Ponder this!

Problem set 9

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As usual, the purpose of this section is to supply teachers and students with a selection of interesting problems. In this issue we invite readers to deal with determinants that remain a core topic of the first course on linear algebra at the undergraduate level. The structure and beauty of determinants have been widely recognised for a long time—"it is difficult to imagine a more fundamental single scalar to associate with a square matrix" (Carlson, Johnson, Lay, & Porter, 2002, p. 25). It was Leibnitz who came to the idea of determinant and gave the first formalisation while solving systems of linear equations. The history of determinants is given in the extensive monograph by Muir (2011/1923). The problems below first appeared in Proskuryakov (1978) and present good examples where the concrete nature of determinants still requires non-standard approaches to come to the solution.

1. Find the maximum value of the 3rd order determinant, if its entries are either 1 or –1.
2. Find the maximum value of the 3rd order determinant, if its entries are either 1 or 0.
3. Find the following nth order determinant:

$$\begin{vmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 0 & 1 & \dots & 1 \\ 1 & 1 & 0 & \dots & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 1 & 1 & \dots & 0 \end{vmatrix}$$

4. Show that the following nth order determinant is the nth Fibonacci number

$$\begin{vmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 \\ -1 & 1 & 1 & 0 & \dots & 0 & 0 \\ 0 & -1 & 1 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & -1 & 1 \end{vmatrix}$$

5. Find:

$$\begin{vmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 \\ -1 & 1 & 1 & 0 & \dots & 0 & 0 \\ 0 & -1 & 1 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & -1 & 1 \end{vmatrix}$$

References

Carlson, D., Johnson, C. R., Lay, D. C., & Porter, A. D. (Eds.) (2002). *Linear algebra gems: Assets for undergraduate mathematics.* Washington, DC: Mathematical Association of America.

Muir, T. (2011). *The contributions to the history of determinants 1900–1920* (reprint 1923). Charleston, SC: BiblioBazaar.

Proskuryakov, I. V. (1978). *Problems in linear algebra.* Moscow: Mir Publishers.

Solutions to this set of problems for publication should be submitted to:
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Solutions to this set will be made available on the AAMT website (www.aamt.edu.au) after 1 October 2012.