APPLYING MODERN PORTFOLIO THEORY TO THE ANALYSIS OF TERRORISM

Computing the Set of Attack Method Combinations from which the Rational Terrorist Group will choose in Order to Maximise Injuries and Fatalities

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ABSTRACT

In this paper, terrorism is analysed using the tools of modern portfolio theory. This approach permits the analysis of the returns that a terrorist group can expect from their activities as well as the risk that they face. The analysis sheds new light on the nature of the terrorist group's (attack method) choice set and the efficiency properties of that set. If terrorist groups are, on average, more risk averse, the economist can expect the terrorist group to exhibit a bias towards bombing and armed attack. Also, even the riskiest (from the terrorist group's point of view) combinations of attack methods have maximum expected returns of less than 70 injuries and fatalities per attack per year.

Key Words: Terrorism, Modern Portfolio Theory, Mean-Variance Analysis, Efficient Choice Set.

JEL Codes: D00, G11, H56

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INTRODUCTION

In this paper, one of financial economics' most important analytical frameworks is utilised in the analysis of terrorism. This analytical framework is modern portfolio theory (MPT). MPT reduces the problem of capital allocation (in an investment setting) to a mathematical process for a utility maximising economic agent during which he or she makes decisions solely on the basis of the mean return and variance of returns (risk) exhibited by particular combinations (portfolios) of assets. It is the objective of this analysis to apply these mean-variance methods to the analysis of terrorism. In so doing we can derive important results, most significantly the efficient set of combinations of terrorist attack methods that yield the maximum injuries/fatalities for any given level of risk. A utility maximising terrorist group will only choose combinations of attack methods that lie in the efficient set. Knowing what the set looks like may be useful.

"Terrorism is the premeditated use or threat of use of extra-normal violence or brutality by sub-national groups to obtain a political, religious, or ideological objective through intimidation of a huge audience, usually not directly involved with the policy making that terrorists seek to influence" (Enders and Sandler, 2002). This definition is quite similar to that formulated in various parts of the literature and by various research groups, including RAND. Whilst terrorists may appear to behave irrationally, the behaviour they exhibit is cold and calculating in many ways (Hoffman, 1998). An objective is clearly formulated and resources are deployed, subject to constraints, in order to achieve that objective. These characteristics of terrorist behaviour make the economic analysis of terrorism both possible and appropriate. Such analysis has a long history dating to at least the 1970s. During the course of this long application of economic analysis to terrorism, many different methods and tools have been deployed.

The key element of any basic economic model of terrorist behaviour is a rational actor expected utility framework where utility is derived from some variable that the terrorist or terrorist group is assumed to desire[i], usually political influence. From the axioms of expected utility theory (completeness, transitivity, continuity *etc*) a utility function is derived that expresses the terrorist group's feelings about a choice involving risk. The terrorist group is assumed to attempt to maximise the function subject to various constraints on time and resources (money, suicide bombers and materials). Even on its own, the basic prototype model yields some important results. An enhanced probability of failure or a diminished payoff decreases the utility or satisfaction that a terrorist group can expect from a particular attack. Such analysis lends support to deterrence policies that increase the costs or decrease the benefits of terrorism. This is the starting point of most economic analysis of terrorism.

Beginning from this basic expected utility framework, economists have brought to bear a wide range of analytical tools on the problem of terrorism. Econometric tools and time series techniques have been used to analyse the empirical data that has been gathered on terrorism and terrorist behaviour. For example, Enders and Sandler (1993; 2002) use vector autoregression (VAR) and threshold autoregression (TAR) to examine patterns in terrorist incidents; Im, Cauley and Sandler (1987) use spectral analysis to analyse the cyclical behaviour of terrorist activities; Cauley and Im (1988) and Brophy-Baermann and Conybeare (1994) deploy ARIMA modelling methods to investigate the effectiveness of government retaliation policies following terrorist attacks; and Bapat (2006) uses probit modelling to analyse the negotiation between states and trans-national terrorists. Some of the newest research in this area is surveyed by Enders (2007). This paper continues the research programme of analysing time series terrorism data[ii].

Another component of the economics of defence literature is that which is concerned with the impact of terrorism on the financial markets. Following the 9/11 terrorist attacks, financial markets exhibited substantial volatility. A number of researchers have examined the impact of terrorism on financial markets. Eldor and Melnick (2004) analysed the reaction of Israeli stock markets and foreign exchange markets to terror attacks. Chen and Siems (2004) utilised the event study methodology that has been used in financial economics for many years to examine the impact of terrorism on global capital markets. Interestingly, they discovered that, whilst terror attacks can significantly shock financial markets in the short term, US capital markets are able to absorb such shocks and display considerable resilience in recovering quite quickly. Finally, Drakos (2004) utilises the ‘market model’ developed in financial economics to examine the effects of the 9/11 attacks on airline stocks. Whilst there are many more studies extant in this emerging branch of the defence economics literature, these studies provide a flavour of the main trends. The present study builds on this literature by recruiting another tool of financial economics to analyse terrorism.

This paper departs from the existing literature in some important ways. First, and most importantly, we deploy mean-variance analysis to analyse time series terrorism data. Whilst the time series methods mentioned above permit the analysis of patterns, cycles and the impact of particular policies, mean-variance analysis, which proceeds from the same starting point (the basic expected utility framework), permits the derivation of the set of attack method combinations that yield the highest number of injuries and fatalities (returns to the terrorist group) for each level of risk (variance of the returns series). In some ways, this may also be viewed as carrying on the process of analysing terrorism using cost-benefit techniques, which are similar in principle to the methods used herein (see John and Rosoff (2007)). Second, whilst some studies focus on the risk of terrorism (its likelihood and consequences) from the policy making point of view (Willis, Morral, Kelly and Medby (2005)), the analysis presented herein takes the terrorist group's perspective and defines risk as the variability in the number of injuries and fatalities generated by various combinations of attack methods (returns). The more variability (variance) in this returns series, the greater the chance that the actual number of injuries and fatalities will diverge from that which was expected. This is the risk facing our terrorist group.

It may already be clear that we will assume, in our theoretical model, that the utility terrorists derive from terrorist attacks is linked to the number of injuries and fatalities generated by that attack. This assumption may be justified as follows. First, the assumption follows indirectly from our definition of terrorism. According to the definition provided above, a political (or other) objective is pursued through violent intimidation. If injuries and fatalities are an important part of this violent intimidation, as is reasonable to expect, then the utility derived from the achievement of the political, religious or ideological objective is, at least implicitly, some function of the injuries and fatalities that are generated by a particular terrorist attack. Under such circumstances, terrorists may be assumed to behave *as if* they wish to injure or kill as many individuals as possible. More injuries and fatalities mean more political, religious or ideological influence which means more utility.

Second, in a world where media attention centres upon the number of people killed and injured (not just by acts of terrorism but in accidents, murders *etc*) it seems quite sensible to assume that terrorists' utility is linked to the amount of injury and death that they can cause and, therefore, that terrorists behave as if they attempt to maximise the injuries and fatalities generated by their attacks. Third, injuries and fatalities are often given high weightings in studies that attempt to measure the objectives and the benefits to terrorists of their methods (John and Rosoff, 2007) and high weightings in studies that attempt to measure and predict the consequences of terrorism for civilians (Willis, Morral, Kelly and Medby (2005)). Fourth, Eldor and Melnick (2004) found that the number of victims killed or injured in an attack had permanent economic consequences for the stock and foreign exchange markets in Israel. Finally, assuming that terrorists behave as if they try to maximise injuries and fatalities from attacks is no less plausible than the assumption that terrorists obtain a wealth payoff from a successful attack (Enders and Sandler, 2002).

This paper is organised as follows. In the next section, an overview of modern portfolio theory and mean-variance analysis is provided. In the third section, the first pieces of the economic theoretical structure deployed in the analysis are constructed. This involves the configuration of the representative terrorist group whose activities are the focus of the investigation. In the fourth section, the mean and variance characteristics of the various terrorist attack methods identified by the RAND Corporation's MIPT Terrorism Knowledge Database are examined. In the fifth section, it is demonstrated how the utility maximising terrorist group may benefit from combining attack methods in a 'portfolio' in order to take advantage of higher returns and lower risks that result from diversification. The efficient set of attack combinations is derived. This set, from which the utility maximising terrorist group will choose, is the set of attack combinations that have the highest returns for given levels of risk. In the sixth section, the question concerning which combination of attacks the terrorist group will choose is addressed. In seventh section, some preliminary analysis is undertaken concerning the terrorist group's (dynamic) portfolio selection over time. The eighth section concludes the paper.

MEAN-VARIANCE ANALYSIS: A BRIEF OVERVIEW

The purpose of this section is to provide a brief overview of modern portfolio theory and tools. Essentially, mean-variance analysis is a method for constructing and selecting combinations (portfolios) of assets that reduces the exercise to a purely mathematical exercise. All of the facets of potential portfolios and their constituent assets are assumed to be reflected in their mean return and risk (variance of returns). Mean return is a good thing and risk is a bad thing. As such, utility maximising economic agents attempt to maximise the utility function:

$$U = f(E_R, \sigma_R) \qquad \textbf{(1)}$$

where U is the agents's total utility, E_R is the expected return of a portfolio or asset ($dU / dE_R > 0$) and σ_R is the standard deviation of the possible divergence of actual returns from expected returns ($dU / d\sigma_R < 0$) (Sharpe, 1964, p.428). Markowitz (1952) was among the first to realise that agents do not care solely about the return of a portfolio. If that were true, agents would simply choose the portfolio with the highest expected return. Rather, (risk averse) agents also care about the risk of the portfolio. Markowitz's definition of risk as the variability of returns (measured by variance or standard deviation) has long been accepted in financial economics and it does appear to capture riskiness in the sense that greater variability of returns increases the likelihood that the actual outcome earned by the agent will be different from that which was expected. Geometrically, the indifference curves that derive from this particular configuration of the (risk averse) individual's utility are concave-upwards in the expected return-risk plane. This is displayed in Figure 1.

INSERT FIGURE 1 HERE

Like most parts of economic theory, modern portfolio theory involves economic agents attempting to maximise utility by making choices. In the context of modern portfolio theory, the choice that must be made is a choice from among a set of possible portfolios. The choice set contains all possible portfolios (combinations) of risky assets that can be constructed from the risky assets in the economic system. For example, if there are only two assets, A and B, we can form various portfolios: (1) 50% in A and B; (2) 40% in A and 60% in B; (3) 30% in A and 70% in B and so on. For every possible portfolio we compute the expected return and variance. Whilst these values are *ex ante* in nature, historical (*ex post*) averages are usually used as proxies.

Formally, the expected return on a portfolio is given by:

$$E(R_P) = \sum_{i=1}^{n} w_i E(R_i) \qquad \textbf{(2)}$$

where w_i is the proportion of total investable funds in asset *i*, $E(R_i)$ is the expected return on asset *i*, and $\sum_{i=1}^{n} w_i = 1$. As mentioned above, the mean historic return is usually used as a proxy for the return that is expected in the future. The calculation of portfolio returns is quite straightforward because the portfolio return is simply the weighted average of the returns on the assets in the portfolio. The calculation of portfolio risk (variance) is not so easy. This is because the portfolio variance is not a weighted average of the variance exhibited by the returns on the assets in the portfolio. Rather, the covariances between the assets must be taken into consideration because they can potentially dampen or reinforce each other depending on the degree of negative or positive correlation they exhibit. Low or negative correlation among assets is the reason why diversification can reduce the risk (and increase the mean return) of a portfolio. Formally, the variance of a portfolio is given by:

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \rho_{ij}\sigma_i\sigma_j$$

$$= \sum_{i=1}^{n} w_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{\substack{j=1 \\ i\neq j}}^{n} w_i w_j \rho_{ij}\sigma_i\sigma_j \qquad \textbf{(3)}$$

where ρ_{ij} is the correlation coefficient that expresses the degree of correlation between asset *i* and asset *j*. It should also be noted that $\rho_{ij}\sigma_i\sigma_j$ is equal to the covariance between assets *i* and *j* (σ_{ij}). The double summation sign simply implies that all possible pairs of assets must be accounted for in the calculation.

The computation of the return and risk of all possible portfolios (combinations of assets) yields the complete opportunity set of portfolios from which the economic agent can make his or her choice. Each point in the opportunity set represents a portfolio that the economic agent could choose. The economic agents will only be interested in portfolios that lie on the upper edge of the opportunity set because these have a higher expected return for each level of risk than those portfolios that lie on the underneath edge. The choice problem that faces the economic agent can be depicted by superimposing the indifference curve map introduced earlier over the opportunity set. This is depicted in Figure 2.

INSERT FIGURE 2 HERE

The opportunity set depicted in Figure 2 along with the indifference map is called the ‘efficient set’. It is efficient because it is the set of all portfolios that have minimum variance given a particular level of expected return. It is sometimes called the efficient frontier because (like a production possibilities frontier (PPF) from economics) it is the farthest to the North West that one can go given the assets available in the economic system. The derivation of the efficient set is a quadratic programming problem:

$$\max E(R_P) = \sum_{i=1}^{n} w_i E(R_i) = R*$$

Subject to a target level of return variance (risk), σ_p^2*:

$$\sigma_p^2* = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \rho_{ij}\sigma_i\sigma_j$$

And the constraints:

$$\sum_{i=1}^{n} w_i = 1$$

$$w_i \geq 0$$

EXHIBIT 1 The Quadratic Programming Problem[iii]

Which point (portfolio) should the agent choose? The answer is: the agent should choose the portfolio that lies on the point of tangency between the efficient set and his or her indifference curve. The indifference curves show that the investor wants to be farther to the North West. The efficient set shows how far he or she can go. The point of tangency is therefore the portfolio that should be chosen.

Modern portfolio theory provides a framework for analysing the choices of (utility maximising) economic agents where such choices are made solely on the basis of two criteria: (1) the mean return; and (2) the variance of returns of the universe of portfolios available. In the following sections of this paper, the tools of modern portfolio theory are applied to the analysis of terrorism and terrorist behaviour. Specifically, mean-variance methods are deployed to analyse the return (injuries and fatalities generated by an attack) and risk (variance of the returns series) of the various types of attack methods that terrorists may choose to deploy. Significantly, mean-variance methods and quadratic programming are deployed to derive the efficient set of terrorist attack types. This permits conclusions to be reached regarding the risk and return characteristics of various terrorist 'portfolios' where methods of attack are assigned different weights. We can then see which combinations of attacks yield the highest return (injuries and fatalities) for the least risk and which combinations of attack methods would be favoured by more risk-averse terrorists and which would be favoured by less risk-averse terrorists.

THE REPRESENTATIVE TERRORIST GROUP

Imagine a world inhabited by a representative terrorist group that acts as a stand in for all the terrorist groups. The representative group is our theoretical portfolio builder and may be best seen as a planner responsible for allocating resources (money, materials, suicide bombers *etc*) in an attempt to maximise a utility function $U(Z)$. The group attempts to gain utility by obtaining political, religious and ideological influence through violent intimidation. It does so by combining different attack methods in a 'portfolio' in order to injure and kill as many individuals as possible. Letting Z denote the economic (utility giving) good[iv] that the terrorist group expects to gain from the choice to undertake an act of terrorism, let us make some assumptions and provide some definitions:

Assumption 1 The axioms of expected utility theory or rational choice (completeness, transitivity and continuity *etc*)[v] apply to terrorist behaviour.

Assumption 2 The economic good Z is solely a function of the expected return and risk (variance) associated with particular combinations (portfolios) of terrorist attack methods.

Definition 1 The *risk* (from the terrorist group's point of view) of a terrorist attack is the standard deviation of the possible divergence of actual returns from expected returns. In practice, *ex post* standard deviation or variance is a proxy for risk. More volatility means a greater chance of an unexpected outcome.

Definition 2 The *expected return* of a terrorist attack is the number of injuries and fatalities that the terrorist group expects to result from the attack. In practice, the *ex post* average return is a proxy for expected return.

These assumptions and the accompanying definitions ensure that it is possible to write a quadratic utility function (which is the type of utility function that permits an agent's first order conditions for utility maximisation to be expressed in terms of mean and variance) for terrorist group as follows:

$$U = (1+b)E_R + bE_R{}^2 - c\sigma_R{}^2 \quad \textbf{(4)}$$

where E_R and $\sigma_R{}^2$ denote expected return and variance of return respectively, *b* is a parameter that adheres to the restriction – 1 < *b* < 0 when terrorists are risk averse and *c* is a parameter that adheres to the restriction $0 < c < 1$ (see Tobin (1958, p.76)). In essence, this is an extension of the standard expected utility rational actor model of terrorist behaviour that has been deployed in the economics of defence literature for many years (see Sandler and Hartley (1995) and Sandler and Enders (2004)).

This characterisation of the terrorist group's utility function gives rise to the indifference curves of Figure 1 (above). This theoretical framework that we have constructed consists of a representative terrorist group (a theoretical planner or portfolio builder) that pursues an economic good through the execution of terrorist attacks. The utility derived from this activity is solely a function of some economic good that the terrorist group desires, Z, which is, in turn, a function of the expected return (expected injuries and fatalities) and risk (variance of returns) of the particular type of attack method that is deployed. The terrorist group, therefore, seeks to maximise utility by making choices between different combinations of possible attack methods on the basis of expected return and risk. This framework permits the mean-variance analysis of terrorism and terrorist behaviour and allows us to determine the combinations of attack methods that yield the highest number of expected injuries and fatalities for any given level of risk.

THE RISK AND RETURN OF TERRORIST ATTACK METHODS

In this section, the risk and return characteristics of various terrorist attack methods are analysed before the 'portfolios' (combinations of attack methods) are constructed and analysed in the following section. The data used in this analysis are data that have been collected in the MIPT Terrorism Knowledge Base database under the auspices of the RAND Corporation. The data collected in this database are extremely detailed and cover the thirty-nine year period between 1968 and 2007. The statistical data is categorised by terrorist tactic and includes the number of attacks per year and the number of injuries and fatalities that the attacks caused. Before 1997, only international attacks (where a terrorist crosses a border to undertake an attack, attacks the domestic interests of a foreign entity or stages an attack on an airline) were recorded in the database. Post 1997, both international and domestic are recorded.

These data are used to compute the time series of returns (injuries and fatalities) for various attack methods and their risks (variance in the returns series). We do this for individual attacks in this section and for combinations of attacks in the next section. The rationale is as follows. Essentially, our representative terrorist group forms expectations about the likely return and risk (variance) for a particular attack or combination of attacks on the basis of the injuries and fatalities and the variability of the injuries and fatalities exhibited by the historical times series. As explained above, historical mean returns and variance are proxies for the expected return and risk of planned attacks. We can imagine, therefore, our representative group using the historical time series data analysed herein to form expectations about the likely return and risk of attack methods and combinations of attack methods that it plans to use in the future. The data for 2001 are provided in Table 1 as an example.

INSERT TABLE 1 HERE

Using the complete data set for 1968 to 2007, a series of returns[vi] generated by each type of attack was generated. This returns series, which forms the basis for the mean-variance analysis, is presented in Table 2.

INSERT TABLE 2 HERE

The interesting feature of the returns series is that the injuries and fatalities generated per attack per year are in most cases quite low with significant 'spikes' occurring relatively infrequently. For example, the returns generated by armed attacks and unconventional attacks are quite low but on two occasions (1995 for armed attacks and 2001 for unconventional attacks) there were significant increases in injuries and fatalities per armed and unconventional attack. The returns series for bombing attacks is quite volatile which indicates that such activity, whilst usually generating steady, low returns, can occasionally deliver substantially higher outcomes. On the other hand, the returns series for assassination, kidnapping and arson are, by and large, relatively stable and yield consistent but low returns for the terrorist involved in such activities. The characteristics of the returns series are displayed in Figures 3 and 4.

INSERT FIGURES 3 AND 4 HERE

The representative terrorist group that seeks to maximise its utility will, according to the theoretical framework introduced in the previous section, not only be interested in the expected return (average return) of a particular type of attack but also its risk (variance of returns). The charts presented above already reveal some of the mean-variance characteristics of terrorist attack methods. Precise calculation yields summary statistics that highlight the risk and return characteristics of particular attack types. These are displayed in Table 3.

INSERT TABLE 3 HERE

Not surprisingly, a higher expected return (higher expected injury and fatality) is usually accompanied by a higher risk (variance of the returns series). In the pursuit of an economic good, the terrorist group finds that there is no free lunch. By and large, the terrorist group that wishes to achieve a higher return from its activity must bear the cost of higher risk and accept the higher probability of a divergence of the actual outcome of the terrorist incident from the outcome that was expected. However, the terrorist group might not be interested in one particular type of attack. Rather, it might decide to 'diversify' and utilise a combination (portfolio) of attack methods. Indeed, modern portfolio theory implies that diversification can reduce the risk and increase the expected return of a portfolio. Our utility-maximising representative terrorist group could therefore be expected to combine attack types. In the following section, we analyse the return and risk characteristics of combinations (portfolios) of attack methods.

MEAN-VARIANCE ANALYSIS OF COMBINATIONS OF ATTACK TYPES

The terrorist group can combine different attack methods into a 'portfolio' and, in so doing, can reduce its exposure to risk (the variance of the returns) whilst at the same time increasing expected returns (the expected injuries and fatalities from the combination of attack methods). The reason why this approach works is that the time series of injuries and fatalities generated by the various terrorist attack methods do not exhibit perfect correlation. There is, therefore, the opportunity for the utility maximising representative terrorist group to reduce risk by combining different attack methods in an overall terrorist portfolio. For example, for the period 1968 to 2007, armed attacks exhibit negative covariance *vis-à-vis* hijacking; bombing exhibited negative covariance *vis-à-vis* unconventional methods; and armed attacks exhibited negative covariance *vis-à-vis* assassination. Combining such methods dampens the variance exhibited by individual methods and yields a 'portfolio' with a lower risk than an individual method taken on its own. The covariance between each possible pair of attack methods can be displayed in the (non-weighted) covariance matrix Table 4.

INSERT TABLE 4 HERE

The non-weighted covariance matrix presents the results of a covariance calculation for each possible pair of attack types. The values along the main diagonal are, of course, simply the variances of the particular attack types. The other values in the remaining cells of the matrix are the covariances between the various pairs of attack methods. Combined into a 'portfolio' the risk (variance) associated with the various individual terrorist attack methods may dampen the overall level of risk faced by the representative terrorist group. The variances and covariances in Table 4 are used, in conjunction with predefined weights w_i, to compute the risk (variance) associated with a combination (portfolio) of terrorist attack methods. That is, the result of Equation 3 (above). For example, an equal weighted combination consisting of a devotion of 10 per cent of time and resources to each attack method yields the weighted covariance matrix presented in Table 5.

INSERT TABLE 5 HERE

This equal weighted combination of terrorist attack methods (where each attack method represents 10 per cent of the overall portfolio) has an expected return of 15.55022 injuries and fatalities per attack in a given year and a risk (standard deviation) of 5.958 injuries and fatalities per attack in a given year (computed using Equations 2 and 3). This is a much higher expected (average) return than any of the terrorist attack methods undertaken individually and a much lower risk than that exhibited by particular individual attack types. This indicates that the economist may expect our representative terrorist group to diversify among attack types and select a combination of methods that maximises its utility given its particular preferences for risk. Since particular combinations will dominate (possess a higher expected return for a given level of risk) other combinations, the utility maximising terrorist group will select from the set of efficient combinations (in the sense of Figure 2).

THE TERRORIST GROUP’S EFFICIENT SET

The computation of the efficient set of terrorist attack combinations is an exercise in quadratic programming, involving the solution of the quadratic programming problem presented in Exhibit 1. This is undertaken as follows: (1) form random combinations (portfolios) of terrorist attack methods by randomly assigning weights to the ten attack types (adhering to the constraint that the weights are positive and sum to one); (2) compute the expected return and variance for each randomly generated portfolio; and (3) solve the quadratic programming problem that results by maximising the expected return subject to the constraints on the weighting scheme and the restriction that the variance be equal to the randomly generated portfolio. The outcome is a set of weights for an efficient portfolio yielding a higher expected return than the random portfolio but having equal variance. Undertaking this exercise many times will permit the derivation of the efficient set.

In order to derive the terrorist group’s efficient set, the procedure outlined in the previous paragraph was undertaken fifty (50) times. That is, fifty randomly assigned weighting schemes were generated and the resulting portfolio expected return and risk calculated. For each portfolio the quadratic programming problem was solved. The geometrical representation of the efficient set is derived by plotting the expected return and standard deviation of the efficient combinations in expected return-risk space. Of course, one could derive and plot the entire opportunity set (including inefficient combinations) but, since utility maximising agents are only interested in the efficient set, that is all that need concern us here. This is displayed in Figure 5.

INSERT FIGURE 5 HERE

The terrorist group's efficient set is analogous to the efficient set described by modern portfolio theory. All combinations of terrorist attack methods that lie on the efficient frontier dominate portfolios that lie in the interior of the choice set. The efficient portfolios have higher expected injuries and fatalities (returns) for each level of risk (variance of the returns series) than their inefficient counterparts. Interestingly, even the riskiest combinations yield a maximum expected return (injuries and fatalities per incident per year) of less than 70 injuries and fatalities per incident per year. This is probably much lower than that which would have been expected by most scholars and the computation of the efficient frontier yields us this piece of information.

WHICH EFFICIENT COMBINATION WILL THE TERRORIST GROUP CHOOSE?

The question that is of obvious importance to governments and policymakers concerns which efficient combination the terrorist group will choose. It is possible to make a number of predictions regarding the behaviour of our representative terrorist group. First and foremost, it is possible to identify which combinations of attack methods will be favoured by less risk-averse groups and which combinations of attack methods will be favoured by more risk-averse groups. Combined with further information concerning the risk aversion coefficients of terrorists, this may one day prove to be a very useful policy tool. For now, however, we simply identify a few combinations that may be favoured by (1) a more risk-averse terrorist group; and (2) a less risk-averse terrorist group. Specifically, more risk-averse groups will prefer combinations that lie on the lower (South West) portion of the efficient set and less risk-averse groups will prefer combinations that lie on the upper (North East) portion of the efficient set. This is displayed in Figure 6.

INSERT FIGURE 6 HERE

An important question concerns the types of attacks that may constitute the portfolio of a more risk-averse terrorist group and the types of attacks that may constitute a less risk-averse terrorist group. These are depicted in Figure 7. Figure 7 may be used to examine the characteristics of portfolios that may be chosen by our utility maximising terrorist group for different levels of risk aversion.

INSERT FIGURE 7 HERE

The portfolios identified are in Figure 7 are just two of the infinite number of possible portfolios that may be chosen. Each point on the efficient set represents a different portfolio and not every portfolio contains every attack method. However, on average, there appears to be a tendency toward a preference for bombing and armed attack in the lower risk portion of the efficient set and a tendency toward a preference for unconventional attack methods in the higher risk portion of the efficient set. On average, higher *expected* returns will be derived from undertaking more risky attack methods. However, the actual returns generated by these risky attack methods may vary considerably from the return the terrorist group expected. If terrorist groups are, on average, more averse to risk, the economist can expect a bias towards bombing and armed attack to be exhibited in the terrorist group's behaviour. This is our only clear result here.

Each combination of attack methods in the efficient set may potentially be chosen by our utility maximising terrorist group. The negative covariance exhibited between the injuries and fatalities generated by various attack methods provides the benefits from diversification that we expect our representative group to exploit. This diversification may lead our terrorist group to devote resources to attack methods that would otherwise seem unusual (such as hijacking in a world with high airport security). Here we must understand that (1) the negative covariance that hijacking exhibits *vis-à-vis* other methods (see Table 4) means that its inclusion in the terrorist group's portfolio reduces the variance of the portfolio's *expected* returns; (2) as the activity becomes more difficult, its expected return increases to compensate for the higher risk[vii]. This is a product of the ex ante nature of this analysis. A particular attack method may seem impossible or implausible to policy makers but a terrorist group making a risk/return trade-off may still devote some resources to it if the expected return compensates for the higher risk. Hijacking (or any other attack method) may be included (as depicted in Figure 7) in an efficient portfolio and any of the efficient portfolios might be chosen by the rational utility maximising agent. Our main conclusion here remains that, if terrorist groups are, on average, more averse to risk, the economist can expect a bias towards bombing and armed attack to be exhibited in the terrorist group's behaviour.

THE DYNAMIC BEHAVIOUR OF OUR REPRESENTATIVE GROUP

Mean-variance analysis is one component of modern portfolio theory. Other components of this body of theoretical work may be deployed in the analysis of terrorism and terrorist behaviour. The theoretical work that has been undertaken on the optimality conditions for portfolio selection under different circumstances may be of some use in developing a purely theoretical response to questions concerning the dynamic behaviour of the terrorist group and how it adjusts its portfolio over time. Whilst a complete theoretical investigation is left for future research, it is possible to identify the behaviour a utility maximising terrorist group might exhibit under certain circumstances in its selection of combinations of attack types.

Terrorist groups may have different types of risk aversion. The type of risk aversion exhibited by the group influences its preferences for more risky attack methods as its stock of the economic good Z changes (see above). There are two ways in which preferences for risky attack methods may be described: (1) absolutely; or (2) relatively. Agents may exhibit increasing, constant or decreasing absolute risk aversion. An agent who exhibits increasing, decreasing or constant absolute risk aversion will devote less, more or the same amount of resources to more risky attack methods as its stock of Z increases. Similarly, agents may exhibit increasing, constant or decreasing relative risk aversion. This will decide the *percentage* of their resources they invest in more risky attack methods as their stock of Z changes. An agent who exhibits increasing, decreasing or constant relative risk aversion will devote a lesser, greater or unchanged percentage of resources to more risky attack methods as its stock of Z increases.

The configuration of terrorist groups' utility functions is important and relevant to the task of constructing optimal combinations of attack methods. Within the context of theoretical financial economics, Merton (1969) derived the 'optimality equations for a multi-asset problem' in a continuous time setting where rates of return are generated by a Wiener process. Analogically, for the case where terrorist groups exhibit constant relative risk aversion (CRRA)—that is, the percentage of resources devoted to more risky attack methods remains constant as the group's stock of Z changes—the optimal consumption (of Z) and portfolio selection rules follow from the solution of the system of nonlinear partial differential equations (Merton, 1969, p.250):

$$
\left\{
\begin{array}{l}
0 = \left[\frac{(1-\gamma)}{\gamma} \frac{\partial I_t}{\partial Z} \right]^{\gamma/\gamma-1} e^{-\rho t/1-\gamma} \\
\quad + \frac{\partial I_t}{\partial_t} + \frac{\partial I_t}{\partial Z} rZ \\
\quad - \frac{(\alpha - r)^2}{2\sigma^2} \frac{[\partial I_t / \partial Z]^2}{\partial^2 I_t / \partial Z^2} \\
\\
C(t) = \left[e^{\rho t} \frac{\partial I_t}{\partial Z} \right]^{1/\gamma-1} \\
\\
w(t) = \frac{-(\alpha - r)\partial I_t / \partial Z}{\sigma^2 Z \partial^2 I_t / \partial Z^2}
\end{array}
\right\} \quad \textbf{(5)}
$$

subject to $I[Z(T),T] = \varepsilon^{1-\gamma} e^{-\rho t}$ and $[Z(T)]\gamma/\gamma, \text{for } 0 < \varepsilon << 1$ where $Z(t)$ is total stock of the terroristic agent's economic good at time t, $t \in [0,T]$, $C(t)$ is consumption per unit of time, $w_i(t)$ is the proportion of total resources devoted to the i^{th} attack method at time t, $I_t \equiv I[Z(t),t]$, r is the return on a 'sure' asset and $1-\gamma$ is Pratt's measure of relative risk aversion, $R_R(Z) = -Z \frac{U''(Z)}{U'(Z)}$.

For terrorist groups with constant relative risk aversion, it is optimal to follow a time-invariant portfolio selection policy with frequent 'rebalancing' of the portfolio after fluctuations in the returns series[viii]. If, for example, the weighting scheme of the terrorist group's portfolio is characterised by 51% hostage, 47% unconventional and 2% armed attacks, the weighting of unconventional attack methods would be reduced following a period where such attacks were particularly successful (because after such a period, more than 47% of total resources would be devoted to unconventional attack methods). Being time-invariant, the policy is followed regardless of the terrorist group's time horizon (Browne, Milevsky and Salisbury, 2003). The economist would expect, therefore, to find a period characterised by the successful deployment of particular attack methods to be followed by a period where such attack methods are less prevalent.

CONCLUSIONS

In this paper, the tools of modern portfolio theory were applied to an analysis of terrorism and terrorist behaviour. Our representative terrorist group pursues some objective, Z, through violent intimidation. The utility derived from this activity is solely a function of some economic good that the terrorist group desires, Z, which is, in turn, a function of the expected return (expected injuries and fatalities) and risk (variance of returns) of the particular type of attack method that is deployed. The terrorist group, therefore, seeks to maximise utility by making choices between different combinations of possible attack methods on the basis of its expected return and risk. This framework permitted the mean-variance analysis of terrorism and terrorist behaviour and allowed us to determine the combinations of attack methods that yield the highest number of expected injuries and fatalities for any given level of risk. The main results and predictions obtained from the analysis presented in this paper may be stated:

1. Armed attacks, barricade/hostage, bombing and unconventional attack methods exhibit the highest expected returns but also exhibit considerably higher risk than most of the other attack methods that the terrorist group may choose from.

2. Hijacking currently has a low expected return relative to armed attacks, barricade/hostage, bombing and unconventional attacks. However, it has a reasonably high risk for the terrorist group (twice as risky as bombing). Presently, hijackings would not appeal to our (risk averse) terrorist group except as a means to dampen the risk of the overall portfolio. This will remain the case until expected returns to hijackings increase or the risk (variance) declines. Of course, a risk seeking agent may be especially attracted to hijacking in the present circumstances.

3. The combination of attack types in a ‘portfolio’ reduces the terrorist group’s risk whilst increasing his or her expected returns. This demonstrates that terrorism is not exempt from benefits derived from diversification.

4. Utility maximising terrorist groups may be expected to choose combinations of attack methods that lie in the efficient set. The identification of the efficient set by this analysis provides the combinations of attack methods that yield the highest expected return for a given level of risk.

5. The exact choice of portfolio depends upon the terrorist group’s risk aversion. More risk-averse groups will choose portfolios more heavily weighted towards armed attacks and

bombing whilst less risk-averse groups will choose portfolios more heavily weighted towards barricade/hostage and unconventional methods.

6. Even the riskiest combinations of attack methods have an expected return of less than 70 injuries and fatalities per attack per year. This result may be surprising to some analysts. The mean-variance method provides this piece of information.

7. If terrorist groups exhibit constant relative risk aversion (CRRA), the optimal portfolio selection and consumption conditions imply a constant proportional strategy where the group re-weights the portfolio following fluctuations in returns. The key prediction that results from this part of the analysis is that CRRA terrorist groups will devote fewer resources to a particular method following the successful deployment of that method.

The predictions generated by the analysis are, at least in principle, empirically testable. It would appear most appropriate to state, in conclusion, that the most important task facing the economist who wishes to analyse terrorist behaviour is the determination of the exact properties of terrorist agents' utility functions, particularly the level and type of risk aversion these individuals exhibit.

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[i] See Anderton and Carter (2005; 2006) for a discussion of the utility maximisation model applied to terrorism.

[ii] And the application of analytical tools developed in finance or financial economics to the analysis of terrorism and terrorist behaviour (see Phillips (2005)). Although of peripheral relevance, this paper also adds to the literature on price and substitution effects (Faria (2006); Frey and Luechinger (2003)).

[iii] *Source*: Strong, R. 2006, p.155.

[iv] An economic good is something that is desired but which costs something to obtain. In this case, it is usually assumed to be political power and resources or a monetary equivalent.

[v] Completeness: given situations A and B an individual can always specify if A is preferred to B, B is preferred to A or A and B are equally attractive; Transitivity: If A is preferred to B and B to C, then A is preferred to C; Continuity: if A is preferred to B then a situation suitably close to A is preferred to B.

[vi] Return on Attack Type $i = R_i = \frac{(\text{injuries} + \text{fatalities})}{\text{incidents}}$

[vii] Also see Phillips (2005) and, for different perspective, Faria (2006).

[viii] Assuming that successful (unsuccessful) attacks increase (decrease) the terrorist's resources and that these resources are 'reinvested' in the same successful (unsuccessful) attack methods. See Enders and Sandler (2002).

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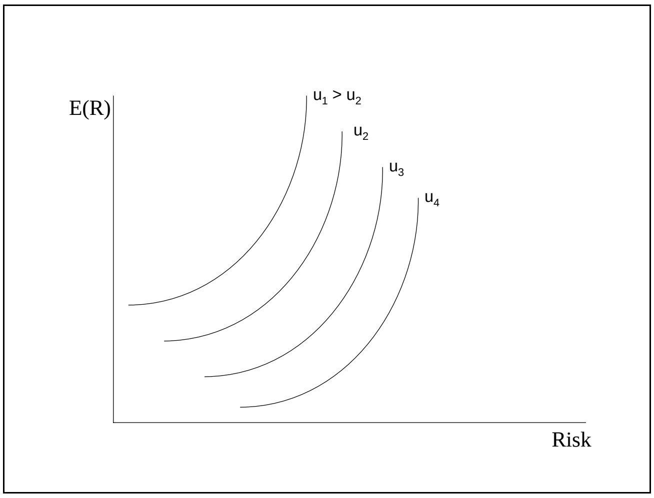

FIG. 1. Risk Averse Indifference Curves

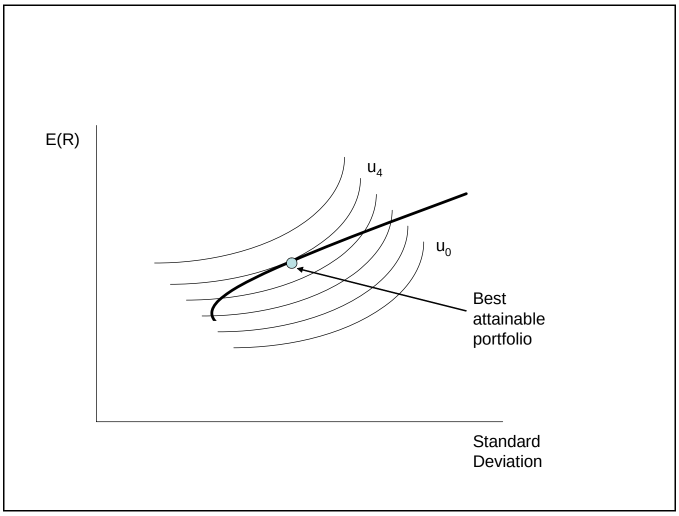

FIG. 2. Optimal Portfolio Choice

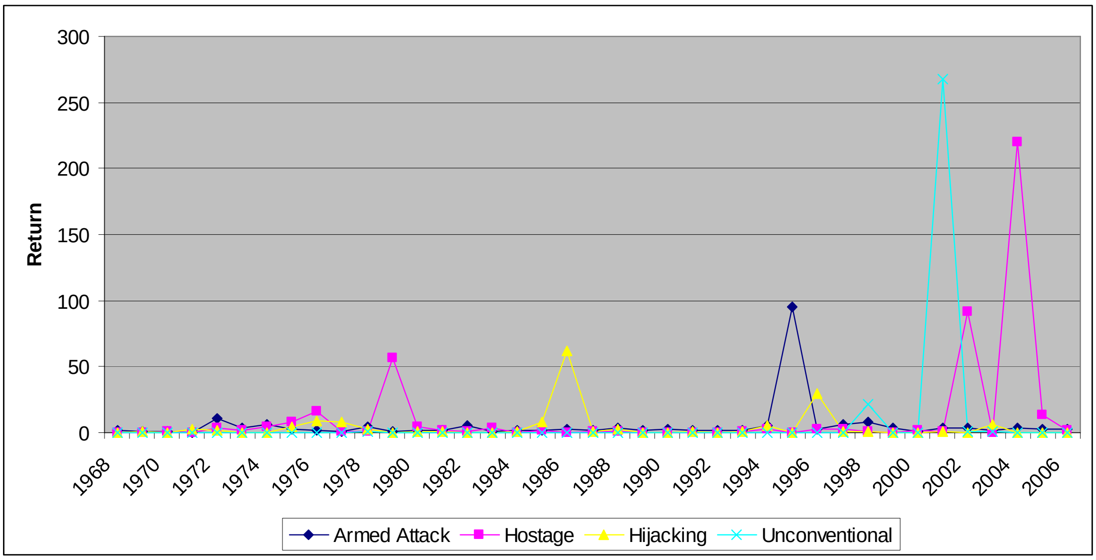

FIG. 3. Returns: Armed Attack, Barricade/Hostage, Hijacking, Unconventional

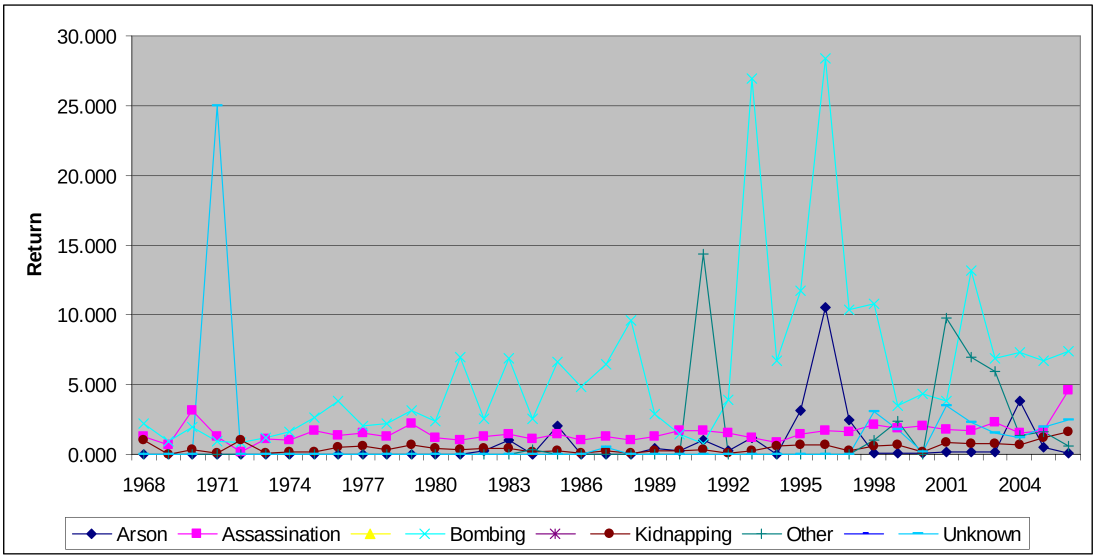

FIG. 4. Returns: Arson, Assassination, Bombing, Kidnapping, Other, Unknown

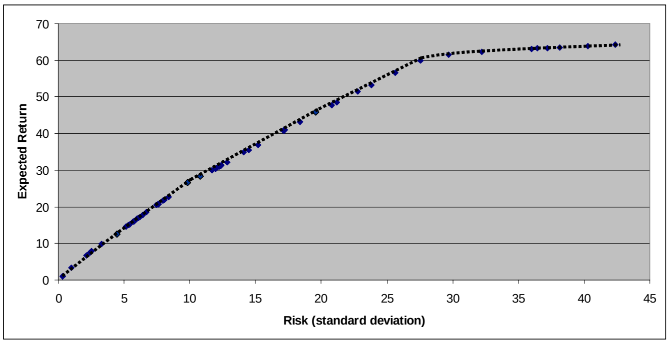

FIG. 5. Terrorist Group's Efficient Set

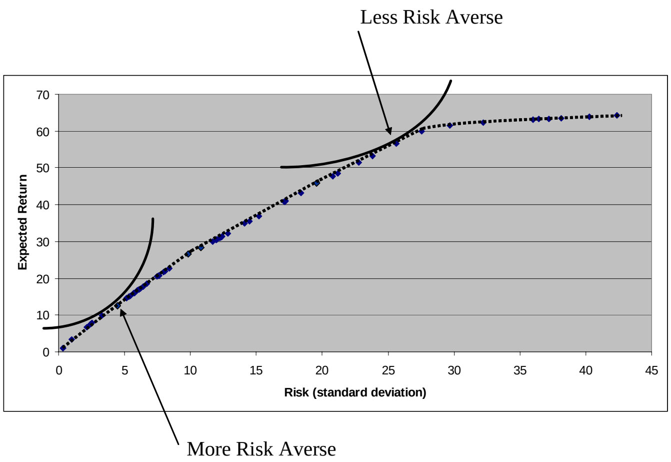

FIG. 6. Choices of Terrorist Groups

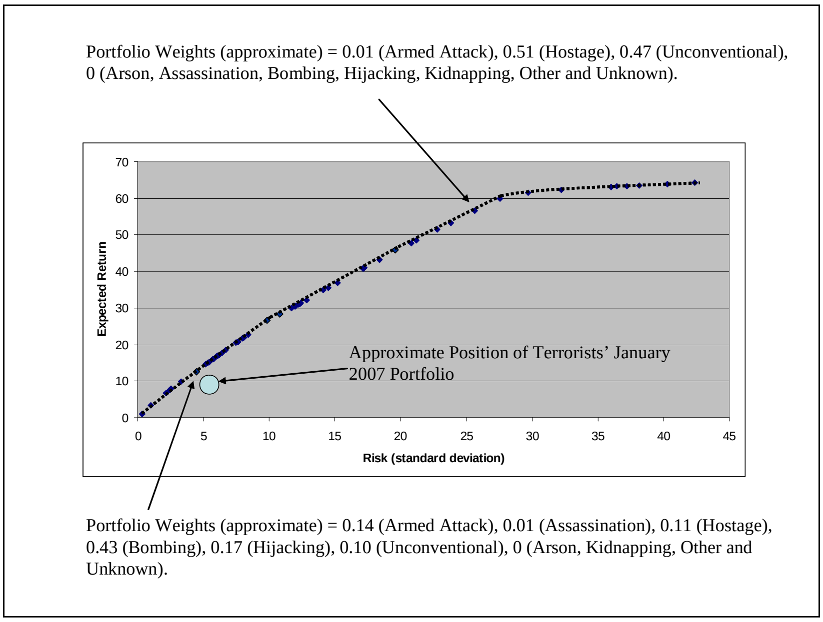

FIG. 7. Portfolio Constituents: More and Less Risk Averse[ix]

TABLE 1. Terrorist Incidents by Tactic 01/01/2001 to 01/01/2002[x]

TACTIC	INCIDENTS	INJURIES	FATALITIES
Armed Attack	294	509	424
Arson	86	14	1
Assassination	174	70	234
Barricade/Hostage	5	0	5
Bombing	1035	3239	762
Hijacking	4	1	3
Kidnapping	69	7	50
Other	21	194	11
Unconventional	20	2360	2999
Unknown	32	21	90
TOTAL	**1740**	**6415**	**4579**

TABLE 2. Returns Generated by Terrorist Attacks 01/01/1968 to 01/01/2007

	Armed Attack	Arson	Assassination	Hostage	Bombing	Hijacking	Kidnapping	Other	Unconventional	Unknown
1968	1.714	0.000	1.250	0.000	2.238	0.000	1.000	0.000	0.000	0.000
1969	1.125	0.000	0.667	0.000	0.962	0.600	0.000	0.000	0.000	0.000
1970	0.588	0.000	3.125	0.500	1.913	0.105	0.308	0.000	0.000	0.000
1971	0.300	0.000	1.250	0.000	0.906	2.429	0.125	0.000	0.000	25.000
1972	11.200	0.000	0.182	3.667	0.833	1.643	1.000	0.000	0.000	0.000
1973	3.938	0.000	1.071	2.000	1.213	0.333	0.100	0.000	0.000	0.000
1974	6.000	0.000	1.000	4.500	1.655	0.286	0.167	0.000	0.000	0.000
1975	3.118	0.000	1.667	7.818	2.612	4.500	0.167	0.000	0.000	0.000
1976	1.381	0.000	1.344	16.000	3.794	9.250	0.526	0.000	0.000	0.000
1977	0.950	0.000	1.524	0.750	2.013	7.769	0.600	0.000	0.000	0.000
1978	4.425	0.000	1.263	1.273	2.217	3.000	0.313	0.000	1.000	0.000
1979	1.243	0.000	2.231	56.615	3.159	0.000	0.692	0.000	0.000	0.000
1980	1.577	0.000	1.175	4.692	2.348	1.333	0.429	0.000	0.000	0.000
1981	1.545	0.000	1.057	1.429	6.984	1.182	0.353	0.000	0.000	0.000
1982	5.778	0.250	1.273	0.833	2.509	0.125	0.385	0.000	0.000	0.000
1983	0.667	1.000	1.407	4.000	6.869	0.000	0.417	0.000	0.000	0.000
1984	1.667	0.000	1.130	0.000	2.560	0.556	0.135	0.333	0.000	0.000
1985	1.615	2.000	1.444	1.667	6.639	8.000	0.220	0.000	0.000	0.000
1986	2.756	0.000	1.050	0.000	4.841	62.000	0.065	0.000	0.000	0.000
1987	1.940	0.000	1.259	0.571	6.435	0.667	0.273	0.000	0.000	0.500
1988	3.147	0.000	1.056	0.600	9.642	2.250	0.063	0.000	0.000	0.000
1989	2.000	0.385	1.304	0.000	2.900	0.000	0.267	0.000	0.000	0.000
1990	2.860	0.231	1.720	0.000	1.481	0.000	0.278	0.000	0.000	0.000
1991	1.571	1.000	1.682	0.000	0.725	0.833	0.350	14.333	0.000	0.000
1992	1.960	0.235	1.538	0.000	3.875	0.000	0.120	0.000	0.000	0.000
1993	2.053	1.167	1.200	0.857	26.944	1.286	0.231	0.000	0.000	0.000
1994	5.404	0.000	0.889	2.333	6.746	5.750	0.615	0.000	0.000	0.000
1995	95.630	3.182	1.471	0.000	11.714	0.000	0.639	0.000	0.000	0.000
1996	3.022	10.500	1.692	2.833	28.421	29.333	0.647	0.000	0.000	0.000
1997	6.088	2.500	1.600	3.000	10.337	0.500	0.255	0.000	0.000	0.000
1998	7.871	0.086	2.149	0.727	10.783	0.600	0.560	1.000	21.333	3.091
1999	3.477	0.049	1.905	0.100	3.505	0.333	0.695	2.400	0.000	1.750
2000	0.971	0.076	2.080	1.833	4.331	0.000	0.162	0.000	0.000	0.182
2001	3.173	0.174	1.747	1.000	3.866	1.000	0.826	9.762	267.950	3.469
2002	3.273	0.192	1.688	91.556	13.197	0.000	0.731	7.000	0.667	2.292
2003	1.832	0.141	2.255	0.000	6.854	6.667	0.737	5.929	0.857	1.500
2004	3.787	3.854	1.492	219.800	7.318	0.000	0.665	1.333	0.000	1.225
2005	2.821	0.500	1.741	13.800	6.732	0.000	1.208	1.595	0.000	1.920
2006	2.943	0.098	4.600	2.000	7.414	0.000	1.601	0.600	0.000	2.474

Notes: Using the MIPT Terrorism Knowledge Database the returns presented in Table 2 were computed for the period 1968 to 2007 by computing the result of the formula: $\text{Return on Attack Type } i = R_i = (\text{injuries} + \text{fatalities}) / \text{incidents}$. In effect, this generates a data series that tells us the injuries and fatalities per type of attack per year.

TABLE 3. Annual Risk and Return Characteristics of Attack Methods

TACTIC	EXPECTED (AVERAGE) RETURN	VARIANCE	STANDARD DEVIATION
Armed Attack	5.318	219.25	14.80
Arson	0.708	3.33	1.82
Assassination	1.543	0.488	0.698
Barricade/Hostage	11.455	1420.66	37.69
Bombing	5.884	36.10	6.00
Hijacking	3.906	114.06	10.67
Kidnapping	0.46	0.1176	0.3429
Other	1.136	8.875	2.97
Unconventional	7.482	1796.69	42.38
Unknown	1.113	15.90	3.98

TABLE 4. (Non-weighted) Covariance Matrix 01/01/1968 to 01/01/2007

Incidents	*Armed Attack*	*Arson*	*Assassination*	*Hostage*	*Bombing*	*Hijacking*	*Kidnapping*	*Other*	*Unconventional*	*Unknown*
Armed Attack	219.2502449									
Arson	6.005033477	3.332313								
Assassination	-0.522198862	0.031811	0.4884912							
Hostage	-23.70661276	15.42445	0.931608855	1420.669						
Bombing	14.38596215	7.492671	0.37337457	20.90283	36.10422365					
Hijacking	-10.10465451	5.664612	-0.874681669	-36.4207	13.85873807	114.0609				
Kidnapping	0.589207485	0.060136	0.11379656	2.386932	0.249632827	-0.51154	0.117689522			
Other	-3.081055916	-0.1782	0.324107298	11.81008	-0.88350004	-2.82455	0.211885121	8.875704		
Unconventional	-13.47359975	-4.04685	1.744647303	-76.8449	-11.13636209	-21.8026	2.580178194	59.37007	1796.695469	
Unknown	-3.927414343	-0.59467	0.131404687	0.51013	-2.438528886	-2.37358	-0.024761939	0.589395	17.26820766	15.90871

TABLE 5. Equal Weighted Covariance Matrix 01/01/1968 to 01/01/2007

Incidents	*Armed Attack*	*Arson*	*Assassination*	*Hostage*	*Bombing*	*Hijacking*	*Kidnapping*	*Other*	*Unconventional*	*Unknown*
Armed Attack	2.192502449									
Arson	0.060050335	0.033323								
Assassination	-0.005221989	0.000318	0.004884912							
Hostage	-0.237066128	0.154244	0.009316089	14.20669						
Bombing	0.143859622	0.074927	0.003733746	0.209028	0.361042236					
Hijacking	-0.101046545	0.056646	-0.008746817	-0.36421	0.138587381	1.140609				
Kidnapping	0.005892075	0.000601	0.001137966	0.023869	0.002496328	-0.00512	0.001176895			
Other	-0.030810559	-0.00178	0.003241073	0.118101	-0.008835	-0.02825	0.002118851	0.088757		
Unconventional	-0.134735998	-0.04047	0.017446473	-0.76845	-0.111363621	-0.21803	0.025801782	0.593701	17.96695469	
Unknown	-0.039274143	-0.00595	0.001314047	0.005101	-0.024385289	-0.02374	-0.000247619	0.005894	0.172682077	0.159087

FIGURE CAPTIONS

FIG. 1. Risk Averse Indifference Curves

FIG. 2. Optimal Portfolio Choice

FIG. 3. Returns: Armed Attack, Barricade/Hostage, Hijacking, Unconventional

FIG. 4. Returns: Arson, Assassination, Bombing, Kidnapping, Other, Unknown

FIG. 5. Terrorist Group's Efficient Set

FIG. 6. Choices of Terrorist Groups

FIG. 7. Portfolio Constituents: More and Less Risk Averse

[ix] This compares to actual weights computed in January 2007 as follows: 0.39 (Armed Attacks), 0.015 (Arson), 0.013 (Assassination), 0.0008 (Hostage), 0.500 (Bombing), 0.00 (Hi-Jacking), 0.0544 (Kidnapping), 0.00085 (Other), 0.00016 (Unconventional), 0.012 (Unknown). **This is a combination with an expected return of 9.55 and a standard deviation (risk) 6.73.**

[x] *Source*: RAND Corporation MIPT Terrorism Knowledge Database