Ponder this!

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Problem set 6

As usual, the purpose of the section is to supply teachers and students with a selection of interesting problems. This issue introduces a special topic of interest, where algebra and geometry meet, with a number of classical results given for readers' consideration. At this time we highlight a few famous geometrical inequalities which have been discovered in the twentieth century. All of them have received considerable attention from researchers in the field of geometrical inequalities despite their elementary nature. Another interesting peculiarity is that these inequalities are linked with each other, although, each can be considered separately as well. Together they give readers the opportunity to make acquaintance with a state-of-art in the area of geometrical applications and, possibly, even find new proofs for some inequalities.

1. Weitzenbock inequality

   Let a, b, and c be the sides of a triangle and S its area. Prove
   $$a^2 + b^2 + c^2 \geq 4\sqrt{3}\,S$$

2. Hadwiger-Finsler inequality

   Let a, b, and c be the sides of a triangle and S its area. Prove
   $$a^2 + b^2 + c^2 \geq 4\sqrt{3}\,S + (a-b)^2 + (b-c)^2 + (c-a)^2$$

3. Reverse Hadwiger-Finsler inequality

   Let a, b, and c be the sides of a triangle and S its area. Prove
   $$a^2 + b^2 + c^2 \leq 4\sqrt{3}\,S + 3[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

4. Oppenheim inequality

   Let a, b, and c be the sides of a triangle and S its area. Prove that there exists a triangle of sides
   $$a^{\left(\frac{1}{p}\right)},\ b^{\left(\frac{1}{p}\right)},\ c^{\left(\frac{1}{p}\right)}\ (p>1)$$
   and area S_p such that
   $$\left(\frac{4S_p}{\sqrt{3}}\right)^p \geq \frac{4S}{\sqrt{3}}$$

5. Garfunkel-Bankoff inequality

   Let α, β, γ be angles of an arbitrary triangle. Prove
   $$\tan^2\frac{\alpha}{2} + \tan^2\frac{\beta}{2} + \tan^2\frac{\gamma}{2} \geq 2 - 8\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}.$$

Solutions to this set of problems for publication should be submitted to:
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Solutions to this set will be made available on the AAMT website (www.aamt.edu.au) after 1 May 2011.