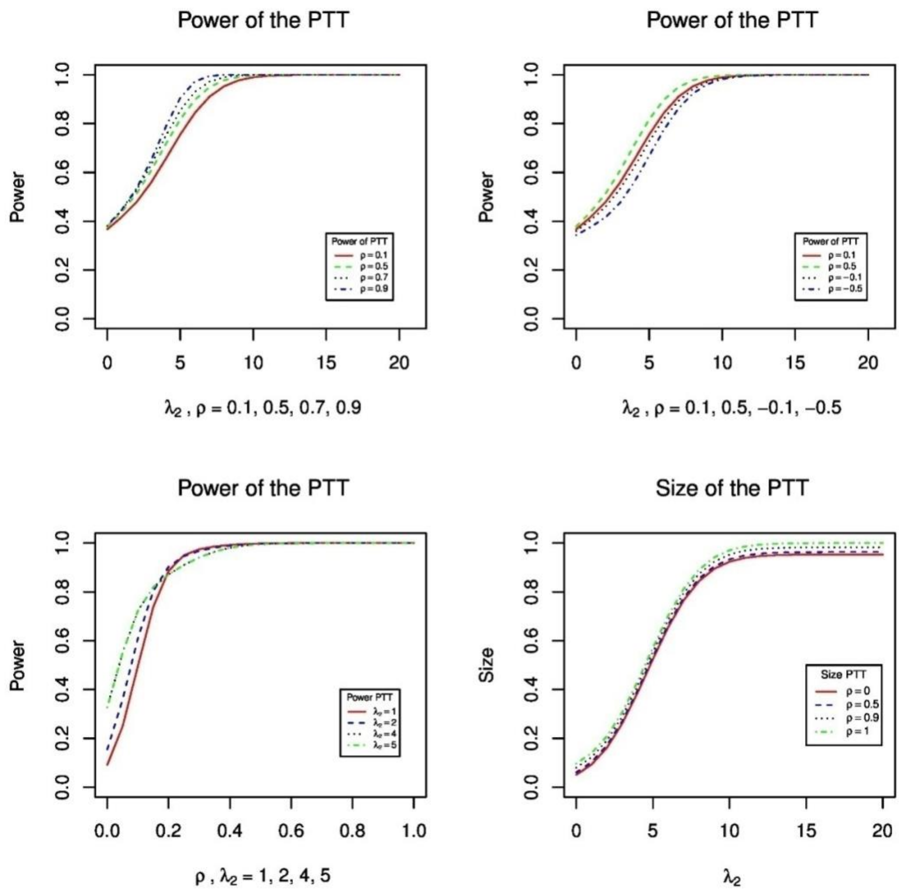

Figure 3: The power curve of the PTT against λ_2, and its power and size curves against ρ.

5 CONCLUDING REMARKS

In practice, the NSPI is obtained from expert knowledge or previous studies, and hence the value of the parameter available from prior information is expected to be close to its true value and the degree of distrust on the null hypothesis is very likely to be close to 0.

Based on the above analyses, it is evident that the power of the RT is always higher than that of the UT and PTT, and the power of the PTT lies between the power of the RT and UT for all values of λ_1, λ_2 and ρ. The size of the UT is smaller than that of the RT and PTT.

Of the three tests, the RT has the maximum power and size, and the UT has minimum power and size. So none of them is achieving the highest power and lowest size. But the PTT protects against maximum size of the RT and minimum power of the UT. As $\lambda_2 \to 0$ the difference between the power of the PTT and RT diminishes for all values of $\lambda_2 \to 0$. That its, if the NSPI is accurate the power of the PTT is about the same as that of the RT. Moreover, the power of the PTT gets closer to that of the RT as $\rho \to 1$. If $\rho = 1$ then the power of the PTT matches with that of the RT. Thus if there is a high (near 1) correlation between the T^{UT} and T^{PT} the power of the PTT is very close to that of the RT.

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