Generating Optimal Strut-and-Tie Models in Prestressed Concrete Beams by Performance-Based Optimization

by Qing Quan Liang, Yi Min Xie and Grant P. Steven

This paper deals with automatic generation of optimal strut-and-tie models in prestressed concrete beams by the performance-based optimization (PBO) method. In the present approach, developing strut-and-tie models in prestressed concrete members is transformed to the topology optimization problem of continuum structures. By treating prestressing forces as external loads, prestressed concrete beams can be analyzed, optimized and dimensioned with strut-and-tie models like reinforced concrete beams. Optimal strut-and-tie models in non-prestressed, partially-prestressed and fully-prestressed concrete beams are investigated by using the performance-based optimization technique. It is demonstrated that the magnitude of prestressing forces significantly affects the layouts of optimal strut-and-tie models in prestressed concrete members. The performance-based optimization method is shown to be effective in developing reliable strut-and-tie models for the design of prestressed concrete beams.

Keywords: performance-based optimization; prestressed concrete; strut-and-tie model; topology optimization.

INTRODUCTION

Strut-and-tie models are conceptual tools for the design of structural concrete. Struts, ties, nodes, fans and arches have been proposed by Marti[1] as the basic tools for the design and detailing of reinforced concrete beams. Schlaich et al.[2] developed the strut-and-tie model theory into a consistent design approach for reinforced and prestressed concrete members.

Some guidelines for the strut-and-tie design of pretensioned concrete members were given by Ramirez.[3] Experimental and analytical study on the use of strut-and-tie models for the design of post-tensioned anchorage zones has been conducted by Sanders and Breen.[4] Their results show that strut-and-tie models can successfully and conservatively estimate the strength performance of specimens. The strut-and-tie model approaches and related theories for the design of structural concrete can be found in the state-of-the-art report by the ASCE-ACI Committee 445 on Shear and Torsion.[5]

The development of strut-and-tie models in structural concrete members using conventional methods usually involves a trial-and-error iterative process based on the designer's intuition and previous experience. It is a difficult task for concrete designers to find correct strut-and-tie models in members with complicated geometry and loading conditions. As a result of this, computer graphics have been used as a design aid for the strut-and-tie modeling of structural concrete members by Alshegeir and Ramirez[6] and by Mish.[7] Recently, the performance-based optimization (PBO) method has been proposed by Liang et al.[8,9] as a rational and efficient tool for automatically developing optimal strut-and-tie models in reinforced concrete members and low-rise shearwalls.

Continuum topology optimization, which is the selection of the best configuration for the design of continuum structures, has become increasingly popular in engineering mechanics in recent years. A comprehensive survey on structural layout optimization has been given by Rozvany et al.[10] The homogenization-based optimization methods[11-14] treat topology optimization as a redistribution problem of composite material with microstructures in a continuum design domain. In the density function approach, optimal topology is generated by varying the element density.[15,16] The discrete variable optimization methods searches for the optimum by removing underutilized material from the continuum design domain.[17,18] Continuum topology optimization has been used in conceptual design of structures by Ramm et al.[19] However, no performance-based optimization criteria are incorporated in most of the existing continuum topology optimization methods to guarantee success in obtaining the global optimum, as pointed out by Liang et al.[20] The performance-based optimization (PBO) method formulated on the basis of system performance criteria has been developed by Liang et al.[21] for topology design of bracing systems for multistory steel building frameworks.

A procedure for optimizing the member size, initial prestressing force and tendon profile of a prestressed concrete system has been proposed by Kirsch.[22] However, no work has been undertaken so far on optimization of strut-and-tie models in prestressed concrete members by continuum topology optimization methods. This paper extends the previously cited work[8] to prestressed concrete beams. The performance-based design optimization concepts, optimality criteria and procedure are briefly outlined. The effects of prestressing forces on optimal strut-and-tie models in non-prestressed, partially-prestressed and fully-prestressed concrete beams are then investigated by using the performance-based optimization (PBO) technique. Optimal stut-and-tie models obtained by the present study are compared with existing analytical solutions.

RESEARCH SIGNIFICANCE

Automating the major and difficult design tasks in the design process of prestressed concrete members based on the strut-and-tie theory is of significant practical importance. Performance-based optimization concepts are consistent with performance-based design concepts being adopted in current building codes of practice. The research reported in this paper shows that the performance-based optimization technique can automatically generate reliable strut-and-tie models for the design of both reinforced and prestressed concrete members. The performance-based optimization method for strut-and-tie modeling would significantly improve the performance of structural concrete, and thus are appropriate to be adopted in concrete model codes. Design rules for strut-and-tie models are given in concrete model codes in some countries.[23-25]

PERFORMANCE-BASED CONCEPT AND OBJECTIVE

The performance-based design has become a popular design concept in the field of structural engineering in recent years. The building codes of practice in many countries are currently changing from prescriptive specifications to performance-based provisions for technical, economic, social and environmental reasons. The intent is to provide owners and designers with the capability to select alternative performance objectives for different structures. Performance objectives are qualitatively expressed by non-engineering terms, which can be easily understood by the owners and community. The performance-based optimal design is to design a structure or structural component that can perform physical functions in a specified manner throughout its design life at minimum cost or weight. The minimum-weight for given requirements and constraints is an important measure of the performance of an optimized design. The advantages of minimum-weight structures are low material cost, high technical performance and low environmental impact. It should be noted that the minimum-weight design is not always the cheapest design. The weight of a structure is commonly used as the objective function in structural optimization because it is readily quantified and it is difficult to construct an efficient cost function, which depends on many factors.

Struts and ties are used to ideally represent stress fields in cracked structural concrete members under ultimate conditions. The effects of both shear and moment can be accounted for simultaneously and directly in the strut-and-tie model. Finding an appropriate strut-and-tie model in a structural concrete member can be treated as a continuum topology optimization problem.[8,9] Since concrete permits only limited plastic deformations relating to the system performance, the best strut-and-tie model in a structural concrete member is the one with the maximum stiffness performance at minimum weight.[2,26] Therefore, the performance objective of topology optimization is minimizing the weight of a continuum structure while maintaining deformations within acceptable levels. The performance objective can be expressed in mathematical forms as follows:

$$\text{minimize } W = \sum_{e=1}^{n} w_{\mathrm{e}}(t) \tag{1}$$

$$\text{subject to } u_j - u_j^* \le 0 \qquad j=1,\ldots,m \tag{2}$$

in which W is the total weight of the structure, w_e is the weight of the eth element, t is the thickness of elements, u_j is the absolute value of the jth constrained displacement, u_j^* is the prescribed limit of u_j, m is the total number of displacement constraints and n is the total number of elements in the design. It should be noted that maximizing the stiffness of a structure is equivalent to minimizing its deflections. The width of the beam is also treated as one of the design variables.

CRITERIA FOR IMPROVING PERFORMANCE

An initial design domain is usually used as the starting point for deriving the optimum in continuum topology optimization methods. The goal of performance-based topology optimization is to maximize the performance of the initial design domain in terms of the efficiency of material usage without violating displacement constraints. In the present methodology, the criteria for improving performance are used in optimization algorithms to decide which element should be eliminated from the design domain so as to achieve the performance objective. The detailed derivation of the performance improving criteria can be found in previous paper.[8]

The effect of element removal on the constrained displacements can be evaluated by utilizing the virtual unit load method. By applying the virtual unit load to the direction of the constrained displacement u_j, the change of displacement due to the removal of the *e*th element can be approximately calculated by the virtual strain energy of the *e*th element, which is represented by

$$c_e = \{u_{ej}\}^T [k_e]\{u_e\} \tag{3}$$

where $\{u_{ej}\}$ is the nodal displacement vector of the *e*th element under the virtual unit load and $\{u_e\}$ is the displacement vector of the *e*th element under real loads. If a structure is divided into different size elements, the virtual strain energy density referred to mass should be calculated for element removal. The virtual strain energy density (VSED) of the *e*th element is defined as $\gamma_e = c_e / w_e$.

For a structure with multiple displacement constraints under multiple loading conditions, the virtual strain energy density of the *e*th element can be evaluated by using either a weighted average approach[20] or a logical AND scheme.[21] In the weighted average approach, the virtual strain energy density of the *e*th element is calculated by

$$\gamma_e^m = \sum_{l=1}^{p}\sum_{j=1}^{m}\beta_j\gamma_e \tag{4}$$

where the weighting parameter β_j is defined as u_j^l / u_j^{l*}, which is the ratio of the jth constrained displacement to the prescribed limit under the lth load case, and p is the total number of load cases.

To achieve the performance objective, it is obvious that elements with the lowest virtual strain energy density should be eliminated from the continuum design domain. The number of removed elements at each iteration is specified by the element removal ratio (ERR), which is the ratio of the number of elements to be removed to the total number of elements in the initial design domain. The element removal ratio is unchanged in the optimization process. The effects of element removal ratios on the optimal topologies by the performance-based optimization method have been investigated by Liang et al.[20]

PERFORMANCE-BASED OPTIMALITY CRITERIA

In performance-based optimization, the structure is gradually modified by removing underutilized elements from the structure. In order to obtain the global optimum, the performance of the resulting topology at each iteration in the optimization process must be quantitatively evaluated by performance indices. In performance-based design, structural responses such as stresses and displacements are used as performance indices to quantify the performance of structures.[27] In performance-based optimal design, the performance objective is to minimize the weight of the structure while maintaining structural responses within acceptable levels. Therefore, displacements alone are not sufficient for evaluating the performance of optimal designs. The minimum weight of a structure with acceptable structural responses is a sound measure of the performance of optimal designs for stiffness. Based on this optimal design concept, performance indices have been proposed by Liang et al.[8,9,,20,21,28,29] for quantifying the performance of structural topologies.

Since the stiffness matrix of a plane stress continuum concrete beam is a linear function of the beam width, the width can be uniformly changed at each iteration to keep the most critical constrained displacement at the prescribed limit.[30] For structures with displacement constraints, the performance-based optimality criterion is proposed as

$$\text{maximize } PI = \frac{u_{0j} W_0}{u_{ij} W_i} \tag{5}$$

in which u_{0j} is absolute value of the most critical constrained displacement in the initial structure under real loads, u_{ij} is absolute value of the most critical constrained displacement in the current structure at the *i*th iteration under real loads, W_0 is the actual weight of the initial design domain and W_i is the weight of the current structure at the *i*th iteration. It is seen from Eq. (5) that the performance objective can be achieved by maximizing the performance index in the optimization process.

PERFORMANCE OPTIMIZATION PROCEDURE

Numerical structural optimization approaches utilize the finite element method as the modeling and analytical tool. Based on the information obtained from the results of the finite element analysis (FEA), inefficiently used elements can be identified. The performance of a design is then improved by removing underutilized elements from the structure. The process of FEA and performance improvement is repeated until the performance index is less than unity. The main steps of the performance optimization procedure are illustrated in the flowchart given in Fig. 1.

The estimation of the initial size of a prestressed concrete beam should be based on serviceability performance criteria, and satisfies restrictions on design spaces. The effects of height constraints imposed on the beams on optimal topologies have been investigated by Liang et al.[20] The results indicate that increasing the height of the beam usually improves the performance of the final optimal design, but alters the load transfer mechanism in the beam. However, changing the width of the beam does not affect the layout of strut-and-tie model in the beam with a fixed height. Therefore, the displacement performance can be satisfied by adjusting the beam width after obtaining the optimal strut-and-tie model by the proposed method. In the model connectivity checking of the optimization procedure, two elements are connected together if they have at least a common edge. Any element that does not meet this requirement is treated as a singular element, which will be removed from the model.

In nature, the loads are transmitted so that the associated strain energy is a minimum. This means that strut-and-tie systems in structural concrete should be developed on the basis of system performance criteria (stiffness) rather than component performance criteria (strength). The component performance criteria are met by dimensioning the component. In the modeling of prestressed concrete members, the prestressing forces are treated as external loads as suggested by Schlaich et al.[2] Concrete beams are modeled using plane stress elements in the present study. The linear elastic behavior of cracked concrete members is assumed. The modeling of structural concrete members in the topology optimization for strut-and-tie models has proved to be effective in producing reliable solutions for design.[8,9] It should be noted that strut-and-tie models correspond to a lower-bound limiting performance analysis. The key importance is the detailing of the reinforcement such that the load transfer mechanism predicted by the PBO technique can be realized at ultimate.

NON-PRESTRESSED CONCRETE BEAM

The performance-based optimization approach is used to develop optimal strut-and-tie models in a non-prestressed concrete beam. Fig. 2 shows a simply supported prestressed concrete beam with rectangular cross section under two concentrated loads of F = 500 kN and the prestressing force P at a distance of 150 mm from the bottom of the beam. When the prestressing force $P = 0$, the beam is simply a non-prestressed concrete beam, which is considered herein for comparison purposes. This concrete beam is discretized into a 160 × 20 mesh using four-node plane stress elements. Values of the Young's modulus of concrete E = 31940 MPa, Poisson's ratio $v = 0.15$ and the initial width of the beam b_0 = 300 mm are assumed. Two displacement constraints of the same limit are imposed on the points of load F in the vertical direction. The element removal ratio ERR = 1% is used in the optimization process.

The performance index history of the nonprestressed concrete beam is presented in Fig. 3. It can be seen that the performance index is equal to unity at the initial iteration because no element is removed from the structure at this stage. When a small number of elements with lowest virtual strain energy density is deleted from the beam, the performance index increases from unity to the maximum value of 1.38. After reaching the peak, it drops sharply and this means that further element removal will cause large deflections. The performance index may jump in the optimization history as shown in Fig. 3 because the element removal ratio used is still high. The research undertaken by Liang et al.[20] indicates that a smoother solution to nonprestressed members may be obtained by using a smaller element removal ratio but the computational time will be considerably increased.

The optimization history of the strut-and-tie model in the nonprestressed concrete beam is presented in Fig. 4, where only half of the model is shown due to symmetry. It can be observed from Fig. 4 that when inefficiently used concrete is removed from the beam at each iteration, the strut-and-tie model is gradually characterized by the remaining material. The strut-and-tie idealization is illustrated in Fig. 4(d), where the solid bold lines represent ties and dotted lines represent struts. This optimal strut-and-tie model suggests that inclined steel reinforcements bent up from the bottom steel bars are most efficient in resisting tensile forces developed in shear spans. A similar solution to this reinforced concrete beam as shown in Fig. 4(e) was obtained by Schlaich et al.[2] using the load path method. In their strut-and-tie model, vertical ties were assumed to form the model.

PARTIALLY-PRESTRESSED CONCRETE BEAM

By applying prestressing forces to a concrete beam, cracking and deflections can be reduced or eliminated. The tendon profile and the type and magnitude of prestressing force can favorably and efficiently change the load transfer mechanism. Prestressing reduces the length of the tension chord along the bottom of a concrete beam. By treating the prestressing force as external load, prestressed concrete beams can be designed, analyzed and dimensioned with strut-and-tie models in the same manners as reinforced concrete beams.[2,3]

A prestressing load of P = 1650 kN is applied to the concrete beam shown in Fig. 2. The performance optimization procedure is carried out to find the load transfer mechanism in this prestressed concrete beam. Two equal vertical displacement constraints are imposed at the points of load application because the deflections of the beam are to be reduced. The ERR = 1% is adopted in the optimization process. The performance index history of this prestressed concrete beam is also presented in Fig. 3. The maximum performance index of this beam is 1.85. The optimization history of this prestressed concrete beam is shown in Fig. 5 where shows that the strut-and-tie model in the prestressed concrete beam gradually evolves towards the optimum. It can be seen from Fig. 5(d) that there is a tension chord at the bottom of the beam that, because of prestressing, is shorter than that of the nonprestressed concrete beam shown in Fig. 4(d). It is apparent that this concrete beam is partially prestressed. It can be observed from a comparison of Figs. 4(d) and 5(d) that the prestressing loads significantly affect the strut-and-tie model in the concrete beam and the loads transmit along more a direct load path. Furthermore, Fig. 3 shows that the partially-prestressed concrete member has the highest performance index. This means that the most economic design can be achieved by using partial prestressing. The strut-and-tie model of a partially prestressed concrete beam given by Schlaich et al.[2] is illustrated in Fig. 5(e). In Schlaich et al.'s model, however, the strut at the bottom of the beam is absent.

FULLY-PRESTRESSED CONCRETE BEAM

The strut-and-tie model of a fully prestressed concrete beam has no tension chord at the bottom of the beam.[2] By choosing the prestressing force P = 2500 kN, the performance optimization procedure is applied to the prestressed concrete beam shown in Fig. 2. Fig. 3 shows that the maximum performance index is 1.62. It can be observed from Fig. 3 that the performance index may increase after decreasing at a few iterations because further element removal results in a more direct load transfer mechanism.

The optimization history is shown in Fig. 6, from which it can be seen that the strut-and-tie model has no tensile chord at the bottom of the member since the full prestressing transforms the beam under applied loads into a beam-column. However, the inclined tensile tie still exists in the shear span since a tensile force is developed in shear spans. Fig. 6(e) shows the strut-and-tie model of a fully prestressed concrete beam developed by Schlaich et al.[2] where the resultant of the prestressed force and the support force meet the line of action of the load F within the kern of the section. The optimal strut-and-tie model obtained by the present study as shown in Fig. 6(d) indicates that full prestress may be achieved without the resultant meeting the action line of the load F, and is characterized by absence of a tension chord along the bottom of the concrete beam.

SUMMARY

The automatic generation of optimal strut-and-tie models in prestressed concrete beams by the performance-based optimization (PBO) technique has been described in this paper. The performance-based design concepts are incopertated into structural topology optimization, which is treated as the problem of improving the performance of the design domain. A performance index is used to quantify the performance of resulting topology in the optimization process. Maximization of the performance index is proposed as the performance-based optimality criteria. It has been shown that by treating prestressing forces as external loads, prestressed concrete beams can be analyzed, optimized and designed like reinforced concrete beams in terms of strut-and-tie models.[2,3]

Strut-and-tie models in non-prestressed, partially-prestressed and fully-prestressed concrete beams generated by the performance-based optimization method are supported by existing analytical solutions. It has been demonstrated that prestressing forces significantly alter the load transfer mechanism in structural concrete members. Moreover, partial prestressing offers a more economic design than conventional reinforcement or full prestressing. Inclined ties are present in optimal strut-and-tie models produced by the PBO method. It has been proved that inclined reinforcement bent up from bottom steel reinforcing bars are most efficient in resisting inclined tension force developed in members especially non-flexural concrete members although they may need extra effort to construct.[8]

The study presented in this paper and previous work on optimization of strut-and-tie models in structural concrete[8,9] demonstrate that performance-based optimization techniques are efficient and reliable tools for generating strut-and-tie models for the design of structural concrete. Further research is still needed to incorporate modules for post-processing and dimensioning struts, ties and nodes, and construction constraints into the design optimization procedure to make it an integrated design tool for practicing concrete designers. It is believed that with the advance of modern performance-based optimization techniques, the simple strut-and-tie theory will be widely adopted in performance-based concrete model codes, and be used in practice by concrete designers with confidence.

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CONVERSION FACTORS

1 mm	=	0.039 in.
1 kN	=	0.2248 kips
1 MPa	=	145 psi

NOTATION

b_0	=	initial width of beam
c_e	=	virtual strain energy of the *e*th element
E	=	Young's modulus of concrete
F	=	applied load
$[k_e]$	=	stiffness matrix of the *e*th element
m	=	total number of displacement constraints
n	=	total number of elements
P	=	prestressing force
PI	=	performance index
t	=	thickness of elements
u_j	=	absolute value of the *j*th constrained displacement
u_j^*	=	prescribed limit of u_j
u_j^{l*}	=	prescribed limit of under the *l*th load case
u_{0j}	=	the *j*th constrained displacement that is the most critical in initial design

u_{ij} = the *j*th constrained displacement that is the most critical in current design

u_j^l = the *j*th constrained displacement under the *l*th load case

$\{u_e\}$ = displacement vector of the *e*th element under real loads

$\{u_{ej}\}$ = displacement vector of the *e*th element under the virtual unit load

W = total weight of a structure

w_e = weigth of the *e*th element

W_0 = actual weight of the initial design

W_i = actual weight of the current design at the *i*th iteration

v = Poisson's ratio

β_j = weighting parameter

γ_e = virtual strain energy density of the *e*th element

γ_e^m = virtual strain energy density of the *e*th element under multiple constraints and load cases

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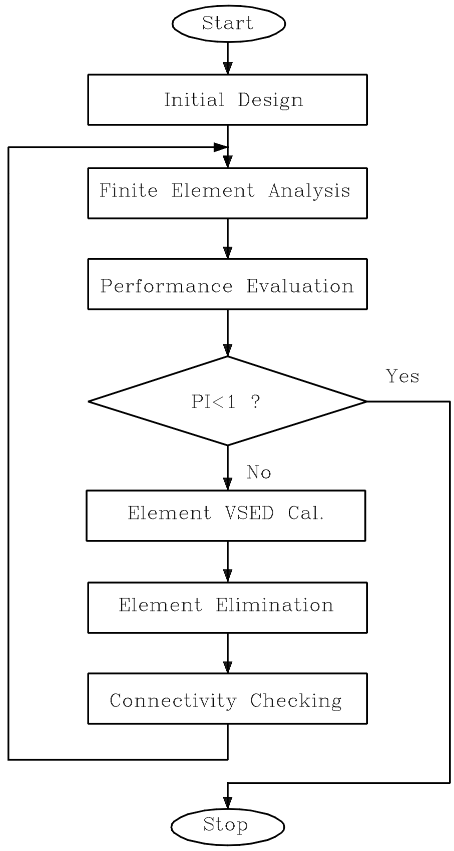


Fig. 1—Flowchart of performance-based optimization procedure

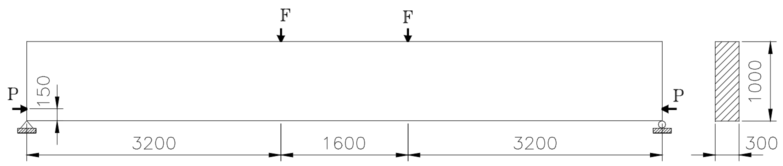


Fig. 2—Simply supported prestressed concrete beam

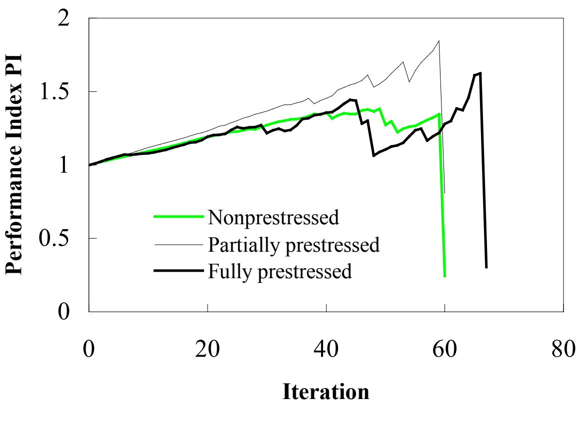
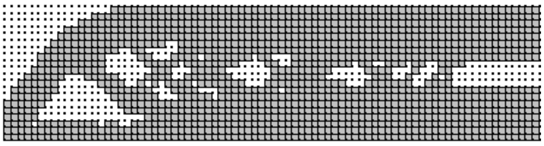
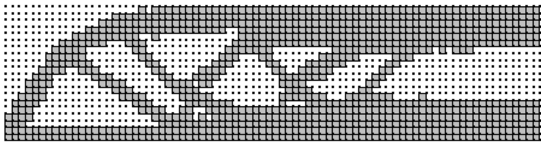


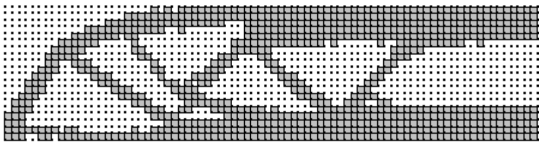
Fig. 3— Performance index history



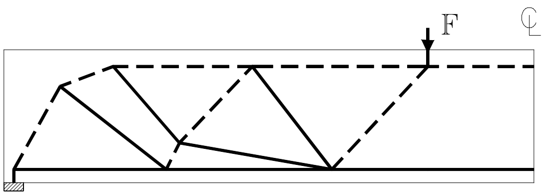
(a) Topology at iteration 20



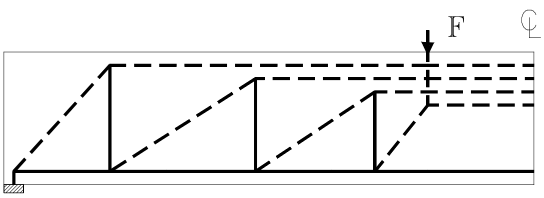
(b) Topology at Iteration 40



(c) optimal topology

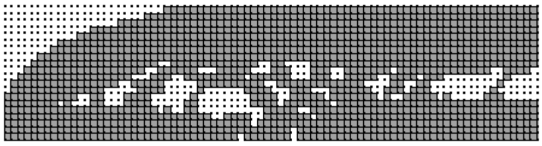


(d) Optimal strut-and-tie model

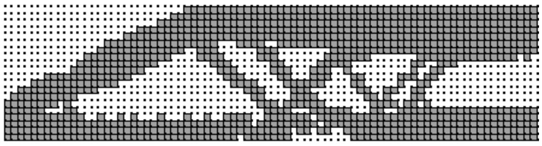


(e) Strut-and-tie model by Schlaich et al.[2]

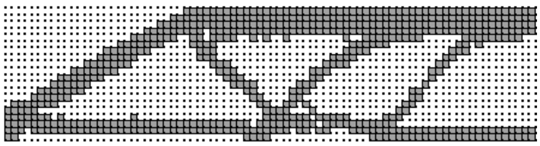
Fig. 4— Optimization history of strut-and-tie model in non-prestressed concrete beam



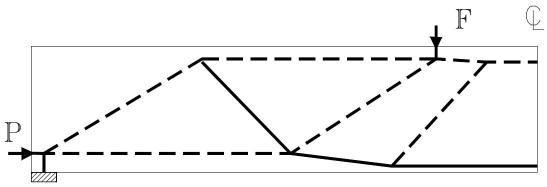
(a) Topology at iteration 20



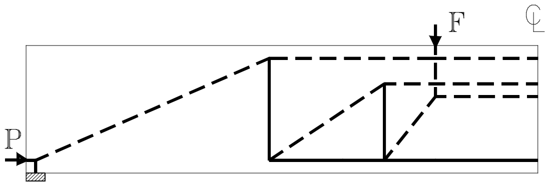
(b) Topology at iteration 40



(c) Optimal topology

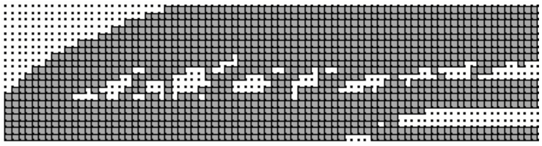


(d) optimal strut-and-tie model

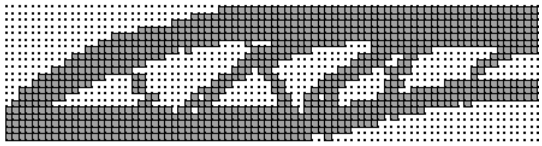


(e) Strut-and-tie model by Schlaich et al.[2]

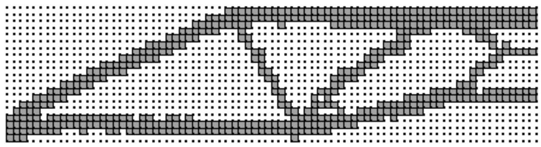
Fig. 5—Optimization history of strut-and-tie model in partially-prestressed concrete beam



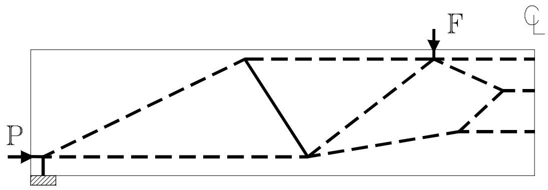
(a) Topology at iteration 20



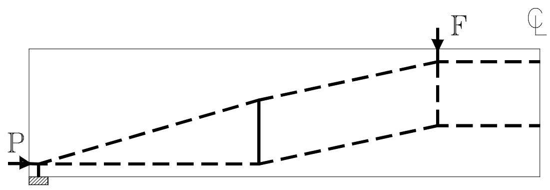
(b) Topology at iteration 40



(c) Optimal topology



(d) Optimal strut-and-tie model



(e) Strut-and-tie model by Schlaich et al.[2]

Fig. 6—Optimization history of strut-and-tie model in fully-prestressed concrete beam