PERFORMANCE-BASED OPTIMIZATION FOR STRUT-TIE MODELING OF STRUCTURAL CONCRETE

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ABSTRACT: Conventional trial-and-error methods are not efficient in developing appropriate strut-and-tie models in complex structural concrete members. This paper describes a Performance-Based Optimization (PBO) technique for automatically producing optimal strut-and-tie models for the design and detailing of structural concrete. The PBO algorithm utilizes the finite element method as a modeling and analytical tool. Developing strut-and-tie models in structural concrete is treated as an optimal topology design problem of continuum structures. The optimal strut-and-tie model that idealizes the load transfer mechanism in cracked structural concrete is generated by gradually removing regions that are ineffective in carrying loads from a structural concrete member based on overall stiffness performance criteria. A performance index is derived for evaluating the performance of strut-and-tie systems in an optimization process. Fundamental concepts underlying the development of strut-and-tie models are introduced. Design examples of a low-rise concrete shearwall with openings and a bridge pier are presented to demonstrate the validity and effectiveness of the PBO technique as a rational and reliable design tool for structural concrete.

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Key words: finite element analysis, performance, pier, strut-and-tie model, structural concrete, shearwall, topology optimization

INTRODUCTION

The shear design of structural concrete members is a complex problem, which has not been solved fully. The adoption of empirical equations in current concrete model codes leads to complex design procedures for shear. Empirical equations generally yield shear strength predictions that deviate considerably from experimental results. In addition, they need to be continuously evaluated for new materials. These highlight the limitations of empirical equations and the need for a rational approach to structural concrete. Strut-and-tie modeling has been proved to be a rational, unified and safe approach for the design and detailing of structural concrete under combined load effects (ASCE-ACI Committee 445 on Shear and Torsion 1998). By strut-and-tie modeling, the influence of shear and moment can be taken into account simultaneously and directly in one model.

Truss models were introduced by Ritter (1899) for the shear design of reinforced concrete beams, and extended by Mörsch (1909) to beams under torsion. The truss analogy method received considerable studies in the 1960s and 1970s (Kupfer 1964; Leonhardt 1965; Lampert and Thurlimann 1971). Collins and Mitchell (1980) proposed the truss model approach that considers deformations for design of reinforced and prestressed concrete. It is noted that truss models can only be used to design regions of a concrete structure where the Bernoulli hypothesis of plane strain distribution is assumed valid. At regions where the strain distribution is significantly nonlinear, the truss model theory is not applicable. The strut-and-tie model, which is a generalization of the truss analogy method for beams, can be used to design disturbed regions of structural concrete as demonstrated by Marti (1985). Schlaich et al. (1987, 1991) extended the truss model theory to a consistent strut-and-tie model approach for the design and detailing of any part of reinforced and prestressed concrete structures. Ramirez and Breen (1991) reported that a modified truss model approach with variable angle of inclination diagonals and a concrete contribution could be used for designing reinforced and prestressed concrete beams. Strut-and-tie modeling has been applied to the design of pretensioned concrete members (Ramirez 1994) and post-tensioned anchorage zones (Sanders and Breen 1997). Strut-and-tie model approach and related theories for the shear design of structural concrete were summarized in the state-of-the-art report by the ASCE-ACI Committee 445 on Shear and Torsion (1998). Moreover, the strut-and-tie model design method has recently been incorporated in the ACI 318-02 for the design of structural concrete (Cagley 2001).

Conventional methods for developing strut-and-tie models in structural concrete involves a trial-and-error iterative process based on the designer's intuition and past experience. It is a challenging task for the designer to select an appropriate strut-and-tie system for a concrete structure with complex geometry and loading conditions from many possible equilibrium configurations. As a result of this, computer programs based on the truss topology optimization theory have been developed for generating truss models in reinforced concrete structures (Anderheggen and Schlaich 1990; Ali and White 2001). Computer graphics as useful design aids have been employed to develop strut-and-tie models in structural concrete (Alshegeir and Ramirez 1992; Mish 1994; Yun 2000). The Performance-Based Optimization (PBO) method for continuum structures with displacement constraints proposed by Liang et al. (2000a, 2001) has been shown to be a rational and efficient tool for automatically generating optimal strut-and-tie models in reinforced and prestressed concrete structures.

Shape and topology optimization of continuum structure has been reviewed by Hafka and Grandhi (1986) and Rozvany et al. (1995). Several continuum topology optimization methods have been developed in the last two decades. The homogenization-based optimization (HBO) method (Bendsøe and Kikuchi 1988; Suzuki and Kikuchi 1991; Díaz and Bendsøe 1992; Díaz and Kikuchi 1992; Tenek and Hagiwara 1993; Bendsøe et al. 1995; Ma et al. 1995; Krog and Olhoff 1999) treats topology optimization of continuum structures as a problem of material redistribution within a design domain composed of composite material with microstructures. The homogenization theory is used in the HBO method to calculate the effective properties of composite material. Simple approaches to topology optimization of continuum structures are also available, such as the density function approach (Mlejnek and Schirrmacher 1993; Yang and Chuang 1994), the hard kill optimization (HKO) method (Rodriguez and Seireg 1985; Atrek 1989; Rozvany et al. 1992; Xie and Steven 1993), and the soft kill optimization (SKO) method (Baumgartner et al. 1992). It should be noted that these continuum topology optimization methods could lead to many locally optimal solutions. To overcome this problem, performance-based optimality criteria have been proposed by Liang et al. (1999, 2000b, 2000c) and Liang (2001) to identify the global optimum as opposed to the local.

This paper extends the PBO method proposed by Liang et al. (2000b) for topology design of continuum structures with mean compliance constraints to the strut-and-tie modeling of structural concrete. The development of strut-and-tie models in structural concrete is transformed to the topology optimization problem of continuum structures. Optimization criteria for strut-and-tie models are described. An integrated design optimization procedure is proposed for strut-and-tie design of structural concrete. Optimal strut-and-tie models in a low-rise concrete shearwall with openings and a bridge pier are automatically generated by the PBO technique, and compared with analytical solutions.

STRUT-AND-TIE MODEL OPTIMIZATION PROBLEM

Strut-and-tie models are used to idealize the load transfer mechanism in cracked structural concrete at ultimate limit states. The design task is mainly to identify the load transfer mechanism in a structural concrete member and reinforce the member such that this load path will safely transfer applied loads to the supports. In reality, some regions of a structural concrete member are not as effective in carrying loads as others. By eliminating underutilized portions from a structural concrete member, the actual load path in the member can be found. The PBO method has the capacity to find the underutilized portions of a member and remove them from the member to improve its performance. Therefore, the strut-and-tie modeling of structural concrete can be transformed to a topology optimization problem of continuum structures.

In nature, loads are transmitted by the principle of minimum strain energy (Kumar 1978). Minimizing the strain energy of a structure is equivalent to maximizing its overall stiffness. Thus, strut-and-tie systems in structural concrete should be developed on the basis of system performance criteria (overall stiffness) rather than component performance criteria (strength). Dimensioning the components of a structural system easily satisfies component performance criteria. It should be noted that the enhanced ductility design approach should be used to detail the strut-and-tie model obtained. Based on these design criteria, the performance objective of the strut-and-tie model optimization is to minimize the weight of a structural concrete member while maintaining its overall stiffness within an acceptable performance level. For a structural member modeled with plane stress elements, the performance objective can be expressed in mathematical forms as follows:

$$\text{minimize} \quad W = \sum_{e=1}^{n} w_e(t) \tag{1}$$

$$subject\ to \quad C \le C^* \tag{2}$$

$$t^{\mathrm{L}} \le t \le t^{\mathrm{U}} \tag{3}$$

where W = the total weight of a structural concrete member; w_e = the weight of the eth element; t = the thickness of elements (or the width of member cross-section); C = the absolute value of the mean compliance of the member; C^* = the prescribed limit of C; n = the total number of elements in the member; t^{L} = the lower limit of element thickness; and t^{U} = the upper limit of element thickness. To simplify the optimization problem, only uniform sizing of the element thickness is considered in the PBO method.

LIMIT ANALYSIS AND FINITE ELEMENT MODELING

The behavior of structural concrete members under applied loads can be well approximated by the uncracked linear, cracked linear and limit analysis (Marti 1999). Strength performance predictions based on a limit analysis will be reliable if structural concrete members are designed with adequate ductility and detailing. The limit analysis can be divided into lower-bound and upper-bound methods (Nielsen 1984). Lower-bound methods require the designer to design a structural concrete member by strengthening its load transfer mechanism. They are particularly suitable for designing new concrete structures. On the other hand, upper-bound methods allow for quick checks for ultimate strength, dimensions and details of existing structures. They are suitable for the performance evaluation of existing structures. Strut-and-tie models correspond to the lower-bound limit analysis, and can indicate the necessary amount, the correct locations and the required detailing of the steel reinforcement.

After extensive cracking of concrete, loads applied to a structural concrete member are mainly carried by concrete struts and steel reinforcement, which form the load transfer mechanism. The failure of a structural concrete member is mainly caused by the breakdown of the load transfer mechanism, such as the yielding of steel reinforcement in ductile structural concrete members (ASCE-ACI Committee 445 on Shear and Torsion 1998). Before designing a structural concrete member, the locations of tensile ties and the amounts of steel reinforcement are unknown. Actually, it is the designer's task to identify an appropriate strut-and-tie system in a structural concrete member in order to reinforce it. As a result of this, the nonlinear behavior of reinforced concrete cannot be taken into account in the finite element model for developing strut-and-tie systems.

It is proposed here to develop strut-and-tie systems in structural concrete based on the linear elastic theory of cracked concrete for system performance criteria and to design the structural concrete member based on the theory of plasticity for component performance criteria. Only two-dimensional models are considered here. In the finite element analysis, plain concrete members are treated as homogenization continuum structures, and modeled using plane stress elements. The PBO algorithm has been written to link with the finite element STRAND6 codes (1993) to perform the finite element analysis and optimization tasks in an iterative manner. The progressive cracking of a concrete member is characterized by gradually removing concrete from the member, which is fully cracked at the optimum. It is noted that load-deformation responses of a structural concrete member in an optimization process are highly nonlinear because the topology of the member is changing at each iteration.

ELEMENT REMOVAL CRITERIA

Element removal criteria can be derived by undertaking a design sensitivity analysis on the mean compliance of a structural concrete member with respect to element removal. A detailed derivation has been given in a previous paper by Liang et al. (2000b). Element removal criteria are such that elements with the lowest strain energy densities should gradually be removed from the continuum design domain to achieve the performance objective. The strain energy density of the *e*th element is defined as (Liang et al. 2000b)

$$\zeta_e = \frac{\left|\mathbf{u}_e^{\mathrm{T}} \mathbf{k}_e \mathbf{u}_e\right|}{2w_e} \tag{4}$$

in which $\mathbf{u}_e$ = nodal displacement vector of the *e*th element; and $\mathbf{k}_e$ = stiffness matrix of the *e*th element.

For a concrete member under multiple load cases, a logical AND scheme is employed in the calculation of element strain energy densities for elimination (Liang et al. 2000b). In the logical AND scheme, an element is deleted from the structural concrete member only if its strain energy density is the lowest for all load cases. By removing elements with the lowest strain energy densities from a concrete member, the maximum stiffness design at minimum weight can be achieved. In order to obtain a smooth solution, however, only a small number of elements are removed from the discretized concrete member. The element removal ratio (R) for each iteration is defined as the ratio of the number of elements to be removed to the total number of elements in the initial design domain.

PERFORMANCE-BASED OPTIMALITY CRITERIA

The performance evaluation of strut-and-tie systems in an optimization process is required in order to determine the optimum. A performance index has been proposed by Liang et al. (2000b) for quantifying the performance of bracing systems for multistory steel building frameworks with an overall stiffness constraint. This performance index is also applicable to strut-and-tie systems, and its mathematical derivation is presented here.

The strain energy or mean compliance of a structure is expressed by

$$C = \frac{1}{2}\mathbf{P}^{\mathrm{T}}\mathbf{u} \tag{5}$$

where $\mathbf{P}$ = nodal load vector; and $\mathbf{u}$ = nodal displacement vector.

In problems with the element thickness as design variables, an infeasible design in an optimization process can be converted into a feasible one by the scaling design procedure (Kirsch 1982; Liang et al. 1999, 2000c). Since the stiffness matrix of a plane stress continuum structure is a linear function of the element thickness, the element thickness can be uniformly scaled to keep the mean compliance constraint active at each iteration in the optimization process. By scaling the initial structural concrete member with a factor of C_0 / C^*, the scaled weight of the initial design is represented by

$$W_0^{\mathrm{s}} = \left(\frac{C_0}{C^*}\right) W_0 \tag{6}$$

where $W_0 =$ the actual weight of the initial design domain; and C_0 = the absolute value of the strain energy of the initial design under applied loads. Similarly, by scaling the current design with respect to the mean compliance limit, the scaled weight of the current design at the *i*th iteration can be determined by

$$W_i^{s} = \left(\frac{C_i}{C^*}\right)W_i \tag{7}$$

in which C_i = the absolute value of the strain energy of the current design under applied loads at the *i*th iteration; and W_i = the actual weight of the current design at the *i*th iteration.

The performance index at the *i*th iteration is proposed as

$$PI_{ES} = \frac{W_0^{s}}{W_i^{s}} = \frac{(C_0 / C^*)W_0}{(C_i / C^*)W_i} = \frac{C_0 W_0}{C_i W_i} \tag{8}$$

The performance index is a measure of structural responses and the weight of a structural member in an optimization process, and thus quantifies the performance of structural topologies. By gradually eliminating elements with the lowest strain energy densities from a concrete member, its performance in terms of the efficiency of material and overall stiffness can be improved. To obtain the optimal topology, performance-based optimality criteria for structures with mean compliance constraints can be proposed as

$$maximize\ PI_{ES} = \frac{C_0 W_0}{C_i W_i} \tag{9}$$

The optimal topology obtained represents the most efficient load-carrying mechanism in the continuum design domain. Thus, optimal topologies generated by the PBO technique can be treated as optimal strut-and-tie models in structural concrete members. The physical meaning of the performance-based optimality criteria is that the optimal strut-and-tie model transmits loads in a way such that the associated strain energy and material consumption are a minimum. For a concrete member subject to multiple loading cases, the performance index can be calculated by using the strain energy of the member under the most critical loading case in an optimization process.

DESIGN OPTIMIZATION PROCEDURE

The design of a structural concrete member using strut-and-tie modeling involves the estimation of an initial member size, developing an appropriate strut-and-tie model and dimensioning struts, ties and nodes. The finite element STRAND6 codes (1993) are used in the PBO method as a modeling and analytical tool. The PBO algorithm has been written to link with STRAND6 to automatically carry out the finite element analysis and optimization tasks. Once the user has set up the finite element model, the computer would automatically generate the optimal strut-and-tie model. The main steps of the design optimization procedure are given as follows:

1. Select an appropriate size for the concrete structure based on serviceability performance criteria and design space constraints.
2. Model the two-dimensional concrete member using finite element programs. Applied loads, support conditions and material properties of the concrete member are specified.

Prestressing forces can be treated as external loads (Schlaich et al. 1987; Ramirez 1994; Liang et al. 2001).

3. Perform a linear elastic finite element analysis on the concrete member.
4. Evaluate the performance of the resulting system by using Eq. (8).
5. Calculate the strain energy densities of elements for each loading case.
6. Remove R (%) elements with the lowest strain energy densities from the concrete member.
7. Check model continuity. This is to ensure that the strut-and-tie model generated by the PBO technique is a continuous structure that satisfies the equilibrium condition.
8. Check model symmetry for a concrete member with an initially symmetrical loading, geometry and support condition.
9. Save current model. The models generated at each iteration are automatically saved to files for use in latter stage.
10. Repeat step (3) to (9) until the performance index is less than unity.
11. Select the optimal strut-and-tie model, which corresponds to the maximum performance index.
12. Analyze the discrete strut-and-tie model to determine internal forces in members.
13. Dimension struts, ties and nodes.
14. Detail steel reinforcement based on the optimal strut-and-tie model obtained.

Dimensioning a strut-and-tie model, which includes sizing the concrete struts, reinforcing the ties, and checking the bearing capacities of nodal zones, is of significant importance to the overall performance of a structural concrete member. The detailing of nodes and steel reinforcement influences the flow of forces in a structural concrete member, and thus directly affects the strength performance of concrete struts and ties connected with them. The key importance is to ensure that the optimal strut-and-tie model generated by the PBO technique can be realized at ultimate after detailing. It should be noted that the optimal strut-and-tie model produced by the PBO technique indicates the locations of struts, ties and nodes but not necessarily their exact dimensions. This is because it is developed on the basis of overall stiffness performance criteria without consideration of strength performance criteria. Dimensioning strut-and-tie models should be based on the bearing conditions and strength requirements.

The compressive strength of concrete in struts is influenced by its state of stresses, cracks and the arrangement of steel reinforcement. For safety, the effective compressive strength of concrete should be used in designing concrete struts. Marti (1985) suggested that the effective compressive strength of concrete in struts should be taken as $0.6f_c^{'}$, whereas Ramirez and Breen (1991) suggested a value of $2.5\sqrt{f_c^{'}}$ (MPa). Different values of the effective compressive strength of struts could be used in design, depending on the state of stresses, cracks and the arrangement of steel reinforcement in the structural concrete member (Schlaich et al. 1987). Design rules for the effective compressive strength of struts have been proposed for ACI 318-02 (Cagley 2001).

Reinforcing steel should be provided to carry tensile forces in ties in strut-and-tie models. The cross-sectional area of reinforcing steel for each tensile tie can be determined from the following expression

$$\phi(A_s f_{yr} + A_p f_{yp}) \geq T \qquad (10)$$

where ϕ = the capacity reduction factor; A_s = the cross-sectional area of reinforcing bars; f_{yr} = the yield strength of reinforcing bars; A_p = the cross-sectional area of prestressing steel; f_{yp} = the effective yield strength of prestressing steel for tensile ties; and T = the tensile force in a tensile tie.

ILLUSTRATIVE DESIGN EXAMPLES

Low-Rise Shearwall with Openings

In this example, the PBO technique is used to automatically generate an optimal strut-and-tie model in a low-rise concrete shearwall with openings, and numerical results are compared with analytical solutions. Fig. 1 shows the geometry and loading of a low-rise concrete shearwall with openings based on the example presented by Marti (1985). The shearwall is fixed on the foundation. In the present study, the values of the point loads P_1 = 1000 kN and P_2 = 500 kN are assumed. A compressive cylinder strength of concrete $f_c^{'}$ = 32 MPa, Young's modulus of concrete E_c = 28600 MPa, Poisson's ratio ν = 0.15 and the initial thickness of the shearwall t_0 = 200 mm are used in the analysis. The concrete shearwall is modeled using 100-mm square, four-node, plane stress elements. A mean compliance constraint is considered. The element removal ratio R = 1% is used.

The performance characteristics of the shearwall in the optimization process are presented in Fig. 2. It is seen that by gradually eliminating elements from the shearwall, the mean compliance of the shearwall increases with reductions in its weight. In addition, rapid increases are observed after more and more elements are deleted from the model. The performance characteristic curve indicates whether a proposed design for required performance is feasible. Fig. 3 shows the performance index history of the shearwall with openings. It can be seen from Fig. 3 that the performance of the shearwall in terms of the efficiency of material and overall stiffness is still gradually improved by eliminating elements with the lowest strain energy densities from the model even if there are a large portion of openings. The maximum performance index of 1.2 occurs at iteration 35.

The optimization history of strut-and-tie model in the shearwall is presented in Fig. 4. When elements with the lowest strain energy densities are removed from the shearwall, the resulting topology evolves to a frame-like structure. Fig. 4(d) shows the optimal topology obtained at iteration 35. This optimal topology represents the load transfer mechanism in the concrete shearwall under given loading and support conditions, and can be idealized as the discrete model illustrated in Fig. 4(e). This model is composed of only struts. The optimal strut model of the shearwall with openings generated by the PBO technique agrees extremely well with the analytical solution given by Marti (1985), as shown in Fig. 4(f).

In detailing the strut-and-tie model, the depths of concrete struts can be based on either the optimal topology shown in Fig. 4(d) or the model given in Fig. 4(f). The final thickness of concrete struts (or the shearwall) can then be determined by using the effective compressive strength of concrete based on the forces they carry and bearing conditions. Since the strut-and-tie model obtained has no tensile ties, the main steel reinforcement is not required to carry tensile forces in the shearwall. However, a minimum amount of steel reinforcement in a form of reinforcing meshes in compliance with codes of practice should be provided in the concrete shearwall to control cracking, which may be induced by shrinkage and temperature effects. For a completed design, the bearing capacities of nodal zones in the model should be checked.

Design of a Bridge Pier

The design domain for a bridge pier is shown in Fig. 5. The bridge pier fixed on the foundation is required to support four concentrated loads of 2750 kN transferred from four steel girders. An initial thickness of 1.5 m is assumed for this bridge pier. The PBO technique is employed to produce an optimal strut-and-tie model for the design and detailing of the bridge pier. A compressive cylinder strength of concrete $f_c^{'}$ = 32 MPa, Young's modulus of concrete E_c = 28600 MPa, and Poisson's ratio ν = 0.15 are used in the finite element analysis. The bridge pier is modeled using 125-mm square, four-node, plane stress elements. Plane stress conditions and the mean compliance constraint are considered. The element removal ratio R = 1% is used in optimization process.

The performance characteristics of the pier structure in the optimization process are fully captured by a weight-compliance curve shown in Fig. 6. Since the overall stiffness of the pier structure is gradually reduced by element elimination, structural responses such as the mean compliance are increased. The topology performance of the bridge pier in optimization process is monitored by the performance index shown in Fig. 7. By removing a small number of elements with the least contribution to the structural stiffness from the pier structure at each iteration, the performance index increases from unity to a maximum value of 1.17. It is observed from Fig. 7 that after iteration 69, the performance index decreases rapidly. This indicates that further element elimination leads to the breakdown of the load-carrying mechanism in this bridge pier.

Fig. 8 demonstrates the optimization history of strut-and-tie model in the bridge pier. It is seen that the load transfer mechanism characterized by remaining elements in the pier structure becomes more and more clear when lowly strained elements are systematically deleted from the finite element model. The optimal topology was obtained at iteration 49, as shown in Fig. 8(c). Applied loads are mainly carried by this optimal structure, which represents the most efficient load transfer mechanism in the design domain considered. The optimal topology shown in Fig. 8(c) is transformed to the discrete strut-and-tie model for the bridge pier illustrated in Fig. 9, where solid lines represent tensile ties and dotted lines represent compression struts. To achieve better force flows within the pier structure and economical designs, a final design proposal for the bridge pier is presented in Fig. 9. It is seen from Fig. 8(c) that the pier wall can be designed as two separated columns to further improve economical construction.

After the strut-and-tie model has been developed, it is a straightforward matter to dimension it. Forces in members of the strut-and-tie model shown in Fig. 9 are given in Table 1. It is important to provide steel reinforcement to carry tensile forces in inclined tensile ties shown in Fig. 9. The locations of these inclined tensile ties are difficult to be predicted by using conventional trial-and-error methods (Warner et al. 1998). An arrangement of the main steel reinforcement for resisting tensile forces in the bridge pier is illustrated in Fig. 10. Additional reinforcing meshes that are not shown in Fig. 10 should be provided in the bridge pier in accordance with the minimum requirements of the codes of practice for crack control.

CONCLUSIONS

The performance-based optimization (PBO) technique formulated on the basis of system performance criteria for automatically generating optimal strut-and-tie models in structural concrete has been described in this paper. Developing strut-and-tie models in structural concrete is transformed to a topology optimization problem of continuum structures. Optimal topologies produced by the PBO technique are treated as optimal strut-and-tie models for the design and detailing of structural concrete. Performance-based optimality criteria for determining optimal strut-and-tie models have been developed. An integrated design optimization procedure has been proposed for optimizing and dimensioning structural concrete with strut-and-tie systems.

The PBO algorithm has been used to generate optimal strut-and-tie models in a low-rise concrete shearwall with openings and a bridge pier, and numerical results has been verified by existing analytical solutions. It has been demonstrated that it is appropriate to develop strut-and-tie systems in structural concrete based on the linear elastic theory of cracked concrete for overall stiffness performance criteria and to design concrete members based on the theory of plasticity for strength performance criteria. The PBO technique presented overcomes the limitations of conventional trial-and-error methods for developing strut-and-tie models in structural concrete, and provides concrete designers with an efficient automated design tool for complex design situations.

APPENDIX. REFERENCES

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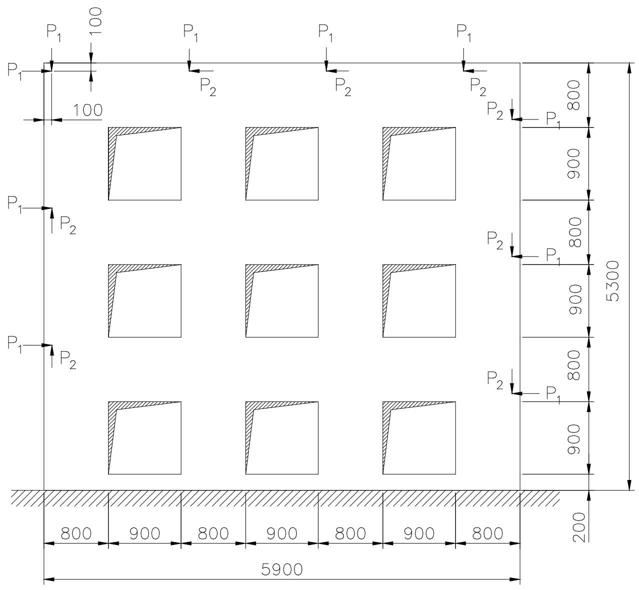


Fig. 1. Low-Rise Concrete Shearwall with Openings

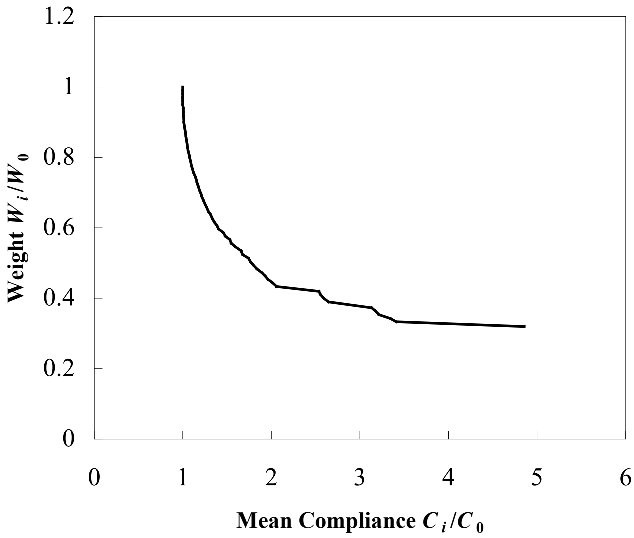


Fig. 2. Performance Characteristics of the Shearwall with Openings

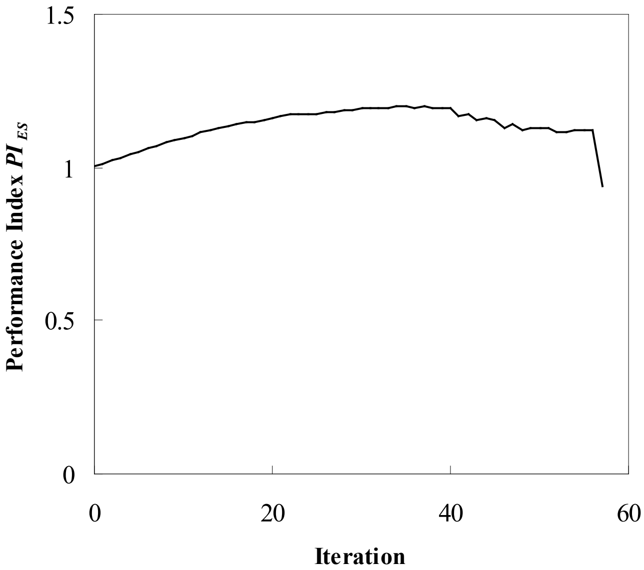
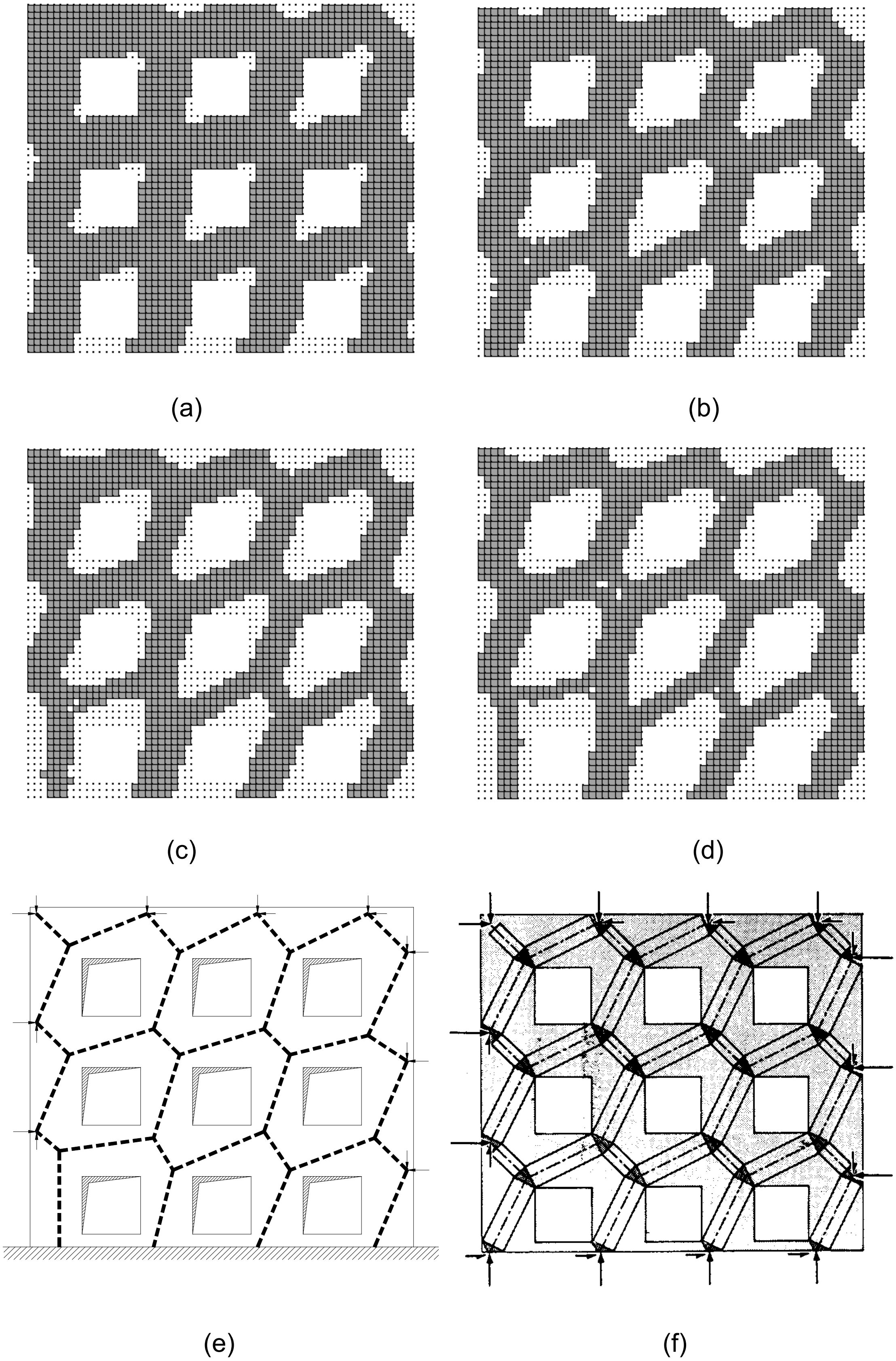


Fig. 3. Performance Index History of the Shearwall with Openings



(a) (b) (c) (d) (e) (f)

Fig. 4. Optimization History of Strut-and-Tie Model in Shearwall with Openings:
(a) Topology at Iteration 10; (b) Topology at Iteration 20; (c) Topology at Iteration 30;
(d) Optimal Topology at Iteration 35; (e) Optimal Model; (f) Model by Marti (1985)

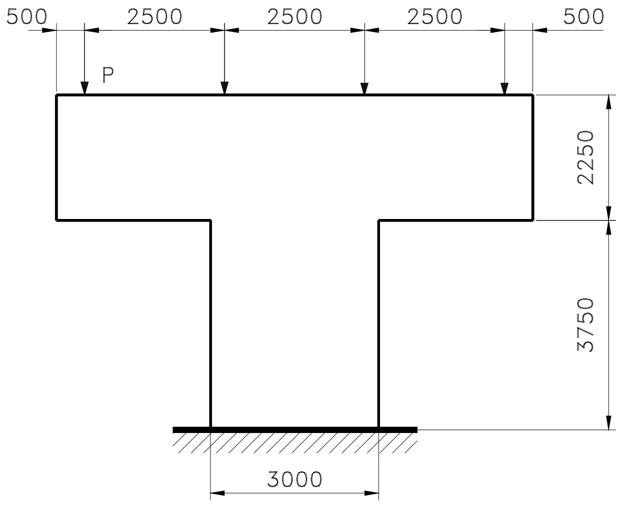


Fig. 5. Design domain for a Bridge Pier

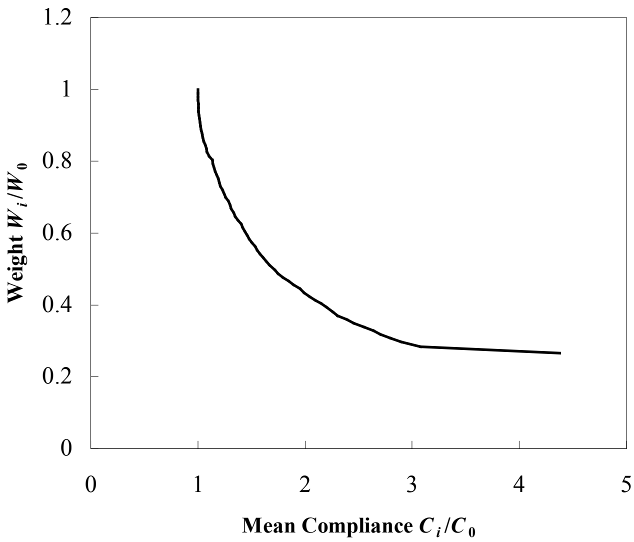


Fig. 6. Performance Characteristics of the Bridge Pier

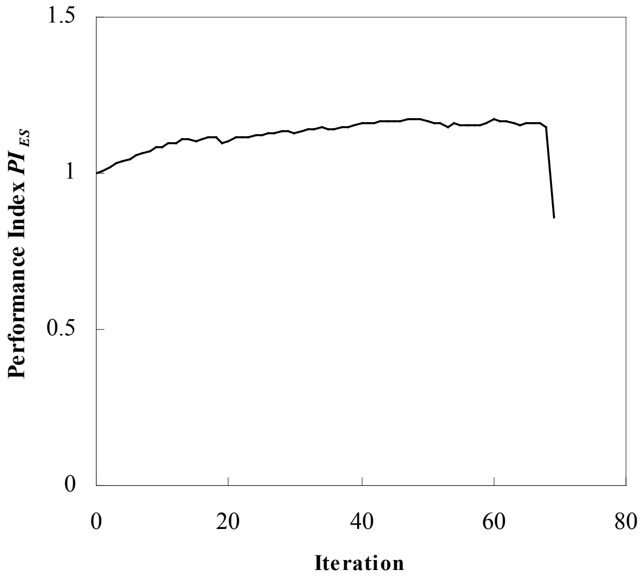
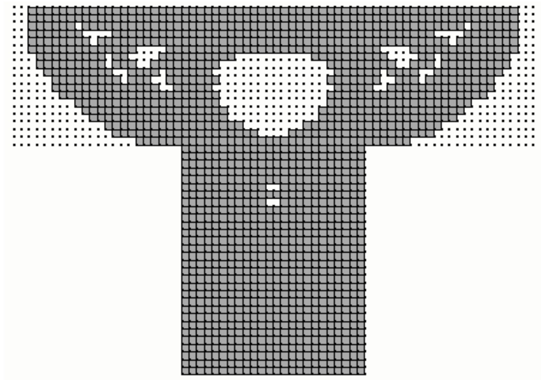
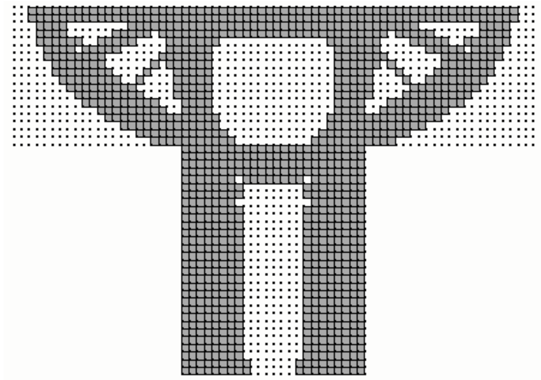


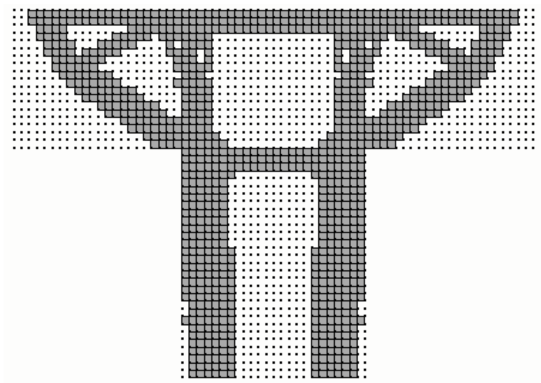
Fig. 7. Performance Index History of the Bridge Pier



(a)



(b)



(c)

Fig. 8. Optimization History of Strut-and-Tie Model in the Bridge Pier: (a) Topology at Iteration 20; (b) Topology at Iteration 40; (c) Optimal Topology at Iteration 49

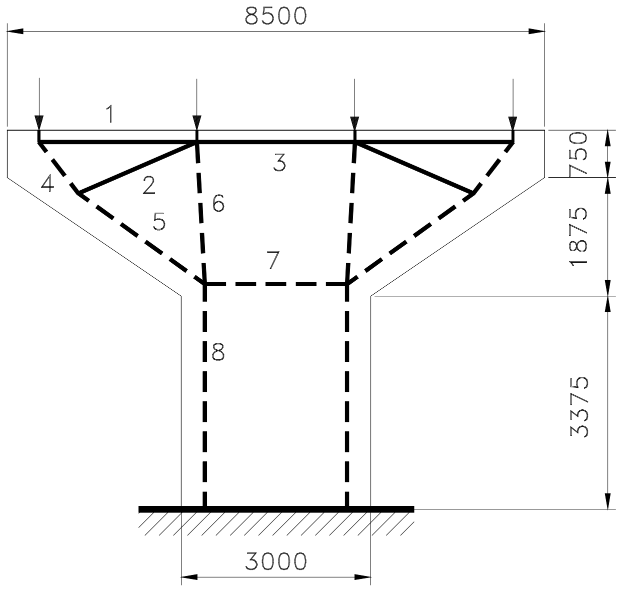


Fig. 9. Optimal Strut-and-Tie Model and Final Design Proposal of the Bridge Pier

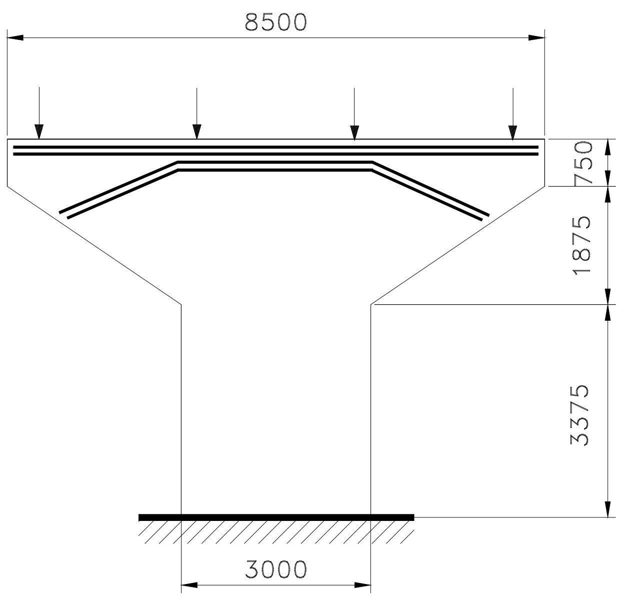


Fig. 10. Arrangement of Main Steel Reinforcement in the Bridge Pier

Table 1. Strut and Tie Forces in Strut-and-Tie Model for the Bridge Pier

Member number (1)	Force (KN) (2)
1	2114
2	1162
3	3363
4	-3470
5	-3919
6	-3219
7	-3363
8	-5500