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**Testing of Intercept when Slope is Under Suspicion**

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Abstract

In simple regression analyses, the inference on the intercept depends on the knowledge of the slope. This paper studies the problem of testing the intercept of a simple regression model when slope is under suspicion. Depending on the situation the slope may be unknown or unspecified, known or specified, and uncertain if the suspected value is unsure. The three different scenarios on the slope lead to three different tests of the intercept. Here we define the unrestricted test (UT), restricted test (RT) and pre-test test (PTT) for the intercept parameter depending on the level of knowledge on the slope. The test statistics, their sampling distributions, and power functions of the tests are derived and compared when the error variance is assumed to be known.

*Keywords and phrases:* Linear regression, test of intercept, power function, normal and bivariate normal distributions.

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# 1 Introduction

The simple regression mode describes the linear relationship between dependent variable ($y$) and independent variable ($x$). For an $n$ pairs of observations, $(x_i, y_i)$, for $i = 1, 2, \cdots, n$ the model is written as

$$y_i = \beta_0 + \beta_1 x_i + e_i, \tag{1.1}$$

where $e_i^{'}s$ are assumed to be normally distributed with mean 0 and variance $\sigma^2$, $x_i^{'}s$ are known real values of the independent variable, and $\beta_0$ and $\beta_1$ are the unknown intercept and slope parameters respectively. We consider the problem of testing $H_0 : \beta_0 = \beta_{00}$ (a fixed value) when there is uncertain information available on the value of $\beta_1$.

Inferences about population parameters could be improved using non-sample prior information (NSPI) from trusted sources (cf Bancroft, 1944). Such information, which is usually provided by previous studies or expert knowledge or experience of the researchers, and is not related to the sample data. An appropriate statistical test on the value (expressed in

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the form) null hypothesis will be useful to eliminate the uncertainty on this suspected information. Then the outcome of the preliminary testing on the uncertain NSPI is used in the hypothesis testing to improve the performance of the statistical test (Khan and Saleh, 2001; Saleh, 2006; Yunus and Khan, 2010).

The suspected value of the slope may be (i) unknown or unspecified if NSPI is not available, (ii) known or specified if the exact value is available from NSPI, and (iii) uncertain if the suspected value is unsure. For the three different scenarios, three different of statistical tests, namely the (i) unrestricted test (UT), (ii) restricted test (RT) and (iii) pre-test test (PTT) are defined.

In the area of estimation with NSPI there has been a lot of work, notably Bancroft (1944, 1964), Hand and Bancroft (1968), and Judge and Bock (1978) introduced a preliminary test estimation of parameters to estimate the parameters of a model with uncertain prior information. Khan (2003, 2008), Khan and Saleh (1997, 2001, 2005, 2008), Khan et al (2002), Khan and Hoque (2003), Saleh (2006) and Yunus (2010) covered various work in the area of improved estimation using NSPI, but there is a very limited number of studies on the testing of parameters in the presence of uncertain NSPI. Although Tamura (1965), Saleh and Sen (1978, 1982), Yunus and Khan (2007, 2010), and Yunus (2010) used the NSPI for testing hypothesis using nonparametric methods, the problem has not been addressed in the parametric context.

This paper considers statistical tests with NSPI and the criteria that are used to compare the performance of the UT, RT and PTT are the size and power of the tests. A statistical test that has a minimum size is preferable because it will give a smaller probability of a Type I error. Furthermore, a test that has maximum power is preferred over any other tests because it guarantees the highest probability of rejecting any false null hypothesis. A test that minimizes the size and maximizes the power is preferred over any other tests. In reality, the size of a test is (kept) fixed, and then the choice of the best test is based on its maximum power.

We define the following three different tests:
For the UT, let $\phi^{UT}$ be the test function and $T^{UT}$ be the test statistic for testing $H_0 : \beta_0 = \beta_{00}$ (known constant) against $H_a : \beta_0 > \beta_{00}$ when $\beta_1$ is unspecified.
For the RT, let $\phi^{RT}$ be the test function and $T^{RT}$ be the test statistic for testing $H_0 : \beta_0 = \beta_{00}$ against $H_a : \beta_0 > \beta_{00}$ when $\beta_1$ is $\beta_{10} = 0$ (specified).
For the PTT, let $\phi^{PTT}$ be the test function and $T^{PTT}$ be the test statistic for testing $H_0 : \beta_0 = \beta_{00}$ against $H_a : \beta_0 > \beta_{00}$ following a pre-test (PT) on the slope. For the PT, let $\phi^{PT}$ be the test function for testing $H_0^* : \beta_1 = \beta_{10}$ (a suspected constant) against $H_a^* : \beta_1 > \beta_{10}$. If the $H_0^*$ is rejected in the PT, then the UT is used to test the intercept, otherwise the RT is used to test $H_0$. Thus, the PTT depends on the PT which is a choice between the UT and RT.

For the above tests we define following unrestricted estimator of $\beta_1$ and intercept $\beta_0$: $\widetilde{\beta_1} = \frac{\sum_{i=1}^n (X_i - \overline{X})((Y_i - \overline{Y})}{\sum_{i=1}^n (X_i - \overline{X})^2} = \frac{S_{xy}}{S_{xx}}$ and $\widetilde{\beta_0} = \overline{Y} - \widetilde{\beta_1}\overline{X}$, where $\overline{X} = \frac{1}{n}\sum X$ and $\overline{Y} = \frac{1}{n}\sum Y$. The restricted estimator (under $H_0^*$) of the slope and intercept are $\widehat{\beta_1} = \beta_{10}$ and $\widehat{\beta_0} =$

$\overline{Y} - \widehat{\beta_1}\overline{X}$.

The following section provides the three tests. Section 3 derives the distribution of the test statistics. The power functions of the tests are obtained in Section 4. An illustrative example is given in Section 5. The comparison of the power of the tests and concluding remarks are provided in Sections 6 and 7.

# 2 The Three Tests

For testing the intercept parameter under three different scenarios of the slope, the test statistics of the UT, RT and PTT for known $\sigma^2$ are given as follows. The test statistic of the UT for testing $H_0 : \beta_0 = \beta_{00}$ against $H_a : \beta_0 > \beta_{00}$ is defined by

$$T_z^{UT} = \frac{\sqrt{n}(\widetilde{\beta_0} - \beta_{00})}{SE(\widetilde{\beta_0})} = \frac{(\widetilde{\beta_0} - \beta_{00})}{\frac{\sigma}{\sqrt{n}}(1 + \frac{n\overline{x}^2}{S_{xx}})^{1/2}} = \frac{\sqrt{n}(\overline{y} - \widetilde{\beta_1}\overline{x} - \beta_{00})}{\sigma(1 + \frac{n\overline{x}^2}{Sxx})^{1/2}}, \tag{2.1}$$

where standard error (SE) of $\widetilde{\beta_0}$ is $\frac{\sigma}{\sqrt{n}}\left(1 + \frac{n\overline{x}^2}{Sxx}\right)^{1/2}$. Under $H_0$, $T_z^{UT}$ follows standard normal distribution $N(0,1)$, and under $H_a$ the distribution is $N\left(\frac{\sqrt{n}(\beta_0 - \beta_{00})}{\sigma(1+\frac{n\overline{x}^2}{Sxx})^{1/2}}, 1\right)$ with $\beta_0 - \beta_{00} > 0$ and $\beta_{00}$ is the value of $\beta_0$.

The test statistic of the RT is given by

$$T_z^{RT} = \frac{(\widehat{\beta_0} - \beta_{00})}{SE(\widehat{\beta_0})} = \frac{\widehat{\beta_0} - \beta_{00}}{\sigma/\sqrt{n}} = \frac{\sqrt{n}(\overline{y} - \beta_{00})}{\sigma} \sim N(0,1). \tag{2.2}$$

Note that $\widehat{\beta_0} = \overline{y} - \widehat{\beta_1}\overline{x} = \overline{y} - \beta_{10}\overline{x}$ and $SE(\widehat{\beta_0}) = \sqrt{Var(\widehat{\beta_0})} = \frac{\sigma}{\sqrt{n}}$. Under $H_a$ the $T_z^{RT}$ follows a normal distribution with mean $\frac{\beta_0 - \beta_{00}}{\sigma/\sqrt{n}}$ and variance 1. If $\beta_1$ is specified to be $\beta_{10}$ then under $H_a$ it follows a normal distribution with mean $\frac{(\beta_0 - \beta_{00}) + (\beta_1 - \beta_{10})\overline{x}}{\sigma/\sqrt{n}}$ and variance 1, where $\beta_0 - \beta_{00} > 0$ and $\beta_1 - \beta_{10} > 0$.

For the preliminary test (PT) $H_0^* : \beta_1 = \beta_{10}$ against $H_a^* : \beta_1 > \beta_{10}$, the test statistic of the PT is given by

$$T_z^{PT} = \frac{\widetilde{\beta_1} - \beta_{10}}{SE(\widetilde{\beta_1})} = \frac{\widetilde{\beta_1} - \beta_{10}}{\sigma/\sqrt{S_{xx}}} \sim N\left(\frac{\beta_1 - \beta_{10}}{\sigma/\sqrt{Sxx}}, 1\right), \tag{2.3}$$

where $SE(\widetilde{\beta_1}) = \sigma/\sqrt{S_{xx}}$. Under the null hypothesis the above test statistic follows the standard normal distribution.

Furthermore, we propose the PTT for testing $H_0$, following the PT on $\beta_1$. Let us choose positive number $\alpha_j$ $(0 < \alpha_j < 1)$ and real values $(z_{\alpha_j})$, such that $P\left(T_z^{UT} > z_{\alpha_1} \mid \beta_0 = \beta_{00}\right) = \alpha_1$, $P\left(T_z^{RT} > z_{\alpha_2} \mid \beta_0 = \beta_{00}\right) = \alpha_2$ and $P\left(T_z^{PT} > z_{\alpha_3} \mid \beta_1 = \beta_{10}\right) = \alpha_3$. Then, the PTT for testing $H_0$ when $\beta_1$ is uncertain, it is given by the test function

$$\Phi_z = \begin{cases} 1, & if\left[T_z^{PT} \leq z_{\alpha_3}, T_z^{RT} > z_{\alpha_2}\right] or \left[T_z^{PT} > z_{\alpha_3}, T_z^{UT} > z_{\alpha_1}\right]. \\ 0, & \text{otherwise.} \end{cases} \tag{2.4}$$

The size of the PTT is then

$$\alpha_z = P\left\{T_z^{PT} \le z_{\alpha_3}, T_z^{RT} > z_{\alpha_2}\right\} + \left\{T_z^{PT} > z_{\alpha_3}, T_z^{UT} > z_{\alpha_1}\right\} \tag{2.5}$$

# 3 Distribution of Test Statistics

For the derivation of the power function of the UT and RT we need the sampling distributions of the $T^{UT}$ and $T^{PT}$, and that of the PTT the joint distribution of $(T^{UT}, T^{PT})$ and $(T^{RT}, T^{PT})$ are essential. Let $\{K_n\}$ be a sequence of alternative hypotheses defined as

$$K_n : (\beta_0 - \beta_{00}, \beta_1 - \beta_{10}) = \left(\frac{\lambda_1}{\sqrt{n}}, \frac{\lambda_2}{\sqrt{n}}\right) = n^{-1/2}\boldsymbol{\lambda}, \tag{3.1}$$

where $\boldsymbol{\lambda} = (\lambda_1, \lambda_2)$ are fixed real numbers, $\beta_0$ is true value and $\beta_{00}$ is NSPI. Under $K_n$ the value of $\beta_0 - \beta_{00}$ is greater than zero (or $\beta_0 - \beta_{00} > 0$), and under $H_0$ the value of $\beta_0 - \beta_{00} = 0$.

Following Yunus and Khan (2011), the test statistic of the UT, under $K_n$, is $T_z^{UT} \sim N\left(\frac{\sqrt{n}(\beta_0-\beta_{00})}{\sigma(1+\frac{n\overline{x}^2}{Sxx})^{1/2}}, 1\right)$. Under alternative hypothesis we then derive $Z_i$, $i = 1, 2, 3$ as follows,

$$Z_1 = T_z^{UT} - \frac{\sqrt{n}(\beta_0 - \beta_{00})}{\sigma(1 + n\frac{\overline{x}^2}{S_{xx}})^{1/2}} = T_z^{UT} - \frac{\lambda_1}{k_1} \sim N(0, 1), \tag{3.2}$$

where $k_1 = \sigma(1 + \frac{n\overline{x}^2}{Sxx})^{1/2}$. Similarly, from equation (2.2) and (2.3), under $K_n$ we obtain

$$Z_2 = T_z^{RT} - \frac{(\beta_0 - \beta_{00}) + (\beta_1 - \beta_{10})\overline{x}}{\sigma/\sqrt{n}} \sim N(0, 1), \tag{3.3}$$

and

$$Z_3 = T_z^{PT} - \frac{\beta_1 - \beta_{10}}{\sigma/\sqrt{Sxx}} \sim N(0, 1). \tag{3.4}$$

Since $T_z^{RT}$ and $T_z^{PT}$ are independent, the joint distribution under $H_a$ is

$$\begin{aligned}\begin{pmatrix} T_z^{RT} \\ T_z^{PT} \end{pmatrix} &\sim N_2\left[\begin{pmatrix} \frac{(\beta_0-\beta_{00})+(\beta_1-\beta_{10})\overline{x}}{\sigma/\sqrt{n}} \\ \frac{\beta_1-\beta_{10}}{\sigma/\sqrt{S_{xx}}} \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right] \\ &= N_2\left[\begin{pmatrix} \frac{\lambda_1+\lambda_2\overline{x}}{\sigma} \\ \frac{\lambda_2\sqrt{S_{xx}}}{\sigma\sqrt{n}} \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right].\end{aligned} \tag{3.5}$$

In the same manner, we have

$$\begin{pmatrix} T_z^{UT} \\ T_z^{PT} \end{pmatrix} \sim N_2\left[\begin{pmatrix} \frac{\sqrt{n}(\beta_0-\beta_{00})}{\sigma(1+\frac{n\overline{x}^2}{S_{xx}})^{1/2}} \\ \frac{\beta_1-\beta_{10}}{\sigma/\sqrt{S_{xx}}} \end{pmatrix}, \begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix}\right]$$

$$= N_2 \left[ \left( \begin{array}{c} \frac{\lambda_1}{\sigma(1+\frac{n\bar{x}^2}{S_{xx}})^{1/2}} \\ \frac{\lambda_2\sqrt{S_{xx}}}{\sigma\sqrt{n}} \end{array} \right), \left( \begin{array}{cc} 1 & -\rho \\ -\rho & 1 \end{array} \right) \right], \tag{3.6}$$

where $\rho$ is correlation coefficient between $T_z^{UT}$ and $T_z^{PT}$.

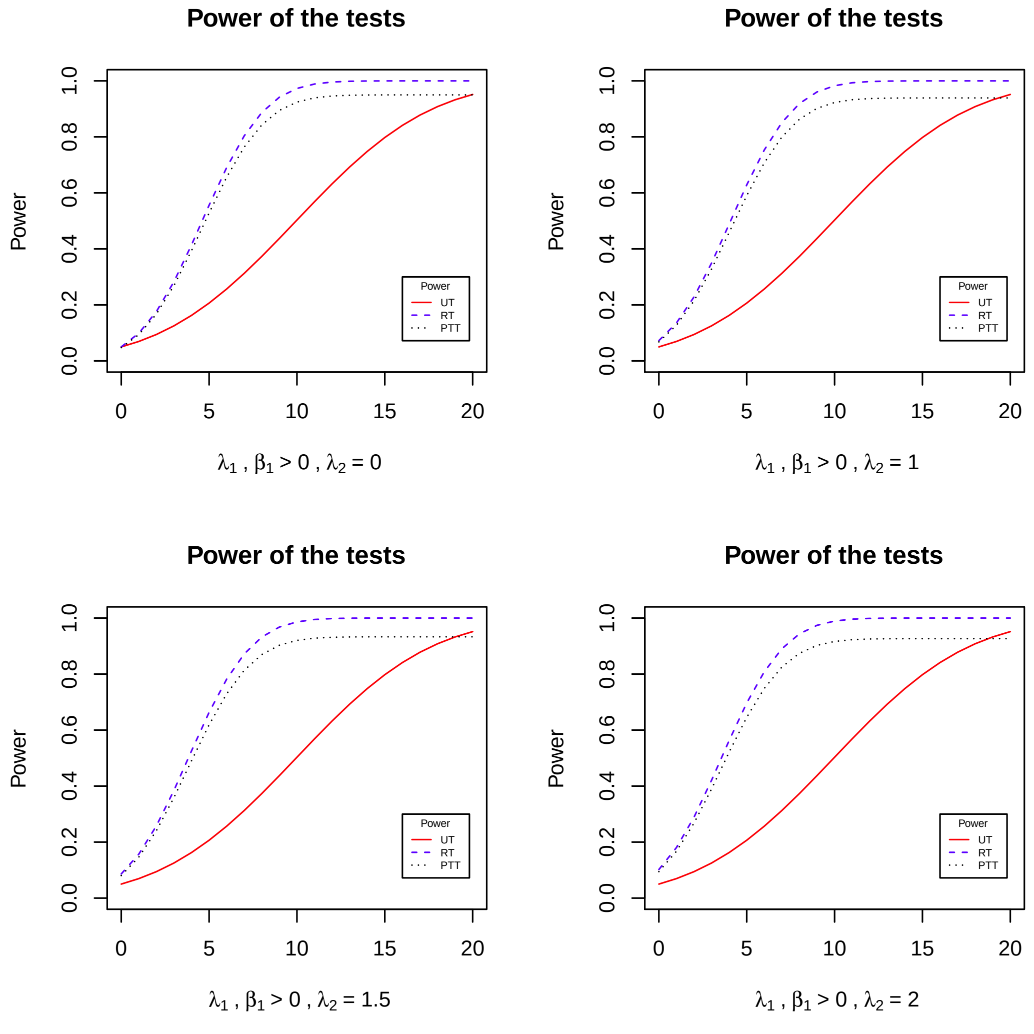


Figure 1: The power of the UT, RT and PTT against $\lambda_1$ with $\beta_1 > 0$ and $\lambda_2 = 0, 1, 1.5, 2$.

# 4 Power Functions and Size of Tests

The power function for the UT, RT and PTT for known variance are derived as below. The power function of the UT is

$$\begin{aligned}\prod_{z_1}(\boldsymbol{\lambda}) &= \prod^{UT}(\boldsymbol{\lambda}) = P(T_z^{UT} > z_{\alpha_1} \mid K_n) \\ &= P\left(Z_1 > z_{\alpha_1} - \frac{\lambda_1}{k_1}\right) = 1 - G\left(z_{\alpha_1} - \frac{\lambda_1}{k_1}\right), \qquad (4.1)\end{aligned}$$

and that of the RT is

$$\begin{aligned}\prod_{z_2}(\boldsymbol{\lambda}) &= \prod^{RT}(\boldsymbol{\lambda}) = P(T_z^{RT} > z_{\alpha_2} \mid K_n) \\ &= P\left(Z_2 > z_{\alpha_2} - \frac{\lambda_1 + \lambda_2\overline{x}}{\sigma}\right) = 1 - G\left(z_{\alpha_2} - \frac{\lambda_1 + \lambda_2\overline{x}}{\sigma}\right). \qquad (4.2)\end{aligned}$$

When $\lambda_1$ grows larger the power of the UT becomes higher. The power grows higher as $\lambda_1$ becomes larger. Similarly, the power function of the PTT is given as

$$\begin{aligned}\prod_{z}(\boldsymbol{\lambda}) &= \prod_{z}^{PTT}(\boldsymbol{\lambda}) = P(\text{rejecting H}_0) \\ &= P\left(T_z^{PT} \le z_{\alpha_3}, T_z^{RT} > z_{\alpha_2}\right) + \left(T_z^{PT} > z_{\alpha_3}, T_z^{UT} > z_{\alpha_1}\right) \\ &= P\left(T_z^{PT} \le z_{\alpha_3}\right) P\left(T_z^{RT} > z_{\alpha_2}\right) + P\left(T_z^{PT} > z_{\alpha_3}, T_z^{UT} > z_{\alpha_1}\right) \\ &= G\left(z_{\alpha_3} - \frac{\lambda_2\sqrt{S_{xx}}}{\sigma\sqrt{n}}\right)\left(1 - G\left(z_{\alpha_2} - \frac{\lambda_1 + \lambda_2\overline{x}}{\sigma}\right)\right) \\ &\quad + d_{1\rho}\left(z_{\alpha_3} - \frac{\lambda_2\sqrt{S_{xx}}}{\sigma\sqrt{n}}, z_{\alpha_1} - \frac{\lambda_1}{k_1}, \rho \ne 0\right), \qquad (4.3)\end{aligned}$$

where $d_{1\rho}$ are bivariate normal probability integral. Here $d_{1\rho}$ is defined for every real $p$, $q$ and $-1 < \rho < 1$ as

$$d_{1\rho}(p,q,\rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_p^\infty \int_q^\infty exp\left[-\frac{1}{2(1-\rho^2)}(x^2 + y^2 - 2\rho xy)\right] dxdy, \qquad (4.4)$$

where $p = z_{\alpha_3} - \frac{\lambda_2\sqrt{S_{xx}}}{\sigma\sqrt{n}}$, $q = z_{\alpha_1} - \frac{\lambda_1}{k_1}$ and $G(x)$ is a cumulative distribution function (cdf) of the standard normal distribution.

Furthermore, the size of the UT, RT and PTT are given as

$$\begin{aligned}\alpha_z^{UT} &= P\left(T_z^{UT} > z_{\alpha_1} \mid H_0\right) = 1 - G\left(z_{\alpha_1} - \frac{\sqrt{n}(\beta_0 - \beta_{00})}{\sigma\sqrt{[(1+n\overline{x})^2/S_{xx}]}} \mid H_0 : \beta_0 = \beta_{00}\right) \\ &= 1 - G\left(z_{\alpha_1} - \frac{\sqrt{n}(\beta_{00} - \beta_{00})}{k_1}\right) = 1 - G\left(z_{\alpha_1}\right), \qquad (4.5)\end{aligned}$$

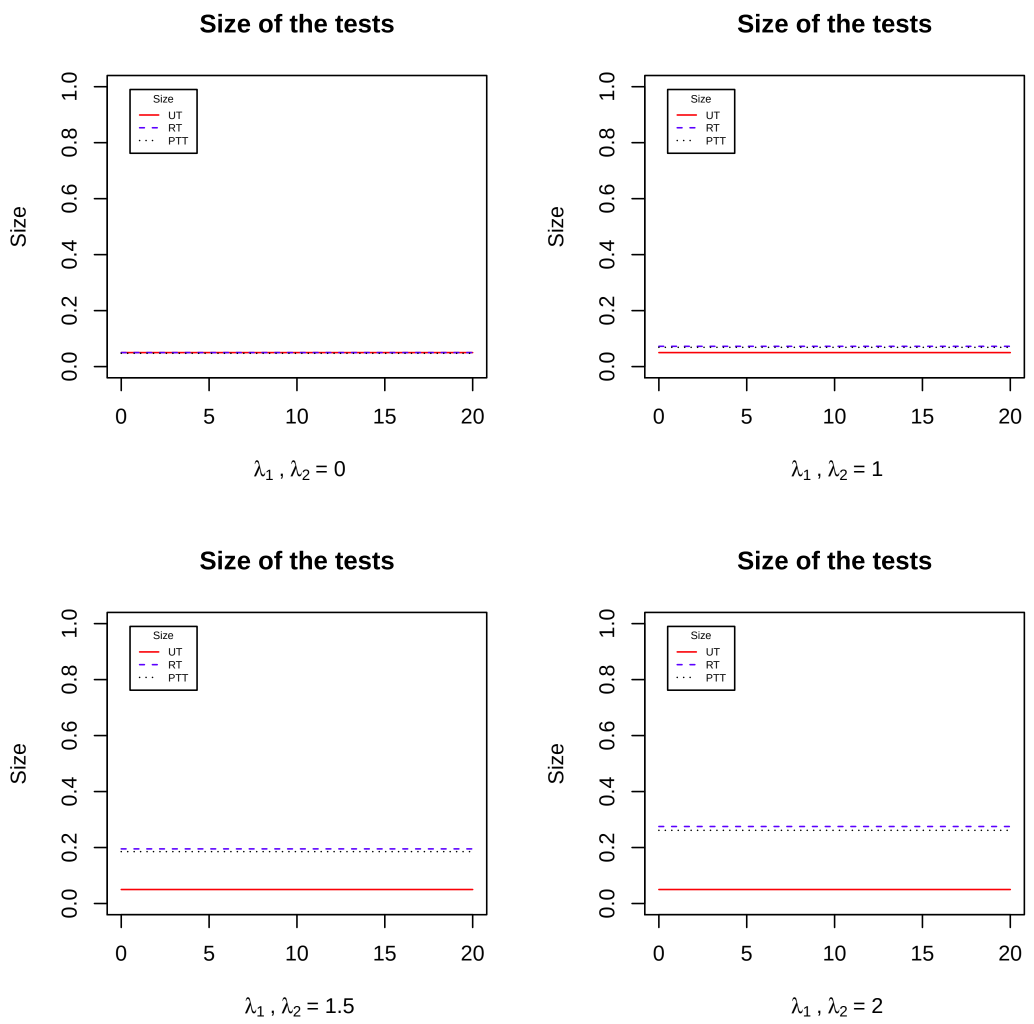


Figure 2: The size of the UT, RT and PTT against $\lambda_1$ with $\lambda_2 = 0, 1, 1.5, 2$.

$$\alpha_z^{RT} = P\left(T_z^{RT} > z_{\alpha_2} \mid H_0\right) = 1 - G\left(z_{\alpha_2} - \frac{\lambda_2 \overline{x}}{\sigma}\right), \text{ and} \tag{4.6}$$

$$\alpha_z^{PTT} = G\left(z_{\alpha_3}\right)\left(1 - G\left(z_{\alpha_2} - \frac{\lambda_2 \overline{x}}{\sigma}\right)\right) + d_{1\rho}\left(z_{\alpha_3}, z_{\alpha_1}, \rho \neq 0\right). \tag{4.7}$$

# 5 A Simulation Example

To study the properties of the three tests we conduct a simulation study. The main aim is to compute the power function of the tests and compare them graphically. In this simulated example we generate random data using R package. The independent variable ($x$) and error ($e$) are generated from the uniform distribution with $a = 0$, and $b = 1$ and from normal distribution with $\mu = 0$ and $\sigma^2 = 1$, respectively. In each case $n = 20$ random variates were generated. The dependent variable ($y$) is then determined by $y = \beta_0 + \beta_1 x + e$ for $\beta_0 = 5$ and $\beta_1 = \pm 2.5$. For the computation of the power functions of the tests we set $\alpha_1 = \alpha_2 = \alpha_3 = \alpha = 0.05$. The graphs for the power functions and size of the three tests for known error variance are provided by using the formulas in (4.1), (4.2) (4.3), (4.5), (4.6) and (4.7). Identical graphs for the power and size curves are observed when the slope is negative.

# 6 Comparison of Power and Size

From Figure 1, as well as from equation (4.1) we see that the power of the UT does not depend on $\lambda_2$ and $\rho$ but it increases as the value of $\lambda_1$ increases. Its form is sigmoid, starting from a very small value of near zero at $\lambda_1 = 0$, it approaches 1 when $\lambda_1$ is large (about 20 in Figure 1 and 2). Thus the power of the UT changed significantly for any value of $\lambda_1$ from 0 to 20. The minimum power of the UT is around 0.05 for $\lambda_1 = 0$. The power curve of the RT is also sigmoid for all values of $\lambda_1$ and $\lambda_2$. The power of the RT increases as the values of $\lambda_1$ and/or $\lambda_2$ increase. Moreover, the power of the RT is always larger than that of the UT and PTT for all values of $\lambda_1$ and/or $\lambda_2$. The minimum power of the RT is around 0.05 for $\lambda_2 = 0$ (as well as for $\lambda_1 = 0$) and increases to be around 0.1 for $\lambda_2 = 2$. The maximum power the RT is around 1 for $\lambda_1$ around 10 or above. The power of the PTT also depends on the values of $\lambda_1$ and $\lambda_2$. Like the power of the RT, the power of the PTT increases for large value of $\lambda_1$ and tend to decrease as $\lambda_2$ grows larger. Moreover, the power of the PTT tend to be larger than that of the UT. The minimum power of the PTT is around 0.05 for $\lambda_2 = 0$ and $\lambda_1 = 0$, and it increases to be around 0.1 for $\lambda_2 = 2$. The gap between the power curves of the RT and PTT is obviously clear for all values of $\lambda_1$ and $\lambda_2$. Like the power of RT, the power of PTT depends on any values of $\lambda_1$ and $\lambda_2$.

Figure 2 or equation (4.5) shows the size of the UT does not depend on $\lambda_2$. It is constant and remains unchanged for all values of $\lambda_1$ and $\lambda_2$. The size of the RT is also constant for all values of $\lambda_1$. However, the size of the RT increases as the value of $\lambda_2$ increases. Moreover, the size of the RT is always larger than that of the UT for all values of $\lambda_2$ except for $\lambda_2 = 0$ when both tests have the same size. Like the size of the RT, the size of the PTT increases as $\lambda_2$ grows larger. The difference between the size of the RT and PTT does not change much as the value of $\lambda_2$ increases. The size of the RT and PTT increase as the value of $\lambda_2$ increases. Also, the size of the RT is larger than that of the UT and PTT except for $\lambda_2 = 0$, when they are the same.

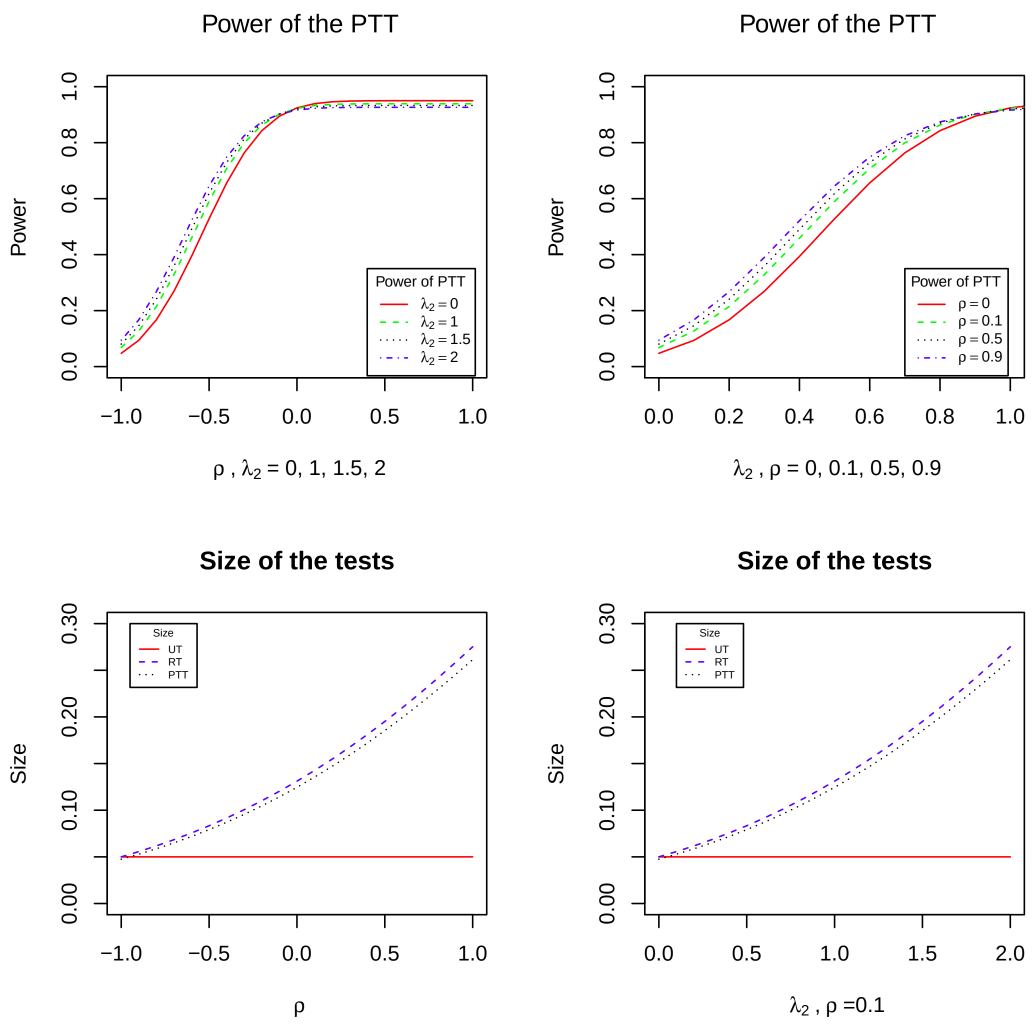


Figure 3: The power of the PTT and size against $\rho$ and $\lambda_2$

Figure 3 shows the power of the PTT depend on $\lambda_2$ and $\rho$. It increases significantly the value of $\rho$ increases from $\rho = -1$ to $\rho = 0$ and stays the same for $\rho$ from zero to 1. The difference of the power of the PTT for $\lambda_2 = 0$ and $\lambda_2 = 2$ is significantly different as the value of $\rho$ increases. From this figure or equation (4.6 and 4.7) the size of the RT and PTT depend on the value of $\lambda_2$. They increase as the value of $\lambda_2$ increases. Unlike the size of the RT and PTT, the size of the UT does not depend on the value of $\lambda_2$ and it remains constants for all the values of $\lambda_2$. The size of the RT is always greater than the size of the

UT and PTT.

# 7 Concluding Remarks

Based on of the above analyses, the power of the RT is always higher than that of the UT and PTT for all values of $\lambda_1$, and the power of the PTT lies between the power of the RT and UT for all values of $\lambda_1$, $\lambda_2$ and $\rho$. The size of the UT is smaller than that of the RT and PTT. The RT has maximum power and size, and the UT has minimum power and size. The PTT has smaller size than the RT and the RT has larger power than the UT. The PTT protects against maximum size of the RT and minimum power of the UT.

As $\lambda_2 \to 0$ the difference between the power of the PTT and RT diminishes for all values of $\lambda_1$. That its, if the NSPI is accurate the power of the PTT is about the same as that of the RT. Moreover, the power of the PTT gets closer to that of the RT as $\rho \longrightarrow 1$.

The size of the PTT becomes smaller as $\lambda_2 \to 0$. Once again if the NSPI is near accurate the size of the PTT approaches that of the UT. Therefore, we recommend PTT when the quality of the NSPI is good (i.e. $\lambda_2 \to 0$) and it performs even better than the UT and RT when $\rho \to 1$.

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